Non-perturbative input in the Parton Branching Evolution

L. Keersmaekers, F. Hautmann, A. Lelek, M. Mendizabal

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Parton Branching equations

Parton branching equations for TMDs:

$$\tilde{\mathcal{A}}_a(x, k_\perp, \mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x, k_\perp, \mu_0^2) + \sum_b \int \frac{d^2\mu_\perp}{\pi\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2)\times$$
$$\times \int_x^{z_M} dz P_{ab}^R(z) \tilde{\mathcal{A}}_b(\frac{x}{z}, k_\perp + (1-z)\mu_\perp, \mu'^2)$$

- AO condition: $q_\perp^2 = (1-z)^2\mu'^2$

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  - Fixed z_M (set1/set2)
  - Dynamical $z_M = 1 - q_0/\mu'$
    q_0 smallest emitted transverse momentum
- Implicit in $P_{ab}^R(z)$ and $\Delta_a(\mu^2)$: $\alpha_s(\mu')$ (set1)/$\alpha_s(q_\perp)$ (set2)

The non-perturbative input

- Starting distribution:

$$\widetilde{A}_a(x, k_{\perp,0}, \mu_0^2) = \tilde{f}_a(x, \mu_0^2) \cdot \frac{1}{q_s^2 \pi} \exp\left(-\frac{k_{\perp,0}^2}{q_s^2}\right)$$

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Study influence of intrinsic $k_{\perp,0}$, distributed according to a Gaussian with width q_s

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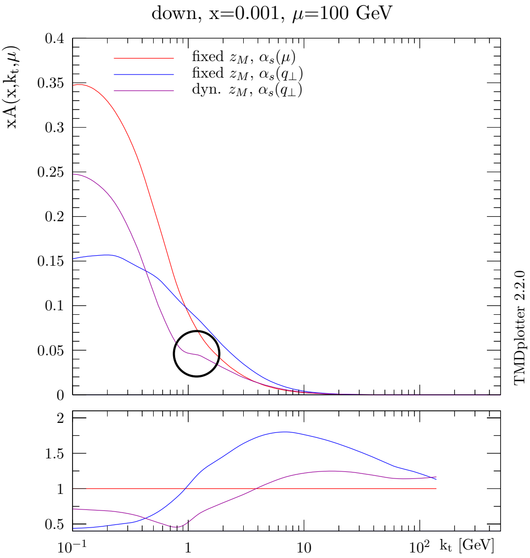
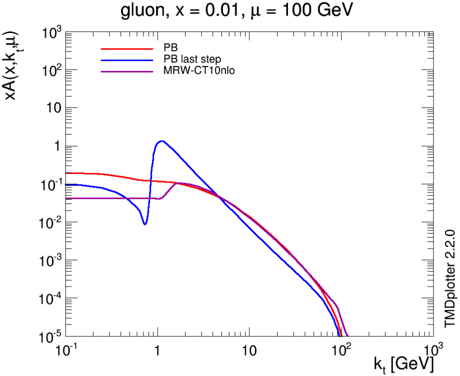
TMD with dynamical resolution scale



TMD with dynamical z_M → bump around $k_\perp = q_0$=1 GeV

Standard PB: $\mathbf{k} = \mathbf{k}_0 - \sum_{i=1}^{n} \mathbf{q}_i$
PB last step: toy model with $\mathbf{k} = \mathbf{k}_0 - \mathbf{q}_n$

$q_\perp > q_0 \rightarrow$ part from evolution
starts to accumualate around q_0
Only one branching$\rightarrow$ large bump



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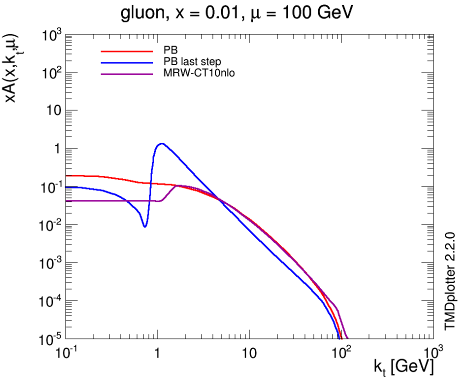
Vector sum $\rightarrow$ with multiple branchings we can reach $k_\perp < q_0$
$\rightarrow$ smooth out bump



Single emission evolution

Standard PB: $\mathbf{k} = \mathbf{k}_0 - \sum_{i=1}^{n} \mathbf{q}_i$
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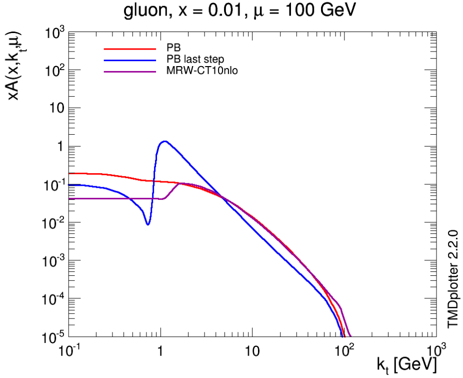
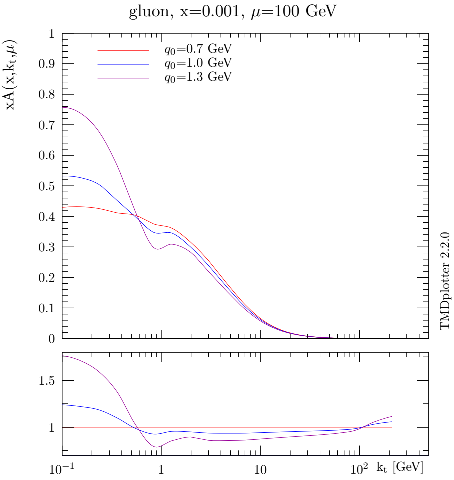
$\rightarrow$ smooth out bump

Often not enough branchings to completely smooth out, small bump left over

(Not visible here because of log-scale)



Different values of q_0

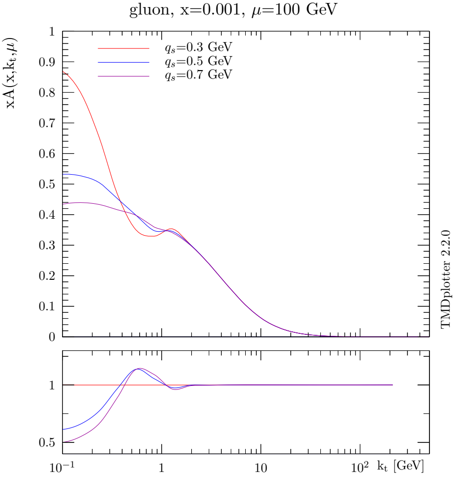


$q_s = 0.5$ GeV

Bump smoother with small values of q_0:

- $q_\perp > q_0$, with smaller q_0 more branchings are allowed
- More overlap between peak of intrinsic $k_\perp$ and peak around q_0

Different values of q_s



$q_0 = 1$ GeV

- Smoother with q_s close to q_0 (more overlap peaks)
- **Almost no effect on** $k_\perp$**-tail** (from +-2 GeV on)

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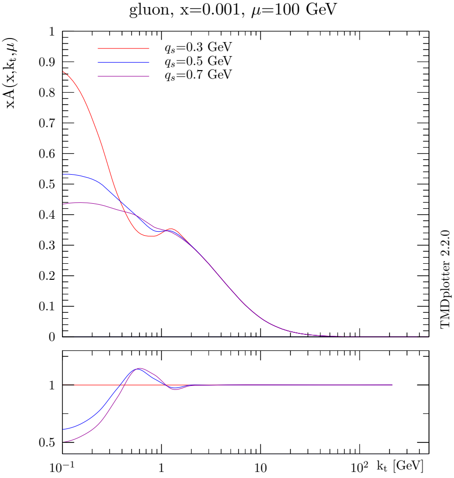

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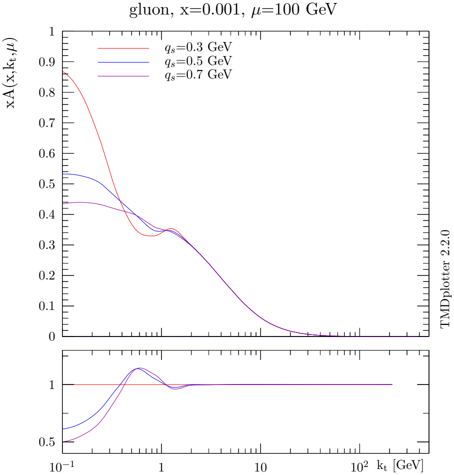

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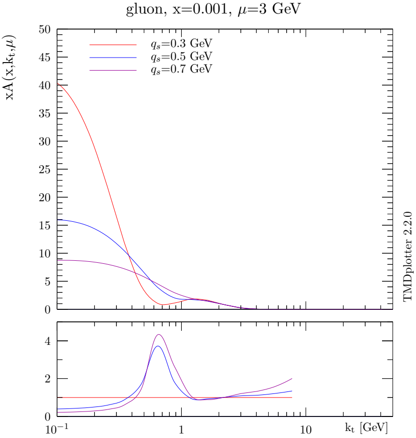
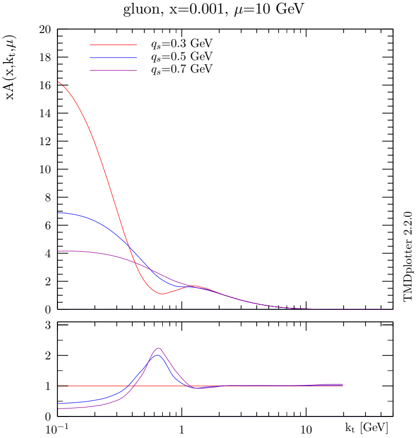
Since $k_\perp$-tail unaffected and $f_a = \int dk_\perp^2 A_a$ independent of q_s, total amount of partons below 2GeV unaffected by q_s

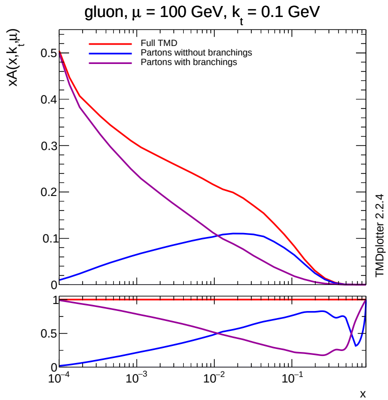
Different values of q_s at low scales



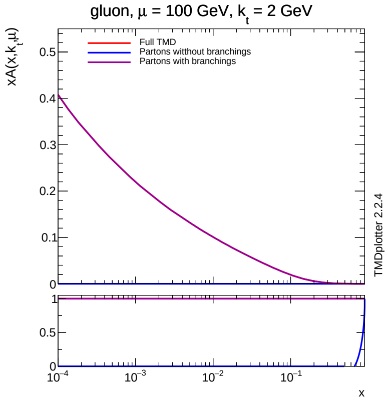


Contributions from partons with or without branchings

Low $k_\perp$ region



$k_\perp = 2\text{GeV}$



At small x: mostly partons that had branchings
At large x: mostly partons that have not branched

Dominated in whole x-region by partons that had branchings

Effects from intrinsic $k_\perp$ at small $k_\perp$ can be important in a large region of x

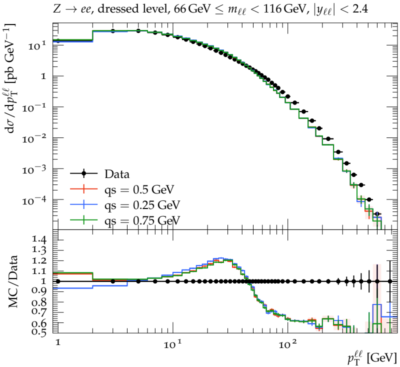
Effects of q_s on data

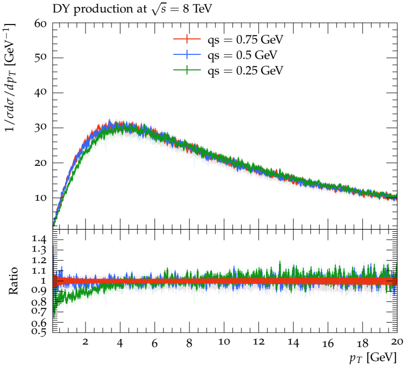
Intrinsic $k_\perp$ important in TMD distribution for $k_\perp < 2$ GeV, but total amount of partons with $k_\perp < 2$ GeV constant:

- Effects of intrinsic $k_\perp$ in $p_\perp$-data might be difficult to see when bin sizes are larger than 2 GeV
- We think data will be most sensitive to intrinsic $k_\perp$ in small $p_\perp$ region:
  Only contributions from two partons with
  - $k_{\perp,1}$ and $k_{\perp,2}$ both small
  - $k_{\perp,1} \approx k_{\perp,2}$ AND $\phi \approx 180^\circ$

Drell Yan with q_0=1

Colours do not correspond on both figures





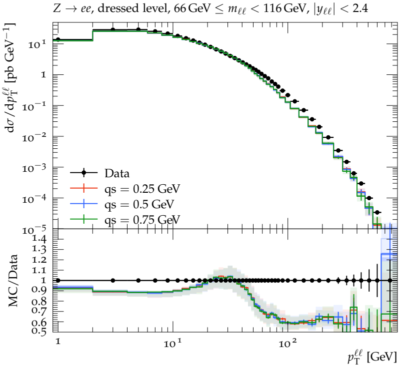
DY at LHC (Atlas), right: fine binning (temporary result)

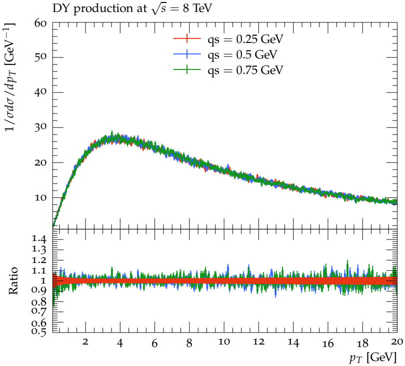
With q_s=0.25 GeV (blue left, green right) we can see the effect of q_s at DY spectrum at LHC
Differences between q_s=0.5 and q_s=0.75 are small

Largest differences at small $p_\perp < 4$ GeV, but also differences at higher $p_\perp \sim$20 GeV

Drell Yan with q_0=0.5

With q_0=0.5 more branchings $\rightarrow$ less sensitivity to q_s

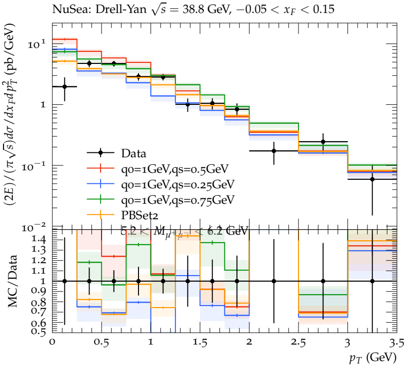




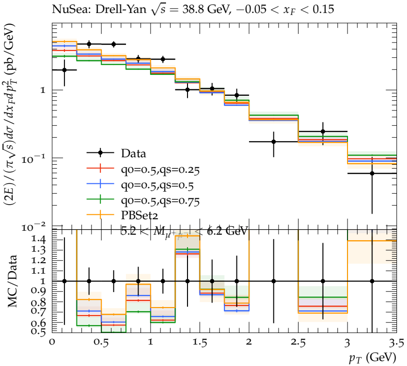
We need higher statistics to see whether we can find some differences with finer binning

Low mass DY

$q_0 = 1$

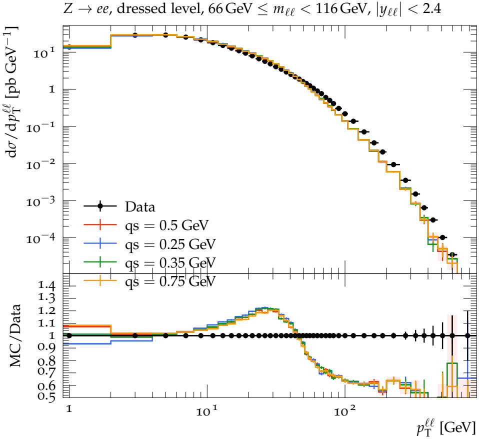


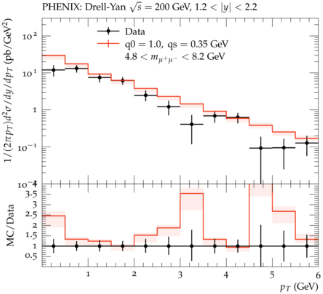
$q_0 = 0.5$

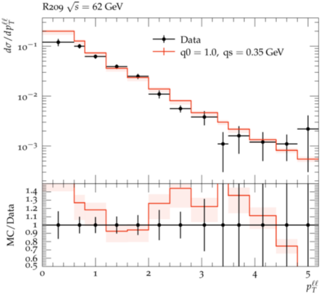


Low mass DY more sensitive to q_s:
Small binnings and less branchings

$q_0 = 1$ GeV, $q_s = 0.35$ GeV (below green)







A set of parameters might describe one experiment best, but might describe another experiment worse

$\Rightarrow$ Further studies needed for best parameters

Conclusions

- The distribution at low $k_\perp$ comes from both intrinsic $k_\perp$ and the effect of multiple branchings in the evolution
- Intrinsic $k_\perp$ affects only small $k_\perp$-region (below 2 GeV)
- In certain cases, low $p_\perp$-region of DY at LHC can have some sensitivity to intrinsic $k_\perp$, but more sensitivity in low mass DY
- Further studies needed for values of q_0, q_s