Nucleon form factors from Lattice QCD for neutrino oscillation experiments

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EuroPLEx

H2020

MARIE CURIE ACTIONS

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Outline

- Introduction and Motivation

  What are neutrino oscillations?

  What are nucleon form factors?

  How do nucleon form factors enter in the neutrino oscillation experiments?

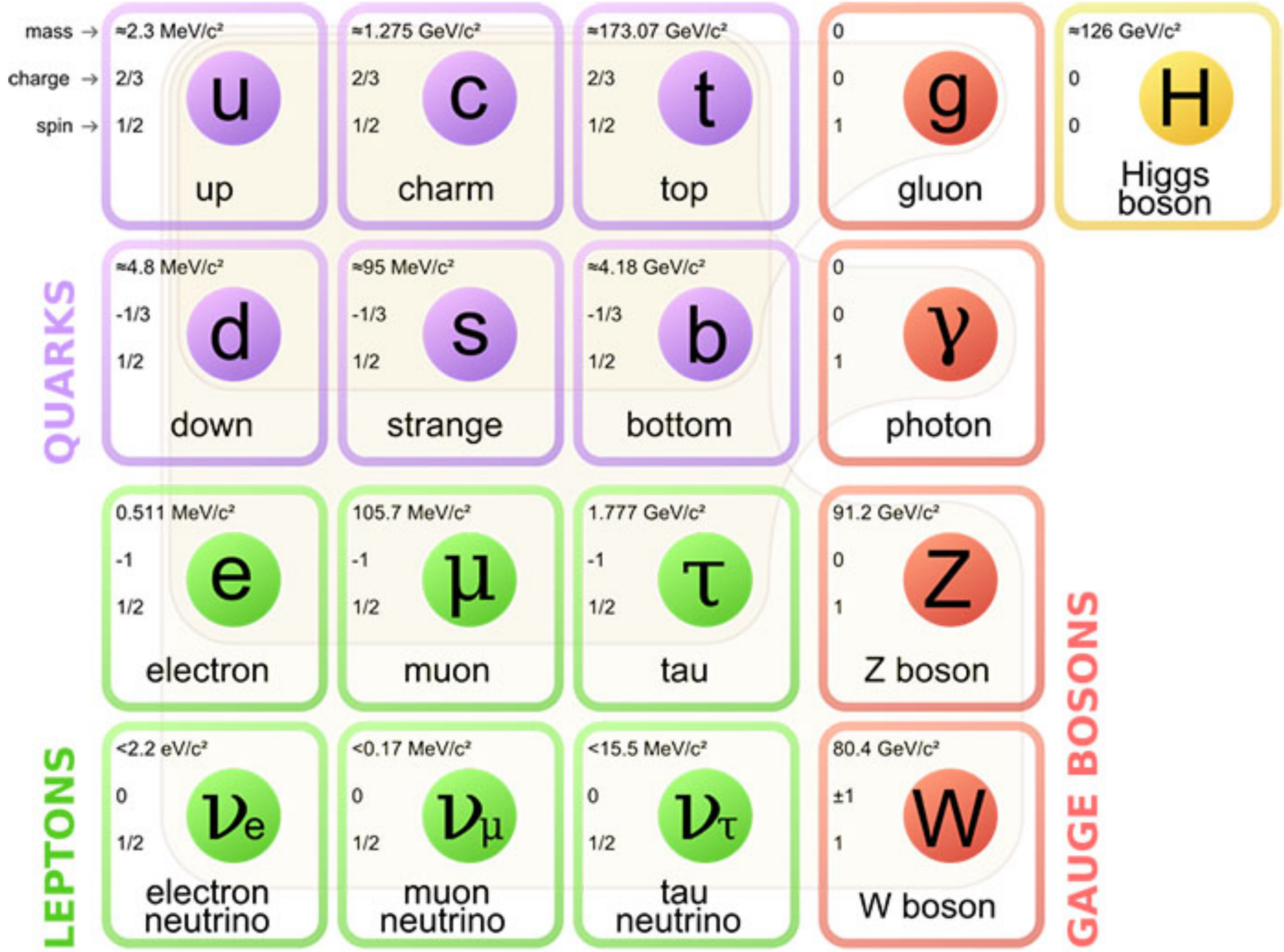
- Extracting nucleon form factors from LQCD with the variational method

- Results

- Conclusions and Outlook

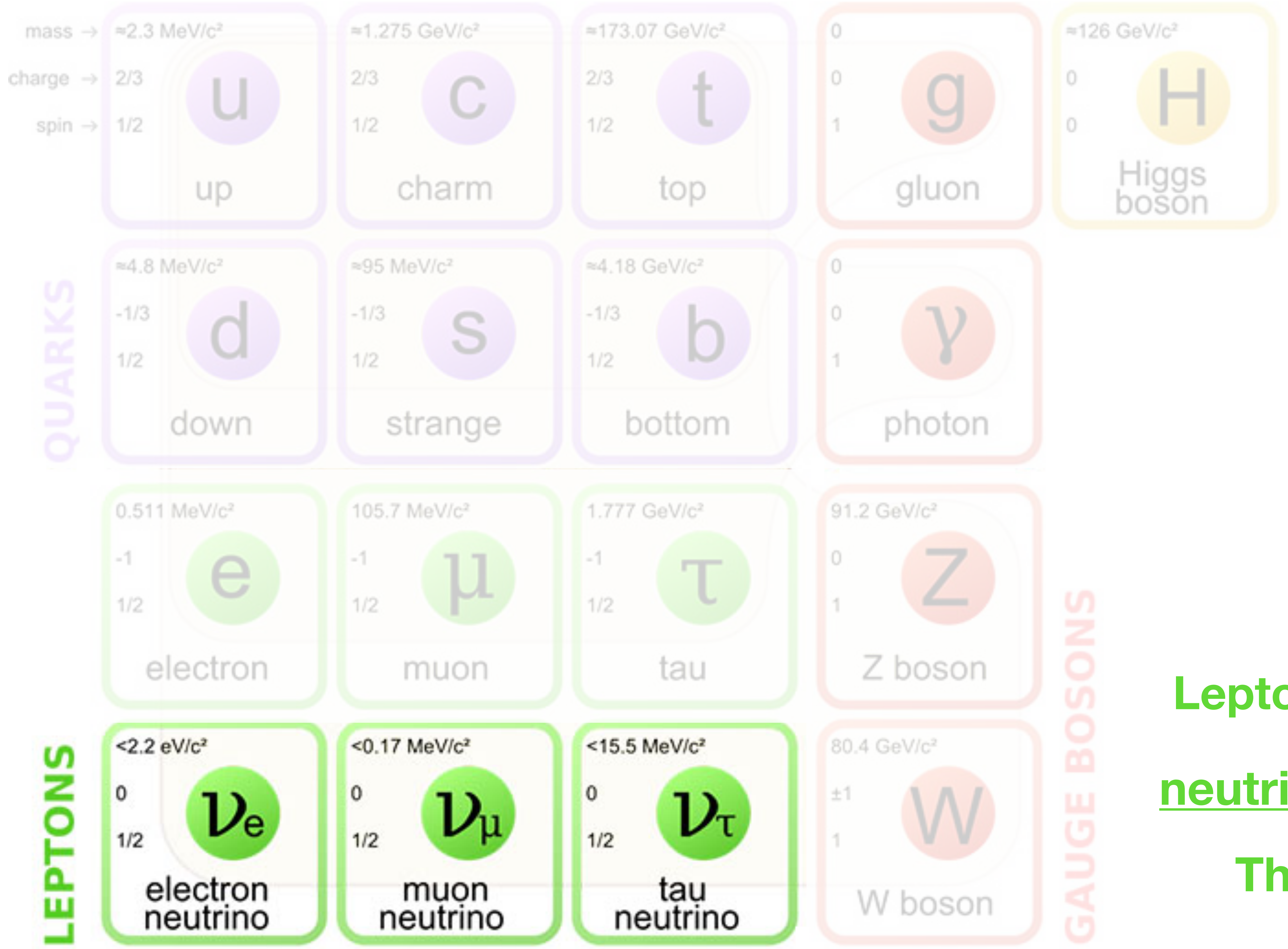
Introduction and Motivation

Classification of all elementary particles



Introduction and Motivation

Classification of all elementary particles



Leptons $(e^-, \mu^-, \tau^-, \nu)$ do not interact via the strong force

neutrinos ν interact only via the weak force and gravity (!)

There is still a lot to understand about their nature…

Introduction and Motivation

Nobel Prize

Prediction of an undetected particle ($\bar{\nu}_e$) in β-decay

Discovery of ν_e *

Discovery of ν_μ *

"Discovery" of ν_τ

1930

1956

1962

2000

DONUT Experiment

W. Pauli

F. Reines and C. Cowan *

L. Lederman *

Introduction and Motivation

*

Nobel Prize

1930 — Prediction of an undetected particle ($\bar{\nu}_e$) in β-decay

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1956 — Discovery of ν_e *

F. Reines and C. Cowan *

1957 — Prediction of neutrino oscillations

B. Pontecorvo
(Z. Maki, M. Nakagawa, S. Sakata)

1962 — Discovery of ν_μ *

L. Lederman *

2000 — “Discovery” of ν_τ

DONUT Experiment

Introduction and Motivation

(ν_e) neutrino flux originated from the sun (pp-chain) and flux detected on earth are in discrepancy.

* Nobel Prize

1930	1956	1957	1962	1960'	2000
Prediction of an undetected particle $(\bar{\nu}_e)$ in β-decay	Discovery of ν_e *	Prediction of neutrino oscillations	Discovery of ν_μ *	**solar neutrino puzzle** *	"Discovery" of ν_τ

W. Pauli

F. Reines and C. Cowan *

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Homestake Experiment *

DONUT Experiment

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- **1930** — Prediction of an undetected particle $(\bar{\nu}_e)$ in β-decay


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- **1956** — Discovery of ν_e *


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- **1957** — Prediction of neutrino oscillations


B. Pontecorvo
(Z. Maki, M. Nakagawa, S. Sakata)

- **1962** — Discovery of ν_μ *


L. Lederman *

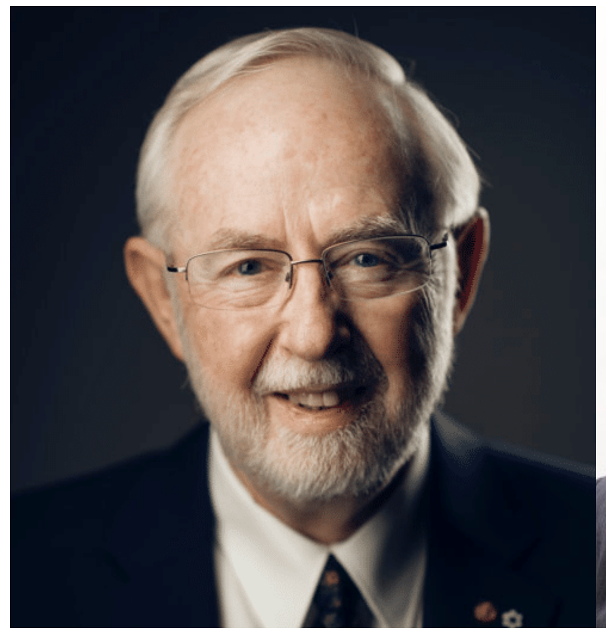
- **1960'** — **solar neutrino puzzle** *


Homestake Experiment *

- **2000** — "Discovery" of ν_τ

DONUT Experiment

- **2015** — (1998/2001) Clear observation of neutrino oscillations *


A. McDonald *
SNO Experiment


T. Kajita *
Super-Kamiokande

Introduction and Motivation

Neutrino oscillations (in vacuum)

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

$$|\nu_i(t)\rangle = e^{-i(E_i t - \mathbf{p}_i \cdot \mathbf{x})} |\nu_i\rangle$$

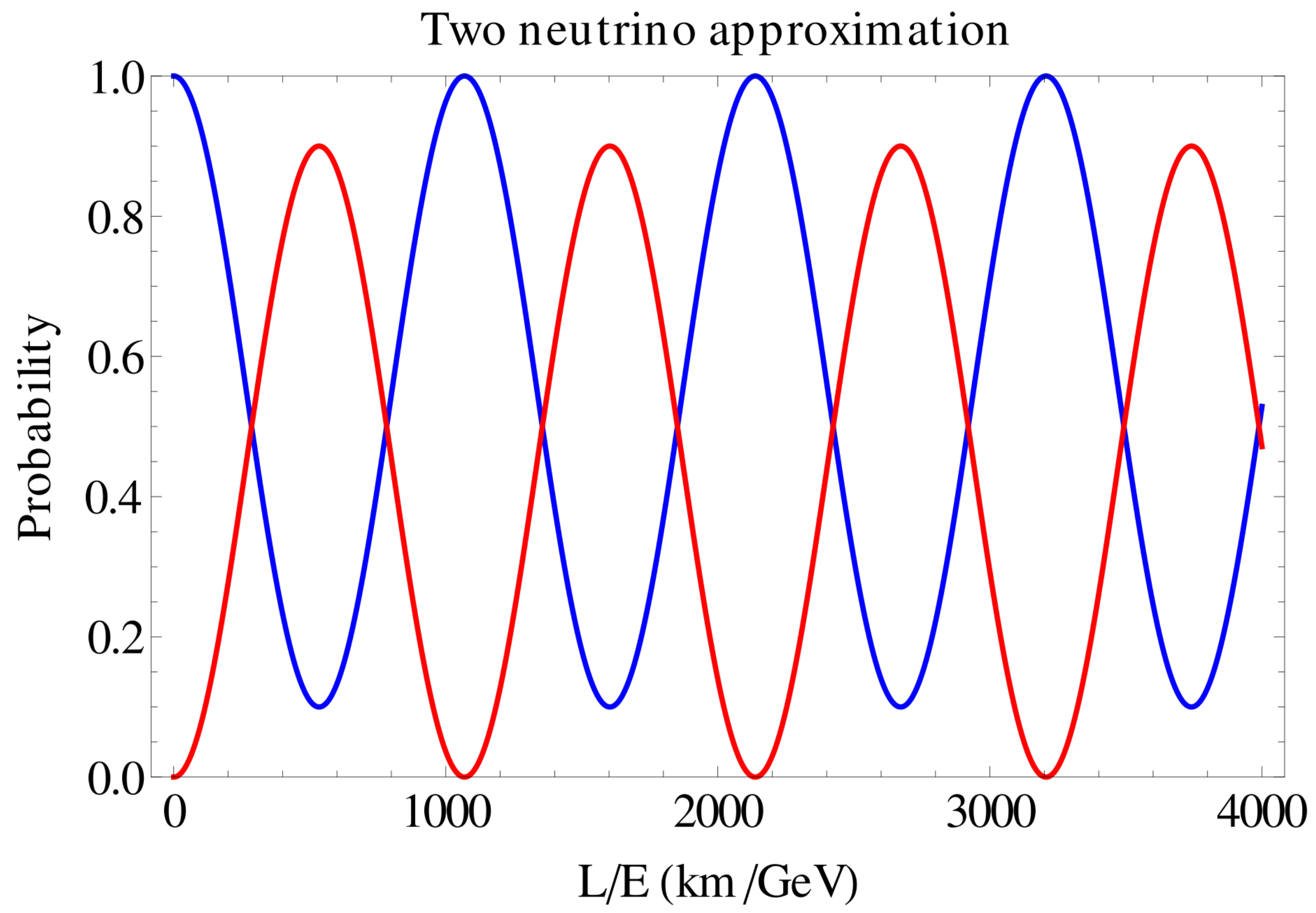
2-flavour case

Ultrarelativistic limit $t \approx L = |\mathbf{x}|$

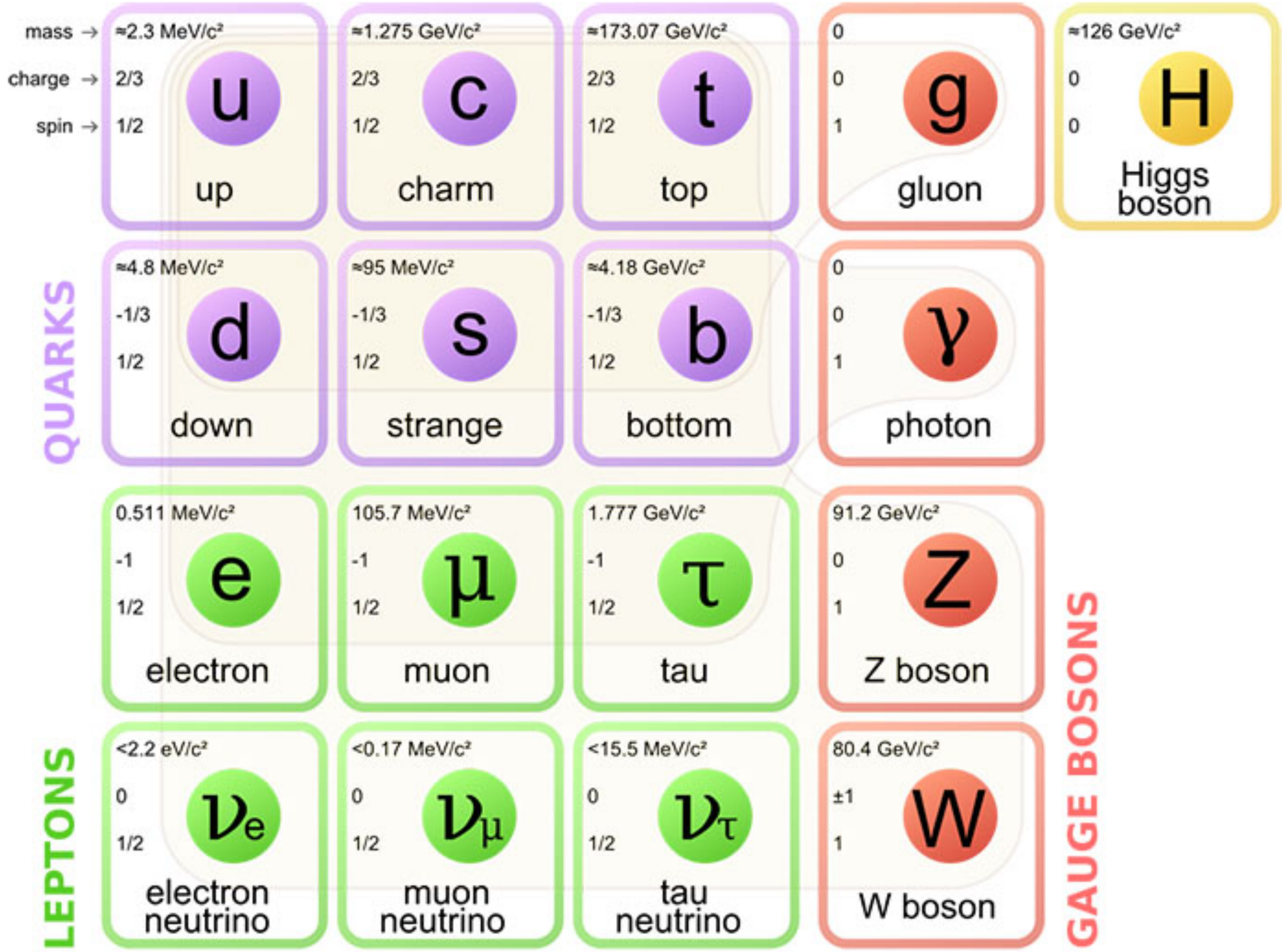
$$P_{\alpha\to\beta} = |\langle \nu_\beta(L) | \nu_\alpha(0)\rangle|^2 = \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\sin^2\theta$$

$$\Delta m^2 = m_1^2 - m_2^2$$

Neutrinos cannot be massless!



Introduction and Motivation



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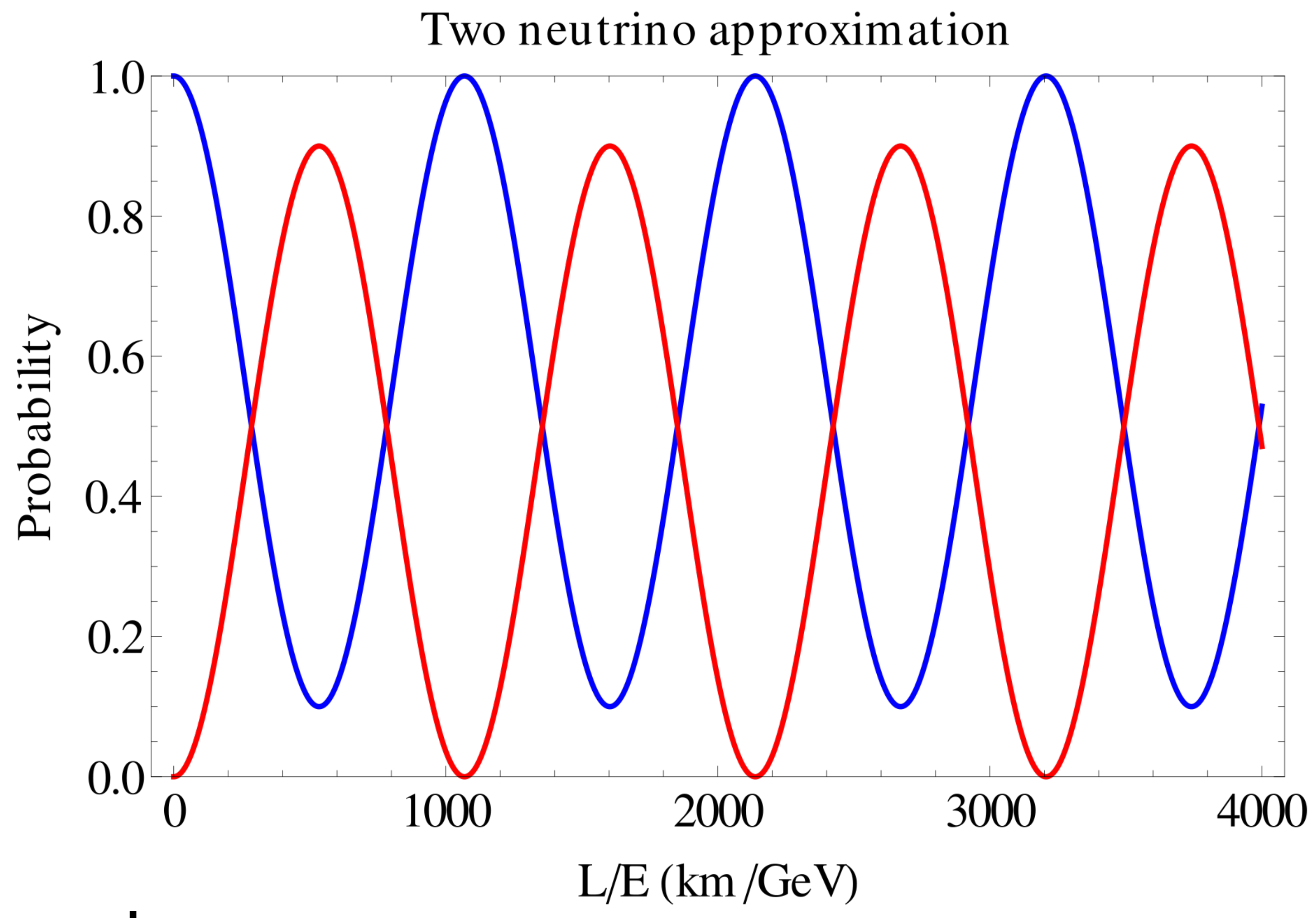
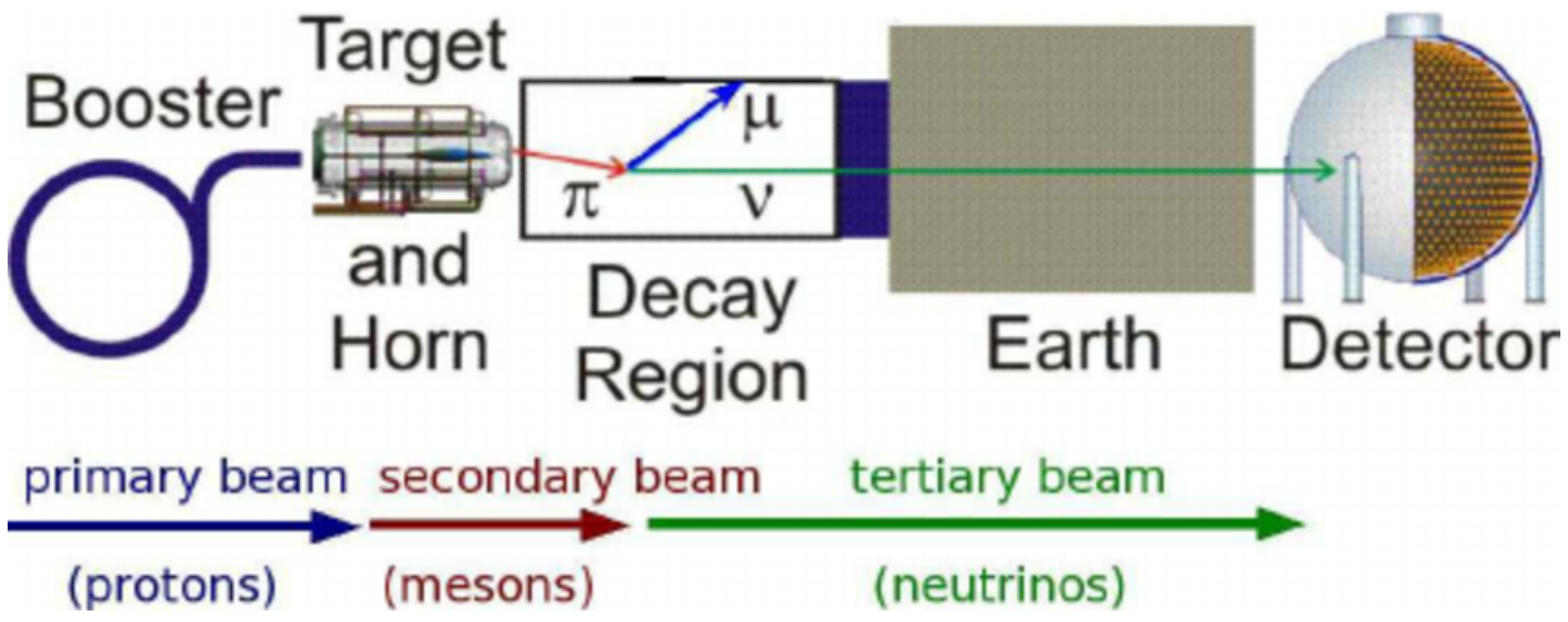


Neutrinos cannot be massless!

- **How do they acquire the mass?**
- **Are they Majorana particles? ($\nu_\ell = \bar{\nu}_\ell$)** ← addressed by $0\nu\beta\beta$ experiments
- **Do $\bar{\nu}_\ell$ oscillate differently than ν_ℓ?** → related to Matter-Antimatter Asymmetry

Introduction and Motivation



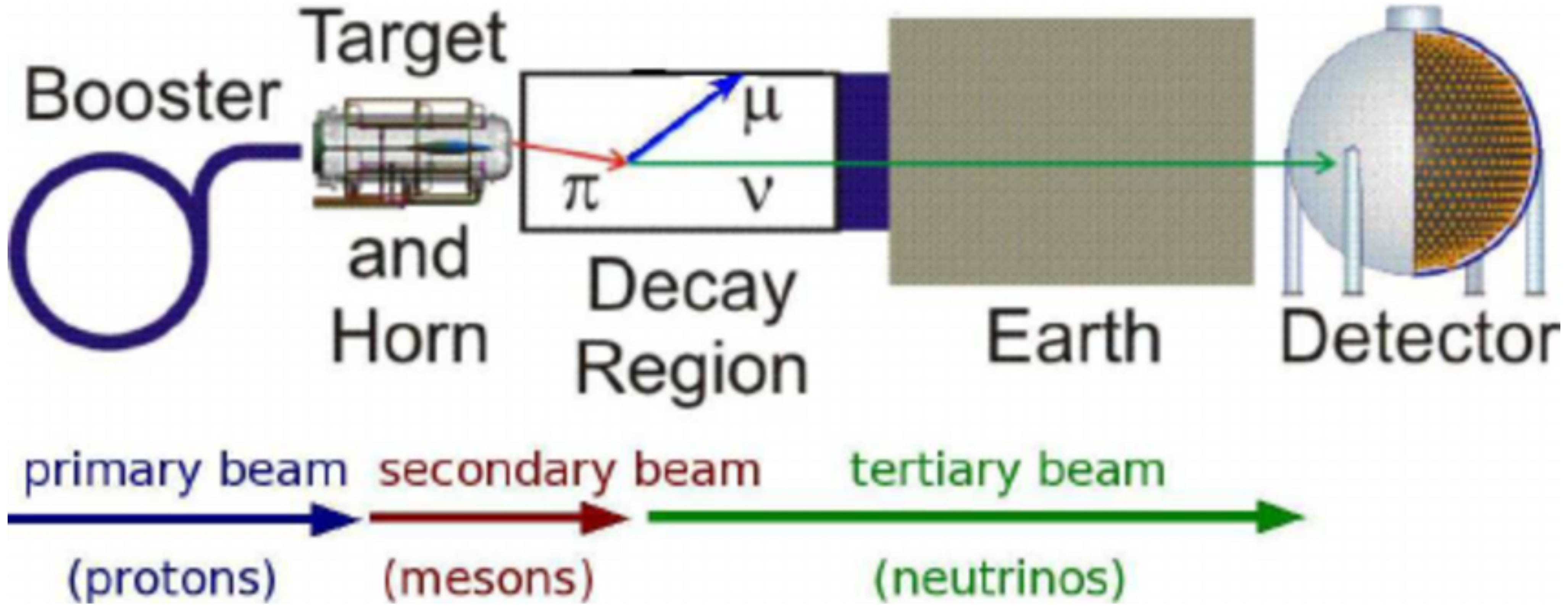
e.g. MiniBooNE (Fermilab)

ν_μ flux is artificially produced and then detected after a travel length L

$$\nu_\mu\, n \xrightarrow{\mathcal{J}^-} \mu^- p \qquad \frac{d\sigma^{(\nu N)}}{dQ^2} \propto |\langle N | \mathcal{J}^- | N \rangle|^2$$

$\mathcal{J}^-$ is the weak current

Introduction and Motivation



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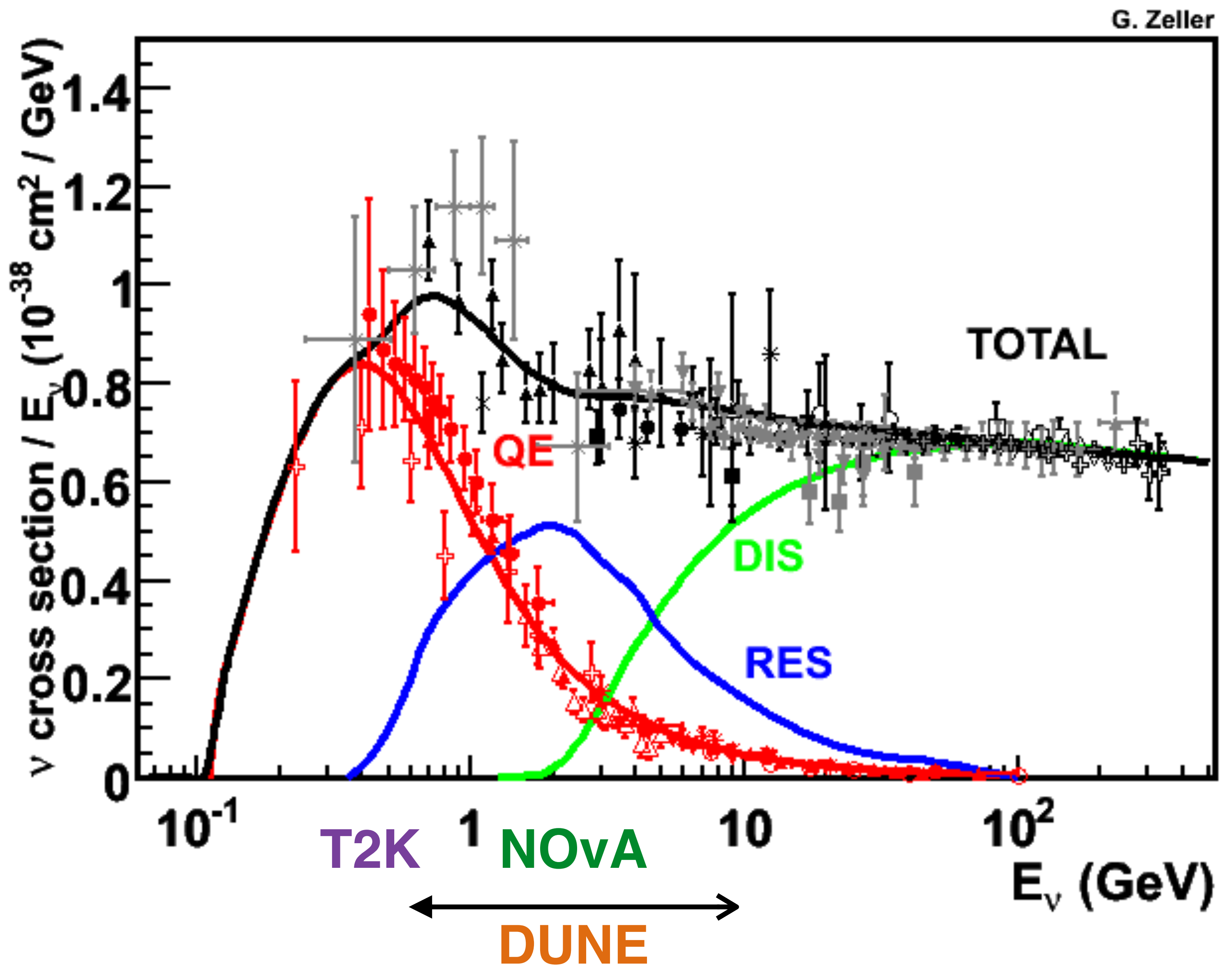
Main challenges

- **Nuclear model to factorise neutrino-nuclei scattering into neutrino-nucleon scattering (known from theory);**
- **Q^2 in a range where excited nucleons are produced!**

[arXiv:2203.09030]



[RMP.84.1307]

Required also knowledge of $\langle N^*|\mathcal{J}^-|N\rangle$, $\langle \Delta|\mathcal{J}^-|N\rangle$ and $\langle N\pi|\mathcal{J}^-|N\rangle$

We are the first to investigate $\langle N\pi|\mathcal{J}^-|N\rangle$ with LQCD

Lorentz decomposition of nucleon matrix elements

$$\langle N(\mathbf{p}') \,|\, \mathcal{J}(\mathbf{q}) \,|\, N(\mathbf{p}) \rangle = \bar{u}_{\mathbf{p}'} \; FF[\mathcal{J}] \; u_{\mathbf{p}}$$

$$\frac{d\sigma^{(\nu N)}}{dQ^2} \propto |\langle N | \mathcal{J}^- | N \rangle|^2$$

$\mathcal{J} = \bar{q}\Gamma q \qquad \Gamma \in \{\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5\}$

$FF[\mathcal{J}]$ = Lorentz decomposition of the nucleon matrix element in terms of nucleon form factors $G(Q^2)$

Pseudoscalar $\quad \Gamma = \gamma_5 \;:\; \mathcal{J} = \mathcal{P}$

$$\langle N(\mathbf{p}') \,|\, \mathcal{P}(\mathbf{q}) \,|\, N(\mathbf{p}) \rangle = \bar{u}_{\mathbf{p}'} \; \gamma_5 G_P(Q^2) \; u_{\mathbf{p}}$$

Axial $\quad \Gamma = \gamma_\mu\gamma_5 \;:\; \mathcal{J} = \mathcal{A}_\mu$

$$\langle N(\mathbf{p}') \,|\, \mathcal{A}_\mu(\mathbf{q}) \,|\, N(\mathbf{p}) \rangle = \bar{u}_{\mathbf{p}'} \left[\gamma_\mu\gamma_5 G_A(Q^2) + \frac{q_\mu}{2m_N}\gamma_5 \widetilde{G}_P(Q^2) \right] u_{\mathbf{p}}$$

$g_A \equiv G_A(0)$

Axial charge

Vector $\quad \Gamma = \gamma_\mu \;:\; \mathcal{J} = \mathcal{V}_\mu$

$$\langle N(\mathbf{p}') \,|\, \mathcal{V}_\mu(\mathbf{q}) \,|\, N(\mathbf{p}) \rangle = \bar{u}_{\mathbf{p}'} \left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2m_N} F_2(Q^2) \right] u_{\mathbf{p}}$$

From nucleon form factors to charge distribution

Vector

$$\langle N(\mathbf{p}')|\mathscr{V}_\mu(\mathbf{q})|N(\mathbf{p})\rangle = \bar{u}_{\mathbf{p}'}\left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2m_N}F_2(Q^2)\right]u_{\mathbf{p}}$$

$$G_E = F_1 - \frac{Q^2}{m_N^2}F_2$$

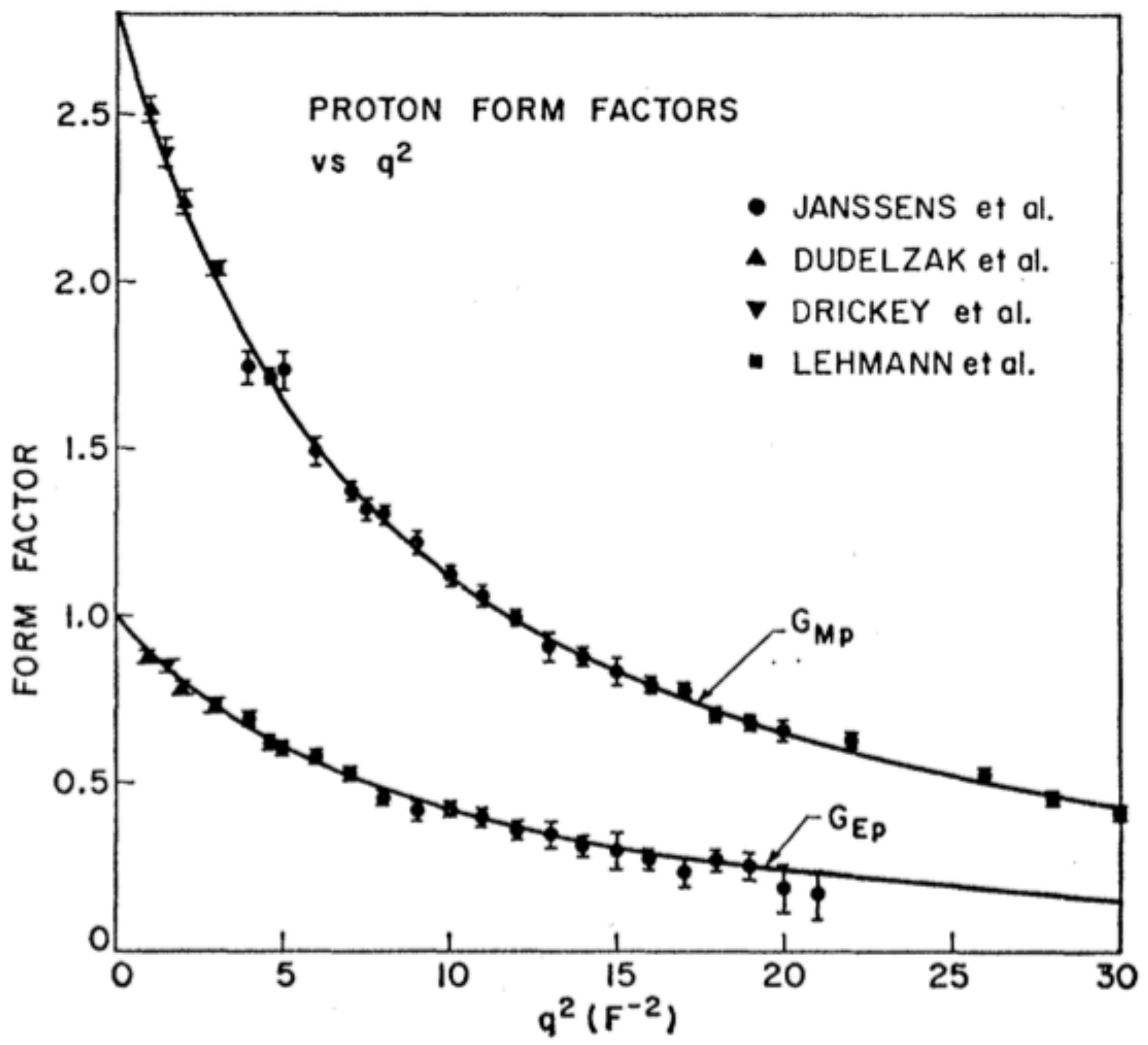
$$G_M = F_1 + F_2$$

Electromagnetic form factors

Form factors are related to charge distribution via Fourier transform

Charge distribution $f(r)$		Form factor $F(q^2)$	
Point	$\delta(r)/4\pi$	1	Constant
Exponential	$(a^3/8\pi)\cdot\exp(-ar)$	$(1+q^2/a^2\hbar^2)^{-2}$	Dipole

Since G_M, G_E are not constant…



Proton is not point-like!

Form factors contain information on the hadron structure

Accessing hadron structure through Lattice QCD

$\langle N' | \mathcal{J}(\mathbf{q}) | N \rangle$

1) Construct operator O_1 with $\mathrm{J}^P = \left(\frac{1}{2}\right)^+$ s.t. $\bar{\mathrm{O}}_1 | \Omega \rangle = c^N | N \rangle + c^{N*} | N* \rangle + c^{N\pi} | N\pi \rangle + \ \ldots$

e.g. $\mathrm{O}_1 \sim uud \sim p$

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2) Compute three-point correlation functions (momentum $\mathbf{p}', \mathbf{p}, \mathbf{q} = \mathbf{p}' - \mathbf{p}$)

$$\langle \mathrm{O}_1(\mathbf{p}', t) \ \mathcal{J}(\mathbf{q}, \tau) \ \bar{\mathrm{O}}_1(\mathbf{p}, 0) \rangle$$

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$$\langle \mathrm{O}_1(\mathbf{p}', t)\ \mathcal{J}(\mathbf{q}, \tau)\ \bar{\mathrm{O}}_1(\mathbf{p}, 0) \rangle$$

$$1 = \sum_n \frac{1}{2E_n} |n\rangle\langle n|$$

(spectral decomposition)

$n = N, N^*, N\pi, \ldots$

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$$\langle \mathrm{O}_1(\mathbf{p}', t) \ \mathcal{J}(\mathbf{q}, \tau) \ \bar{\mathrm{O}}_1(\mathbf{p}, 0) \rangle \propto \langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p}) \rangle \ e^{-E'_N(t-\tau)} e^{-E_N \tau} \ + \ \dots \quad (N*, N\pi, \dots) \qquad E_N < E_{N\pi} \ , E_{N*}$$

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3) Extract $\langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p}) \rangle$ at $t \gg \tau \gg 0$

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3) Extract $\langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p}) \rangle$ at $t \gg \tau \gg 0$

Problem
large t, τ data is noisy w/ current statistics.
We use small t, τ data

Consequence
Contamination from excited nucleons $(N*, \ldots)$
and multiparticle states $(N\pi, \ldots)$

Construct a ratio of correlation functions to extract nucleon matrix element

$$R^{\mathcal{J}}(\mathbf{p}', t; \mathbf{q}, \tau) = \frac{C^{\mathcal{J}}_{3pt}(\mathbf{p}', t; \mathbf{q}, \tau)}{C_{2pt}(\mathbf{p}, t)} \sqrt{\frac{C_{2pt}(\mathbf{p}', t)\ C_{2pt}(\mathbf{p}', \tau)\ C_{2pt}(\mathbf{p}, t-\tau)}{C_{2pt}(\mathbf{p}, t)\ C_{2pt}(\mathbf{p}, \tau)\ C_{2pt}(\mathbf{p}', t-\tau)}} \propto \langle N(\mathbf{p}') \,|\, \mathcal{J}(\mathbf{q}) \,|\, N(\mathbf{p}) \rangle \ + \ \ldots$$

Any t and τ dependence in $R^{\mathcal{J}}$ is sign of ESC (Excited State Contamination)

$$C^{\mathcal{J}}_{3pt}(\mathbf{p}', t; \mathbf{q}, \tau) = \langle \mathrm{O}_1(\mathbf{p}', t)\ \mathcal{J}(\mathbf{q}, \tau)\ \bar{\mathrm{O}}_1(\mathbf{p}, 0) \rangle \qquad C_{2pt}(\mathbf{p}, t) = \langle \mathrm{O}_1(\mathbf{p}, t)\ \bar{\mathrm{O}}_1(\mathbf{p}, 0) \rangle$$

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Forward limit $(\mathbf{q} = \mathbf{0},\ \mathbf{p}' = \mathbf{p})$

$$R^{\mathcal{J}}(\mathbf{p}, t; \mathbf{0}, \tau) = \frac{C^{\mathcal{J}}_{3pt}(\mathbf{p}, t; \mathbf{0}, \tau)}{C_{2pt}(\mathbf{p}, t)} \propto \langle N(\mathbf{p}) \,|\, \mathcal{J}(\mathbf{0}) \,|\, N(\mathbf{p}) \rangle \ + \ \ldots$$

Example

Axial charge g_A

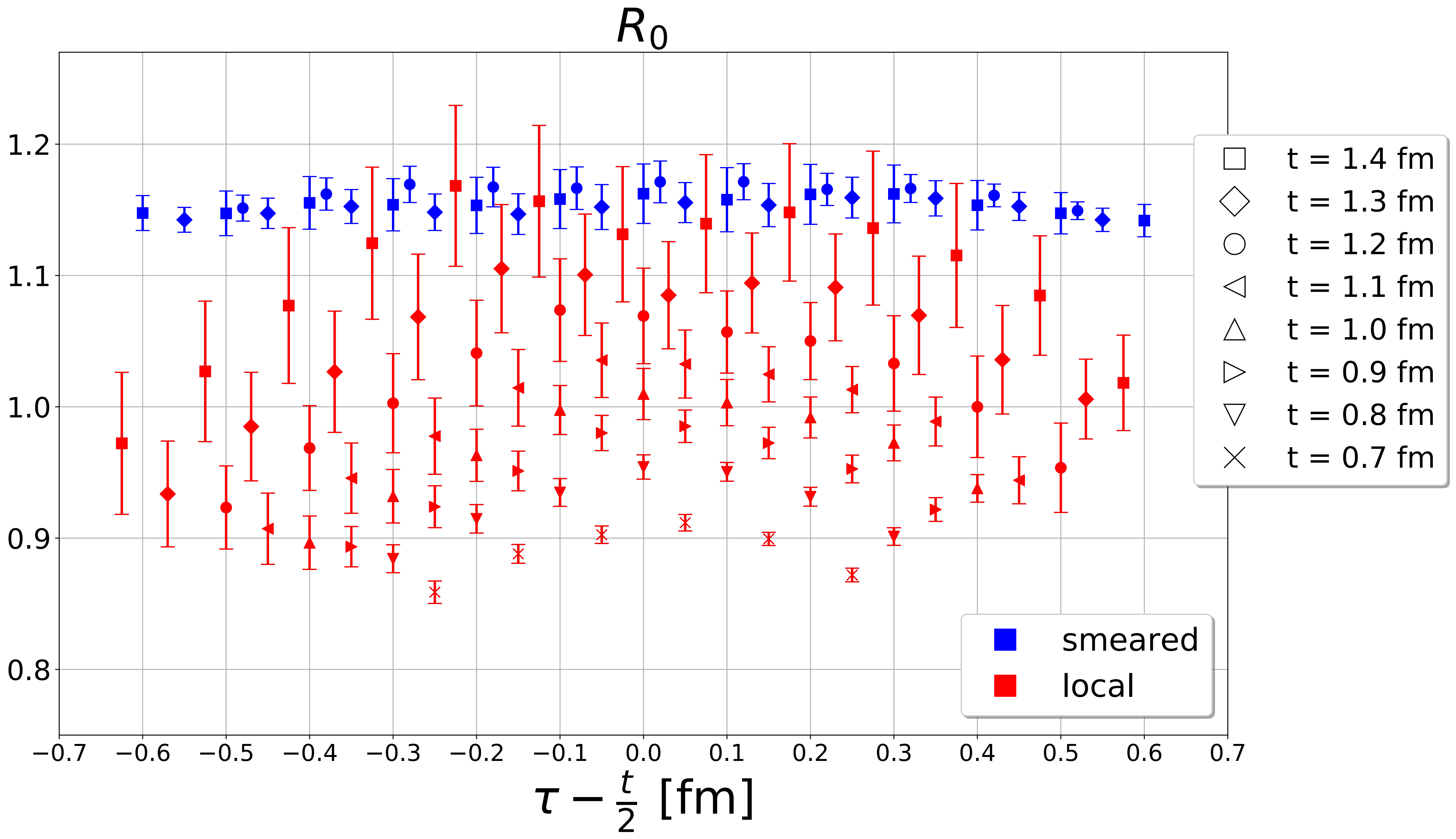
Results for g_A in the following

$$\mathcal{A}_i = \bar{q}\ \gamma_i \gamma_5\ q$$

$$R^{\mathcal{A}_i}(\mathbf{p}, t; \mathbf{0}, \tau) = G_A(Q^2 = 0) \ + \ \ldots$$

The effect of the smearing at zero momentum

O_1 can be iteratively improved with smearing techniques (quark + link smearing)



Axial charge g_A

$$R_0 = \frac{\langle O_1(\mathbf{0},t)\ \mathscr{A}_i(\mathbf{0},\tau)\ \bar{O}_1(\mathbf{0},0)\rangle}{\langle O_1(\mathbf{0},t)\ \bar{O}_1(\mathbf{0},0)\rangle} = g_A$$

$\mathscr{A}_i = \bar{q}\ \gamma_i\gamma_5\ q$ $\qquad$ $\mathbf{p}' = \mathbf{q} = \mathbf{p} = \mathbf{0}$ (Rest frame)

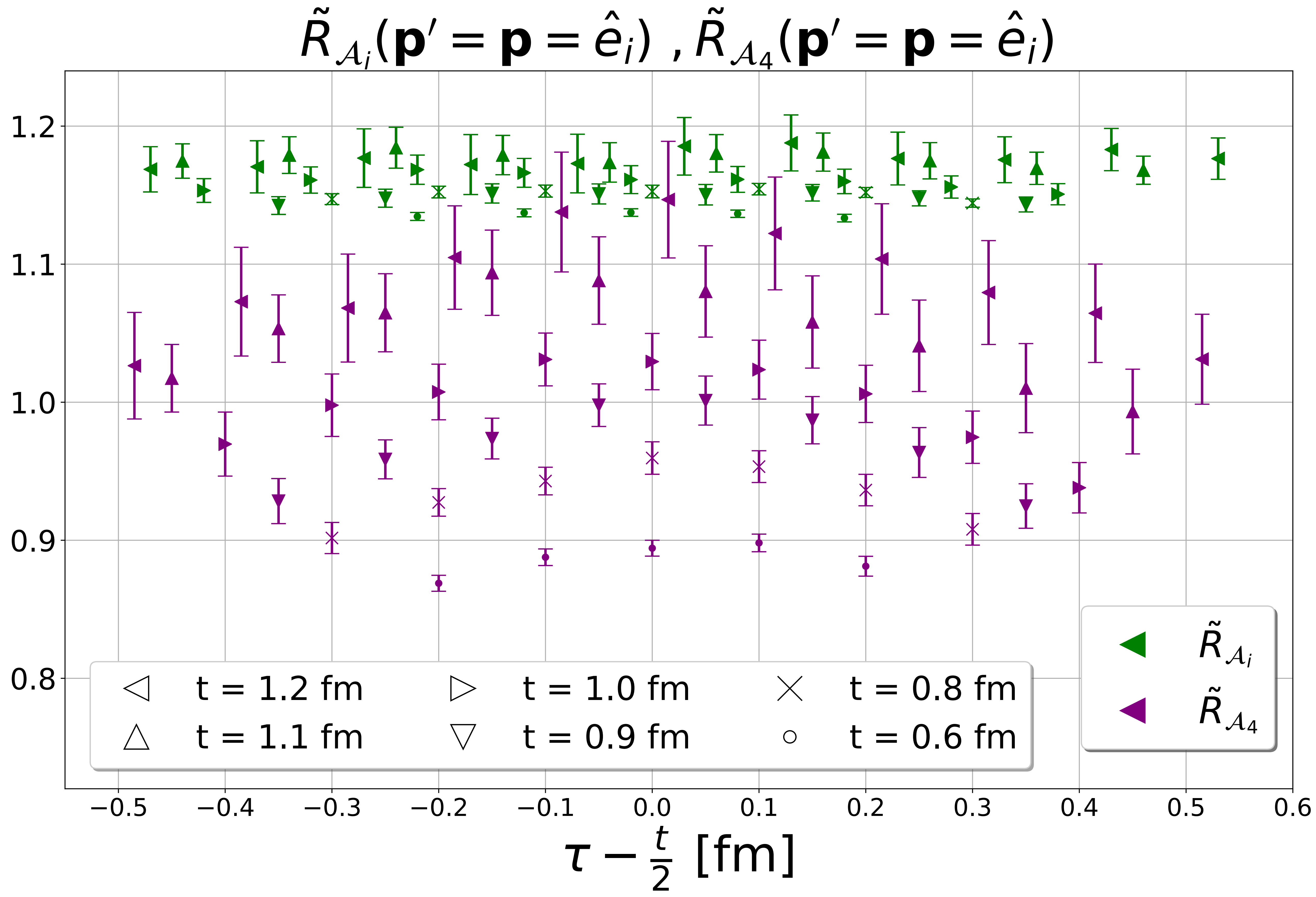
Clear sign of excited state contamination with local nucleon operators

The effect of the smearing is evident

$g_A = 1.16 \pm 0.07$ $\qquad$ at $m_\pi = m_K \approx 426$ MeV, $a \approx 0.098$ fm, $L = 24a, T = 2L$

The axial charge g_A from $\langle N(\mathbf{p}) | \mathscr{A}_\mu(\mathbf{q}=\mathbf{0}) | N(\mathbf{p}) \rangle$

g_A can be also extracted from moving frame $\mathbf{p}' = \mathbf{p} = \hat{e}_i = \frac{2\pi}{L}\hat{n}_i$



(Here only improved/smeared operators)

$$\tilde{R}_{A_i} = \frac{\langle \mathrm{O}_1(\mathbf{p},t)\ \mathscr{A}_i(\mathbf{q}=\mathbf{0},\tau)\ \bar{\mathrm{O}}_1(\mathbf{p},0)\rangle}{\langle \mathrm{O}_1(\mathbf{p},t)\ \bar{\mathrm{O}}_1(\mathbf{p},0)\rangle} = g_A + \ldots$$

$$\tilde{R}_{A_4} = \frac{\langle \mathrm{O}_1(\mathbf{p},t)\ \mathscr{A}_4(\mathbf{q}=\mathbf{0},\tau)\ \bar{\mathrm{O}}_1(\mathbf{p},0)\rangle}{\langle \mathrm{O}_1(\mathbf{p},t)\ \bar{\mathrm{O}}_1(\mathbf{p},0)\rangle}\left(-\frac{E}{p_i}\right) = g_A + \ldots$$

Results with $\mathscr{J} = \mathscr{A}_i$ are consistent with rest frame $g_A = 1.16 \pm 0.07$

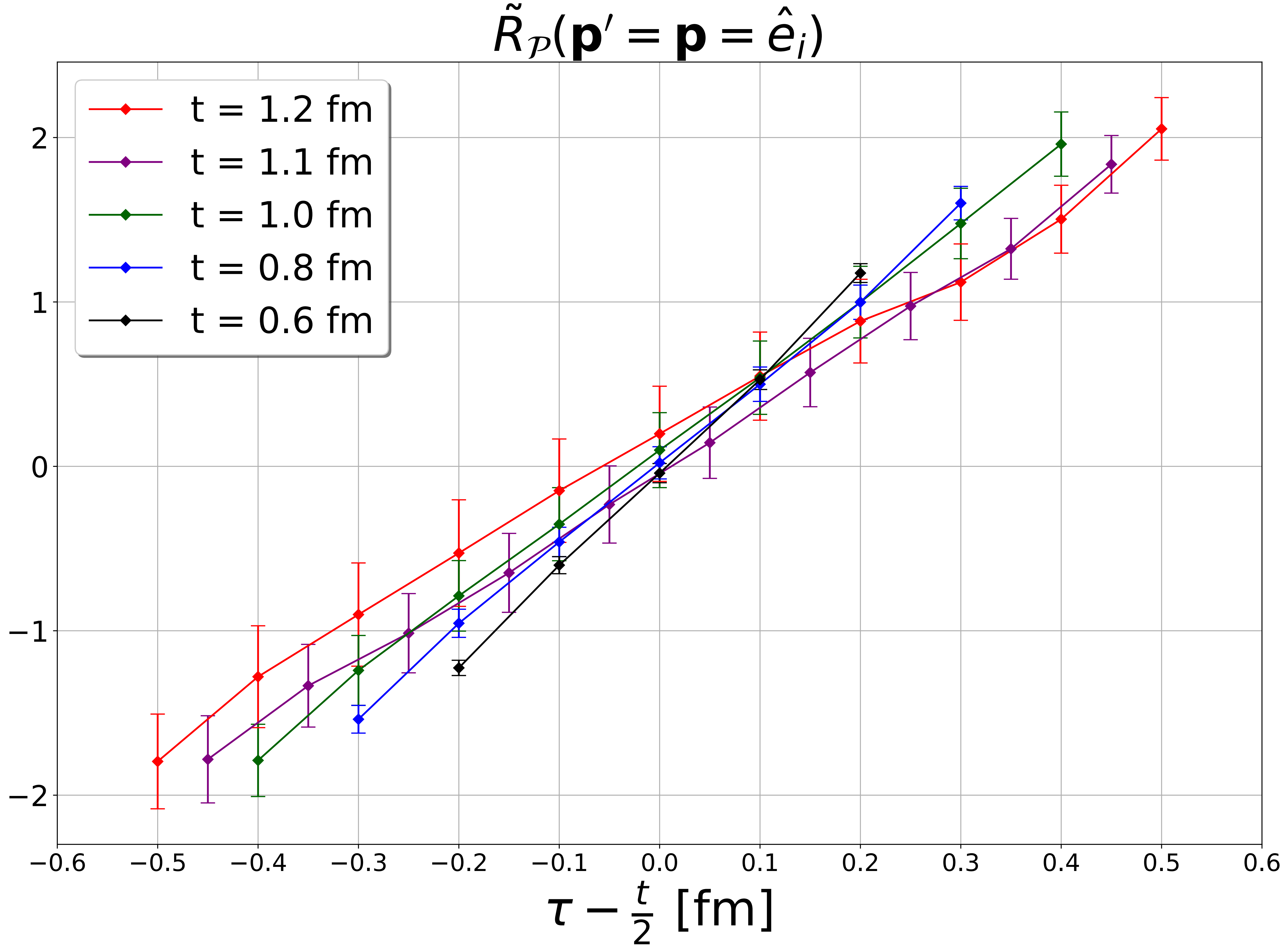
Results with $\mathscr{J} = \mathscr{A}_4$ show 5%-20% discrepancy

Observed also by χPT collaboration *[arXiv:1612.04388]*

$m_\pi = m_K \approx 426$ MeV, $a \approx 0.098$ fm, $L = 24a, T = 2L$

Excited state effect in the pseudoscalar channel ($\mathbf{q}=\mathbf{0}$, but $\mathbf{p}'=\mathbf{p}\neq\mathbf{0}$)

We investigate, for the first time, channels with $\mathcal{J}=\mathcal{P}$ and $\mathbf{p}'=\mathbf{p}=\hat{e}_i=\frac{2\pi}{L}$



$m_\pi = m_K \approx 426$ MeV, $a \approx 0.098$ fm, $L=24a, T=2L$

$$\tilde{R}_P = \frac{\langle \mathrm{O}_1(\mathbf{p},t)\ \mathcal{P}(\mathbf{q}=\mathbf{0},\tau)\ \bar{\mathrm{O}}_1(\mathbf{p},0)\rangle}{\langle \mathrm{O}_1(\mathbf{p},t)\ \bar{\mathrm{O}}_1(\mathbf{p},0)\rangle}\frac{E}{p_i} = 0 + \ldots$$

The signal is purely excited states and in particular $N\pi$.

$$\bar{\mathrm{O}}_1|\Omega\rangle \approx c^N|N> + c^{N\pi}|N\pi>$$

O. Bär predicts with ChPT that terms $\propto \langle N\pi|\mathcal{P},\mathcal{A}_4|N\rangle$ are relevant [PRD.100.054507] [PRD.99.054506]

With LO-ChPT, the correction to the 3pt at tree-level is

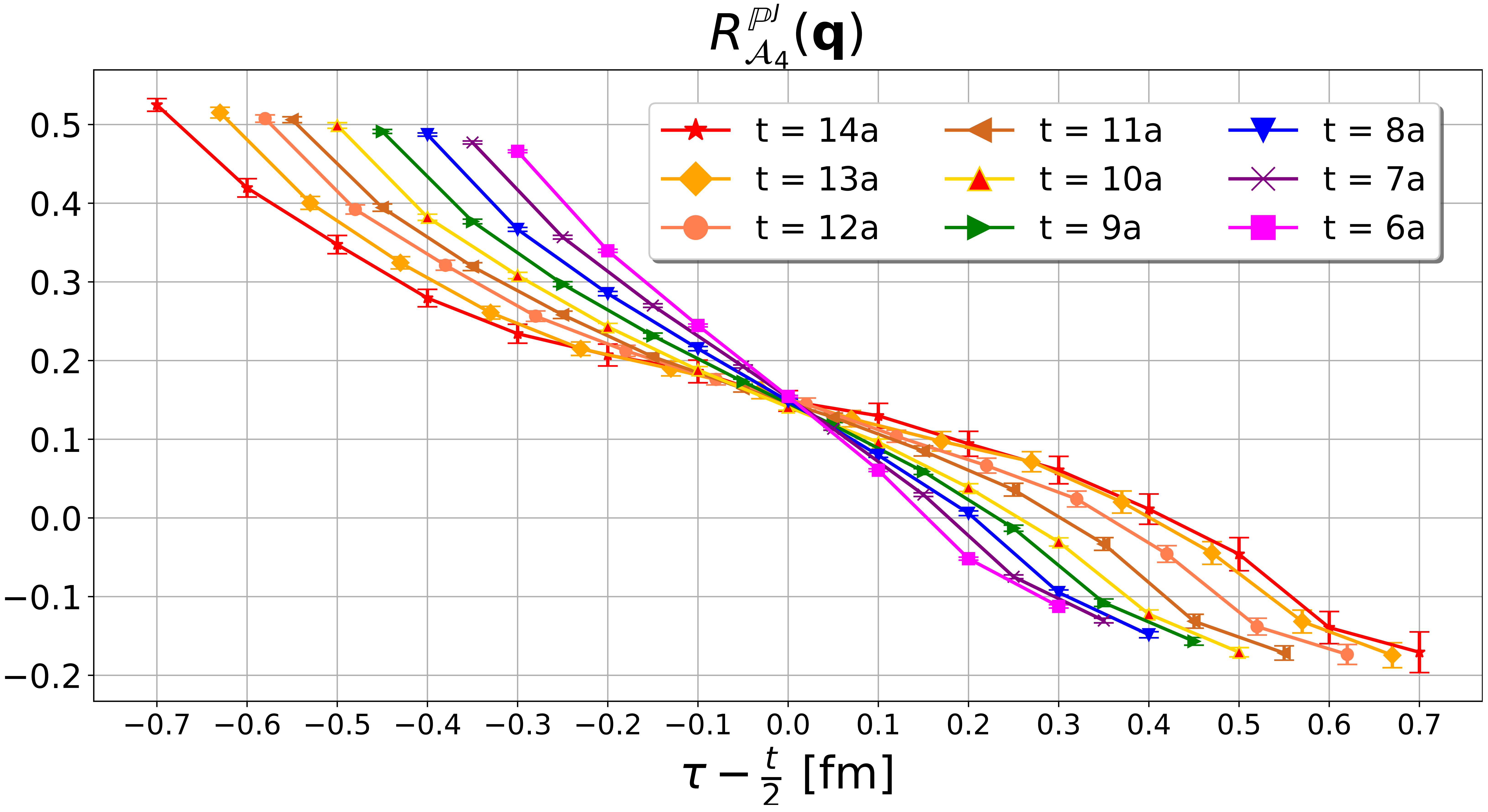
$$\delta^{\mathcal{P}}_{\chi PT} = A\frac{E'}{E_\pi}e^{-(E'-m_\pi/2)t}\sinh\left(m_\pi(\tau-t/2)\right)$$

[JHEP05(2020)126]

where $A \propto g_A, \mathbf{p}$

This channel is the clearest case of $N\pi$ state contamination

Excited state effect at $\mathbf{q} \neq \mathbf{0}$: the $\mathscr{A}_4$ channel



$$q = \hat{e}_j = \frac{2\pi}{L}\hat{n}_j$$

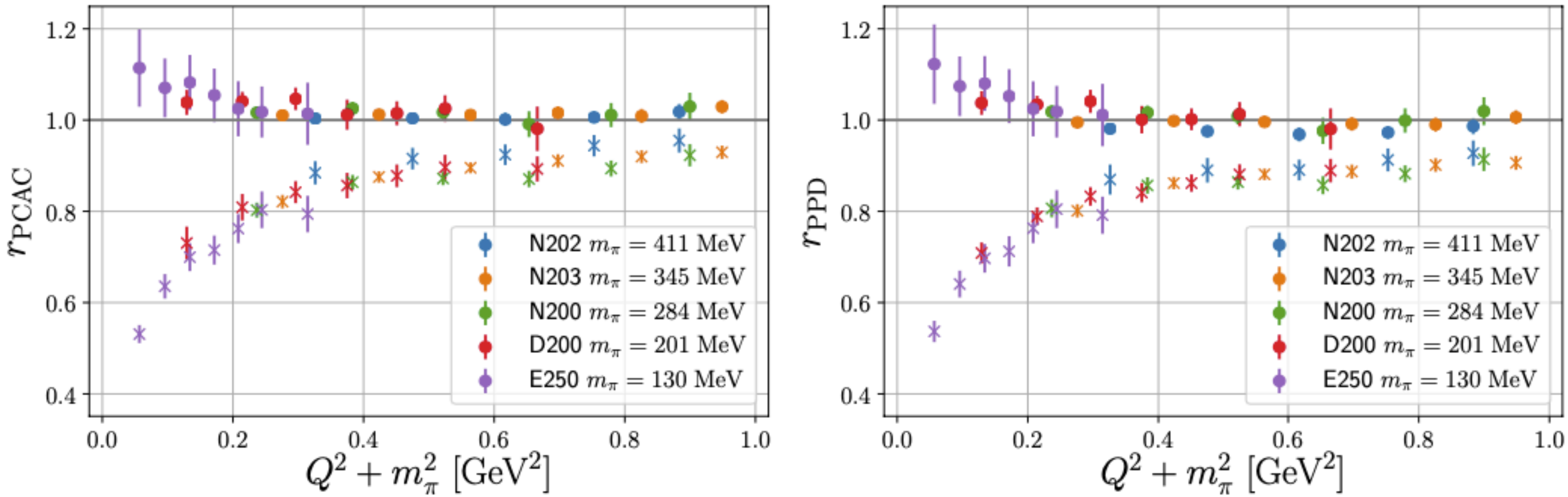
$$R^{\mathscr{A}_4}(\mathbf{p}' = \mathbf{0}, t; \mathbf{q}, \tau) \propto G_A, \widetilde{G}_P$$

Ratio is t- and τ-dependent

G_A and $\widetilde{G}_P$ are extracted unreliably
if ESC is not taken into account carefully

G_A is extracted from another channel

PCAC and PPD ratios



$$r_{\mathrm{PCAC}} = \frac{m_q G_P(Q^2) + \frac{Q^2}{4m_N}\tilde{G}_P(Q^2)}{m_N G_A(Q^2)} = 1\,?$$

$$r_{\mathrm{PPD}} = \frac{(m_\pi^2 + Q^2)\tilde{G}_P(Q^2)}{4m_N^2 G_A(Q^2)} = 1\,?$$

Filled circles are when taking into account $N\pi$ contamination with ChPT-based ansatz

Literature background

Year	Paper	**ES $(N\pi)$ contamination is taken into account…**
2019	[PRL.124.072002 (R. Gupta et al.)]	by extracting energies of the excited states from the 3pt (!) with $\mathscr{J} = \mathscr{A}_4$ and employing a multistate fit
2019	[JHEP05(2020)126 (RQCD: G. Bali, **L. B.** et al)]	by employing a ChPT-based ansatz
2022	[arXiv:2211.12278 (**L.B**., G. Bali, S. Collins)]	by adopting a GEVP analysis with O_N and $\mathrm{O}_{N\pi}$

ChPT reveals significance of $N\pi$ contamination *[PRD.100.054507, PRD.99.054506]* O. Bär

The GEVP approach for matrix elements was treated in *[JHEP04(2009)094]* R. Sommer et al.; *[JHEP01(2012)140]* J. Bulava, R. Sommer, et al.

$\Delta E_n = E_{N\pi^n} - E_N$ depends on the box size and m_π *[PoS Lattice 2018]* J. Green

Literature background

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2022	[arXiv:2211.12278 (**L.B.**, G. Bali, S. Collins)]	by adopting a GEVP analysis with O_N and $\mathrm{O}_{N\pi}$

As a pilot study, we use the same ensemble as before

$$m_\pi = m_K \approx 426 \text{ MeV}, \qquad a \approx 0.098 \text{ fm}, \qquad L = 24a, T = 2L, \qquad m_\pi L = 5.1$$

The Variational Method in a nutshell

Construct a basis $\mathbb{B}_n = \{\mathrm{O}_1, \mathrm{O}_2, \ldots, \mathrm{O}_n\}$ of operators with same quantum numbers $J^P = \left(\frac{1}{2}\right)^+$

$\mathrm{O}_1 \propto (qqq)$ $\mathrm{O}_2 \propto (qqq)(\bar{q}q)$

Suppose we find $n = 2$ operators s.t. :

$$\bar{\mathrm{O}}_1 |\Omega\rangle \approx c_1^N |N\rangle + c_1^{N\pi} |N\pi\rangle$$
$$\bar{\mathrm{O}}_2 |\Omega\rangle \approx c_2^N |N\rangle + c_2^{N\pi} |N\pi\rangle$$

(!) O_2 must be projected to have spin $1/2$ and isospin $1/2$

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(!) O_2 must be projected to have spin $1/2$ and isospin $1/2$

Construct

$$C(t) = \begin{pmatrix} \langle O_1(t)\ \bar{O}_1(0)\rangle & \langle O_1(t)\ \bar{O}_2(0)\rangle \\ \langle O_2(t)\ \bar{O}_1(0)\rangle & \langle O_2(t)\ \bar{O}_2(0)\rangle \end{pmatrix}$$

solve $\quad C(t)v^\alpha(t,t_0) = C(t_0)\ \lambda^\alpha(t,t_0)v^\alpha(t,t_0)$ $\quad$ **GEVP**

$v^\alpha(t_0)$, $\lambda^\alpha(t_0)$ are Generalised Eigenvectors and Eigenvalues

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solve $\quad C(t) v^\alpha(t, t_0) = C(t_0)\, \lambda^\alpha(t, t_0) v^\alpha(t, t_0)$ **GEVP**

$v^\alpha(t_0)$, $\lambda^\alpha(t_0)$ are Generalised Eigenvectors and Eigenvalues

(Amazing) Properties

$$\lambda^\alpha(t_0) = d^\alpha(t_0)\, e^{-E_\alpha(t-t_0)}$$

$$\sum_{i,j} v_i^\alpha(t_0)\, C_{ij}(t_0)\, v_j^\beta(t_0) = \delta^{\alpha\beta}$$

→

$\bar{\mathrm{O}}_\alpha = \sum_i v_i^\alpha(t_0)\, \bar{\mathrm{O}}_i$ $\quad$ s.t. $\quad$ $\bar{\mathrm{O}}_\alpha |\Omega\rangle \approx c_\alpha |\alpha\rangle$

System is diagonalised! e.g. $\bar{\mathrm{O}}_N |\Omega\rangle \approx c_N |N\rangle$

Operators with $J^P = (1/2)^+$ and $I = 1/2,\ I_z = -1/2$ (neutron channel)

Isospin projection with Clebsch-Gordan

$$\mathrm{O}_2(x,y) = \frac{1}{\sqrt{3}}\mathrm{O}_p(x)\ \mathrm{O}_{\pi^-}(y) - \frac{2}{\sqrt{3}}\mathrm{O}_n(x)\ \mathrm{O}_{\pi^0}(y)$$

New correlation functions to be computed

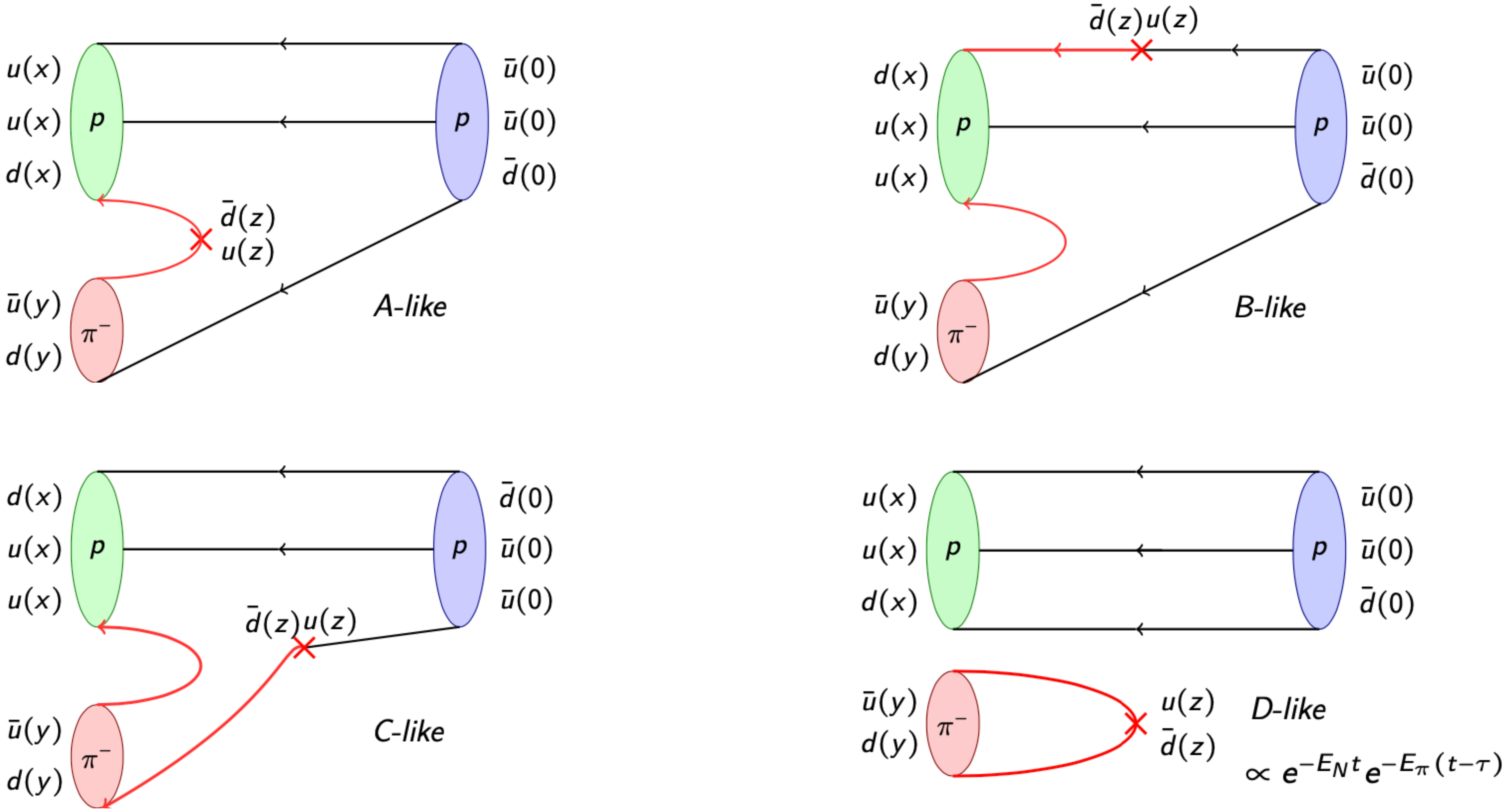
$$\langle \mathrm{O}_{p\pi^-}(\mathbf{p}', t)\ \mathcal{J}^-(\mathbf{q}, \tau)\ \mathrm{O}_p(\mathbf{p},0)\rangle \qquad \langle \mathrm{O}_{n\pi^0}(\mathbf{p}', t)\ \mathcal{J}^-(\mathbf{q}, \tau)\ \mathrm{O}_p(\mathbf{p},0)\rangle$$

Helicity projection with (Lattice) Group Theory

$$\mathrm{O}_{2,\uparrow}(\mathbf{p}'=\mathbf{0}) = \mathrm{O}_{N\downarrow}(-\hat{e}_x)\mathrm{O}_\pi(\hat{e}_x) - \mathrm{O}_{N\downarrow}(\hat{e}_x)\mathrm{O}_\pi(-\hat{e}_x) - i\mathrm{O}_{N\downarrow}(-\hat{e}_y)\mathrm{O}_\pi(\hat{e}_y) + i\mathrm{O}_{N\downarrow}(\hat{e}_y)\mathrm{O}_\pi(-\hat{e}_y) + \mathrm{O}_{N\uparrow}(-\hat{e}_z)\mathrm{O}_\pi(\hat{e}_z) - \mathrm{O}_{N\uparrow}(\hat{e}_z)\mathrm{O}_\pi(-\hat{e}_z)$$

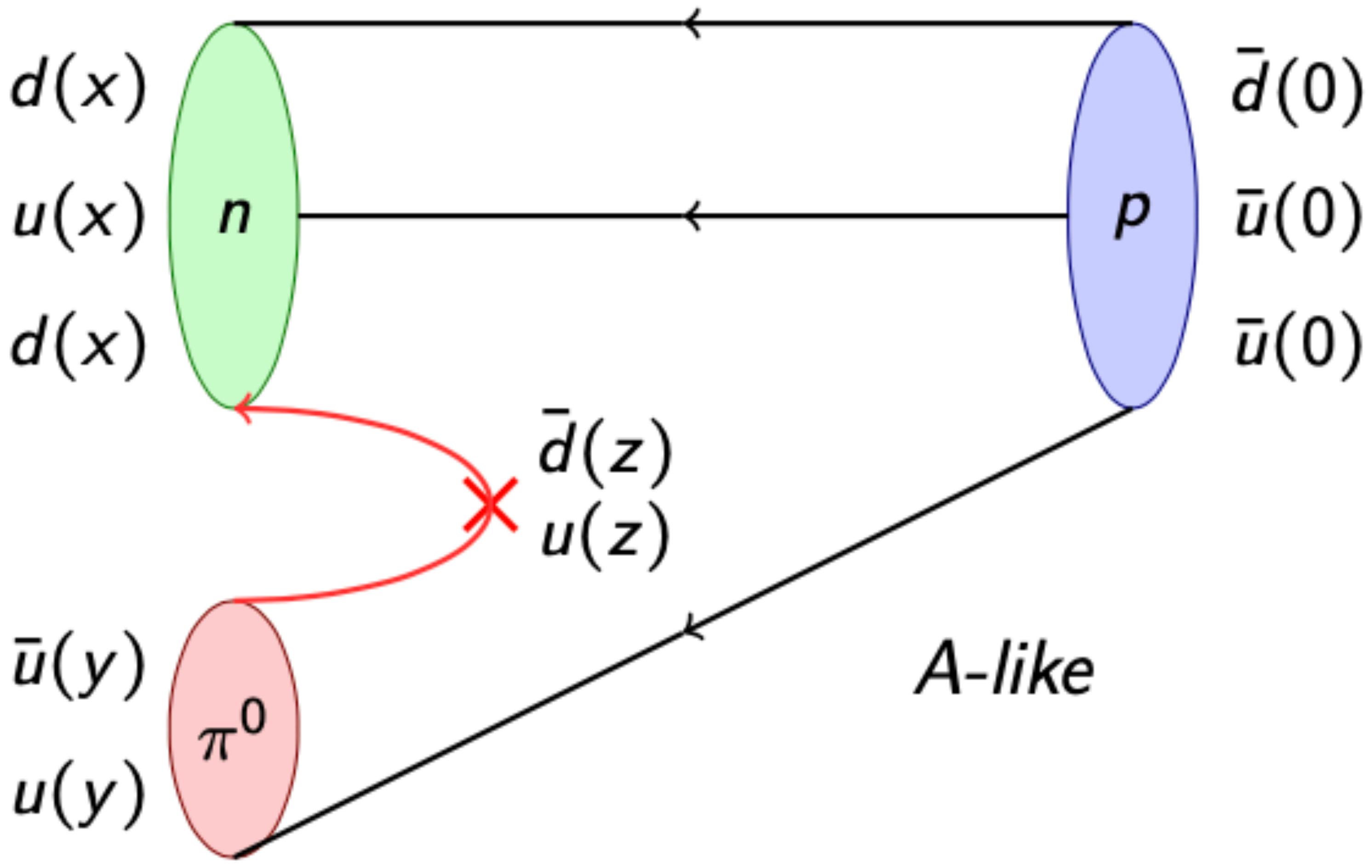
$$\mathrm{O}_2^{(1)}(\mathbf{p}'=\hat{e}_i) = \mathrm{O}_N(\mathbf{0})\mathrm{O}_\pi(\hat{e}_i) \qquad \mathrm{O}_2^{(2)}(\mathbf{p}'=\hat{e}_i) = \mathrm{O}_N(\hat{e}_i)\mathrm{O}_\pi(\mathbf{0}) \qquad \hat{e}_i = \frac{2\pi}{L}\hat{n}_i$$

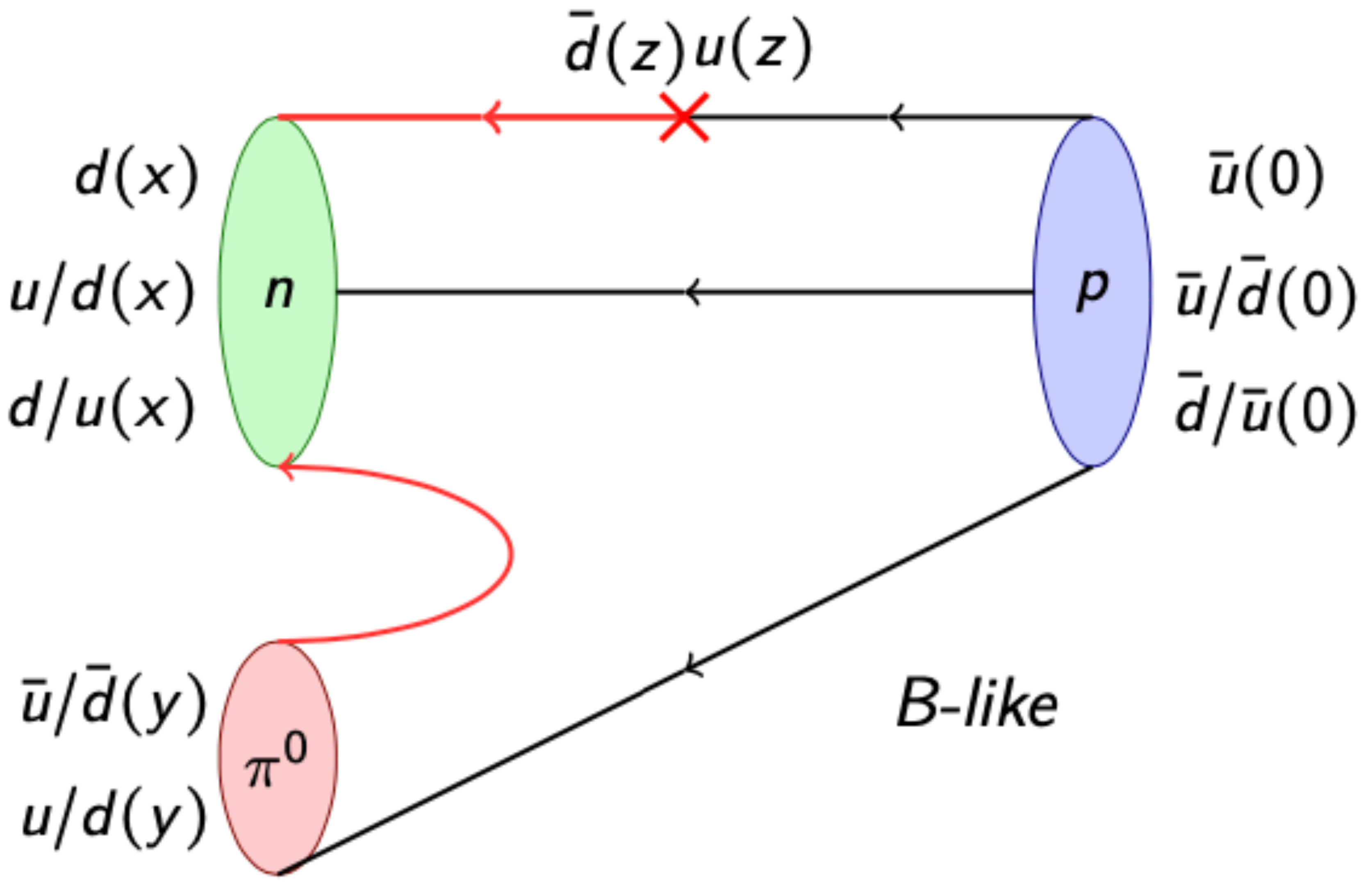
Topologies in $p \xrightarrow{\mathcal{J}^-} p\pi^-$ for $\langle \mathrm{O}_2(\mathbf{p}', t)\ \mathcal{J}^-(\mathbf{q}, \tau)\ \bar{\mathrm{O}}_1(\mathbf{p}, 0)\rangle$

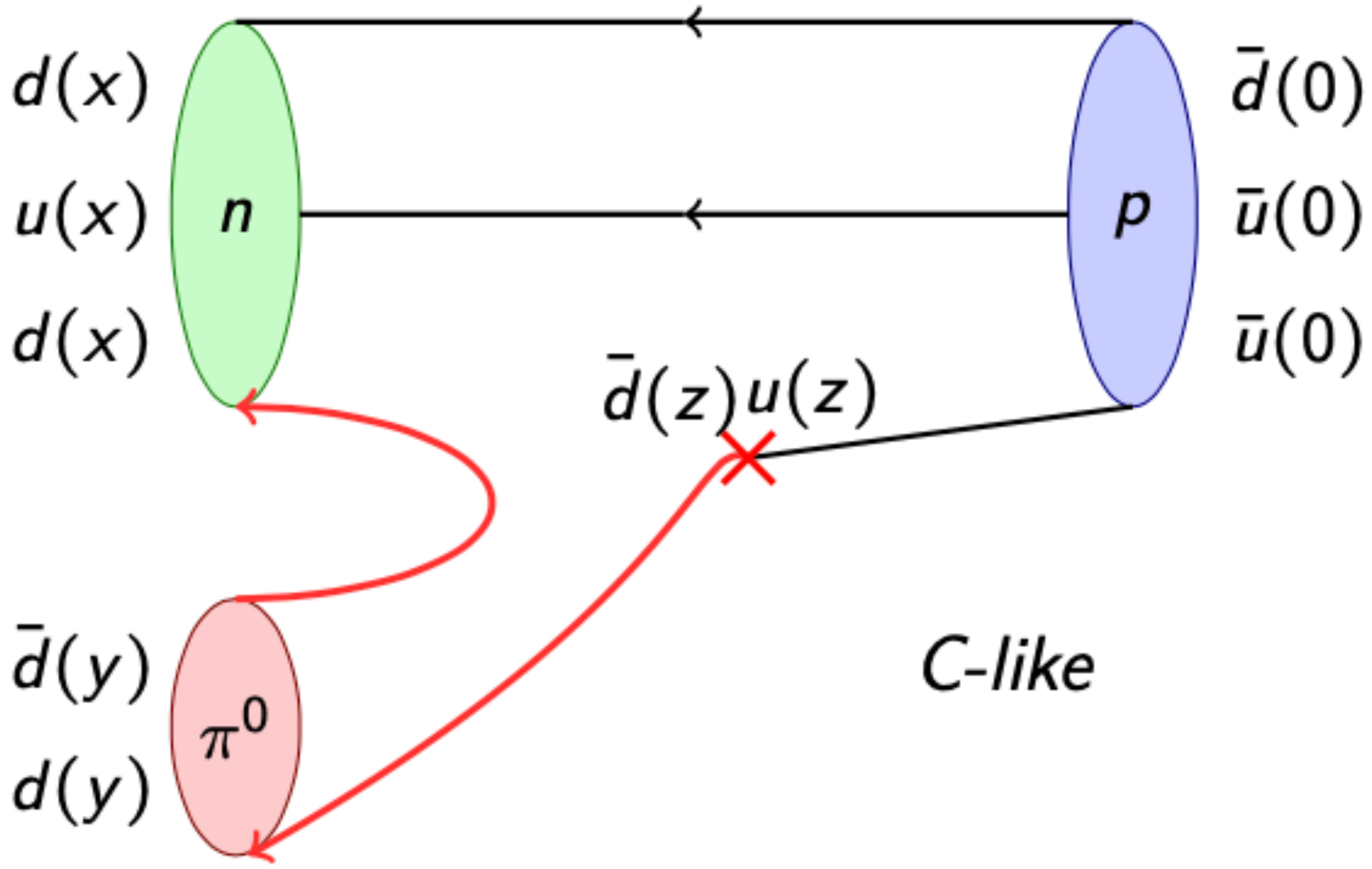


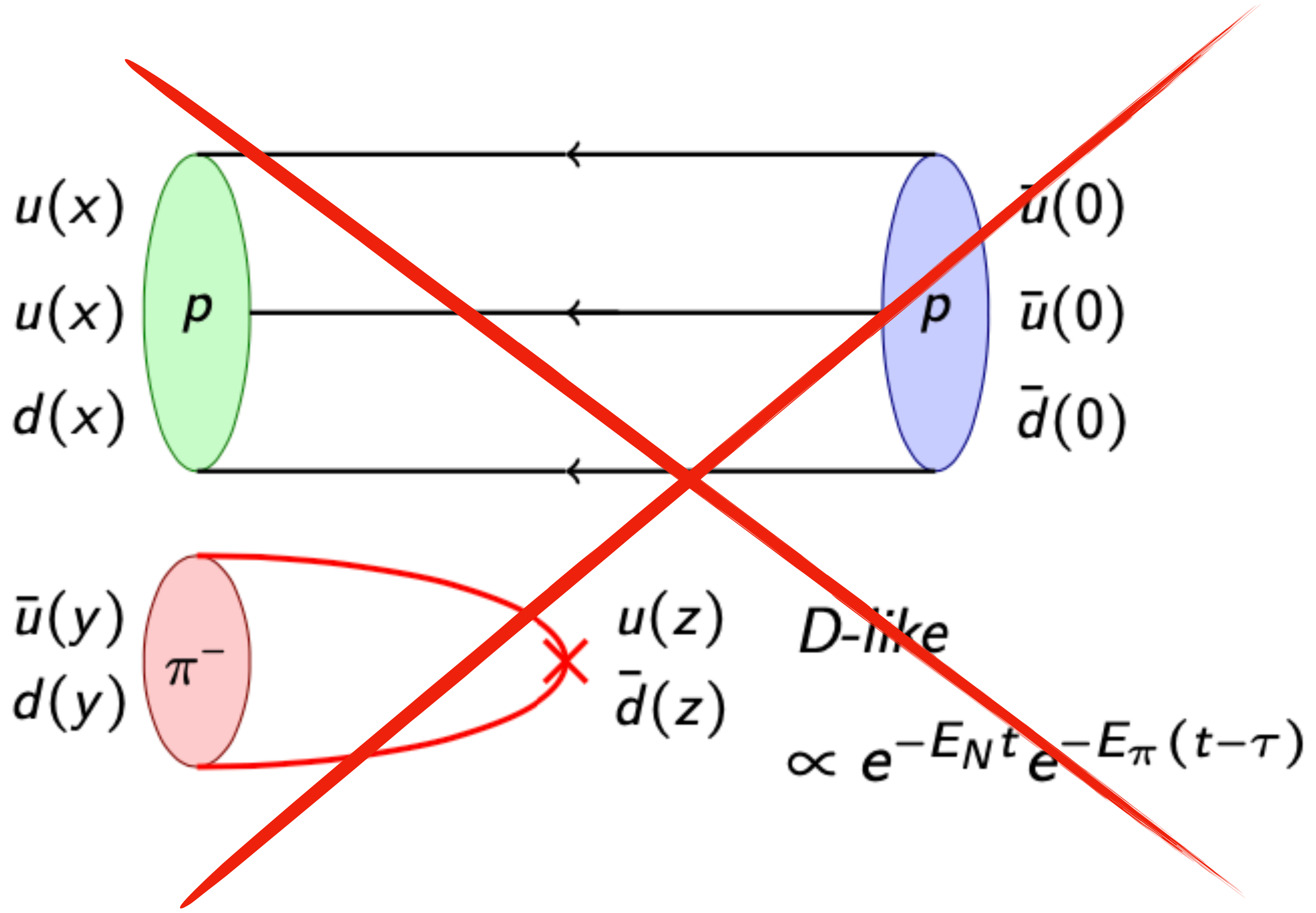
as LO-ChPT predicts!

Topologies in $p \xrightarrow{\mathcal{J}^-} n\pi^0$ for $\langle \mathrm{O}_2(\mathbf{p}', t)\ \mathcal{J}^-(\mathbf{q}, \tau)\ \bar{\mathrm{O}}_1(\mathbf{p}, 0) \rangle$









GEVP results with $\mathbf{p} = \mathbf{0}$

$$C(t) = \begin{pmatrix} \langle O_1(t)\, \bar{O}_1(0)\rangle & \langle O_1(t)\, \bar{O}_2(0)\rangle \\ \langle O_2(t)\, \bar{O}_1(0)\rangle & \langle O_2(t)\, \bar{O}_2(0)\rangle \end{pmatrix}$$

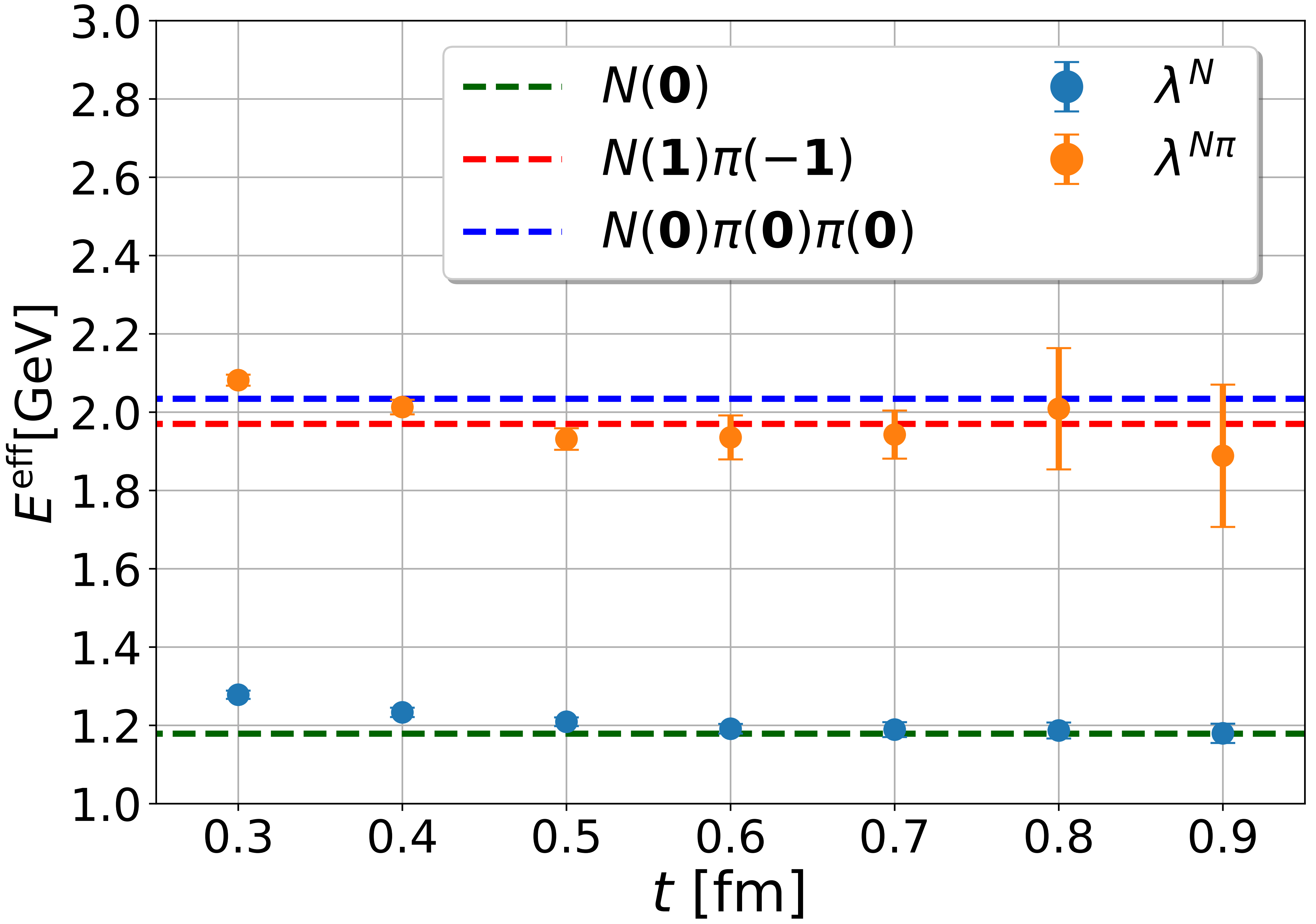
$$C(t)v^\alpha(t,t_0) = C(t_0)\ \lambda^\alpha(t,t_0)v^\alpha(t,t_0)$$

$$\lambda^1 \propto e^{-E_N(t-t_0)} \equiv \lambda^N$$

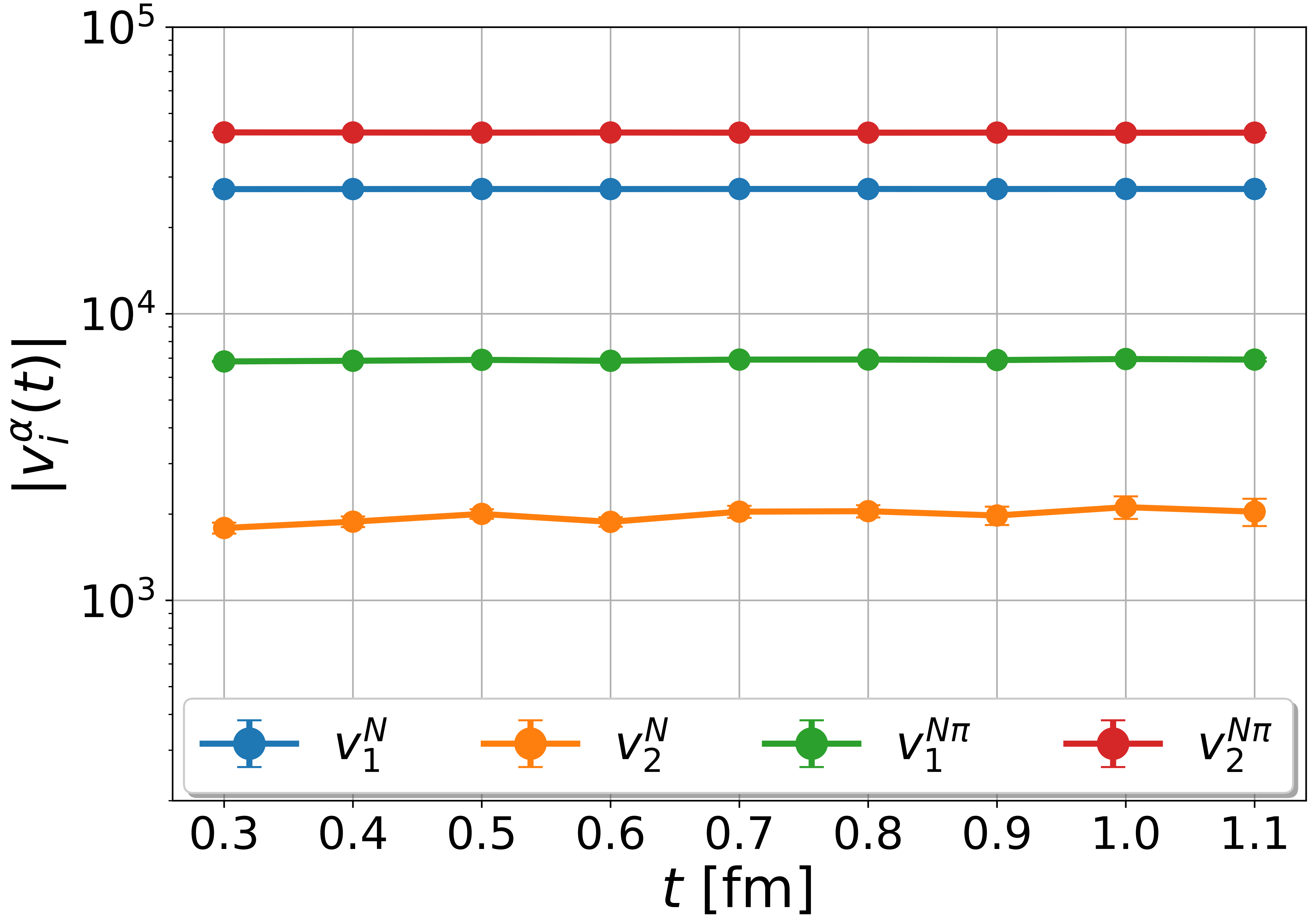
$$\lambda^2 \propto e^{-E_{N\pi}(t-t_0)} \equiv \lambda^{N\pi}$$

We extract the (effective) energies from the eigenvalues: $E^{\text{eff}}_\alpha = \log\left(\lambda^\alpha(t-a)\ /\ \lambda^\alpha(t)\right)$

$$v^1 \equiv v^N,\ v^2 \equiv v^{N\pi}$$



(Dashed lines are non-interacting energy levels)



$v^\alpha(t,t_0)$ normalised s.t. $(v^\alpha(t,t_0), C(t_0)v^\beta(t,t_0)) = \delta^{\alpha\beta}$

GEVP results with $\mathbf{p} = (2\pi/L)\ \hat{n}_z$

$\mathrm{O}_2(\mathbf{p}) = \mathrm{O}_{qqq}(\mathbf{p})\ \mathrm{O}_{\bar{q}q}(\mathbf{0})$

$$C(t) = \begin{pmatrix} \langle \mathrm{O}_1(t)\ \bar{\mathrm{O}}_1(0)\rangle & \langle \mathrm{O}_1(t)\ \bar{\mathrm{O}}_2(0)\rangle \\ \langle \mathrm{O}_2(t)\ \bar{\mathrm{O}}_1(0)\rangle & \langle \mathrm{O}_2(t)\ \bar{\mathrm{O}}_2(0)\rangle \end{pmatrix}$$

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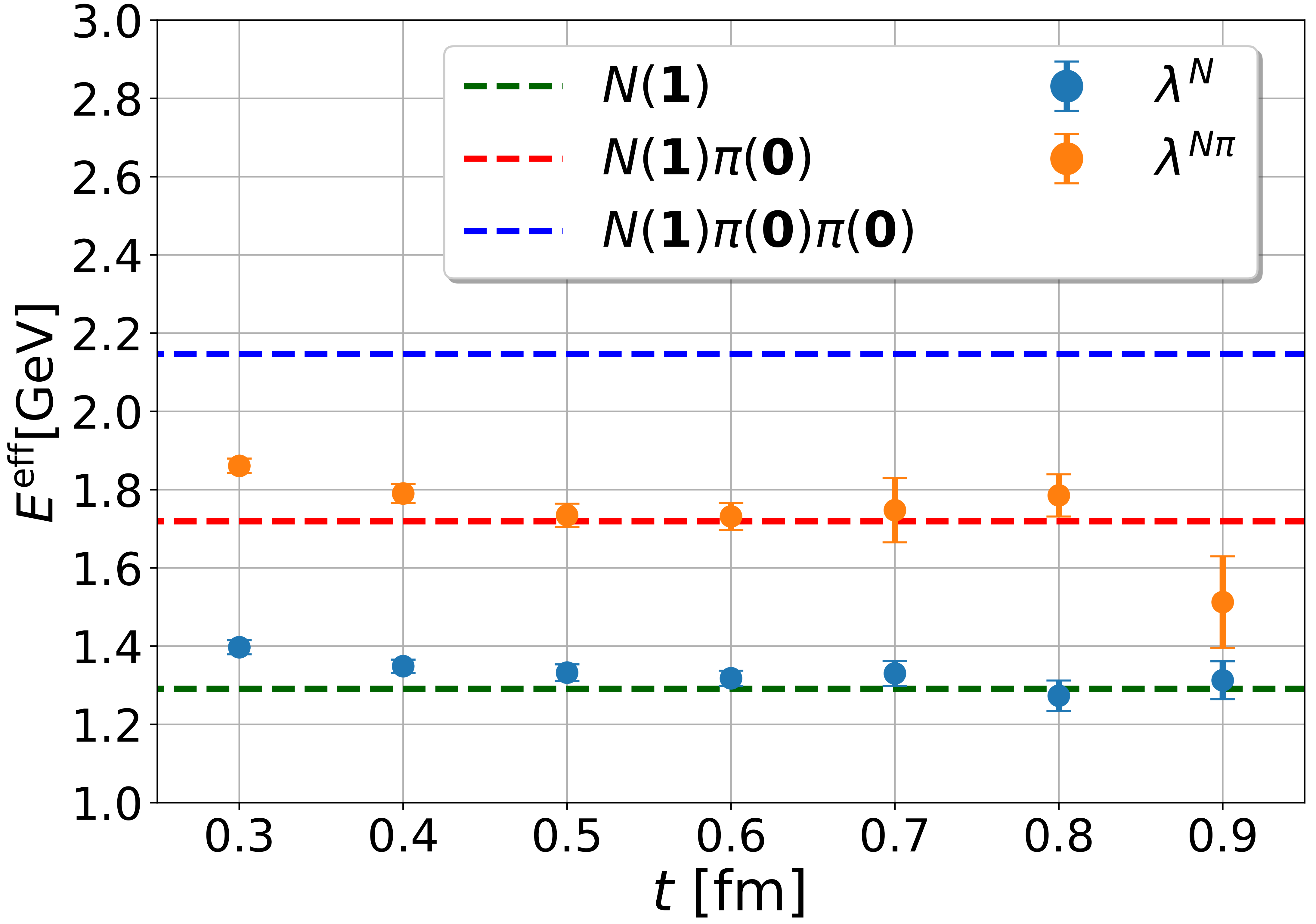
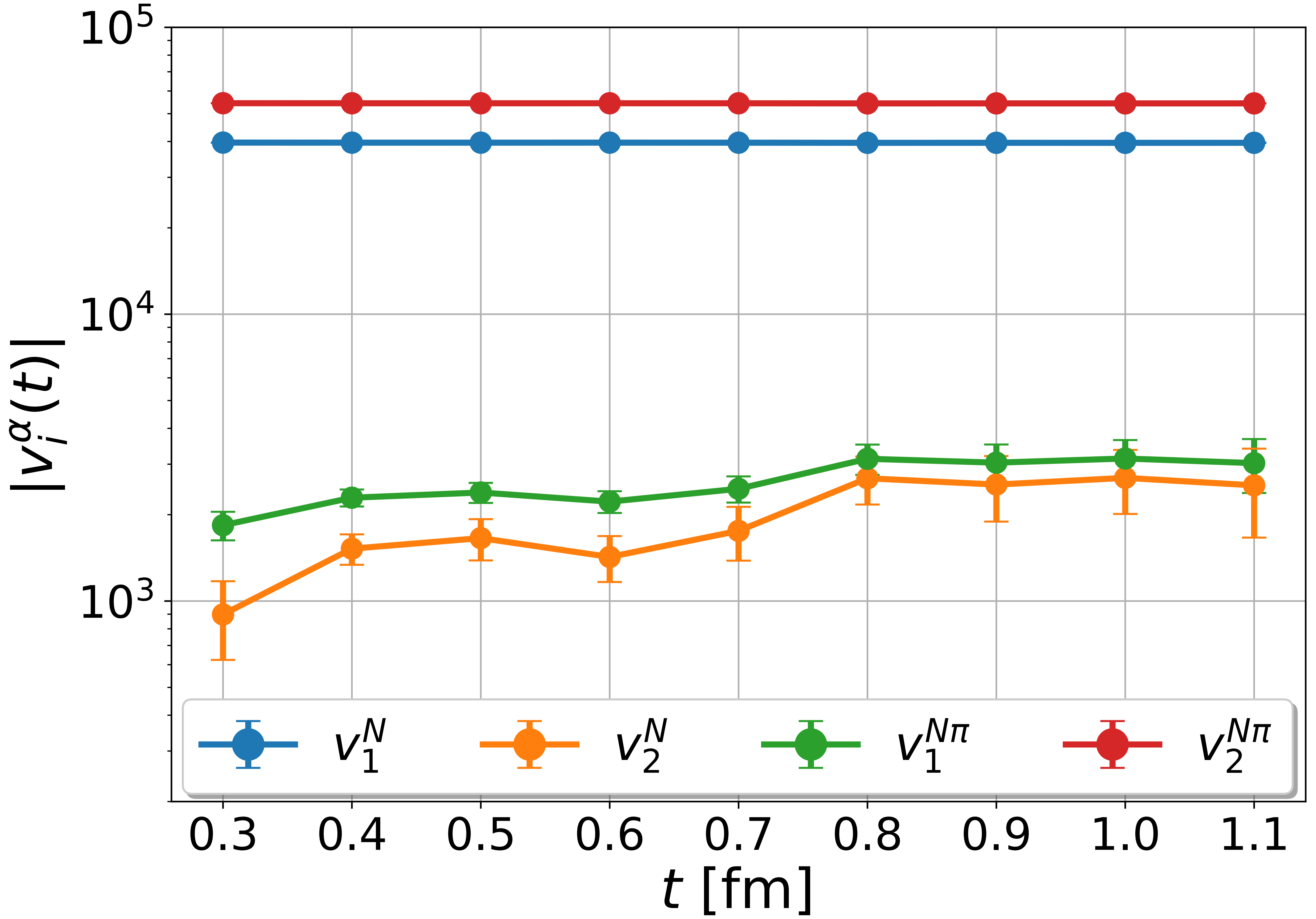


(Dashed lines are non-interacting energy levels)



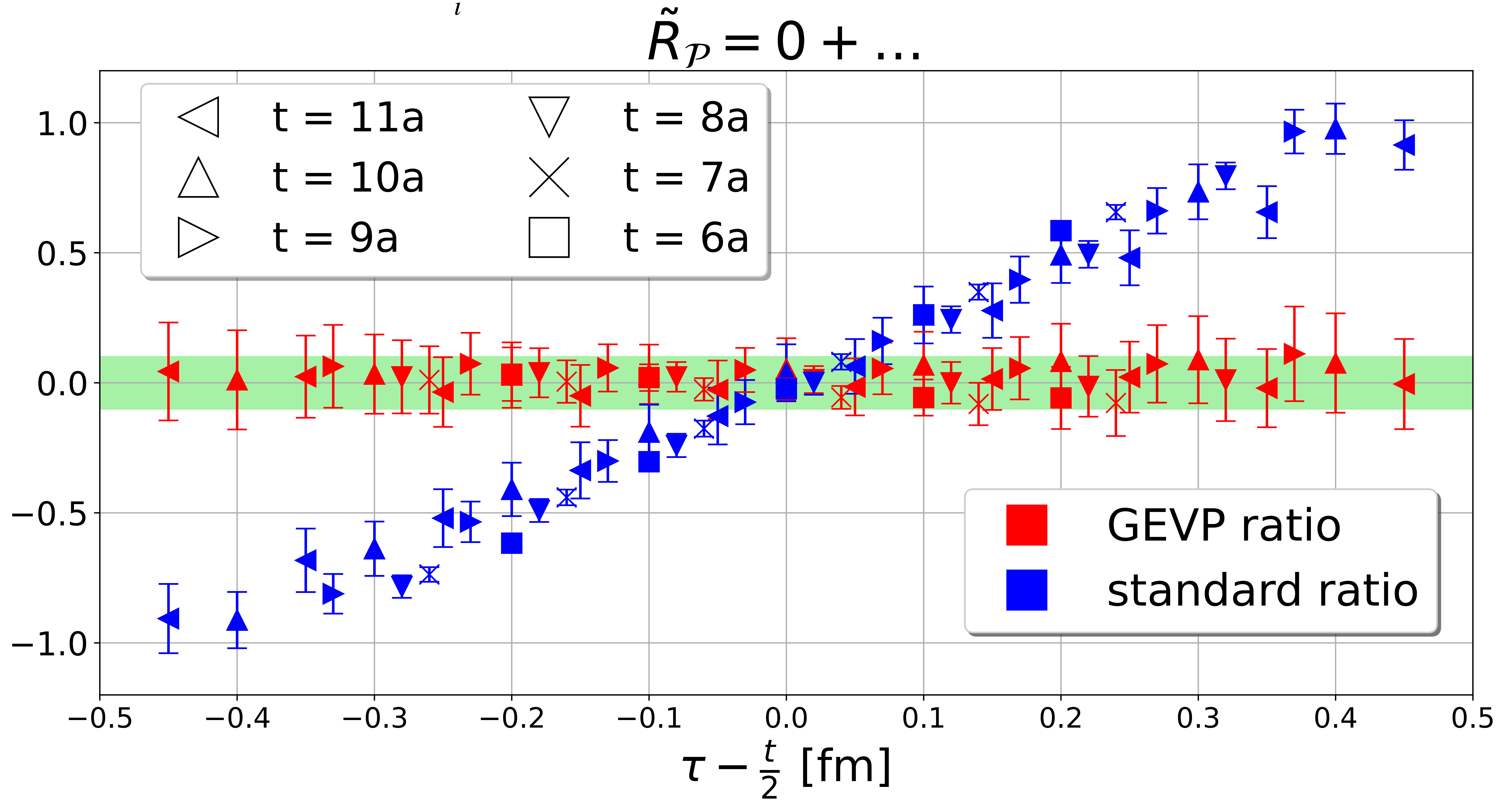
$v^\alpha(t,t_0)$ normalised s.t. $(v^\alpha(t,t_0), C(t_0)v^\beta(t,t_0)) = \delta^{\alpha\beta}$

GEVP ratio in the pseudoscalar channel $(\mathbf{q} = \mathbf{0})$

we replace O_1 with O_N to make the GEVP ratio

$$O_N = \sum_i v_i^N(t_0)\, O_i$$

$$\tilde{R}_{\mathscr{P}} = \frac{\langle O_N(\mathbf{p}', t)\ \mathscr{P}(\mathbf{q} = \mathbf{0}, \tau)\ \bar{O}_N(\mathbf{p}, 0)\rangle}{\langle O_N(\mathbf{p}, t)\ \bar{O}_N(\mathbf{p}, 0)\rangle} \frac{E}{p_i} = 0 + \ldots$$

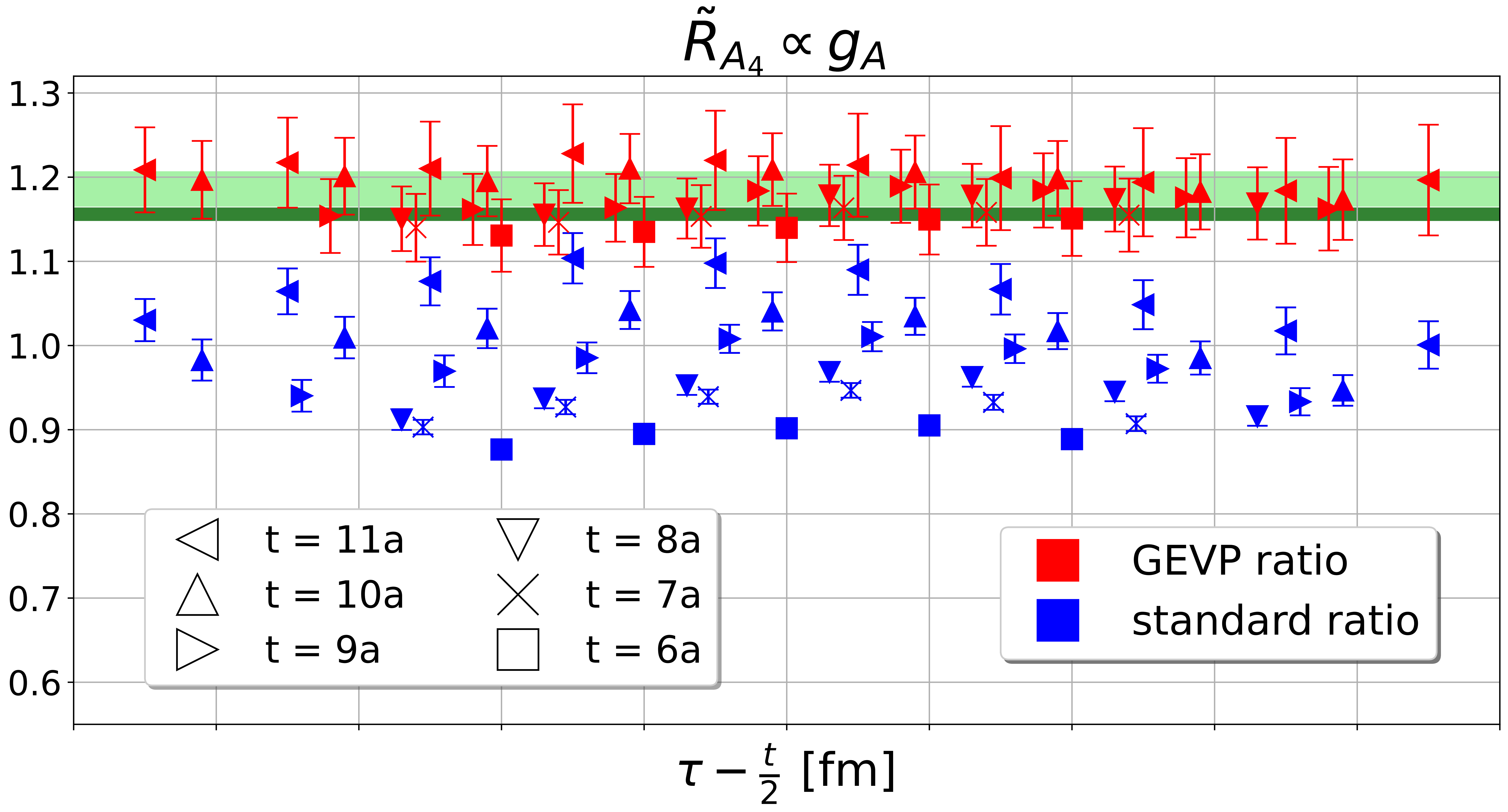


GEVP method removes the $N\pi$ contamination!

Green band in the following plots is the expected result, (0 in this case)

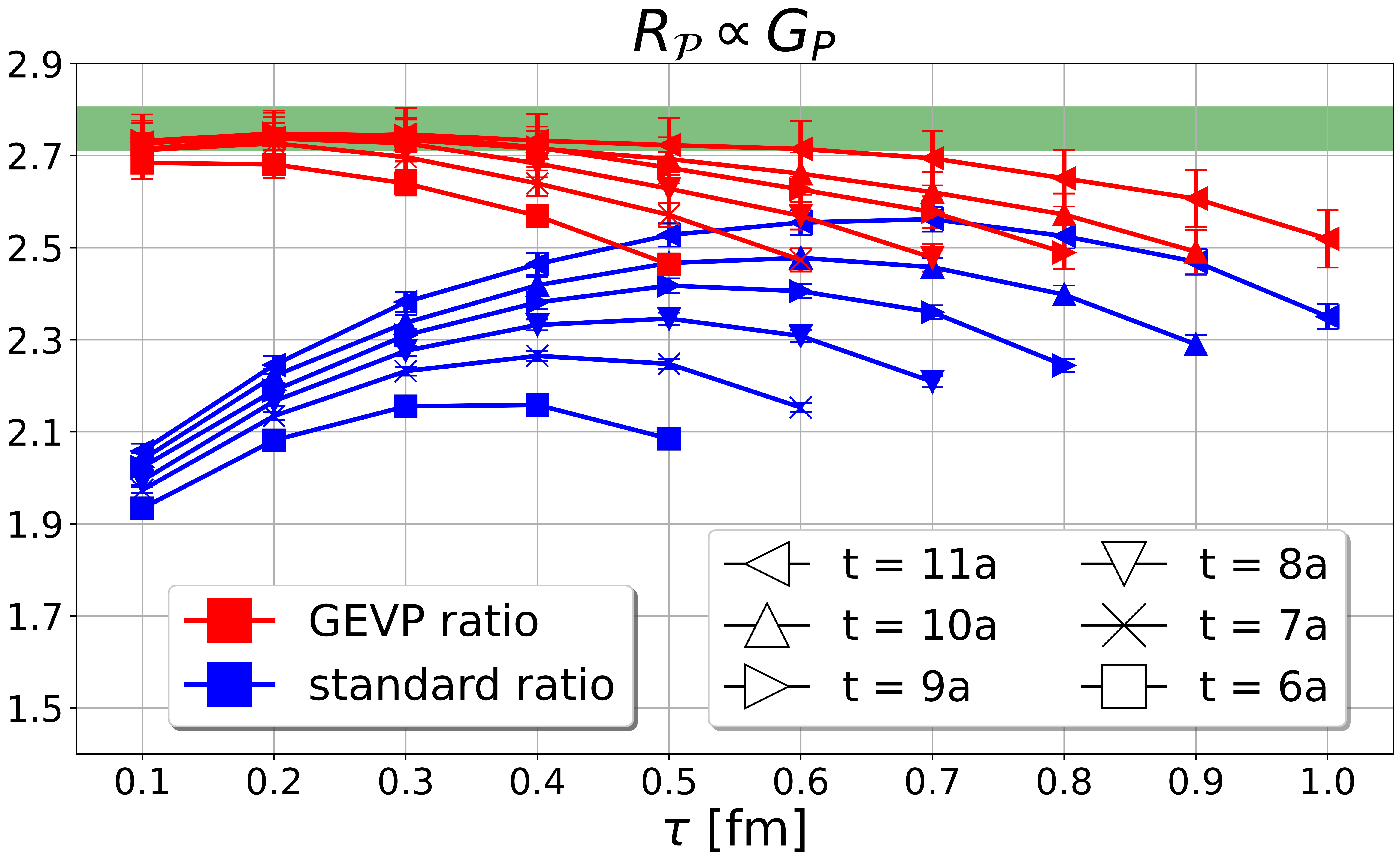
GEVP ratio in the axial temporal channel $(\mathbf{q} = \mathbf{0})$

$$\tilde{R}_{\mathscr{A}_4} = -\frac{\langle \mathrm{O}_N(\mathbf{p}', t)\ \mathscr{A}_4(\mathbf{q} = \mathbf{0}, \tau)\ \bar{\mathrm{O}}_N(\mathbf{p}, 0)\rangle}{\langle \mathrm{O}_N(\mathbf{p}, t)\ \bar{\mathrm{O}}_N(\mathbf{p}, 0)\rangle} \left(\frac{E}{p_i}\right) = g_A + \ldots$$



GEVP ratio at $Q^2 \approx 0.3\ \mathrm{GeV}^2$ in the pseudoscalar channel

Phenomenologically more interesting are nucleon form factors $G_A, G_P, \widetilde{G}_P$ at $Q^2 \neq 0\ \mathrm{GeV}^2$
Unfortunately, a traditional fit to lattice data gives unreliable form factors.



ChPT studies* show that $N\pi$ contribution can be quite large!

*[PRD.100.054507, PRD.99.054506]

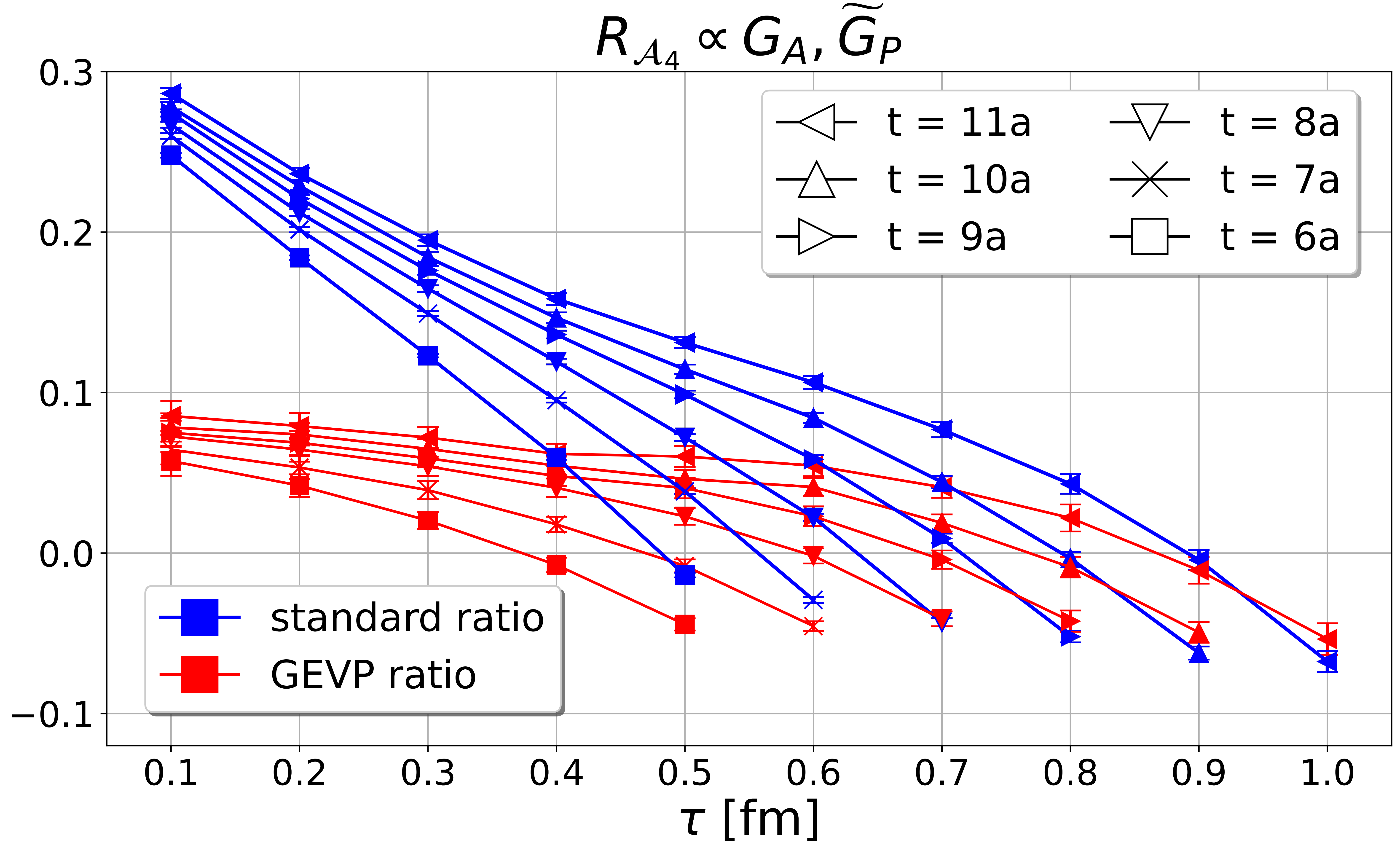
$R_{\mathscr{P}}$ is constructed with $\mathscr{J} = \mathscr{P}$

The GEVP improves significantly the ratios, as they approach the green band (nucleon ground state)

Ideally data points should lie on green band

There is still a trace of contamination left at $\tau = t$ (aka “sink“ = rightmost part)

GEVP ratio at $Q^2 \approx 0.3\ \mathrm{GeV}^2$ in the temporal axial channel



$R_{\mathscr{A}_4}$ **is constructed with** $\mathscr{J} = \mathscr{A}_4$

The GEVP improves significantly the ratios, as they approach the green band (nucleon ground state)

Still trace of contamination left at $\tau = t$

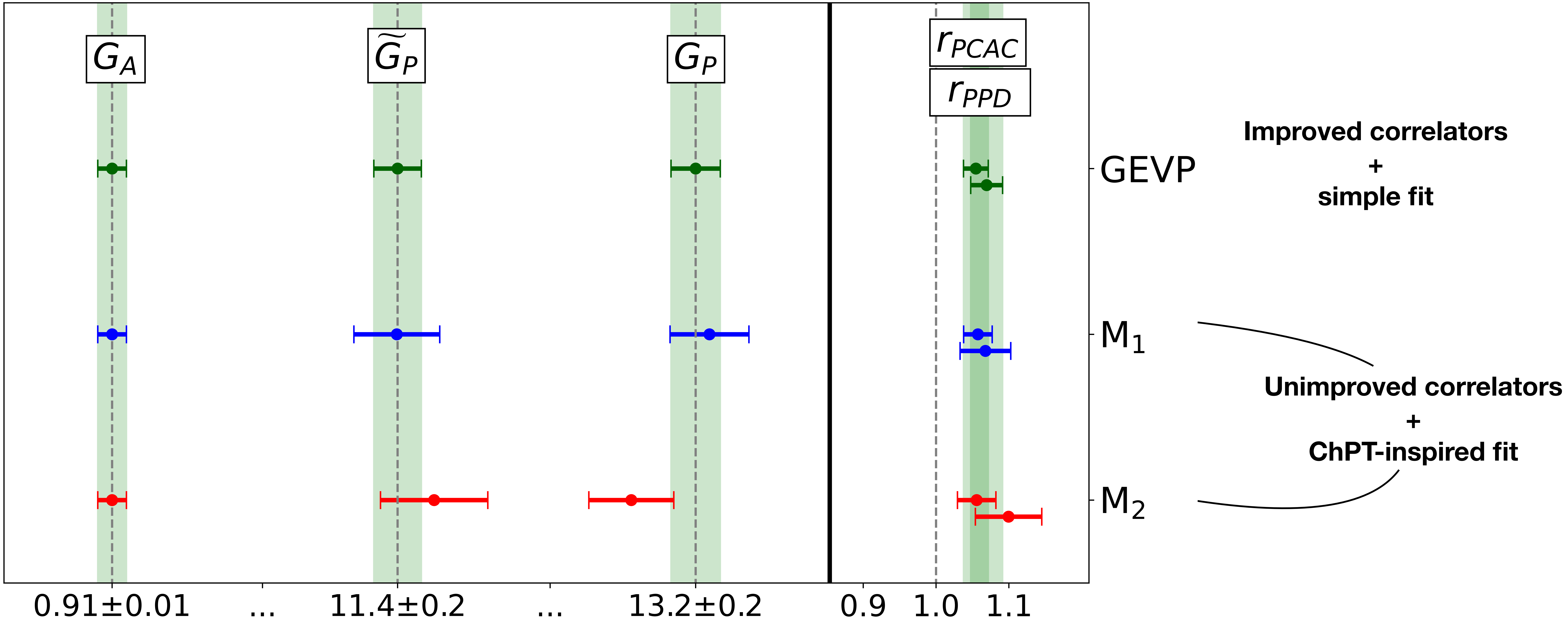
$G_A, G_P, \widetilde{G}_P$ **satisfy PCAC/PPD with a simple fit**

$$m_N G_A(Q^2) = m_\ell G_P(Q^2) + \frac{Q^2}{4m_N}\widetilde{G}_P(Q^2) \qquad \textbf{PCAC}$$

$$G_{\tilde{P}}(Q^2) = \frac{4m_N^2}{Q^2 + m_\pi^2} G_A(Q^2) \qquad \textbf{PPD}$$

[JHEP05(2020)126, PRL.124.072002]

Comparison of final results using different methods



$\mathcal{O}(a)$-effects must be taken into account

Conclusions and outlook

- Structure of nucleons and excited nucleons is relevant for neutrino oscillation experiments
- For the first time, we use the variational method with N & $N\pi$-like operators for 3pts
- It works very well in the forward limit to extract $\langle N(\mathbf{p})\,|\,\mathcal{J}(\mathbf{q}=\mathbf{0})\,|\,N(\mathbf{p})\rangle$ with $\mathcal{J} = \mathcal{P}, \mathcal{A}_\mu$
- It works quite well also in the off-forward limit to extract $\langle N(\mathbf{p}'=\mathbf{0})\,|\,\mathcal{J}(\mathbf{q})\,|\,N(-\mathbf{q})\rangle$
- This project confirms the ChPT picture of $N\pi$ contamination dominance in some $\mathcal{J} = \mathcal{P}, \mathcal{A}_\mu$ channels

The D-like/disconnected diagram has the largest signal, and it depends on the $\mathcal{J} - \pi$ coupling

$$\langle\langle \mathrm{O}_N(\mathbf{p}'_N, t)\ \bar{\mathrm{O}}_N(\mathbf{p}_N, 0)\rangle\ \langle \mathrm{O}_\pi(\mathbf{p}'_\pi, t)\ \mathcal{A}_\mu(\mathbf{q}, \tau)\ \rangle\rangle \qquad \langle \pi(\mathbf{k})\,|\,\mathcal{A}_\mu(\mathbf{q})\,|\,\Omega > =\ iq_\mu\ f_\pi\ \delta_{q,k}$$

Next?

Possible projects along this line...

- Compute $\langle N\pi | \mathcal{J} | N\rangle$ on this ensemble with $\mathcal{J} = \mathcal{A}_\mu \, , \mathcal{P}$
- Investigate $\langle N\pi | \mathcal{V}_\mu | N\rangle$ and $\langle N | \mathcal{V}_\mu | N\rangle$
- Include more ensembles and take physical limit $(m_\pi \to m_\pi^{\rm phys},\ a \to 0,\ V \to \infty)$
- Investigate $\langle N^* | \mathcal{J} | N\rangle$ and $\langle \Delta^+ | \mathcal{J} | N\rangle$ with 3-quark operators *(good for students)*
- Investigate $\langle N^* | \mathcal{J} | N\rangle$ and $\langle \Delta^+ | \mathcal{J} | N\rangle$ with 3-quark and 5-quark operators *(a bunch of postdocs)*

π states contaminate other channels like e.g. $B \to \pi \ell \bar{\nu}$ [PoS Lattice 2022 O. Bär, A. Broll, R. Sommer]

Talk by A. Broll on 13/02/2023 "Excites states in B meson correlation functions"

Based on

- *[JHEP05(2020)126] (G. Bali, **L. B.**, et al.)*
- *[arXiv:2211.12278] (**L. B.**, G. Bali, S. Collins)*
- *[PoS LATTICE2021 (2022) 359] (**L. B.**, G. Bali, S. Collins)*
- *[PoS NSTAR 2022 in preparation] (**L. B.**)*

Thank you!

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 813942

BACKUP SLIDES

For completeness the spectral decomposition of the 3pt is …

$$\langle \mathrm{O}_1(\mathbf{p}', t)\ \mathcal{J}(\mathbf{q}, \tau)\ \bar{\mathrm{O}}_1(\mathbf{p},0)\rangle = \langle \Omega | \mathrm{O}_1 | N(\mathbf{p}')\rangle\ \langle N(\mathbf{p}') | \mathcal{J} | N(\mathbf{p})\rangle\ \langle N(\mathbf{p}) | \bar{\mathrm{O}}_1 | \Omega \rangle\ \frac{e^{-E'_N(t-\tau)} e^{-E_N \tau}}{4E'_N E_N} +$$

(ESC at the sink)

$$+\langle \Omega | \mathrm{O}_1 | N^*(\mathbf{p}')\rangle\ \langle N^*(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p})\rangle\ \langle N(\mathbf{p}) | \bar{\mathrm{O}}_1 | \Omega \rangle\ \frac{e^{-E'_{N^*}(t-\tau)} e^{-E_N \tau}}{4E'_{N^*} E_N} +$$

(ESC at the source)

$$+\langle \Omega | \mathrm{O}_1 | N(\mathbf{p}')\rangle\ \langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N^*(\mathbf{p})\rangle\ \langle N^*(\mathbf{p}) | \bar{\mathrm{O}}_1 | \Omega \rangle\ \frac{e^{-E'_N(t-\tau)} e^{-E_{N^*} \tau}}{4E'_N E_{N^*}} + \ldots$$

$$N^* \to N\pi, \ldots$$

$$\frac{\langle \Omega | \mathrm{O}_1 | N \rangle}{\langle \Omega | \mathrm{O}_1 | N^* \rangle}$$ can be increased with smearing techniques

ESC = Excited State Contamination

New correlation functions to be computed

$\mathrm{O}_1 \propto (qqq)$ $\mathrm{O}_2 \propto (qqq)(\bar{q}q)$

2pts

$$\langle \mathrm{O}_2(\mathbf{p},0)\ \mathrm{O}_1(\mathbf{p},0) \rangle$$

$$\langle \mathrm{O}_2(\mathbf{p},t)\ \mathrm{O}_2(\mathbf{p},t) \rangle$$

Already investigated for $N\pi$ scattering (I=1/2) in

[PRD.87.054502 (C. Lang, V. Verduci)]

[PRD.95.014510 (S. Prelovsek et al.)]

[arXiv:2208.03867 (J. Bulava et al.)]

[PoS Lattice 2022 (ETMC)]

3pts

$$\langle \mathrm{O}_{p\pi^-}(\mathbf{p}',t)\ \mathcal{J}^-(\mathbf{q},\tau)\ \mathrm{O}_p(\mathbf{p},0) \rangle$$

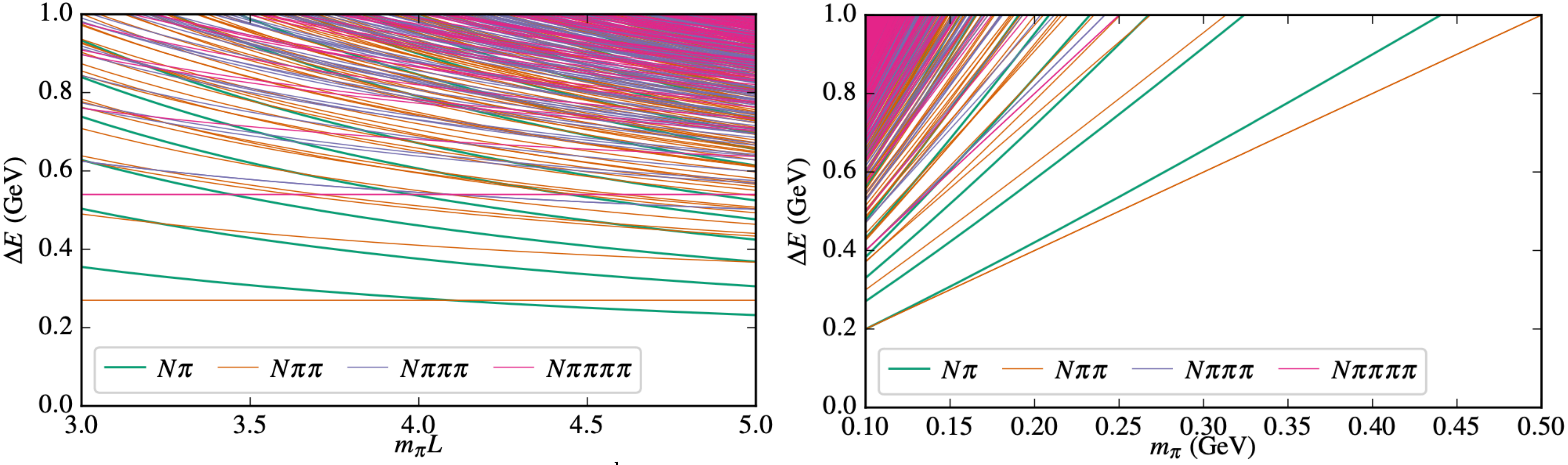
$$\langle \mathrm{O}_{n\pi^0}(\mathbf{p}',t)\ \mathcal{J}^-(\mathbf{q},\tau)\ \mathrm{O}_p(\mathbf{p},0) \rangle$$

[PoS Lattice 2021 (**L. B.**, G. Bali, S. Collins)]

[PoS Lattice 2022 (ETMC)]

“Systematics in nucleon matrix elements” J. Green

[arXiv:1812.10574]



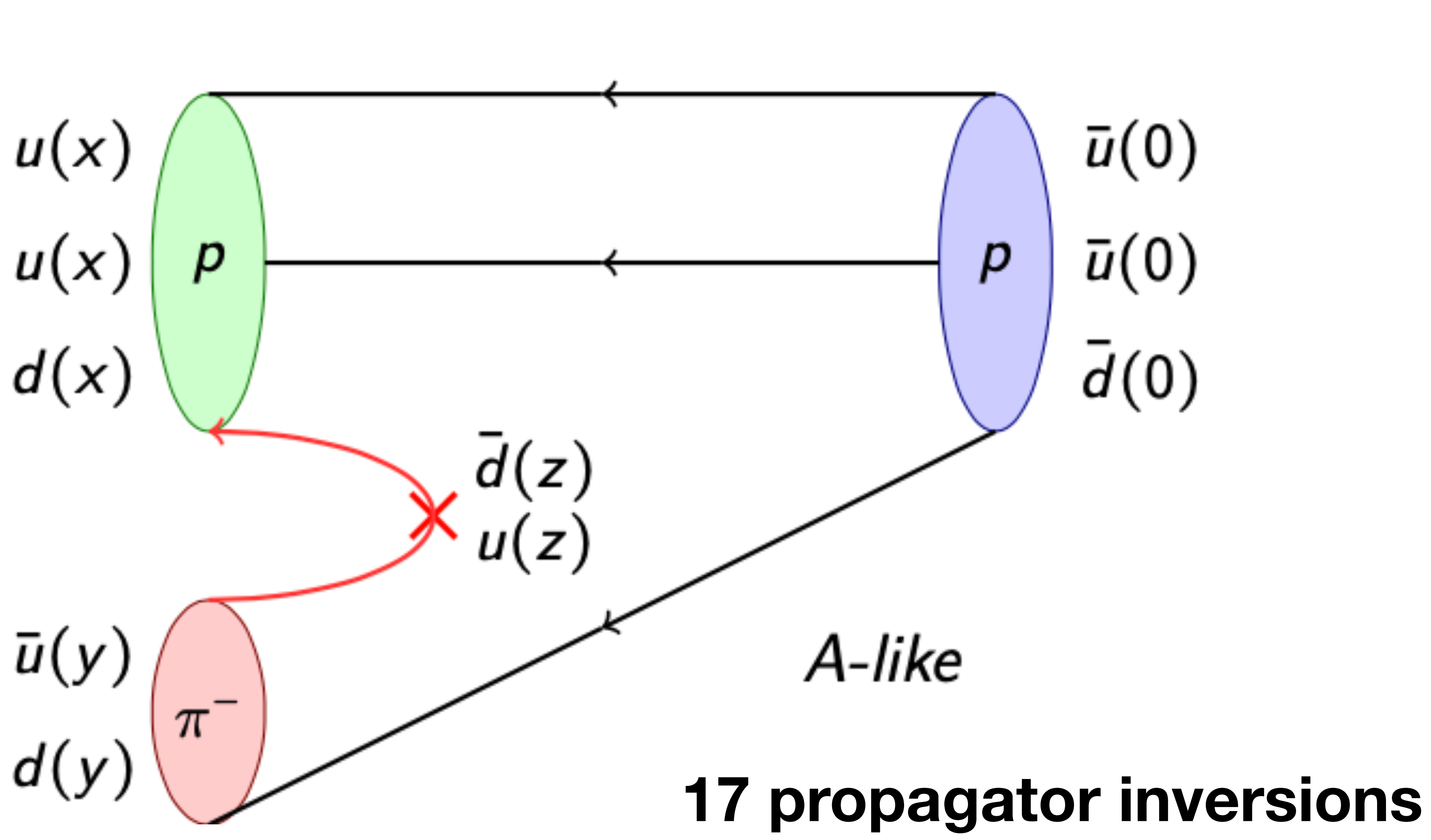
$m_\pi = m_\pi^{\text{phys}}$ $\qquad$ $m_\pi L = 4$

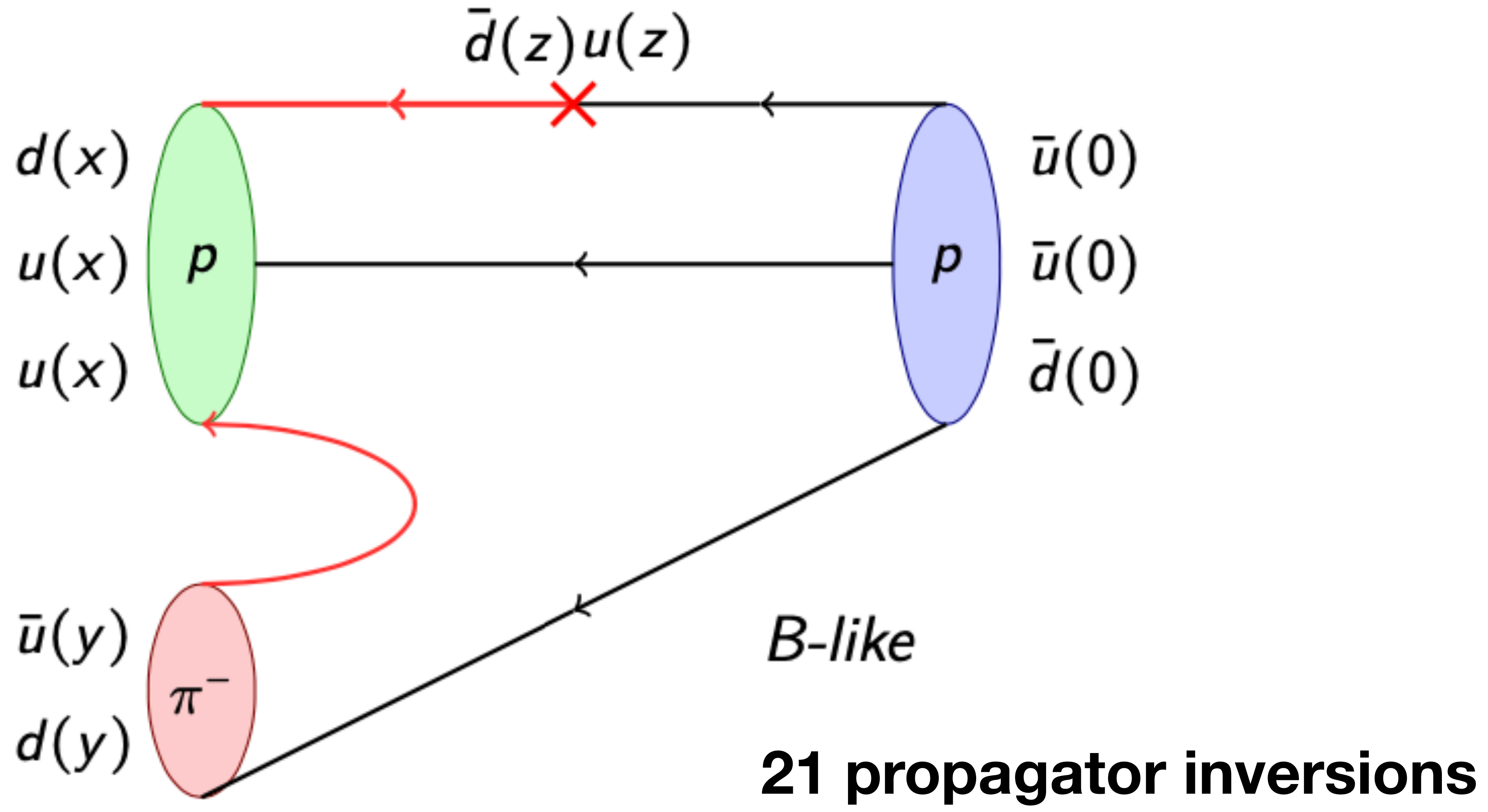
The energy gap $\Delta E = E_{N\pi^n} - E_N$ depends on the box size and m_π

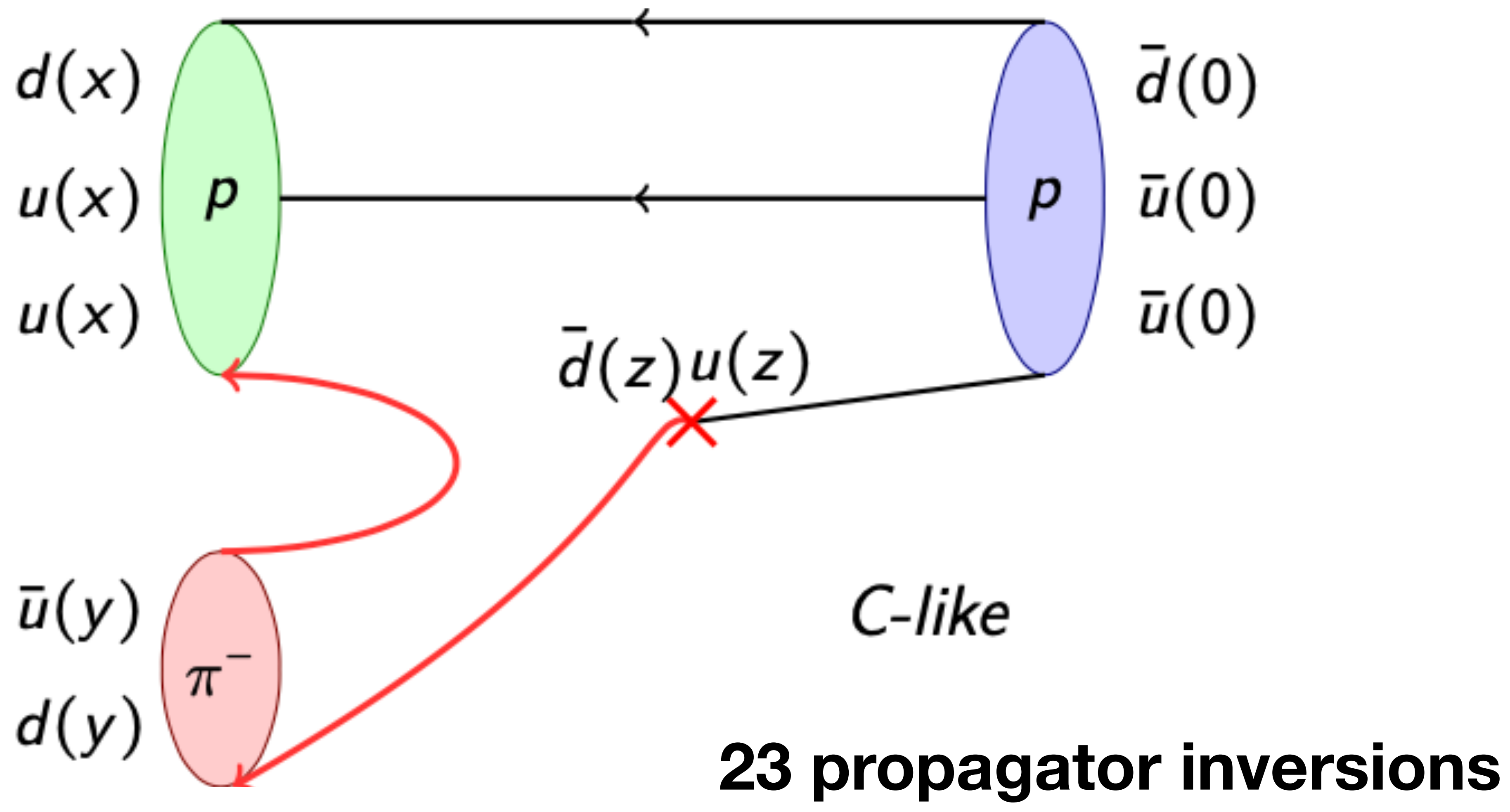
Our ensemble

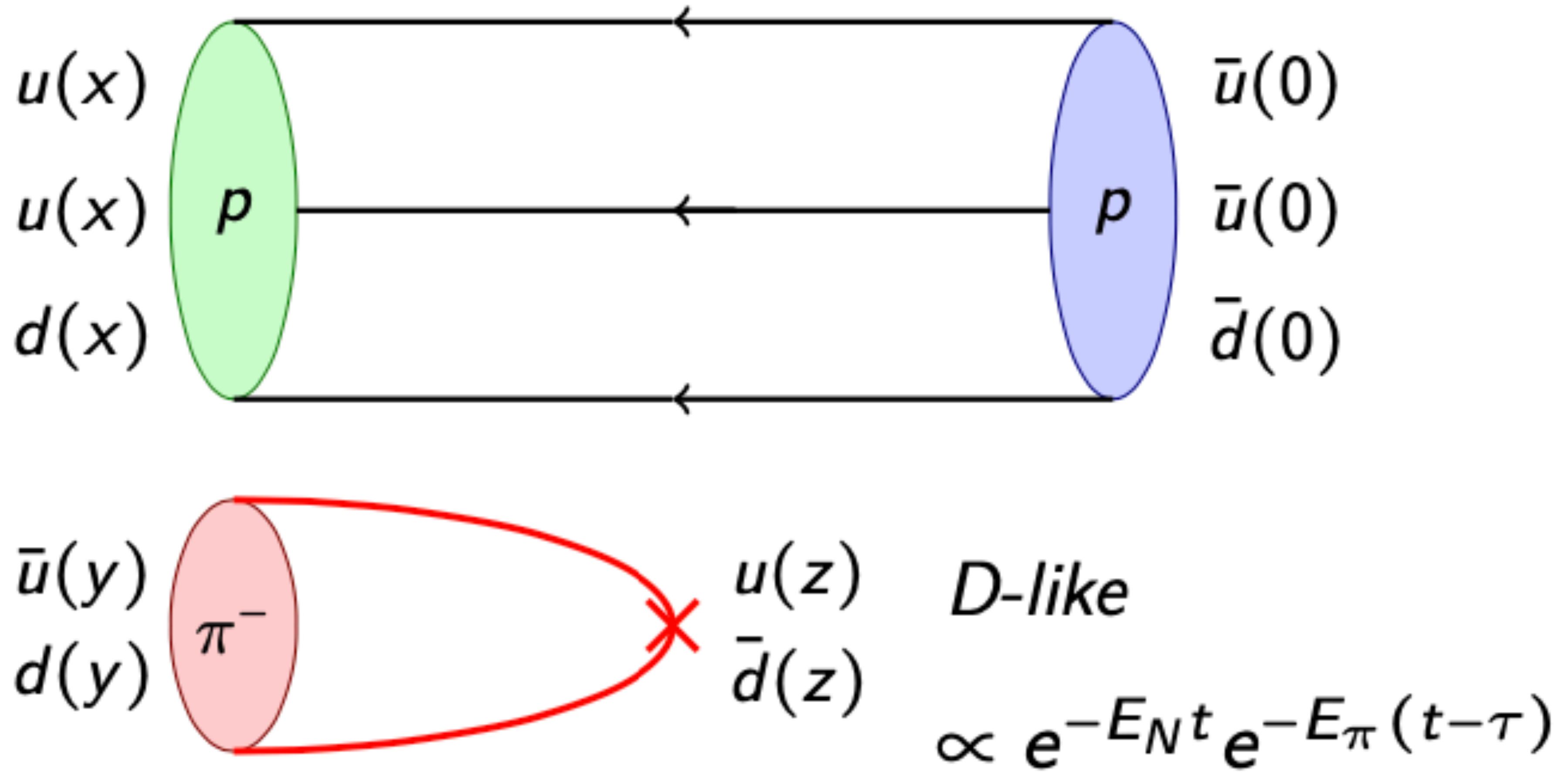
$m_\pi = m_K \approx 420$ MeV $\qquad$ $m_\pi L = 5.1$

Topologies in $p \xrightarrow{\mathcal{J}^-} p\pi^-$ for $\langle \mathrm{O}_2(\mathbf{p}', t)\ \mathcal{J}^-(\mathbf{q}, \tau)\ \bar{\mathrm{O}}_1(\mathbf{p}, 0)\rangle$









as LO-ChPT predicts!

Nucleon spectrum

Nucleon resonances with I = $\frac{1}{2}$

Symbol	J^P	PDG mass average (MeV/c^2)	Full width (MeV/c^2)	Pole position (real part)	Pole position (−2 × imaginary part)	Common decays ($\Gamma_i/\Gamma > 50\%$)
N(939) P_{11} [PDG 3]†	$\frac{1}{2}^+$	939	†	†	†	†
N(1440) P_{11} [PDG 4] (the Roper resonance)	$\frac{1}{2}^+$	1440 (1420–1470)	300 (200–450)	1365 (1350–1380)	190 (160–220)	**N** + π
N(1520) D_{13} [PDG 5]	$\frac{3}{2}^-$	1520 (1515–1525)	115 (100–125)	1510 (1505–1515)	110 (105–120)	**N** + π
N(1535) S_{11} [PDG 6]	$\frac{1}{2}^-$	1535 (1525–1545)	150 (125–175)	1510 (1490–1530)	170 (90–250)	**N** + π or **N** + η
N(1650) S_{11} [PDG 7]	$\frac{1}{2}^-$	1650 (1645–1670)	165 (145–185)	1665 (1640–1670)	165 (150–180)	**N** + π
N(1675) D_{15} [PDG 8]	$\frac{5}{2}^-$	1675 (1670–1680)	150 (135–165)	1660 (1655–1665)	135 (125–150)	**N** + π + π or Δ + π
N(1680) F_{15} [PDG 9]	$\frac{5}{2}^+$	1685 (1680–1690)	130 (120–140)	1675 (1665–1680)	120 (110–135)	**N** + π
N(1700) D_{13} [PDG 10]	$\frac{3}{2}^-$	1700 (1650–1750)	100 (50–150)	1680 (1630–1730)	100 (50–150)	**N** + π + π
N(1710) P_{11} [PDG 11]	$\frac{1}{2}^+$	1710 (1680–1740)	100 (50–250)	1720 (1670–1770)	230 (80–380)	**N** + π + π

GEVP results with $\mathbf{p} = (2\pi/L)\ \hat{n}_z$

$O_2(\mathbf{p}) = O_{qqq}(\mathbf{0})\ O_{\bar{q}q}(\mathbf{p})$

$$C(t) = \begin{pmatrix} \langle O_1(t)\ \bar{O}_1(0)\rangle & \langle O_1(t)\ \bar{O}_2(0)\rangle \\ \langle O_2(t)\ \bar{O}_1(0)\rangle & \langle O_2(t)\ \bar{O}_2(0)\rangle \end{pmatrix}$$

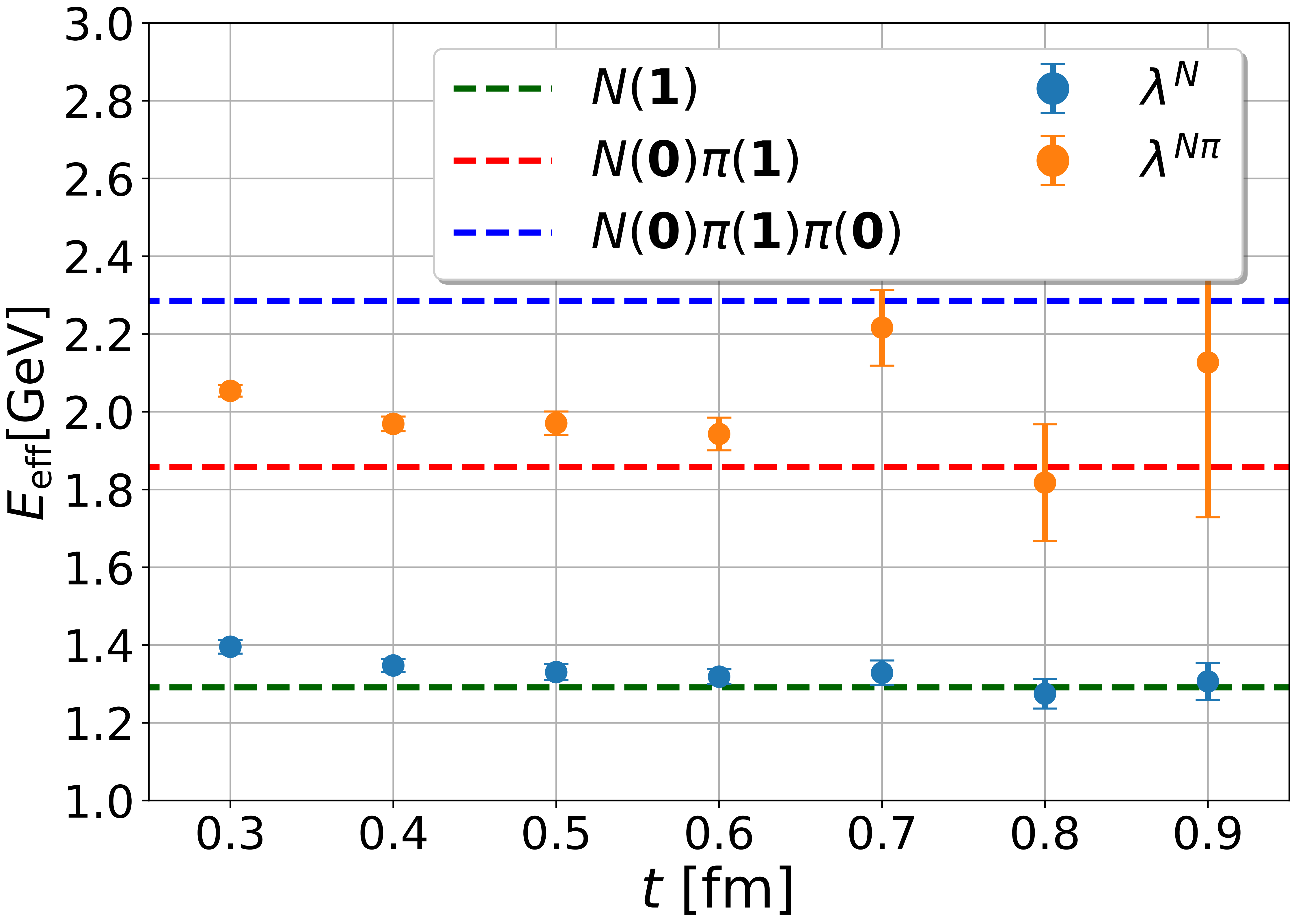
$$C(t)v^\alpha(t,t_0) = C(t_0)\ \lambda^\alpha(t,t_0)v^\alpha(t,t_0)$$

$\lambda^1 \propto e^{-E_N(t-t_0)} \equiv \lambda^N$

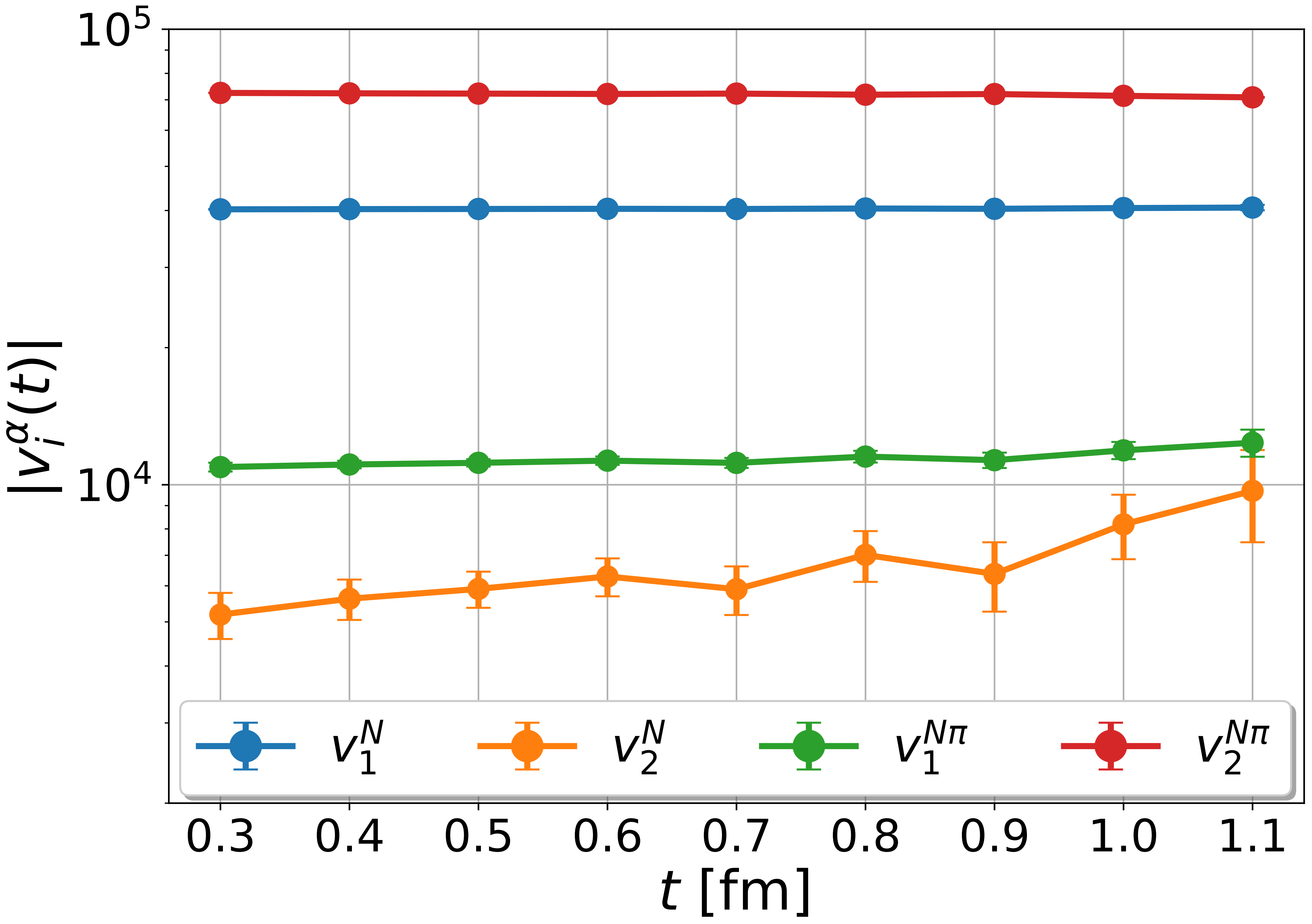
$\lambda^2 \propto e^{-E_{N\pi}(t-t_0)} \equiv \lambda^{N\pi}$

We extract the (effective) energies from the eigenvalues: $E_\alpha^{\text{eff}} = \log\left(\lambda^\alpha(t-a)\ /\ \lambda^\alpha(t)\right)$

$v^1 \equiv v^N,\ v^2 \equiv v^{N\pi}$

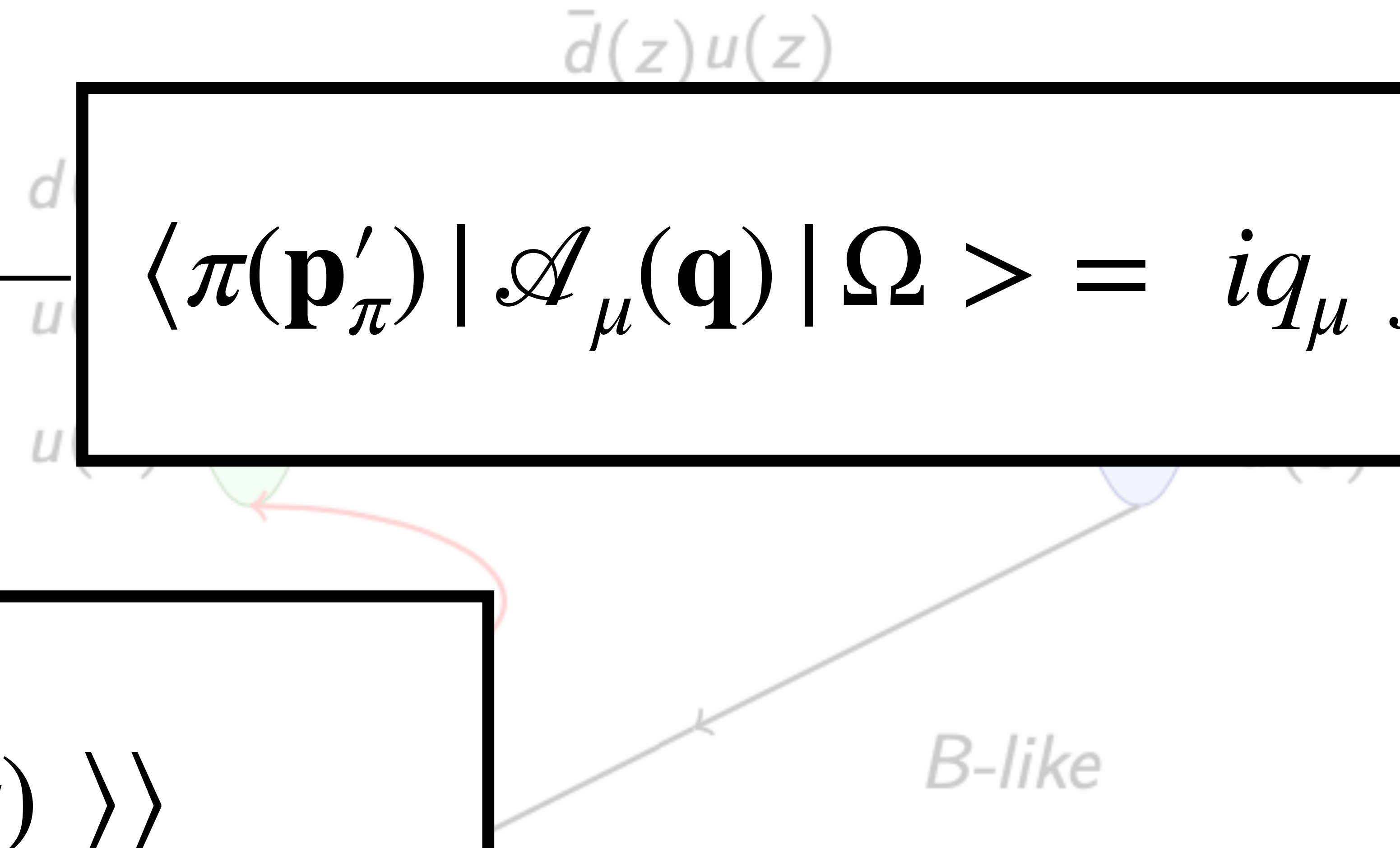


(Dashed lines are non-interacting energy levels)



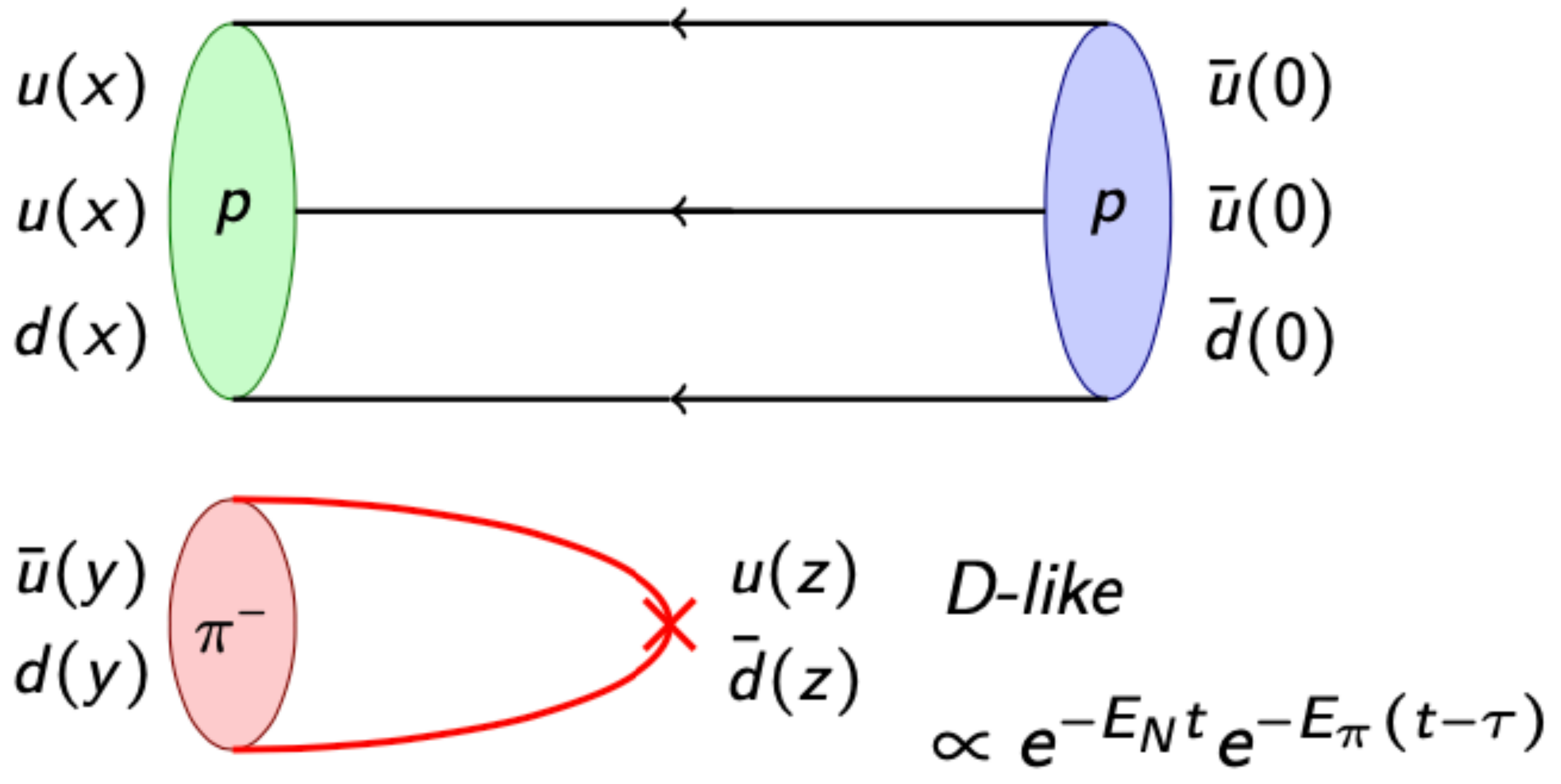
$v^\alpha(t,t_0)$ normalised s.t. $(v^\alpha(t,t_0), C(t_0)v^\beta(t,t_0)) = \delta^{\alpha\beta}$

Topologies in $p \xrightarrow{\mathcal{J}^-} p\pi^-$ for $\langle \mathrm{O}_2(\mathbf{p}', t)\ \mathcal{J}^-(\mathbf{q}, \tau)\ \bar{\mathrm{O}}_1(\mathbf{p}, 0)\rangle$



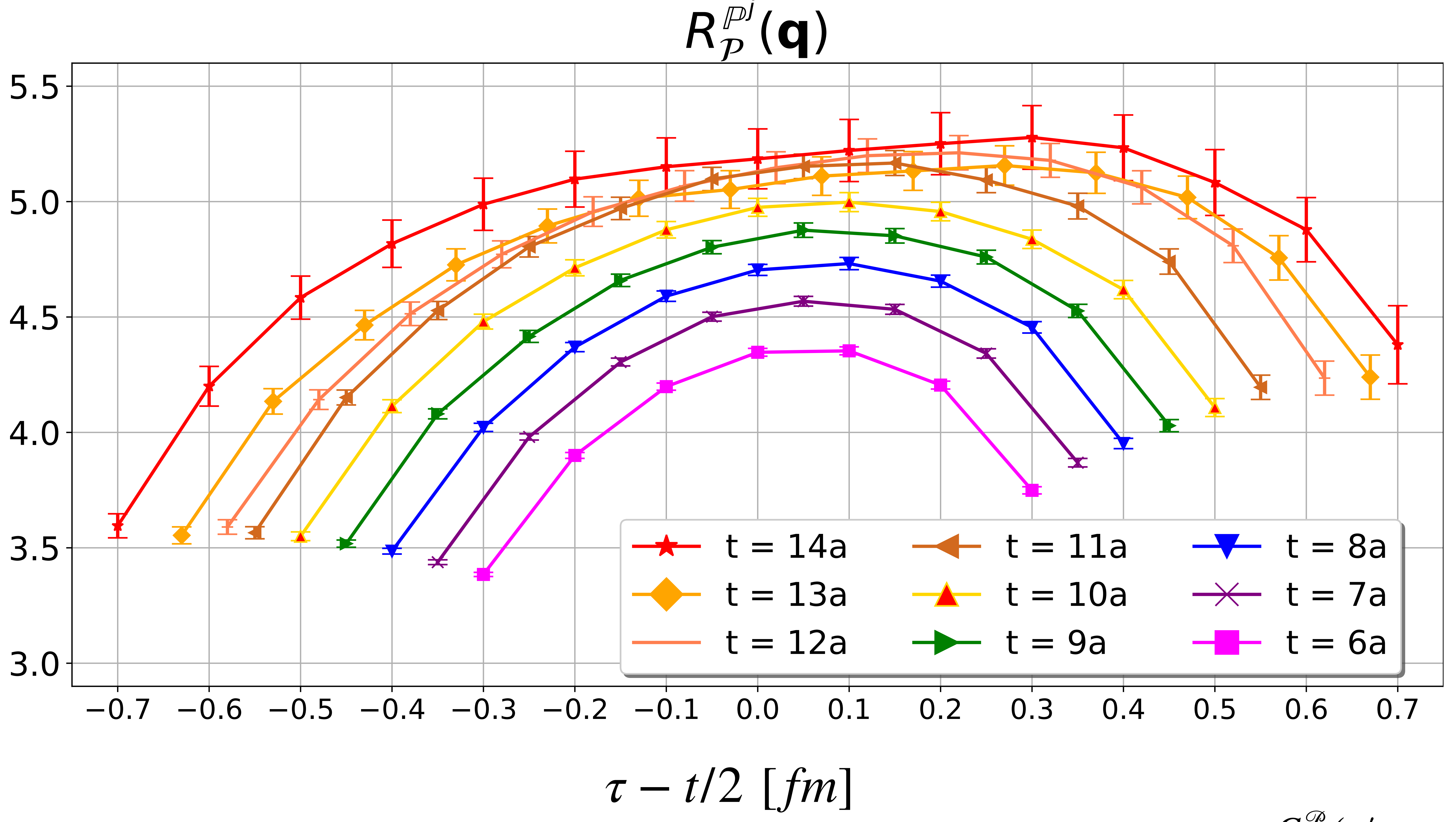
$$\langle \pi(\mathbf{p}'_\pi)\,|\,\mathcal{A}_\mu(\mathbf{q})\,|\,\Omega > \; = \; iq_\mu\ f_\pi\ \delta_{q,p'_\pi}$$

$$\langle\langle \mathrm{O}_N(\mathbf{p}'_N, t)\ \bar{\mathrm{O}}_N(\mathbf{p}_N, 0)\rangle\ \langle \mathrm{O}_\pi(\mathbf{p}'_\pi, t)\ \mathcal{J}_\mu(\mathbf{q}, \tau)\ \rangle\rangle$$



as LO-ChPT predicts!

Excited state effect at $\mathbf{q} \neq \mathbf{0}$: the $\mathscr{P}$ channel



$\mathbf{p}' = \mathbf{0} \qquad q_j \neq 0 \qquad (\mathbf{q} \parallel \mathbb{P})$

$\mathbb{P}$ = spin-parity projector

$$q = \hat{e}_j = \frac{2\pi}{L}\hat{n}_j$$

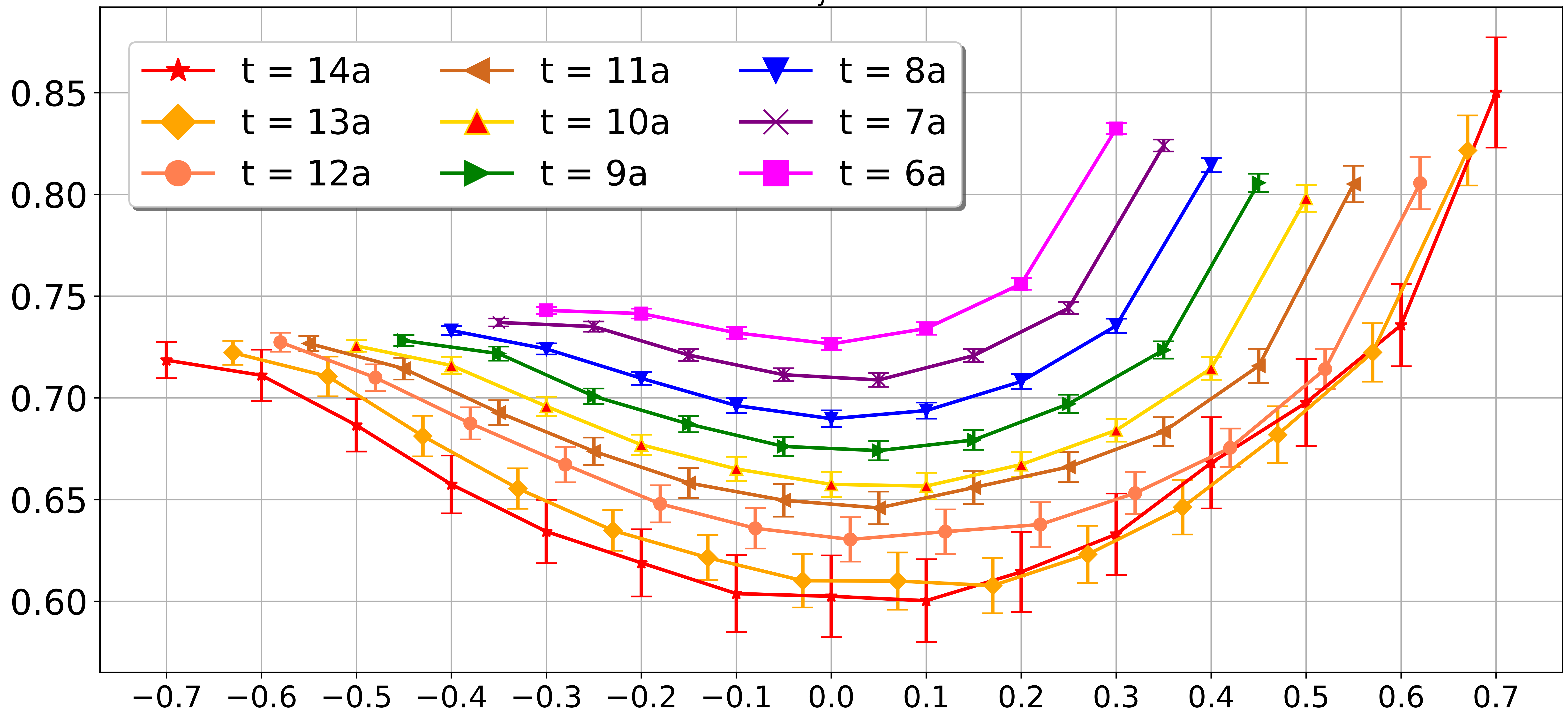
$$R^{\mathscr{P}}(\mathbf{p}' = \mathbf{0}, t; \mathbf{q}, \tau) \propto G_P$$

Ratio is t- and τ-dependent

$$R_{\mathscr{P}}(\mathbf{p}', t; \mathbf{q}, \tau) = \frac{C_{3pt}^{\mathscr{P}}(\mathbf{p}', t; \mathbf{q}, \tau)}{C_{2pt}(\mathbf{p}', t)} \sqrt{\frac{C_{2pt}(\mathbf{p}', \tau)\ C_{2pt}(\mathbf{p}', t)\ C_{2pt}(\mathbf{p}, t-\tau)}{C_{2pt}(\mathbf{p}, \tau)\ C_{2pt}(\mathbf{p}, t)\ C_{2pt}(\mathbf{p}', t-\tau)}}$$

Excited state effect at $\mathbf{q} \neq \mathbf{0}$: the $\mathscr{A}_j$ channel

$R^{\mathbb{P}^j}_{\mathcal{A}_j}(\mathbf{q})$



$\tau - t/2\ [fm]$

$\mathbf{p}' = \mathbf{0} \qquad q_j \neq 0 \qquad (\mathbf{q} \parallel \mathscr{A}_j, \mathbb{P})$

$\mathbb{P}$ = spin-parity projector

$$q = \hat{e}_j = \frac{2\pi}{L}\hat{n}_j$$

$$R^{\mathscr{A}_j}(\mathbf{p}' = \mathbf{0}, t; \mathbf{q}, \tau) \propto G_A, \tilde{G}_P$$

Ratio is t- and τ-dependent

$$R_{\mathscr{A}_j}(\mathbf{p}', t; \mathbf{q}, \tau) = \frac{C_{3pt}^{\mathscr{A}_j}(\mathbf{p}', t; \mathbf{q}, \tau)}{C_{2pt}(\mathbf{p}', t)} \sqrt{\frac{C_{2pt}(\mathbf{p}', \tau)\ C_{2pt}(\mathbf{p}', t)\ C_{2pt}(\mathbf{p}, t-\tau)}{C_{2pt}(\mathbf{p}, \tau)\ C_{2pt}(\mathbf{p}, t)\ C_{2pt}(\mathbf{p}', t-\tau)}}$$

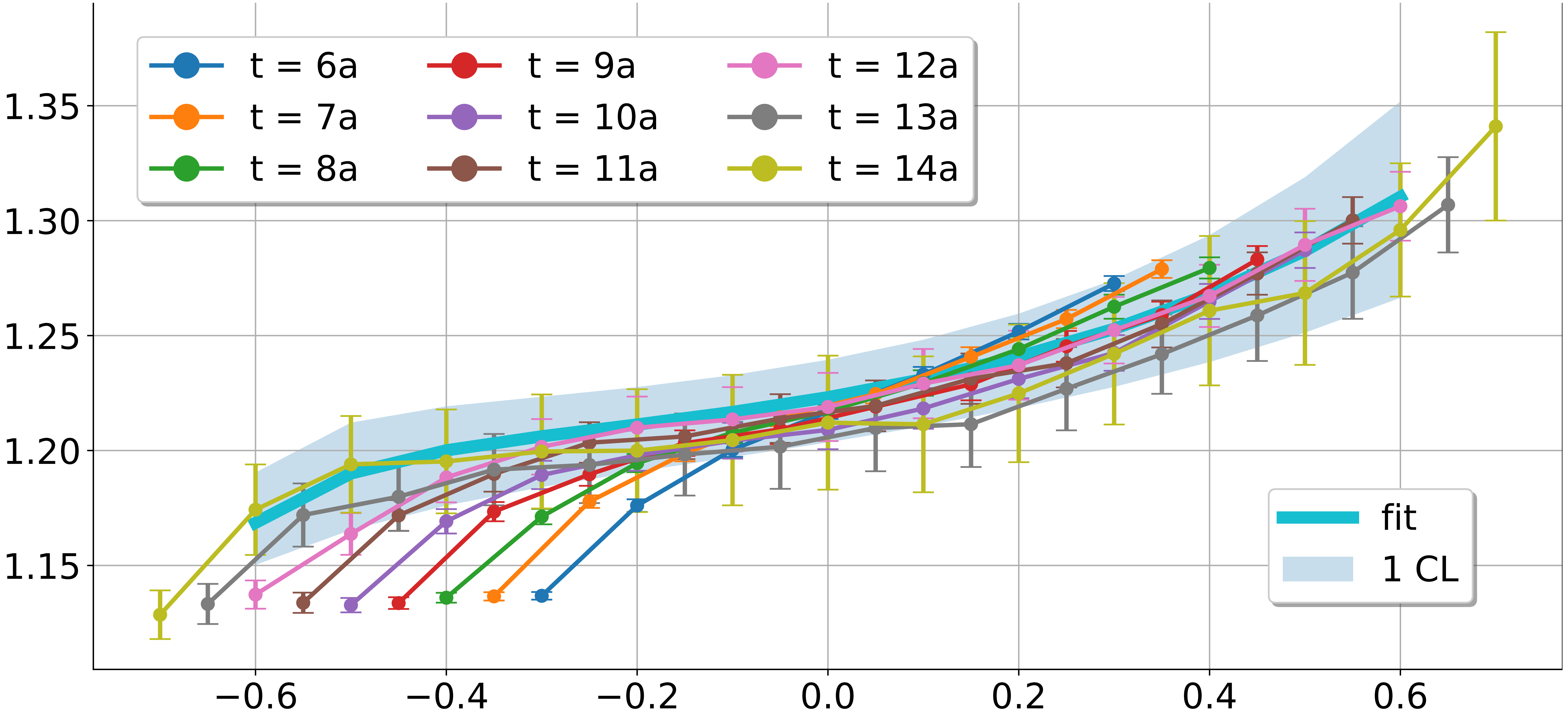
Extraction of $G_A(Q^2 \neq 0)$

$$R_{\mathcal{J}}(\mathbf{p}', t; \mathbf{q}, \tau) = \frac{C^{\mathcal{J}}_{3pt}(\mathbf{p}', t; \mathbf{q}, \tau)}{C_{2pt}(\mathbf{p}', t)} \sqrt{\frac{C_{2pt}(\mathbf{p}', \tau)\ C_{2pt}(\mathbf{p}', t)\ C_{2pt}(\mathbf{p}, t-\tau)}{C_{2pt}(\mathbf{p}, \tau)\ C_{2pt}(\mathbf{p}, t)\ C_{2pt}(\mathbf{p}', t-\tau)}}$$

$\mathcal{J} = \mathcal{A}_j$

$\mathbf{p}' = \mathbf{0}$ $\quad$ $\mathbf{q}_{i \neq j} \neq 0$

$(\mathbf{q} \perp \mathcal{A}_j)$



$\tilde{Q}^2 = 0.297\ \mathrm{GeV}^2$

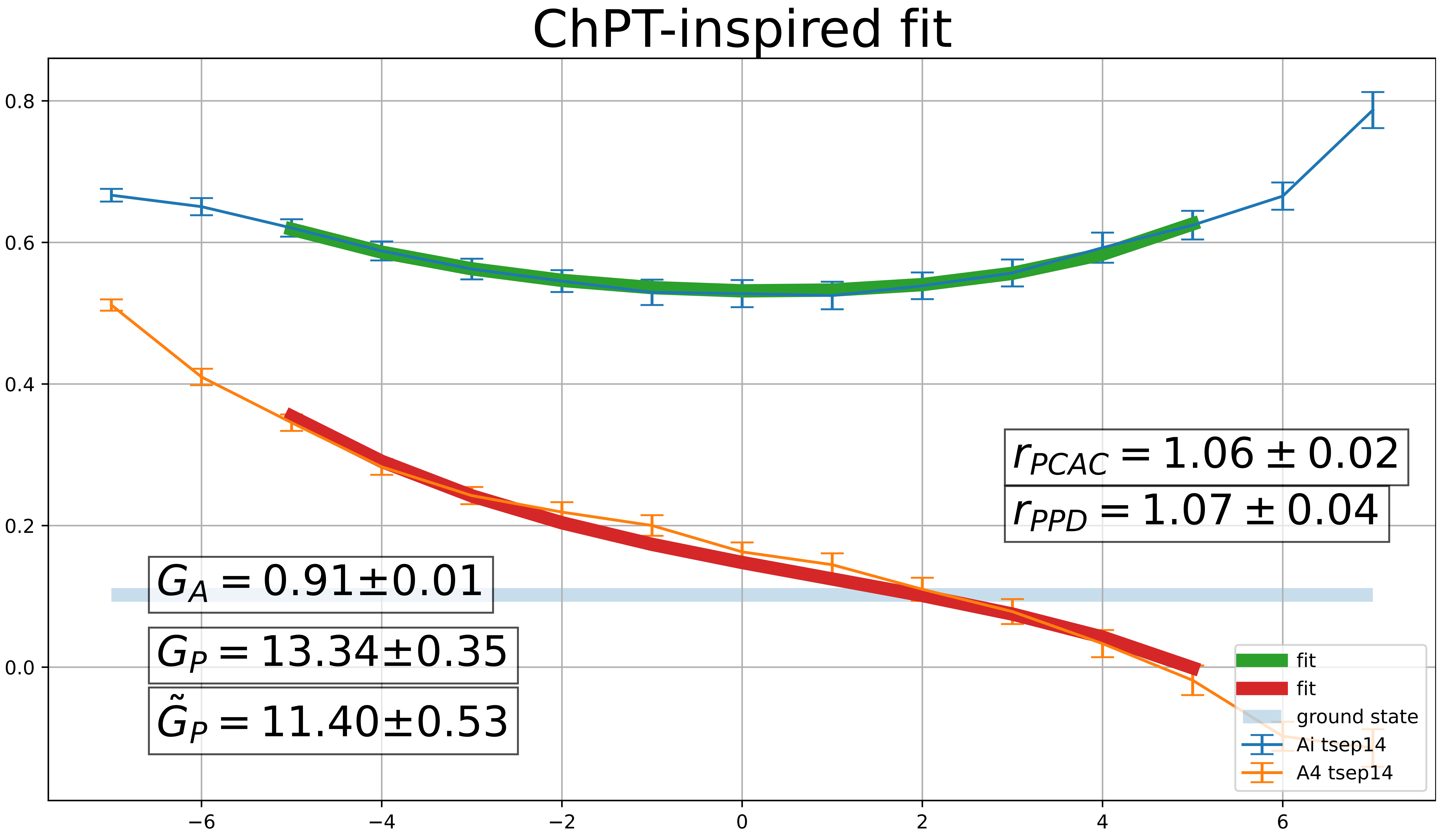
Const. + 2 exp. fit gives

$G_A(\tilde{Q}^2) = 1.22 \pm 0.02$

better way to extract G_A:
Summation method

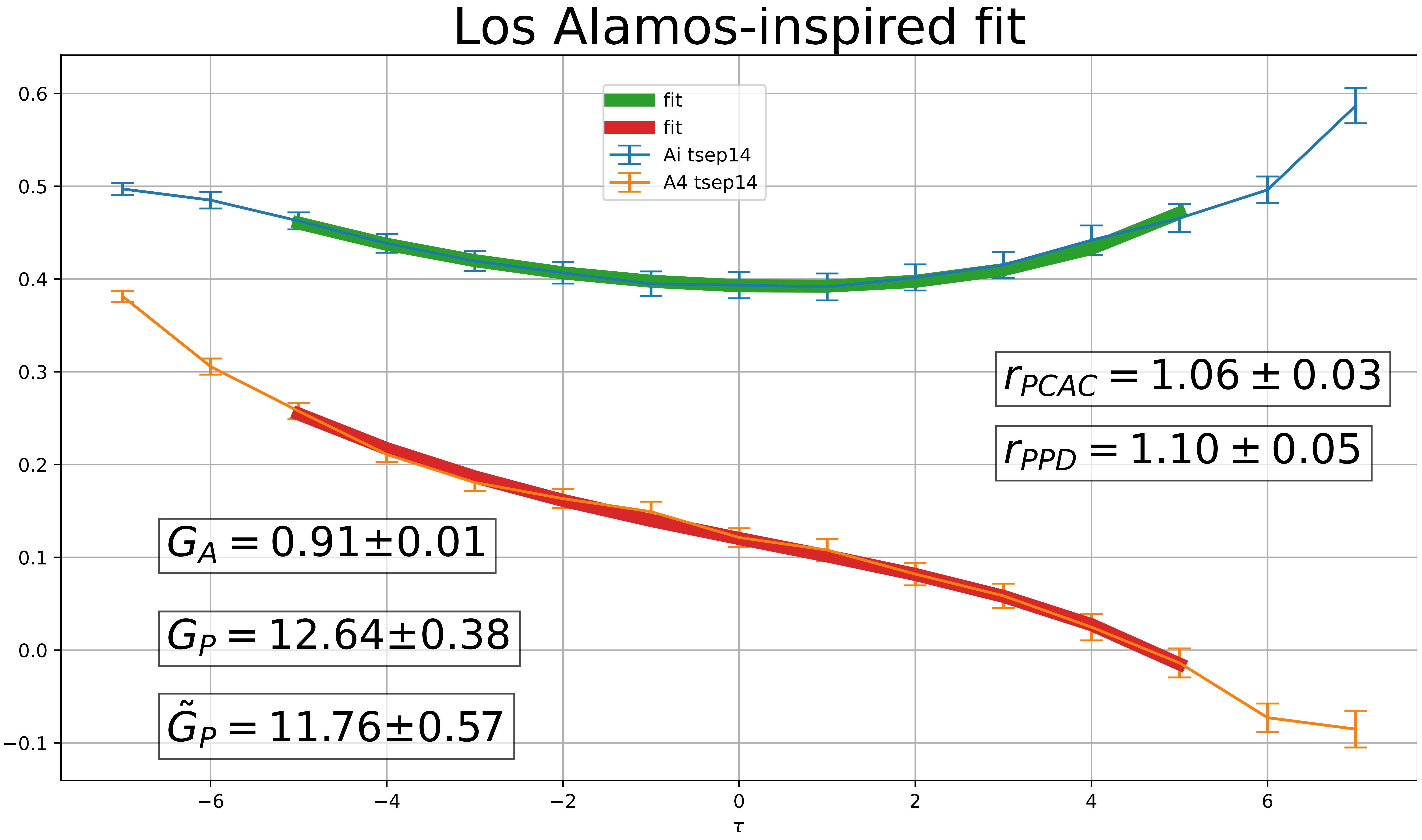
PRD106.074503 Mainz

Fit ChPT

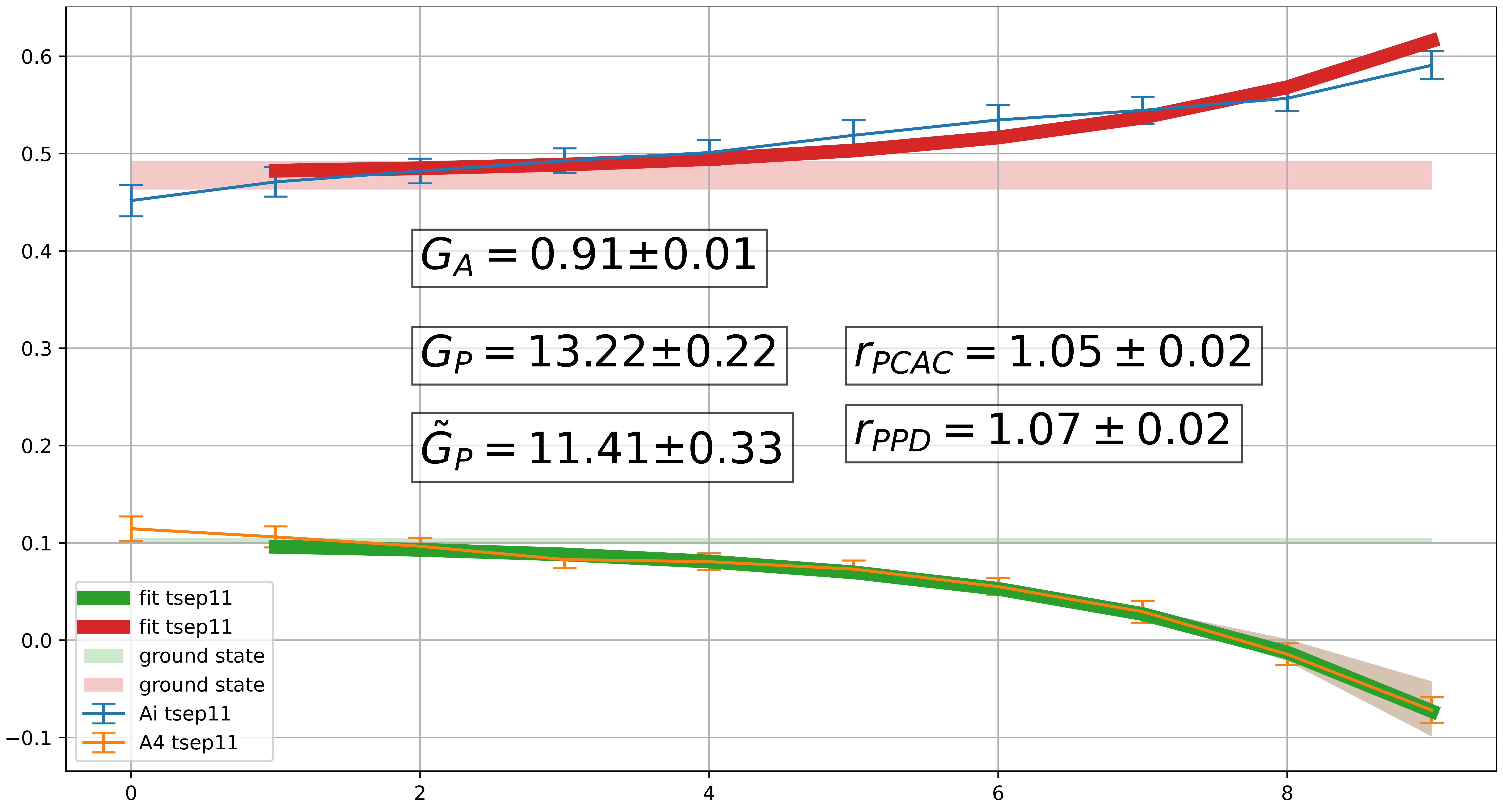


ChPT-inspired fit
0.8
0.6
0.4
0.2
0.0
−6
−4
−2
0
2
4
6
$r_{PCAC} = 1.06 \pm 0.02$
$r_{PPD} = 1.07 \pm 0.04$
$G_A = 0.91 \pm 0.01$
$G_P = 13.34 \pm 0.35$
$\tilde{G}_P = 11.40 \pm 0.53$
fit
fit
ground state
Ai tsep14
A4 tsep14

Fit Los Alamos



Fit GEVP ratios



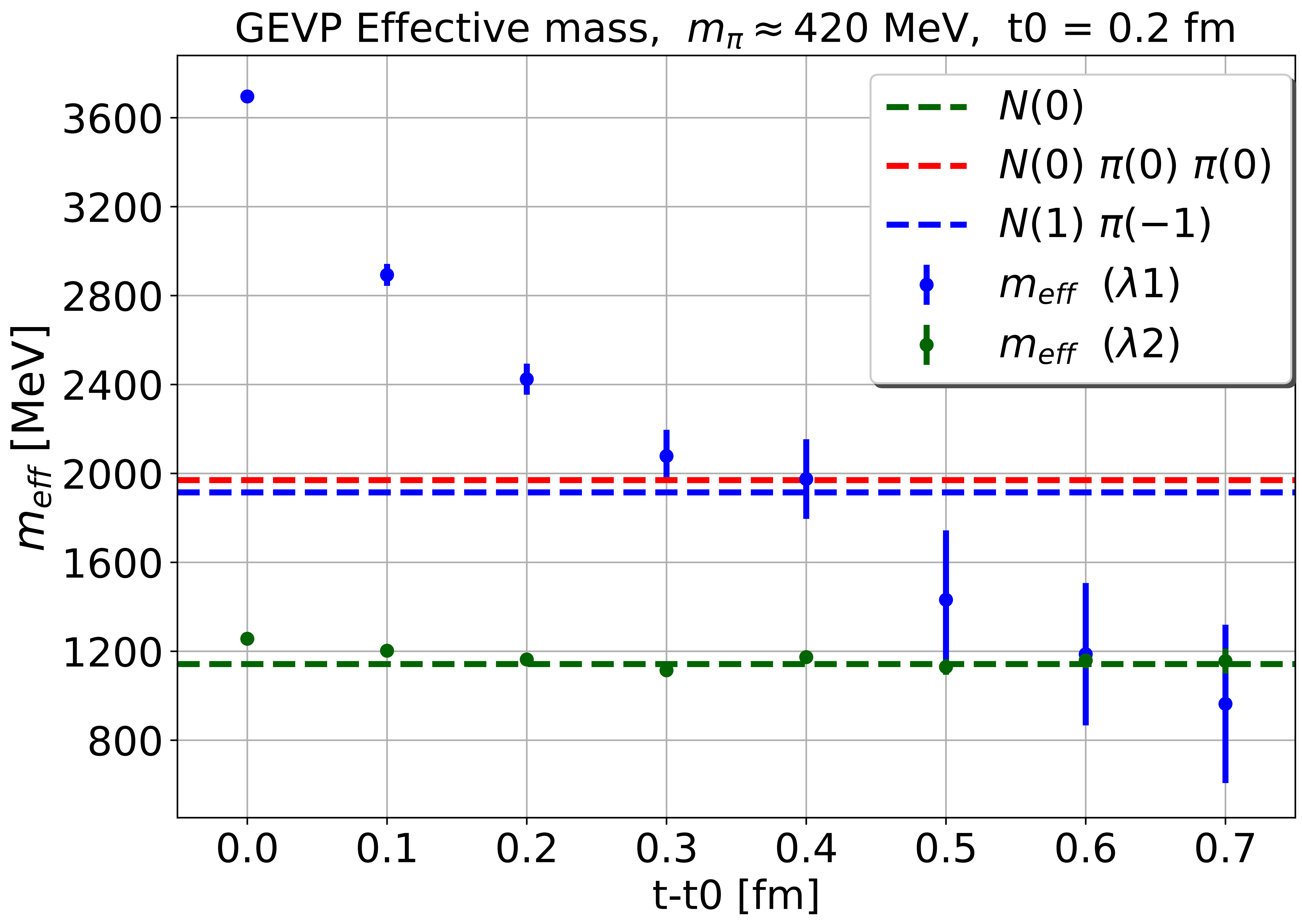
GEVP results with basis $\mathbb{B} = \{O_1, \Phi O_1\}$ and $\mathbf{p} = \mathbf{0}$

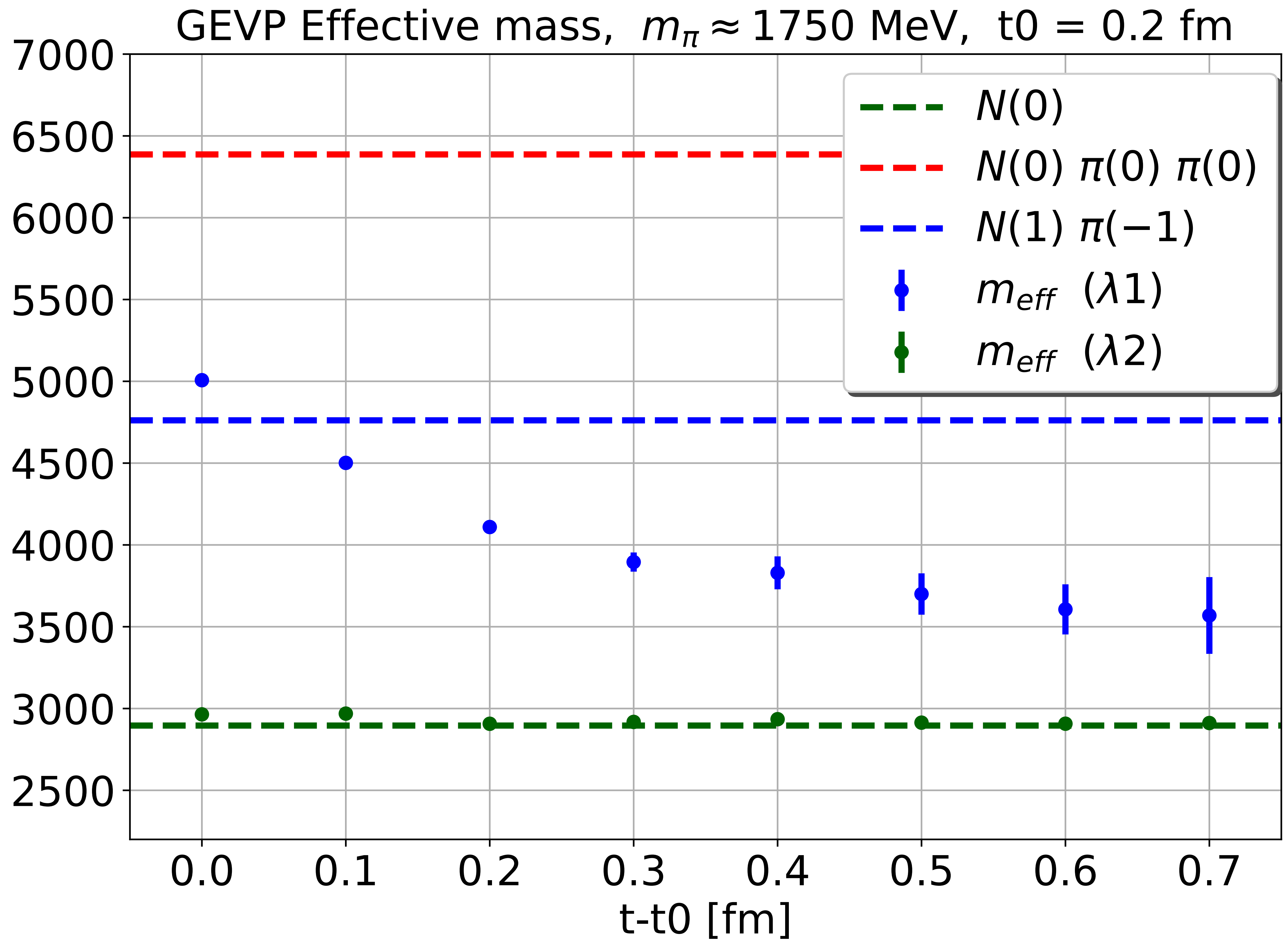
$$C(t) = \begin{pmatrix} \langle O_1(t)\ \bar{O}_1(0)\rangle & \langle O_1(t)\ \Phi\bar{O}_1(0)\rangle \\ \langle \Phi O_1(t)\ \bar{O}_1(0)\rangle & \langle \Phi O_1(t)\ \Phi\bar{O}_1(0)\rangle \end{pmatrix}$$

$$C(t)v^\alpha(t,t_0) = C(t_0)\ \lambda^\alpha(t,t_0)v^\alpha(t,t_0)$$

$$\lambda^1 \propto e^{-E_N(t-t_0)} \equiv \lambda^N$$

$$\lambda^2 \propto e^{-E_{N^*}(t-t_0)} \equiv \lambda^{N^*}$$

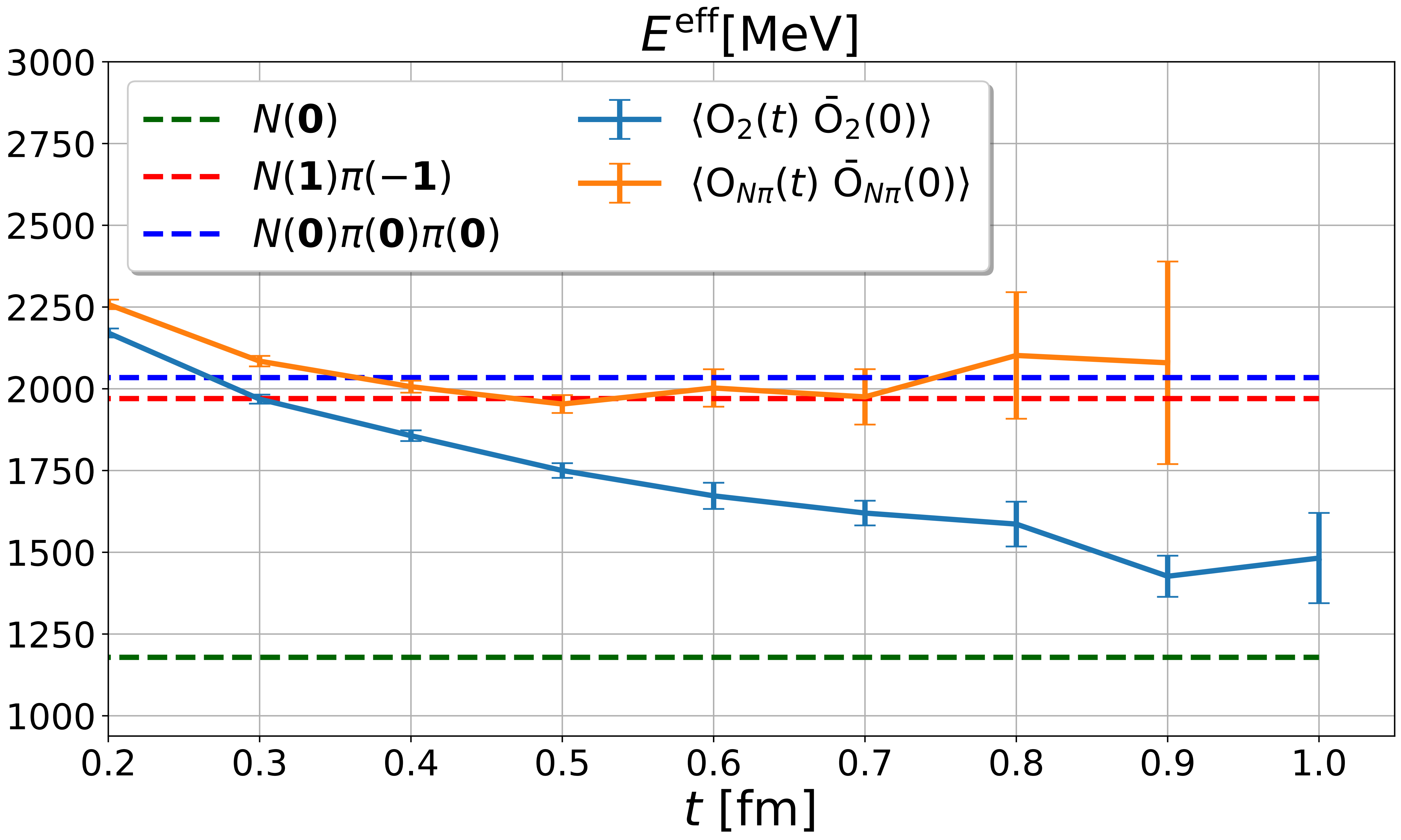




GEVP-projected operators ($\mathbf{p} = 0$)

We use eigenvectors to project operators: $\langle O_2(t)\ \bar{O}_2(0)\rangle \approx c_2^{N\pi} e^{-E_{N\pi}t} + c_2^{N} e^{-E_N t}$ at $t \gg 0$ the dominant term is the nucleon

After GEVP-projection: $O_{N\pi} = \sum_i v_i^{N\pi}\ O_i = v_1^{N\pi}\ O_N + v_2^{N\pi}\ O_{N\pi}$ → $\langle O_{N\pi}(t)\ \bar{O}_{N\pi}(0)\rangle \approx c_{N\pi} e^{-E_{N\pi}t}$



(Dashed lines are non-interacting energy levels)

$$E^{\text{eff}} = \log\left(\frac{\langle O(t-a)\ \bar{O}(0)\rangle}{\langle O(t)\ \bar{O}(0)\rangle}\right)$$

The correlation functions with O_2 don't exhibit a plateau here because of the mixing with N states

New step will be the computation of

$$\langle (N\pi)(\mathbf{p}')|\ \mathcal{J}(\mathbf{q})\ |N(\mathbf{p})\rangle$$

through

$$\langle O_{N\pi}(\mathbf{p}', t)\ \mathcal{J}(\mathbf{q}, \tau)\ \bar{O}_N(\mathbf{p}, 0)\rangle$$

Extraction of form factors

$$C_{2pt}(\mathbf{p},t) = \langle \mathrm{O}_N(\mathbf{p},t)\ \bar{\mathrm{O}}_N(\mathbf{p},0)\rangle \qquad C^{\mathcal{J}}_{3pt}(\mathbf{p}',t;\mathbf{q},\tau) = \langle \mathrm{O}_N(\mathbf{p}',t)\ \mathcal{J}(\mathbf{q},\tau)\ \bar{\mathrm{O}}_N(\mathbf{p},0)\rangle$$

$$R_{\mathcal{J}}(\mathbf{p}',t;\mathbf{q},\tau) = \frac{C^{\mathcal{J}}_{3pt}(\mathbf{p}',t;\mathbf{q},\tau)}{C_{2pt}(\mathbf{p}',t)}\sqrt{\frac{C_{2pt}(\mathbf{p}',\tau)\ C_{2pt}(\mathbf{p}',t)\ C_{2pt}(\mathbf{p},t-\tau)}{C_{2pt}(\mathbf{p},\tau)\ C_{2pt}(\mathbf{p},t)\ C_{2pt}(\mathbf{p}',t-\tau)}}$$

$$\propto\ \mathrm{tr}\left[\mathbb{P}\ (-i\gamma_\mu p'_\mu + m_N)\ FF[\mathcal{J}]\ (-i\gamma_\mu p_\mu + m_N)\right]$$

$\mathcal{J} = \bar{q}\Gamma q$

$\mathcal{J}$	Γ	
$\mathcal{A}_i$	$\gamma_i\gamma_5$	$\propto G_A, G_{\tilde{P}}$
$\mathcal{A}_4$	$\gamma_4\gamma_5$	$\propto G_A, G_{\tilde{P}}$
$\mathcal{P}$	γ_5	$\propto G_P$

$FF[\mathcal{A}_\mu]$

$$\langle N(\mathbf{p}')\,|\,\mathcal{A}_\mu(\mathbf{q})\,|\,N(\mathbf{p})\rangle = u_{\mathbf{p}'}\left[\gamma_\mu\gamma_5 G_A(Q^2) + \frac{q_\mu}{2m_N}\gamma_5 G_{\tilde{P}}(Q^2)\right]u_{\mathbf{p}}$$

$$\langle N(\mathbf{p}')\,|\,\mathcal{P}(\mathbf{q})\,|\,N(\mathbf{p})\rangle = u_{\mathbf{p}'}\ \gamma_5 G_P(Q^2)\ u_{\mathbf{p}}$$

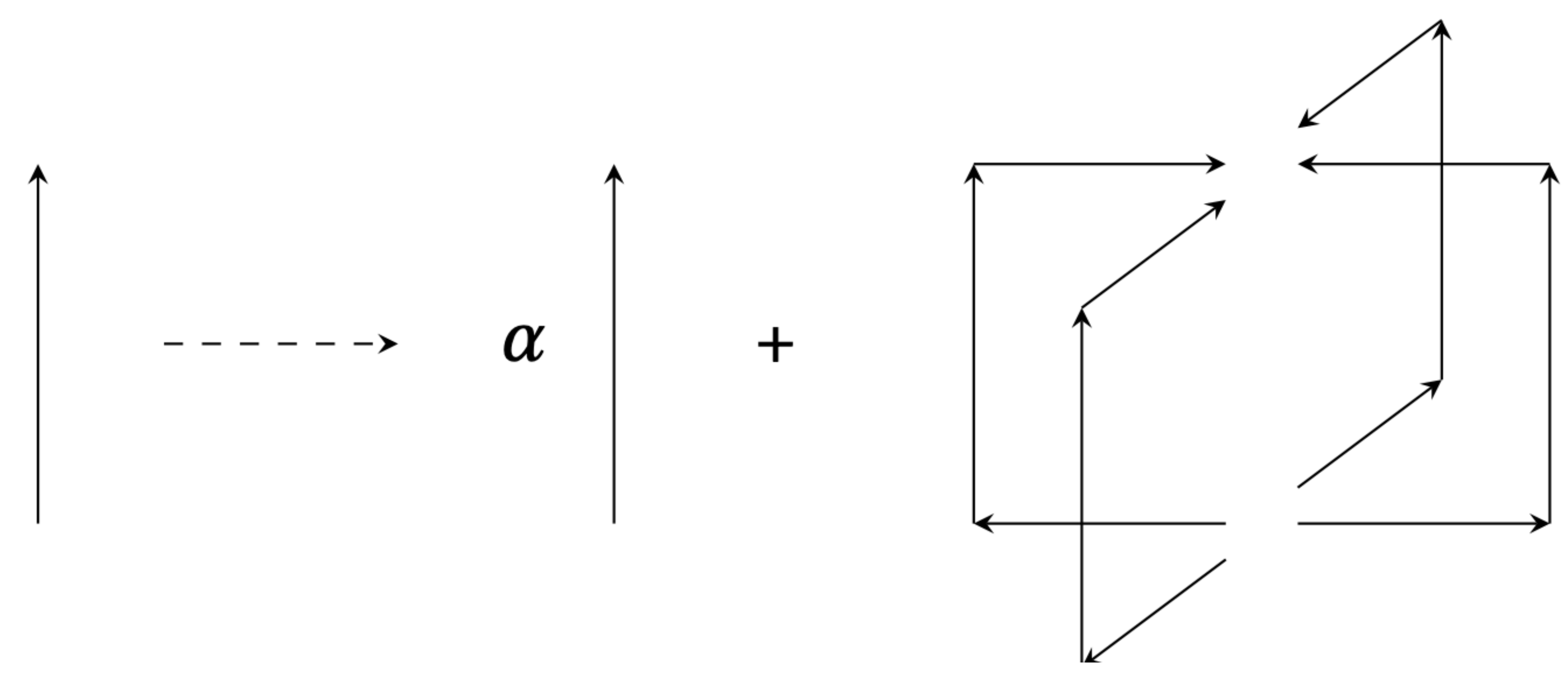
$FF[\mathcal{P}]$

$$\langle N(\mathbf{p}')\,|\,\mathcal{V}_\mu(\mathbf{q})\,|\,N(\mathbf{p})\rangle = u_{\mathbf{p}'}\left[\gamma_\mu F_1(Q^2) + i\frac{\sigma_{\mu\nu}q_\mu}{2m_N}F_2(Q^2)\right]u_{\mathbf{p}}$$

$FF[\mathcal{V}_\mu]$

APE gauge link smearing

$$U_i^{(n+1)} = \mathbb{P}_{SU(3)}\left[\alpha U_i^{(n)}(x) + \sum_{j\neq i} C_{ij}^{(n)}(x)\right]$$



$\alpha = 2.5$ $n = 25$

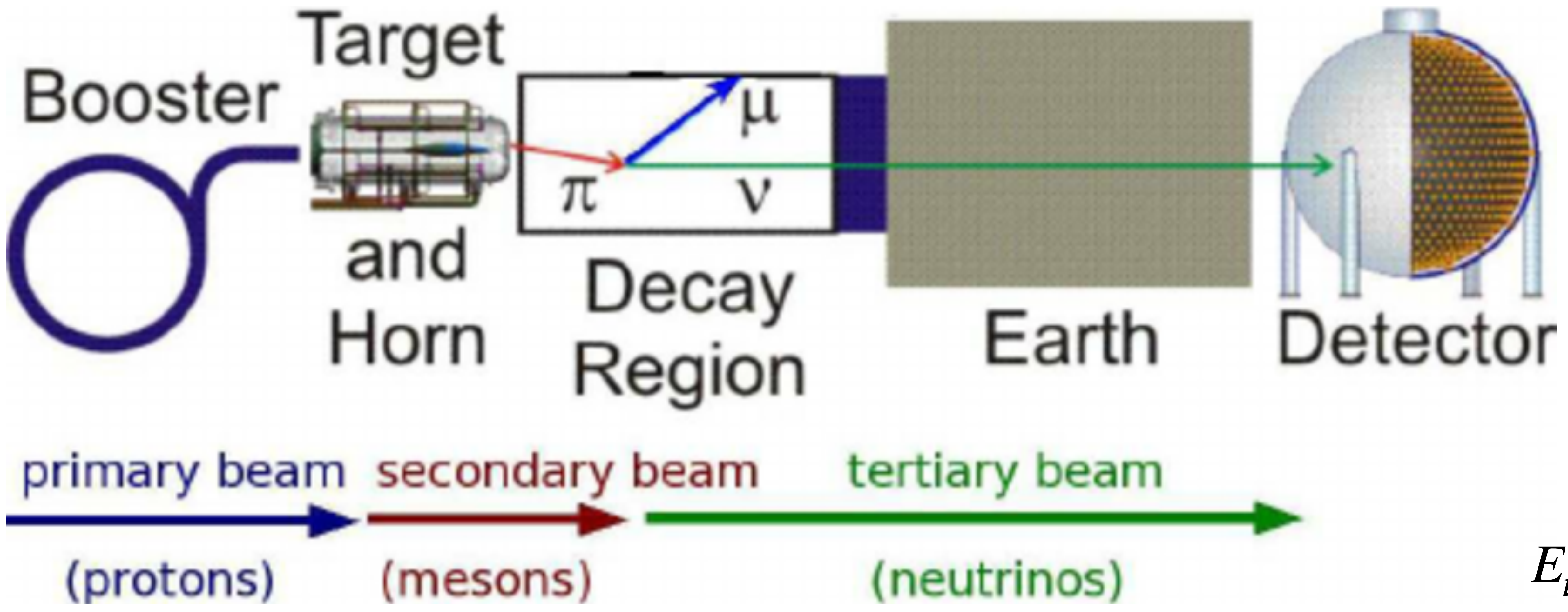
Wuppertal quark smearing

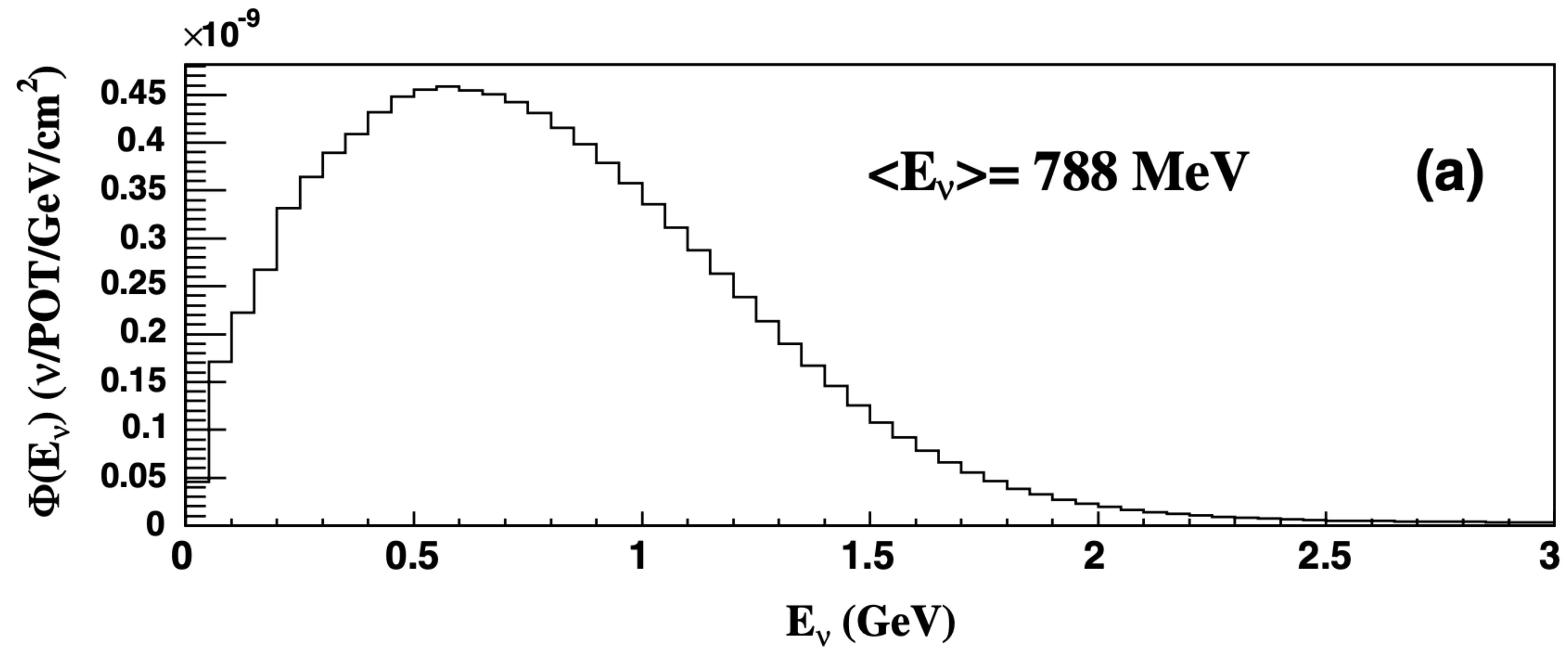
$$q^{(n)}(x) = \left(1 + \frac{\kappa_s}{1+6\kappa_s}\nabla^2\right)^n q(x)$$

$$\nabla^2 q(x) = -6q(x) + \sum_{\mu=\pm 1}^{\pm 3} U_\mu(x) q(x+\hat{\mu})$$

$\kappa_s = 0.25$ $n = 150$

MiniBooNE experiment





Predicted flux at the detector

$$E_\nu^{QE} = \frac{2M_N E_\mu - M_N^2 + m_\mu^2 + M_p^2}{2[M_N - E_\mu + cos\theta_\mu \sqrt{E_\mu^2 - m_\mu^2}]}$$

$$M_N = M_n - E_b$$

(Carbon binding energy)

Detector: spherical steel tank filled with 818 tons of mineral oil $(\mathrm{CH_2})$

Booster: proton beam is accelerated

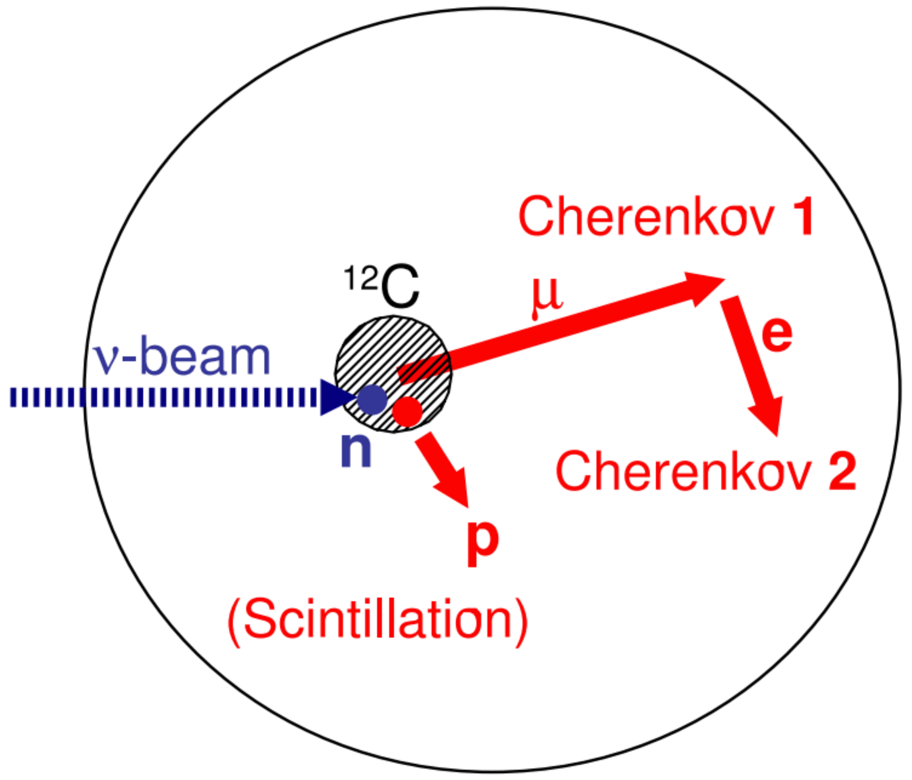
Target: proton beam scatters off a target, producing a meson beam

Mesons' decay: (muon) neutrino beam is produced $\quad \pi^+ \to \mu^+ + \nu_\mu$

Earth: neutrino beam travels a length $L \approx 450\mathrm{m}$

Detector: muons are produced and accelerated in matter $\quad \nu_\mu + n \to \mu^- + p$

—> Cherenkov light (refractive index) $n = \frac{c}{v} = 1.47$ —> Cherenkov threshold: $\beta = \frac{v}{c} > 0.68$



CCQE interaction at the MiniBoone detector

Neutrinoless double beta-decay

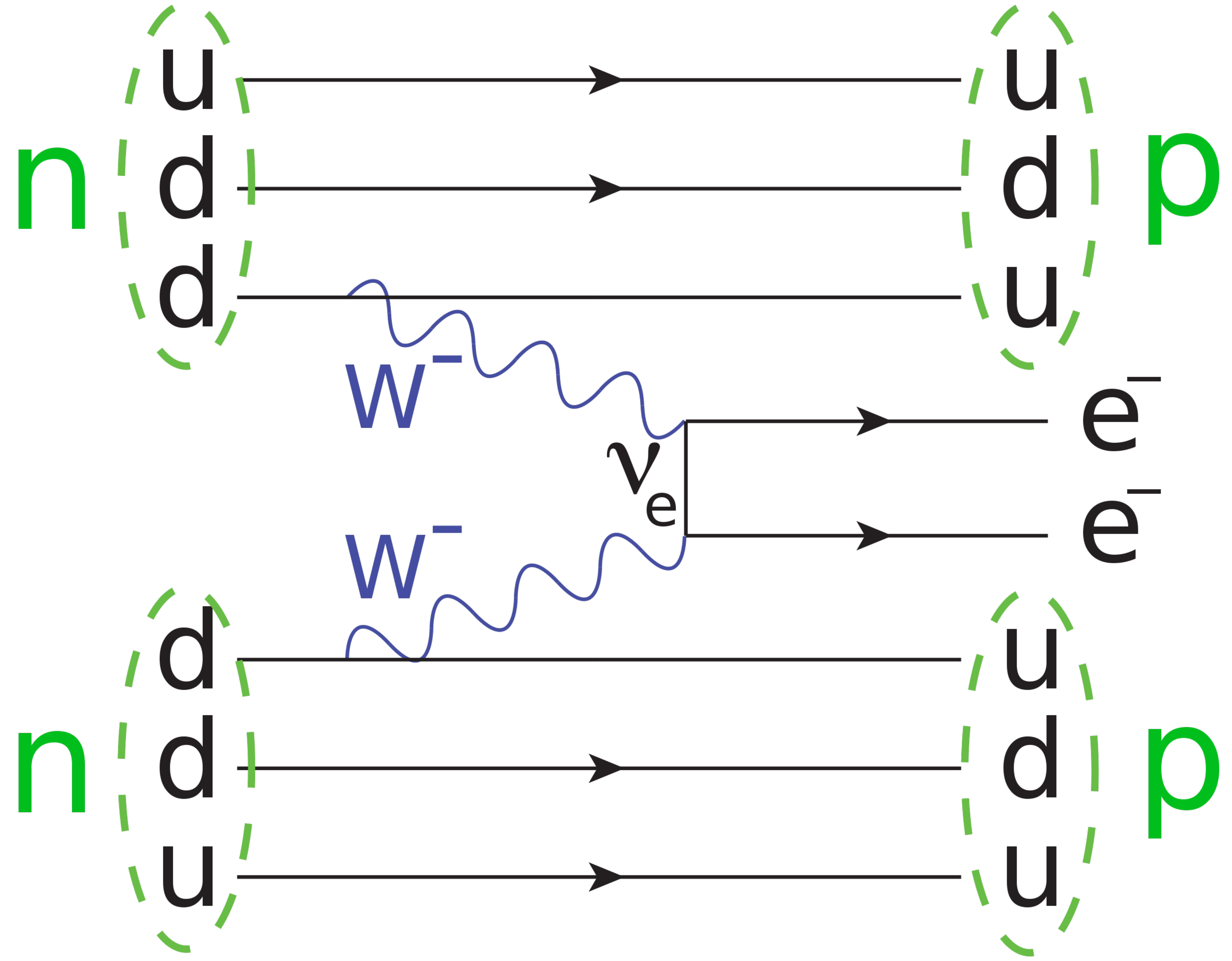
$$(A,Z) \rightarrow (A,Z+2) \; + \; 2e^-$$

$$\Gamma^{0\nu}_{\beta\beta} = \frac{1}{T^{0\nu}_{\beta\beta}} \propto |M^{0\nu}|^2 \langle m_{\beta\beta} \rangle^2$$

$M^{0\nu}$ **is the nuclear matrix element**

$$\langle m_{\beta\beta} \rangle = \sum_i U_{ei}^2 \, m_i$$

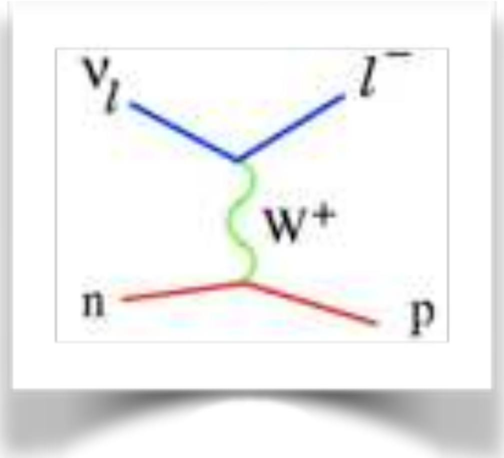
U_{ei} **is a PMNS matrix element**



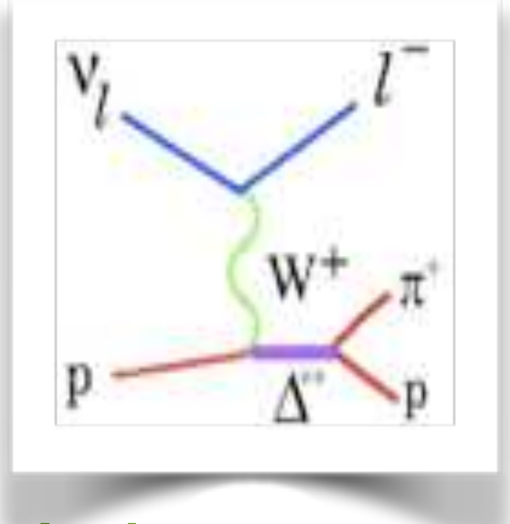
Isotope	Experiment	lifetime $T^{0\nu}_{\beta\beta}$ [years]
^{48}Ca	ELEGANT-VI	$> 1.4 \cdot 10^{22}$
^{76}Ge	Heidelberg-Moscow[14]	$> 1.9 \cdot 10^{25}$ [14]
^{76}Ge	GERDA	$> 1.8 \cdot 10^{26}$ [15]
^{82}Se	NEMO-3	$> 1.0 \cdot 10^{23}$
^{82}Se	CUPID-0	$> 4.6 \cdot 10^{24}$ [16]
^{96}Zr	NEMO-3	$> 9.2 \cdot 10^{21}$
^{100}Mo	NEMO-3	$> 2.1 \cdot 10^{25}$
^{116}Cd	Solotvina	$> 1.7 \cdot 10^{23}$
^{130}Te	CUORE	$> 2.2 \cdot 10^{25}$
^{136}Xe	EXO	$> 3.5 \cdot 10^{25}$ [17]
^{136}Xe	KamLAND-Zen	$> 1.07 \cdot 10^{26}$ [18]
^{150}Nd	NEMO-3	$> 2.1 \cdot 10^{25}$

very rare process

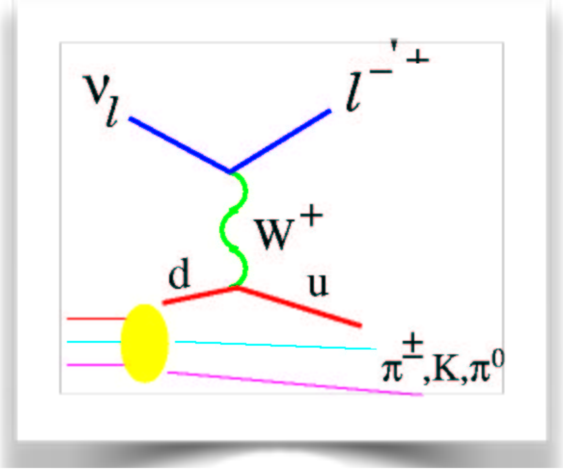
Quasi-elastic scattering (QE)

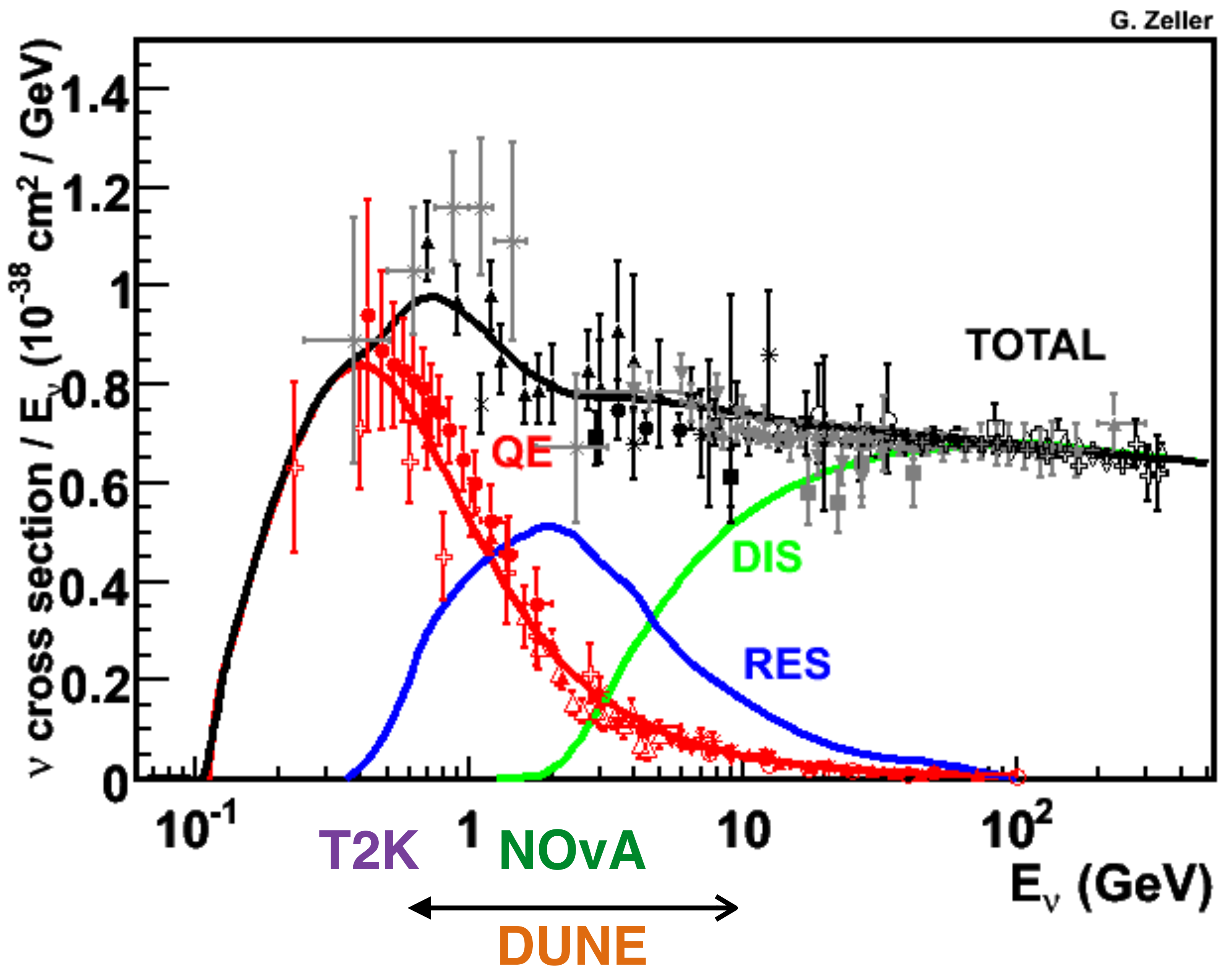


Resonance production (RES)



Deep Inelastic scattering (DIS)





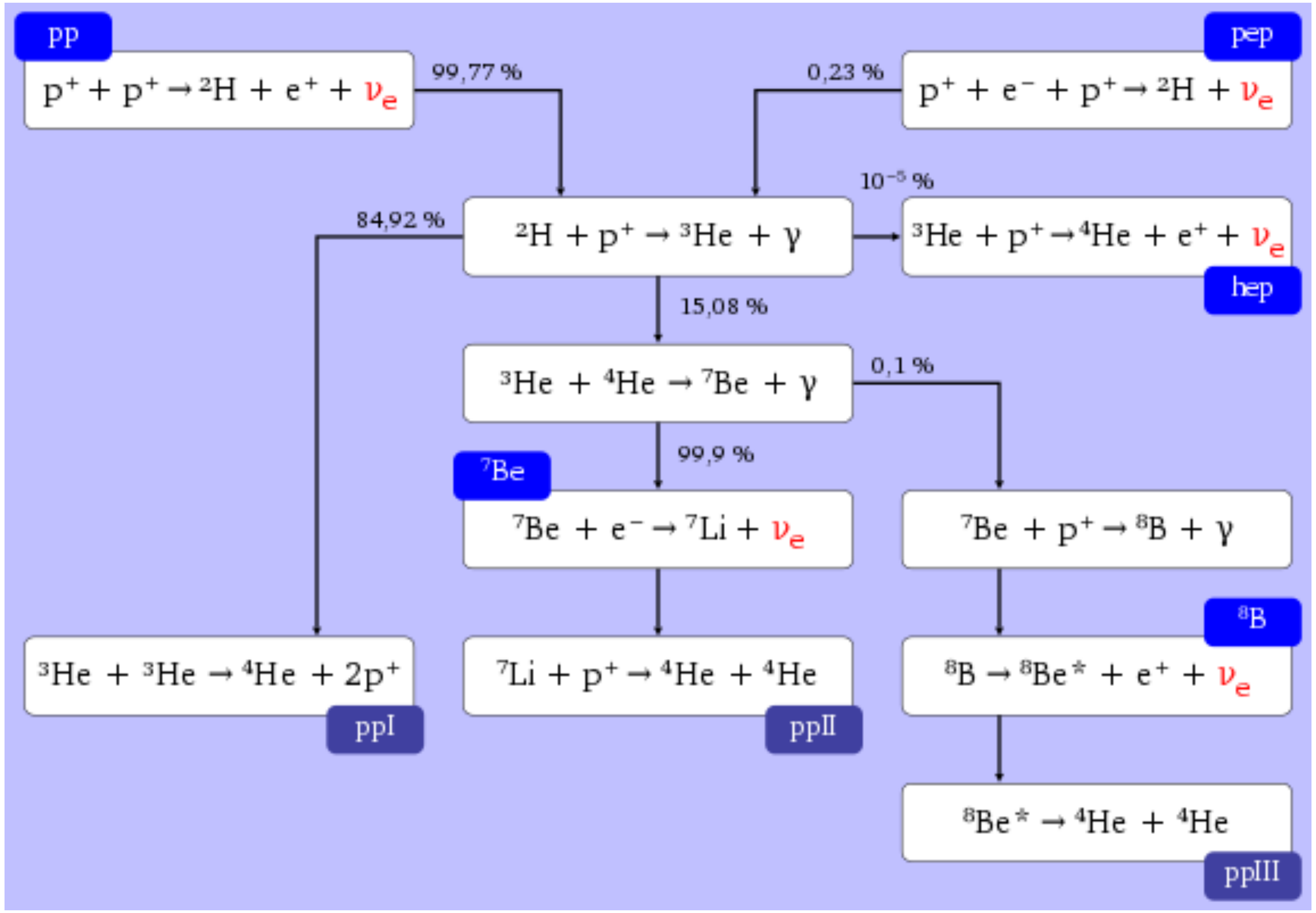
J.A. Formaggio, G. Zeller, Reviews of Modern Physics, 84 (2012)

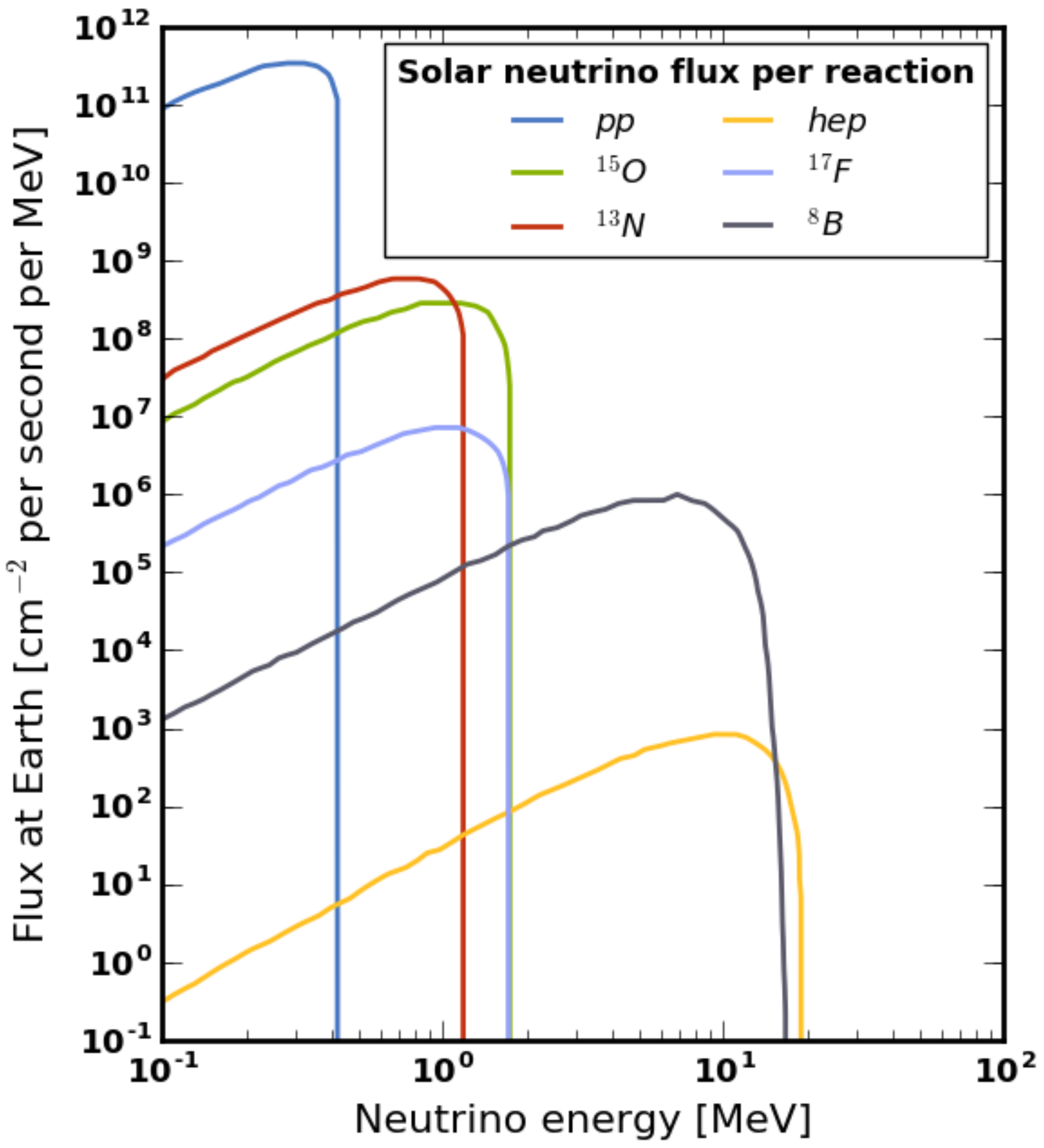
neutrino process	abbreviation	reaction	fraction (%)
CC quasielastic	CCQE	$\nu_\mu + n \to \mu^- + p$	39
NC elastic	NCE	$\nu_\mu + p(n) \to \nu_\mu + p(n)$	16
CC $1\pi^+$ production	CC$1\pi^+$	$\nu_\mu + p(n) \to \mu^- + \pi^+ + p(n)$	25
CC $1\pi^0$ production	CC$1\pi^0$	$\nu_\mu + n \to \mu^- + \pi^0 + p$	4
NC $1\pi^\pm$ production	NC$1\pi^\pm$	$\nu_\mu + p(n) \to \nu_\mu + \pi^+(\pi^-) + n(p)$	4
NC $1\pi^0$ production	NC$1\pi^0$	$\nu_\mu + p(n) \to \nu_\mu + \pi^0 + p(n)$	8
multi pion production, DIS, etc.	other	$\nu_\mu + p(n) \to \mu^- + N\pi^\pm + X$, etc.	4

[PRD.81.092005]

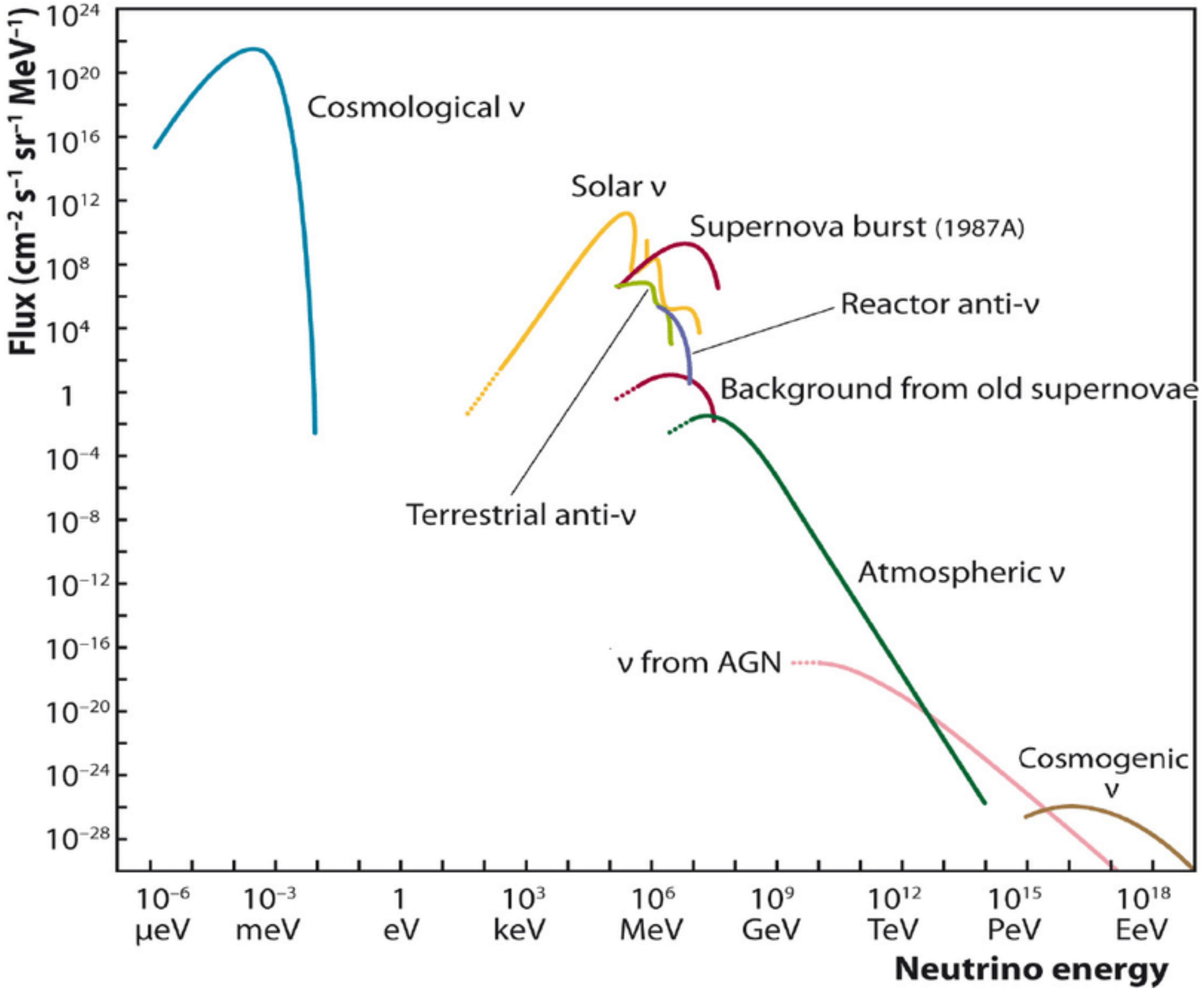
[RMP.84.1307] *[arXiv:2203.09030]*

Solar fusion and neutrino flux





Neutrino flux from different sources



AGN: Active Galactic Nucleus