NNLL resummation with PB using the effective soft-gluon coupling & N_f studies

Cascade developer meeting

Mees van Kampen,
Lissa Keersmaekers, Ola Lelek

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The CSS approach

[Nuclear Physics B250 (1985) 199-224]

Well-established formalism that provides an analytical expression for inclusive processes:

$$h_1(p_1) + h_2(p_2) \to F(M, \boldsymbol{q}_\perp^2) + X$$

The differential cross section is given by:

$$\frac{d\sigma}{dQ^2 dy d\boldsymbol{q}_\perp^2} = \left[\frac{d\sigma^{(\text{res.})}}{dQ^2 dy d\boldsymbol{q}_\perp^2}\right]_{q_T \ll Q} + \left[\frac{d\sigma^{(\text{fin.})}}{dQ^2 dy d\boldsymbol{q}_\perp^2}\right]_{q_T \gg Q}$$

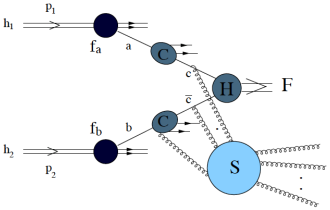
$$= \int d^2\boldsymbol{b} e^{i\boldsymbol{b}\cdot\boldsymbol{q}_\perp} \frac{\sigma^{(0)}(Q^2)}{s} H(\alpha_s(Q^2)) \sum_{a,b} \mathcal{F}_{a/A}(\boldsymbol{b}; Q, \xi_a, \mu)\mathcal{F}_{b/B}(\boldsymbol{b}; Q, \xi_b, \mu)$$

$$+ Y(\boldsymbol{q}_\perp; Q, x_a, x_b)$$

TMDs from CSS are factorized:

$$\mathcal{F}_{q/i} \sim f_{q/i} \otimes C_{jq} \otimes \sqrt{S}$$

- parton distributions: $f_{q/i}(x, b_0/b)$
- coefficient functions: $C_{jq}(\alpha_s(b_0/b), z)$
- Sudakov form factor: $\sqrt{S(Q, b)}$



Schematic representation of factorized TMDs of low-q_T contribution to σ [Catani, de Florian, Grazzini arXiv:0008184v2]

The CSS approach - Sudakov

- Soft-gluon resummation by coefficient functions C_{ab} and Sudakov form factor $\sqrt{S}$
- Perturbative part
- Non-negligible non-perturbative part S_{NP}

$$\sqrt{S(Q,b)} = \exp\left\{ -\frac{1}{2} \int_{b_0/b}^{Q^2} \frac{d\mu^2}{\mu^2} \left[\ln \frac{Q^2}{\mu^2} A_a(\alpha_s(\mu^2)) + B_a(\alpha_s(\mu^2)) \right] \right\} \times S_{NP}(b)$$

Perturbatively calculable coefficients:

$$A_a(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n A_a^{(n)},$$

$$B_a(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n B_a^{(n)},$$

$$C_{ab}(\alpha_s, z) = \delta_{ab}\delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n C_{ab}^{(n)}(z),$$

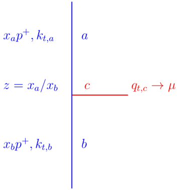
$$H(\alpha_s) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n H^{(n)}$$

The Parton Branching (PB) method

JHEP 01 (2018) 070 [arXiv:1708.03279]

PB evolution equation for TMDs $\tilde{\mathcal{A}}_a(x, k_t^2, \mu^2)$ can be solved iteratively with the Monte Carlo method:

$$\tilde{\mathcal{A}}_a(x, k_t^2, \mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x, k_{t,0}^2, \mu_0^2) +$$
$$+ \sum_b \left[\int \frac{d^2\boldsymbol{\mu}'}{\pi\mu'^2} \int_x^{z_M(\mu')} dz \Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2) \right.$$
$$\left. \times \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} P_{ab}^{(R)}(\alpha_s(q_\perp), z) \tilde{\mathcal{A}}_b\left(\frac{x}{z}, \underbrace{k_{t,b} - q_{t,c}}_{k_{t,a}}, \mu'^2\right) \right]$$



Kinematics in each branching governed by momentum conservation: $k_{t,b} = k_{t,a} + q_{t,c}$

DGLAP splitting functions:

$$P_{ab}(\alpha_s, z) = d_a(\alpha_s)\delta_{ab}\delta(1-z) + \frac{k_a(\alpha_s)\delta_{ab}}{(1-z)_+} + R_{ab}(\alpha_s, z)$$

$$P_{ab}^{(R)}(\alpha_s, z) = \frac{k_a(\alpha_s)\delta_{ab}}{1-z} + R_{ab}(\alpha_s, z) \quad \text{resolvable emission probability}$$

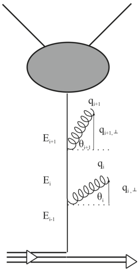
Sudakov form factor (non-resolvable / no-emission probability):

$$\Delta_a^{(PB)}(\mu, \mu_0) = \exp\left\{ -\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M(\mu')} dz\ z\ P_{ba}^{(R)}(\alpha_s(q_t), z) \right\}$$

The Parton Branching (PB) method

Angular ordering condition: $q_t^2 = (1-z)^2\mu'^2$

$$\tilde{\mathcal{A}}_a(x,k_t^2,\mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x,k_{t,0}^2,\mu_0^2) + \\ + \sum_b \int \frac{d^2\boldsymbol{\mu}'}{\pi\mu'^2} \int_x^{1-\frac{q_0}{\mu'}} dz \Theta(\mu^2-\mu'^2)\Theta(\mu'^2-\mu_0^2)\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \\ \times P_{ab}^{(R)}\Big(\alpha_s((1-z)\mu'),z\Big)\tilde{\mathcal{A}}_b\left(\frac{x}{z}, k_{t,b}-(1-z)\mu', \mu'^2\right)$$



Dynamical soft-gluon resolution scale, avoid Landau pole Λ_{QCD}.
Hautmann, Keersmaekers, Lelek, MvK [arXiv:1908.08524]

- Sum over all transverse momenta: $k_t = k_{t,0} - \sum_i q_{t,i}$
- Intrinsic k_t in non-perturbative part: $\tilde{\mathcal{A}}_a(x, k_{t,0}, \mu_0)$
  e.g. collinear with Gaussian $f_a(x,\mu_0)e^{-k_t^2/2\sigma^2} \longrightarrow f_a(x,\mu_0)$ and σ fitted to data

PB implemented in the Monte Carlo event generator CASCADE3 [arXiv:2101.10221]:

- associate k_t to partons in the hard process according to the TMD
- for exclusive observables: perform backward evolution with TMD parton shower (unfolding the TMD)

Analytical comparison CSS and PB: logarithmic accuracy

CSS Sudakov (with scale $\mu = q_t$):

$$S(Q^2, b^2) = \exp\left(-\int_{1/b^2}^{Q^2} \frac{dq_t^2}{q_t^2}\left[\ln\left(\frac{Q^2}{q_t^2}\right) A_a(\alpha_s(q_t^2)) + B_a(\alpha_s(q_t^2))\right]\right)$$

PB Sudakov (*twice in the cross section!*) rewritten with angular ordering and virtual splitting functions:

$$\Delta_a^{(PB)}(Q^2, q_0^2) = \exp\left(-\int_{q_0^2}^{Q^2} \frac{dq_t^2}{q_t^2}\left[\frac{1}{2}\ln\left(\frac{Q^2}{q_t^2}\right) k_a(\alpha_s(q_t^2)) - d_a(\alpha_s(q_t^2))\right]\right)$$

Logarithmic accuracy	**Terms**	**Single log**	**Double log**	
Leading log (LL)	$\alpha_s^n \ln^{n+1}(Q^2/q_t^2)$		$A_a^{(1)} = k_a^{(0)}$	✓
Next-to-leading log (NLL)	$\alpha_s^n \ln^n(Q^2/q_t^2)$	$B_a^{(1)} = -2d_a^{(0)}$	$A_a^{(2)} = k_a^{(1)}$	✓
Next-to-next-to-leading log (NNLL)	$\alpha_s^n \ln^{n-1}(Q^2/q_t^2)$	$B_q^{(2)} = -2d_q^{(1)}$ $+16\pi C_F \beta_0(\zeta_2 - 1)$ $\Downarrow$ *Resummation scheme dependence*	$A_q^{(3)} = k_q^{(2)}$ $+2\beta_0 C_F d_2^q$* $\Downarrow$ *Effective* α_s	✓/✗

Soft-gluon effective coupling: Banfi et al. [arXiv:1807.11487] & Catani et al. [arXiv:1904.10365]

$^*d_2^q = C_F C_A\left(\frac{808}{27} - 28\zeta_3\right) - \frac{112}{27} C_F N_f$

Single logarithmic part

$$B_q^{(2)} - (-2) \cdot d_q^{(1)} = 16\pi C_F \beta_0 (\zeta_2 - 1)$$

- In PB, we use universal splitting functions P_{ab}. These are independent of any scheme choice. So $d_q^{(1)}$ is fixed.
- In CSS, certain perturbative coefficients can mix via renormalization group transformations:

$$\frac{\partial \ln H^F(\alpha_s(\mu^2))}{\partial \ln \mu^2} = \gamma_H(\alpha_s(\mu^2))$$

$$H^F(\alpha_s(c_1 Q^2)) = \exp\left\{ \int_{c_0/b^2}^{c_1 Q^2} \frac{d\mu^2}{\mu^2} \gamma_H(\alpha_s(\mu^2)) \right\} H^F(\alpha_s(c_0/b^2)).$$

- Parts of H can mix with the B in the Sudakov with a different *resummation scheme*

$$B^F(\alpha_s(\mu^2)) = B(\alpha_s(\mu^2)) - \left(-\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 + \mathcal{O}(\alpha_s^4)\right) \left(\frac{H^{F(1)}}{\pi} + \frac{2\alpha_s}{\pi^2} H^{F(2)} + \mathcal{O}(\alpha_s^2) \right).$$

The difference we found is equal to the first term of this expansion of γ_H, the term with α_s^2

Double logarithmic part

$$A_i^{(3)} - k_i^{(2)} = C_i(\beta_0\pi)\left[C_A\left(\frac{808}{27} - 28\zeta_3\right) - N_f\frac{112}{27}\right]$$

- K part from splitting functions is *cusp anomalous dimension*, in other words: *soft-gluon coupling*
- Intensity of soft-gluon radiation is $C_i\alpha_s/\pi$. **To get NLL**, replace it by:

$$C_i\frac{\alpha_s}{\pi} \to \mathcal{A}_i^{CMW}(\alpha_s(q_T^2)) = C_i\frac{\alpha_s(q_T^2)}{\pi}\left(1 + \frac{\alpha_s(q_T^2)}{2\pi}K\right)$$

- **To go to NNLL** (or higher), use the *effective soft-gluon coupling*:

$$\alpha_s^{eff} = \alpha_s\left(1 + \sum_n\left(\frac{\alpha_s}{2\pi}\right)^n K^{(n)}\right).$$

Banfi et al. [arXiv:1807.11487] & Catani et al. [arXiv:1904.10365]

- $A_i^{(3)}$ of CSS is obtained by taking $n = 2$ and multiplication with C_i/π
- NNLL resummation in PB? implement the effective coupling up to $n = 2$ in TMD evolution

Implementation effective soft-gluon coupling in TMD evolution

A start has been made in the implementation of α_s^{eff} into UPDFEVOLV.

Use within:

- $\Delta_a(\mu^2, \mu_0^2, \alpha_s^{eff})$
- $P_{ab}^{(R)}(z, \alpha_s^{eff})$

We have already NLO splitting functions (so k_1 is there) and the CMW coupling for NLL resummation. Only need for $K^{(2)}$!

In the code:

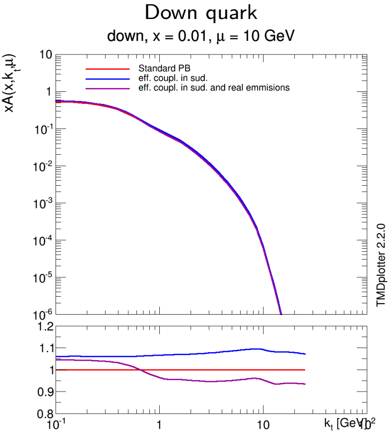
$$\alpha_s \to \alpha_s \cdot \left(1 + \alpha_s^2 K^{(2)}\right)$$

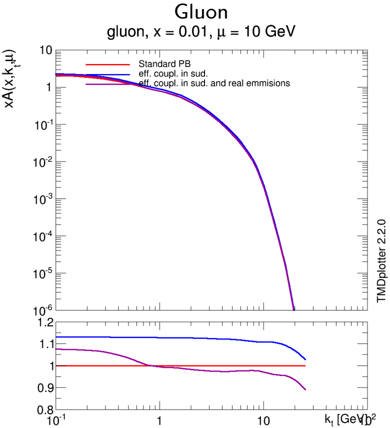
with

$$K^{(2)} = C_A^2\left(\frac{245}{24} - \frac{67}{9}\zeta_2 + \frac{11}{6}\zeta_3 + \frac{11}{5}\zeta_2^2\right) + C_F N_f\left(-\frac{55}{24} + 2\zeta_3\right) + C_A N_f\left(-\frac{209}{108} + \frac{10}{9}\zeta_2 - \frac{7}{3}\zeta_3\right) - \frac{1}{27}N_f^2 + \frac{\pi\beta_0}{2}\left(C_A\left(\frac{808}{27} - 28\zeta_3\right) - \frac{224}{54}N_f\right).$$

Open question: what should N_f be? It depends on the evolution scale μ'.
However: $\alpha_s(q_t)$!

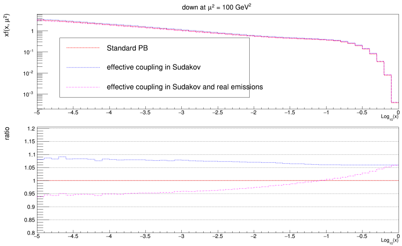
First numerical results: TMDs



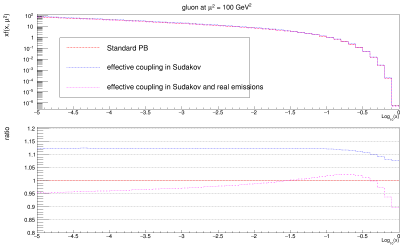


test: PB TMD
test1: α_s^{eff} in Sudakov
test2: α_s^{eff} in Sudakov and splitting functions

First numerical results: iTMDs



Down quark



Gluon

Number of active flavors N_f

- The number of active flavors N_f depends on a certain scale.
- Branching scale μ' or the emitted transverse momentum q_T

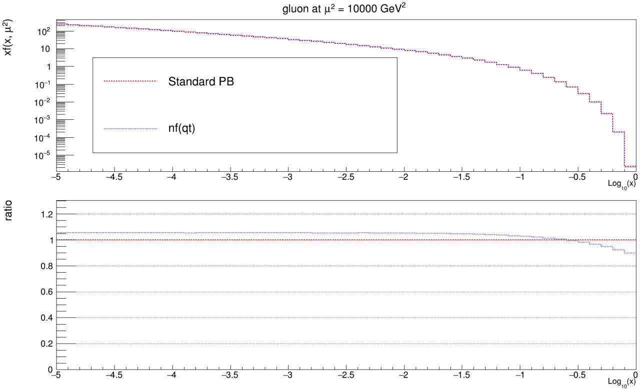
N_f appears in the evolution in:

- Splitting function $P_{ab}(\alpha_s, z, N_f)$
- $\alpha_s(N_f)$ (and later also $K^{(2)}(N_f)$)

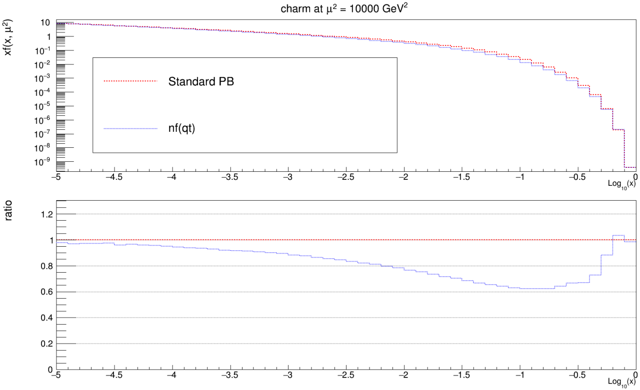
$\rightarrow$ we investigated the numerical effect on (i)TMDs of varying the scale of N_f in the following scenarios, without implementation of the effective soft-gluon coupling:

- Standard evolution: $P_{ab}(\alpha_s(q_T), N_f(\mu'))$, $\alpha_s(N_f(q_T))$
- q_T in all functions where N_f appears

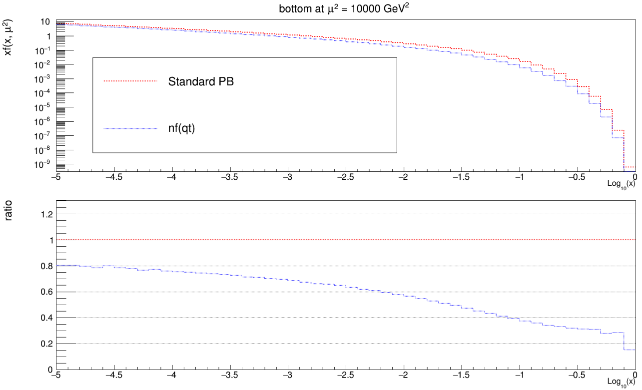
Number of active flavors N_f - gluon distribution



Number of active flavors N_f - charm distribution



Number of active flavors N_f - bottom distribution



Summary & open questions

Efforts have been put in the systematical comparison of the PB and CSS approaches to Sudakov resummation for DY p_T spectra.

- PB and CSS Sudakov form factors coincide at LL and NLL
- Differences at NNLL tracked down to **resummation scheme dependence** (single log) and **effective soft-gluon coupling** (double log)

Open issues and future steps:

- Number of light quarks N_f in $K^{(2)}$ not the same as in calculation α_s
- Numerical comparison with CSS; e.g. generate DY p_T spectrum with MC@NLO+CASCADE3. For this, integrated TMDs are necessary.

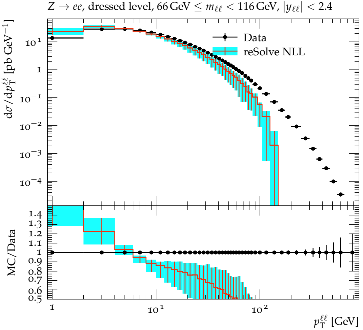
Backup

Inclusive Z boson p_T spectrum @LHC using reSolve

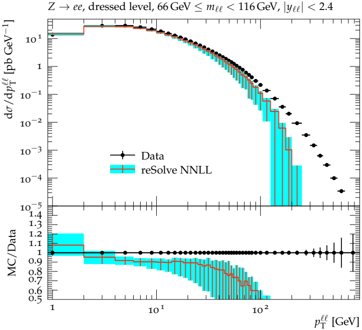
reSolve [Coradeschi, Cridge arXiv:1711.02083] provides a numerical implementation of CSS

- Resummation $d\sigma^{(\text{res.})}/dp_T^2 dQ^2 dy$ up to NNLL
- No matching with finite Y-terms yet (no NLO for Z+0)

Z boson p_T spectrum at $\sqrt{s} = 8$ TeV



NLL prediction with scale uncertainties

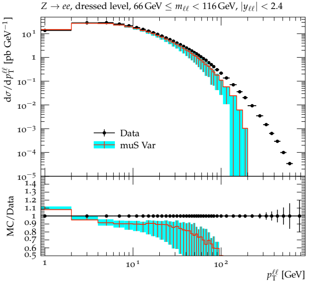


NNLL prediction with scale uncertainties

Estimation of theoretical uncertainties

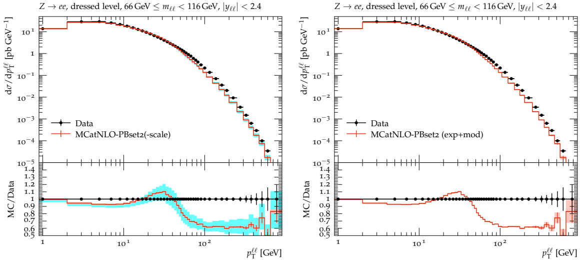
CSS:

- μ_S resummation/matching scale $\longrightarrow$
- μ_R variation mostly influences small p_T
- μ_F variation both at small and large p_T
- 9-point scale variation
  - Vary μ_R and μ_F with factor 2 and $1/2$ (9 combinations)
  - Constraint $1/2 \leq \mu_R/\mu_F \leq 2$ (leaves 7 combinations)
  - Vary μ_S factor 2 and $1/2$ with central value of μ_R, μ_F



PB:

- Scale variations: μ_F, μ_R (9-point)
- Matching scale not varied yet
- TMD variations included but small uncertainties in LHC region around M_Z



Comparisons of theoretical uncertainty bands with different resummation approaches by LHC electroweak working group: https://twiki.cern.ch/twiki/bin/view/LHCPhysics/EWWG1

Parameterization of non-perturbative physics

Gaussian smearing factor in reSolve for non-perturbative physics:

$$S_{NP} = \exp\left(-g_{NP} b^2\right)$$

Commonly g^{NP} $[0.5 - 2.5]$

In the PB method intrinsic k_t:

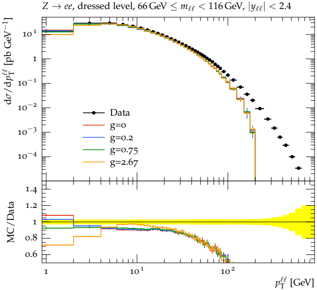
$$\tilde{\mathcal{A}}_a(x, k_{t,0}, \mu_0) \sim f_a(x, \mu_0) e^{-\frac{k_t^2}{2\sigma^2}} \quad (\sigma \sim 350 \text{ MeV})$$

- subject to (perturbative) evolution!
- describe effects in a few GeV range

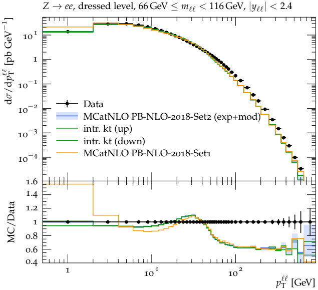
Width of PB Gaussian is small compared to the one used in reSolve and also compared to parton shower event generators.

$$\int e^{-k_t^2/2\sigma^2} e^{ik_t b} dk_t \sim e^{-b^2\sigma^2/4}$$

$$\rightarrow \sigma \sim 350 \text{ MeV} \Leftrightarrow g_i^{NP} \sim 0.03$$

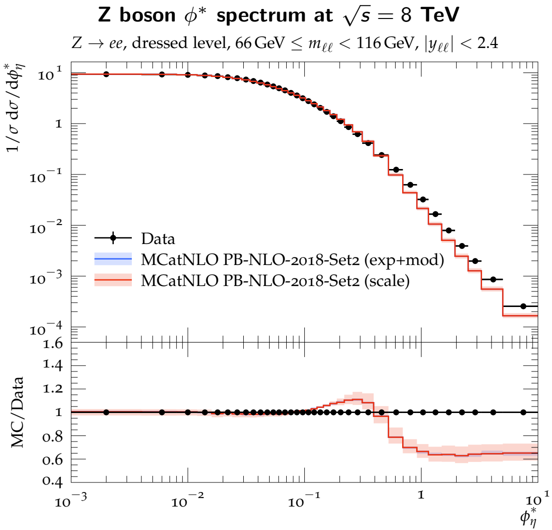


reSolve at NNLL



PB at NLO

Z boson ϕ^* distribution with PB



[arXiv:1906.00919]

Rewrite PB Sudakov

PB Sudakov:

$$\Delta_a^{(PB)}(\mu,\mu_0) = \exp\left\{-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\ z\ P_{ba}^{(R)}(\alpha_s(a(z)\mu'), z)\right\}.$$

Real part of the splitting functions:

$$P_{ab}^{(R)}(\alpha_s, z) = K_{ab}(\alpha_s)\frac{1}{1-z} + R_{ab}(\alpha_s, z)$$

Virtual part of the splitting functions:

$$P_{ab}^{(V)}(\alpha_s, z) = \delta_{ab}\frac{k_a(\alpha_s)}{(1-z)_+} + \delta(1-z)\delta_{ab} d_a(\alpha_s)$$

Use:

- **angular ordering** condition: $q_t = (1-z)\mu'$ and
- **virtual splitting functions**: $P_{ab}^{(R)} = P_{ab} - P_{ab}^{(V)}$ and
- **momentum sum rule**: $\sum_b \int_0^1 dz\ z\ P_{ab}(z, \mu'^2) = 0$

$$\Delta_a^{(PB)}(Q^2, q_0^2) \simeq \exp\left\{-\int_{q_0^2}^{Q^2} \frac{dq_t^2}{q_t^2}\left[\frac{1}{2}\ln\left(\frac{\mu^2}{q_t^2}\right) k_a(\alpha_s(q_t^2)) - d_a(\alpha_s(q_t^2))\right]\right\}$$

Analytical comparison CSS and PB up to NLL

[de Florian, Grazzini; Phys.Rev.Lett. 85 (2000) 4678-4681]
[Hautmann et al.; JHEP 01 (2018) 070]

- LL':

$$A_q^{(1)} = C_F, \qquad k_q^{(0)} = 2C_F$$

- NLL':

$$A_q^{(2)} = \frac{1}{2} C_F C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{18} N_f C_F \qquad B_q^{(1)} = -\frac{3}{2} C_F$$

$$k_q^{(1)} = 2 C_F C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} N_f C_F \qquad d_q^{(0)} = \frac{3}{2} C_F$$

Scale variations reSolve