Excited States in B Meson Correlation Functions

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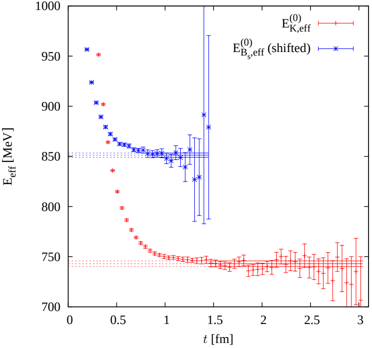
Motivation

- Interpolating field $\mathcal{O}$ for particle p

$$\sum_{\vec{x}} \left\langle \mathcal{O}(\vec{x},t)\mathcal{O}^\dagger(\vec{0},0) \right\rangle \sim |\langle 0|\mathcal{O}|p\rangle|^2 \, e^{-m_p t} + \sum_{e.s.} |\langle 0|\mathcal{O}|e.s.\rangle|^2 \, e^{-E_{e.s.} t}$$

- Systematic error if not taken into account properly
- Increase precision of flavour physics results
- Tension between inclusive and exclusive determinations of $|V_{ub}|$
- Chiral Perturbation Theory successfully predicted excited states in Nucleon correlators



[ALPHA 2019]

Outline

- Chiral Perturbation Theory for heavy mesons
- B meson 2-point function
- $BB^*\pi$ coupling constant g_π
- $B \to \pi$ form factors
- $B^0 - \bar{B}^0$ mixing

Chiral Symmetry

- Massless QCD Lagrangian:

$$\mathcal{L}_{\mathrm{ferm}} = \sum_f \bar{q}_f \not{D} q_f = \sum_f \left(\bar{q}_{f,L} \not{D} q_{f,L} + \bar{q}_{f,R} \not{D} q_{f,R} \right)$$

- Global symmetry $SU(N_f)_L \times SU(N_f)_R$
- Define $Q_V = Q_R + Q_L$ and $Q_A = Q_R - Q_L$
- No parity doubling observed $\to$ **spontaneous** symmetry breaking with order parameter $\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{q}_L q_R + \bar{q}_R q_L|0\rangle \neq 0$

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$$

Chiral Symmetry

- $N_f^2 - 1$ massless Goldstone bosons, SU(3): π, K, η

$$\left\langle 0 \middle| A_\mu^a \middle| \pi^b \right\rangle = \delta^{ab} \mathrm{i} f_\pi p_\mu e^{\mathrm{i}px}$$

- Soft explicit symmetry breaking by small quark masses $\rightarrow$ massive pseudo-Goldstone bosons
- Low energy effective theory with **chiral symmetry**
- Chiral Ward identities; can be derived by demanding **local** chiral symmetry, external fields

$$W^{QCD}[a_\mu, v_\mu, s, p] = \langle 0, \mathrm{out} | 0, \mathrm{in} \rangle_{a_\mu, v_\mu, s, p} = W^{ChPT}[a_\mu, v_\mu, s, p]$$

- QCD mass term: $s = M_q$

Chiral Perturbation Theory

- **Low Energy** Effective Theory for QCD with **light quarks**
  - **Chiral symmetry** $SU(N_F)_L \times SU(N_F)_R$
  - Infinite number of interactions between Pions, Kaons and Eta
  - Expansion parameters: $\mathbf{p}$ and $\mathbf{m_q}$
- CCWZ construction: nonlinear realisation of symmetry

$$\Sigma = e^{\frac{2i}{f}\pi^a T^a} \to L\Sigma R^\dagger$$

- Valid in regime $< \Lambda_\chi = 4\pi f \sim 1$ GeV
- LO (p^2) Lagrangian

$$\mathcal{L} = \frac{f^2}{4}\mathrm{Tr}\left[\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right] - \frac{Bf^2}{2}\mathrm{Tr}\left[\Sigma M_q + M_q\Sigma^\dagger\right]$$

- Baryon Chiral Perturbation Theory

Heavy Mesons

- Mesons with light valence quarks $\Rightarrow$ Chiral Perturbation Theory
- What about heavy mesons (D, B)?
  - Bound states of **heavy** and **light** quark $Q\bar{q}$
  - $m_b \gg \Lambda_\chi$
- Lattice simulations with b quarks? $\Rightarrow$ Heavy Quark Effective Theory
  - Expansion in $1/m_b$
  - Eliminates antiquarks
  - **Heavy quark spin symmetry (HQSS)** ($\mathcal{L} = \bar{Q}D_4Q$)
- Heavy Meson Chiral Perturbation Theory
  - Expansion in $\mathbf{1/m_B}$
  - Heavy quark spin symmetry and chiral symmetry
  - Removes on-shell momentum $m_B v_\mu$ and antiparticles
- Bound states: $(B^-, \bar{B}^0, \bar{B}^0_s)$ & $(B^{*-}, \bar{B}^{*0}, \bar{B}^{*0}_s)$

HM ChPT Lagrangian

- Operators P, P^*_μ destroy $\bar{B}$ mesons
- Form a **doublet** under HQS transformations

$$H_a = \frac{1+\gamma_4}{2}\left[\mathrm{i}P^*_{a,\mu}\gamma_\mu + \mathrm{i}\gamma_5 P_a\right] , \qquad P^*_4 = 0$$

- Transformation under **chiral symmetry** ($\to$ coupling to NGBs) and HQS

$$\Sigma = \xi\xi \, , \xi \to L\xi U^\dagger(x) = U(x)\xi R^\dagger$$

$$H \to SHU^\dagger \, , \qquad \xi_\mu = \frac{\mathrm{i}}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) \to U\xi_\mu U^\dagger$$

- LO (p) Lagrangian (Euclidean Space)

$$\mathcal{L} = -\,\mathrm{Tr}\left[\bar{H}_a D_{ab,4} H_b\right] - \mathrm{i}g\,\mathrm{Tr}\left[\bar{H}_a H_b \gamma_5\gamma_\mu\xi_{ba,\mu}\right]$$

$$= 2(P^{*\dagger}_\mu D_4 P^*_\mu + P^\dagger D_4 P) + 2\mathrm{i}g P^*_\mu \frac{\partial_\mu \pi}{f} P^\dagger + \dots$$

$$\langle B|A_k|B^*\rangle \sim g_\pi = g + \dots , \qquad A^{QCD}_k = \bar{q}\gamma_k\gamma_5 q'$$

Interpolating Fields

- Interpolating fields $\bar{\mathcal{B}}$ and $\bar{\mathcal{B}}^*_k \Leftrightarrow \bar{q}\gamma_5 Q$ and $\bar{q}\gamma_k Q$
- Form dictated by **symmetries** and **quantum numbers**
- Heavy Quark Spin Symmetry relates coefficients of $\bar{\mathcal{B}}$ and $\bar{\mathcal{B}}^*_k$!
- $\alpha = \hat{f} = f_B\sqrt{m_B}$

$$\bar{\mathcal{B}} = \alpha P\left(1 - \frac{1}{f^2}\pi^2\right) + \frac{\mathrm{i}\beta_1}{f}P^*_k\partial_k\pi + \frac{\beta_2}{f^2}P\partial_4\pi\pi + \mathcal{O}(p^2, \pi^3, m_B^{-1})$$

$$\bar{\mathcal{B}}^*_k = \alpha P^*_k\left(1 - \frac{1}{f^2}\pi^2\right) - \frac{\mathrm{i}\beta_1}{f}(\varepsilon_{klm}P^*_m\partial_l\pi + P\partial_k\pi) + \frac{\beta_2}{f^2}P^*_k\partial_4\pi\pi + \mathcal{O}(p^2, \pi^3, m_B^{-1})$$

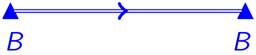
- $\tilde{\alpha}$, $\tilde{\beta}_i$ for smeared fields

Excited States of 2-Point Function

$$C_2(t) = \int_{L^3} \mathrm{d}^3x \left\langle \mathcal{B}(t,\vec{x})\mathcal{B}^\dagger(0,\vec{0}) \right\rangle \sim |\langle 0|\mathcal{B}|B\rangle|^2 + \sum_{\vec{p}} f(\vec{p})|\langle 0|\mathcal{B}|B^*\pi\rangle|^2 e^{-E_{\pi,\vec{p}}t} + \ldots$$

Parametrisation of 2-point function

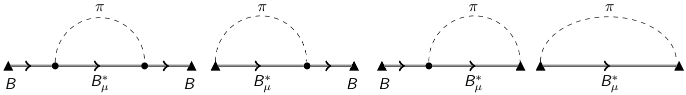
$$C_2(t) = C_2^B(t) + C_2^{B\pi}(t) + \ldots = C_2^B(t)\left(1 + \frac{C_2^{B\pi}(t)}{C_2^B(t)} + \ldots\right)$$
$$= C_2^B(t)(1 + \delta C_2(t))$$



Leading order: $C_2^B = \frac{\tilde{\alpha}^2}{2}$

$$\Delta_B(x) = \frac{\theta(t)}{2L^3}\delta^{(3)}(\vec{x})$$

Excited States of 2-Point Function



$$\Delta_\pi(x) = \sum_{\vec{p}} \frac{e^{-E_{\pi,\vec{p}}|t|}}{2L^3 E_{\pi,\vec{p}}} e^{i\vec{p}\cdot\vec{x}}$$

- Excited states

$$\delta C_2(t) = \sum_{\vec{p}} \frac{3}{8(fL)^2(E_{\pi,\vec{p}}L)} \frac{\vec{p}^2}{E^2_{\pi,\vec{p}}} \left(g + \frac{\tilde{\beta}_1}{\tilde{\alpha}} E_{\pi,\vec{p}}\right)^2 e^{-tE_{\pi,\vec{p}}}$$

- Effective mass and decay constant

$$m_{eff}(t) = m_B - \partial_t \log(C_2(t)) , \qquad \hat{f}^2_{eff}(t) = 2C_2(t)e^{\Delta m_{eff}(t)t}$$

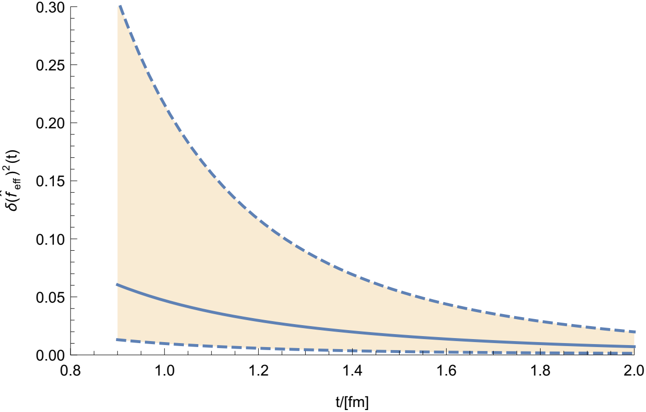
Numerical Evaluation

- Parameters at physical point: $m_\pi = 140\,\mathrm{MeV}$, $f = 93\,\mathrm{MeV}$, $g = 0.5$
- Value for $\tilde{\beta}_1/\tilde{\alpha}$ not known, depends on smearing procedure
- Dimensional analysis

$$\frac{\tilde{\beta}_1}{\tilde{\alpha}} = \frac{\mathrm{const}}{\Lambda_\chi}\,, \qquad \mathrm{const} = \mathcal{O}(1) \Rightarrow -1 < \mathrm{const} < 1$$

- ChPT works only for small energies $\to$ high energy pions must be suppressed in excited states $\to$ $t \gtrsim 1.3\,\mathrm{fm}$
- Infinite volume limit

$\delta \hat{f}^2_{eff}$



How to determine β_1?

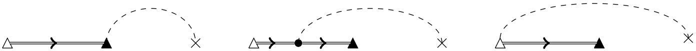
- Light axial current A_μ

$$A_\mu(\vec{x}, t) = \frac{\delta S(a_\mu, v_\mu, p, s)}{\delta a_\mu(\vec{x}, t)}\bigg|_{a_\mu, v_\mu, p=0, s=M_q} \sim \partial_\mu \pi(\vec{x}, t) + \dots$$

- Consider the 3-point function with $t_A > t > 0$

$$C_3(t, t_A, \vec{q}) = \int_{L^3} \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{y}\; e^{\mathrm{i}\vec{q}\vec{z}} \left\langle A_4(t_A, \vec{z})\mathcal{B}(t, \vec{x})\mathcal{B}_k^{*\dagger}(0, \vec{0})\right\rangle$$

$$= \frac{\tilde{\alpha}^2 q_k}{8E_{\pi,\vec{q}}}\left(e^{-t_A E_{\pi,\vec{q}}}\left(g + \frac{\tilde{\beta}_1}{\tilde{\alpha}} E_{\pi,\vec{q}}\right) + e^{-(t_A - t)E_{\pi,\vec{q}}}\left(-g + \frac{\tilde{\beta}_1}{\tilde{\alpha}} E_{\pi,\vec{q}}\right)\right)$$



- Eliminate prefactors by taking ratio with A_μ and B 2-point functions
- Compute ratio for several $\vec{q}$'s and fit to the analytic result

Computation of g_π

$$\mathcal{L} = \mathcal{L}^{kin} + 2\mathrm{i} g P^*_\mu \frac{\partial_\mu \pi}{f} P^\dagger + \dots$$

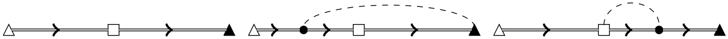
$$\langle B | A_k | B^* \rangle \sim g_\pi = g + \dots , \qquad A_k^{QCD} = \bar{q}\gamma_k\gamma_5 q'$$

- 3-point function

$$C_3(t, t_A) = \int_{L^3} \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{z}\,\left\langle \mathcal{B}(\vec{x}, t) A_k(\vec{z}, t_A) \mathcal{B}_k^{*\dagger}(\vec{0}, 0) \right\rangle$$

- Light axial current

$$A_\mu(\vec{x}, t) = \left. \frac{\delta S(a_\mu, v_\mu, p, s)}{\delta a_\mu(\vec{x}, t)} \right|_{a_\mu, v_\mu, p=0, s=M_q} \sim g P^*_\mu P^\dagger + \dots$$



g_π

- Summed ratio

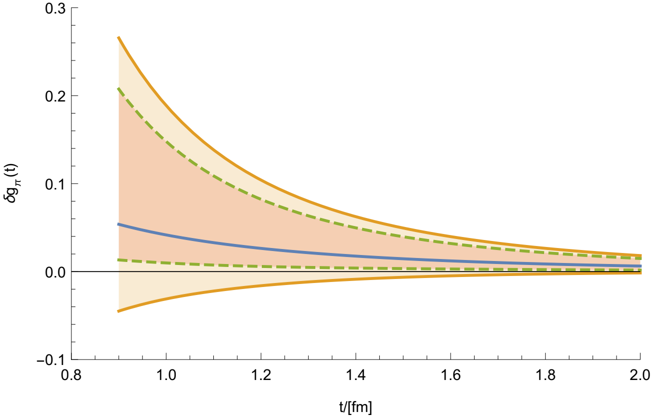
$$g_\pi^{sum}(t) = 2\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}t_A \, \frac{C_3(t, t_A)}{C_2(t)}$$

- Excited states

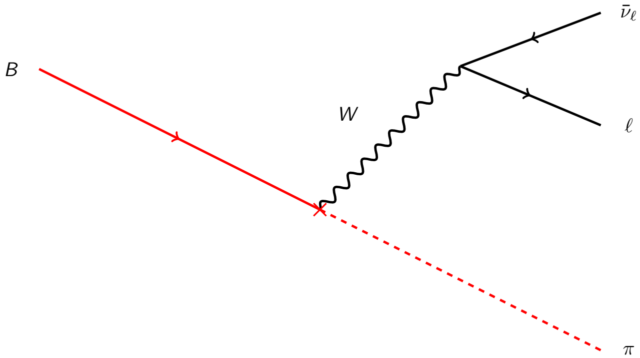
$$\delta g_\pi(t) = \sum_{\vec{p}} \frac{1}{3(fL)^2(E_{\pi,\vec{p}}L)} \frac{\vec{p}^2}{E_{\pi,\vec{p}}^2} \left[2g\left(g + \left(\frac{\tilde{\beta}_1}{\tilde{\alpha}} + \frac{\gamma}{g}\right)E_{\pi,\vec{p}}\right) - \left(g + \frac{\tilde{\beta}_1}{\tilde{\alpha}}E_{\pi,\vec{p}}\right)^2 (1 - E_{\pi,\vec{p}}t)\right] e^{-tE_{\pi,\vec{p}}}$$

- γ: linear combination of NLO LECs of A_μ

Result



Semileptonic Decay



Form Factors

- Definition in QCD:

$$\langle \pi(p_\pi)|V_\mu|B(p_B)\rangle$$
$$= \left((p_B + p_\pi)_\mu - q_\mu \frac{m_B^2 - m_\pi^2}{q^2}\right) f_+(q^2) + q_\mu \frac{m_B^2 - m_\pi^2}{q^2} f_0(q^2)$$
$$q = p_B - p_\pi$$

- For $m_\ell = 0$:

$$\frac{\mathrm{d}\Gamma(B \to \pi\ell\bar{\nu}_\ell)}{\mathrm{d}q^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}|p_\pi|^3\left|f_+(q^2)\right|^2$$

- In Heavy Quark Effective Theory

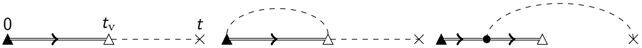
$$(2m_B)^{-\frac{1}{2}}\langle \pi(p)|V_0|B(0)\rangle = h_\parallel(E_\pi)$$
$$(2m_B)^{-\frac{1}{2}}\langle \pi(p)|V_k|B(0)\rangle = p_k h_\perp(E_\pi)$$

Correlators

- 3-point function with momentum of final state pion $\vec{p}$

$$C_{3,\mu}(t,t_{\rm v},\vec{p}) = \int_{L^3} \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{z}\,e^{-\mathrm{i}\vec{p}(\vec{x}-\vec{z})}\left\langle \Pi(\vec{x},t)V_\mu(\vec{z},t_{\rm v})\mathcal{B}^\dagger(\vec{0},0)\right\rangle$$

- Sample Diagrams (only $\langle\pi|V_\mu|B^*\pi\rangle$ excited states)



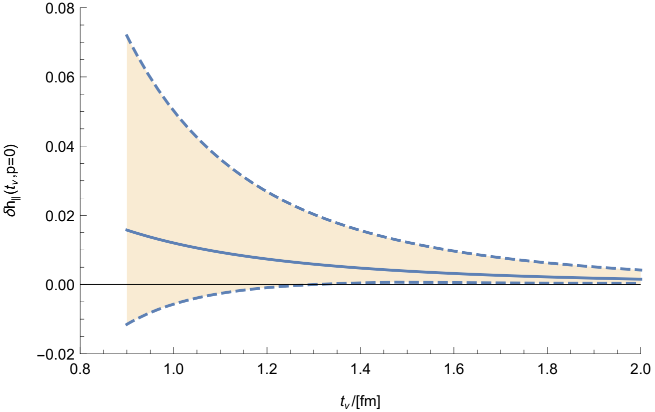
- Results

$$h_\parallel = \frac{\alpha}{\sqrt{2}f}\left(1-\frac{\beta_2 E_{\pi,\vec{p}}}{\alpha}\right), \qquad h_\perp = \frac{\alpha g}{\sqrt{2}fE_{\pi,\vec{p}}}\left(1-\frac{\beta_1 E_{\pi,\vec{p}}}{\alpha g}\right)$$

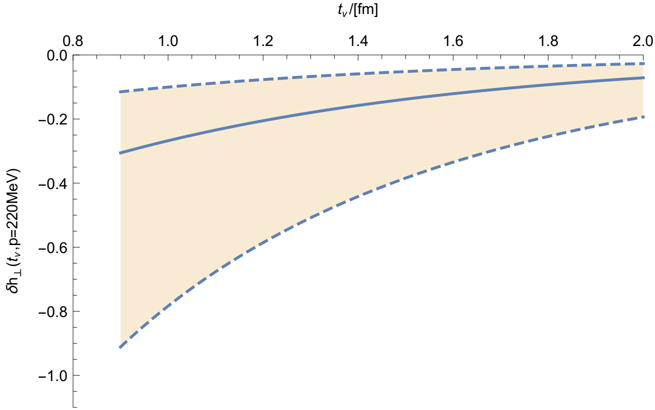
$$\delta h_\parallel = \frac{1}{L^3}\sum_{\vec{l}} A_1(\vec{p},\vec{l},\beta_1,\tilde{\beta}_1,\beta_2)e^{-t_{\rm v}E_{\pi,\vec{l}}}$$

$$\delta h_\perp = -\frac{1+\tilde{\beta}_1 E_{\pi,\vec{p}}/(\tilde{\alpha}g)}{1-\beta_1 E_{\pi,\vec{p}}/(\alpha g)}e^{-t_{\rm v}E_{\pi,\vec{p}}} + \frac{1}{L^3}\sum_{\vec{l}} A_2(\vec{p},\vec{l},\beta_1,\tilde{\beta}_1,\gamma)e^{-t_{\rm v}E_{\pi,\vec{l}}}$$

$\delta h_\parallel$ as Function of Time

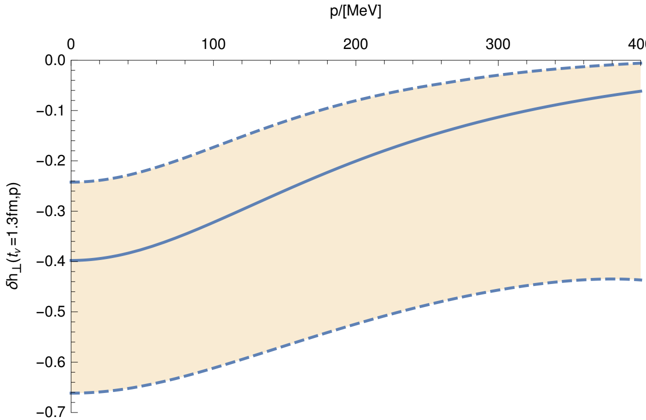


$\delta h_\perp$ as Function of Time



$$\delta h_\perp = -\frac{1 + \tilde{\beta}_1 E_{\pi,\vec{p}}/(\tilde{\alpha} g)}{1 - \beta_1 E_{\pi,\vec{p}}/(\alpha g)} e^{-t_v E_{\pi,\vec{p}}} + \frac{1}{L^3} \sum_{\vec{l}} A_2(\vec{p}, \vec{l}, \beta_1, \tilde{\beta}_1, \gamma) e^{-t_v E_{\pi,\vec{l}}}$$

$\delta h_\perp$ as Function of Pion Momentum



$$\delta h_\perp = -\frac{1+\tilde{\beta}_1 E_{\pi,\vec{p}}/(\tilde{\alpha} g)}{1-\beta_1 E_{\pi,\vec{p}}/(\alpha g)} e^{-t_\mathrm{v} E_{\pi,\vec{p}}} + \frac{1}{L^3}\sum_{\vec{l}} A_2(\vec{p},\vec{l},\beta_1,\tilde{\beta}_1,\gamma) e^{-t_\mathrm{v} E_{\pi,\vec{l}}}$$

How to determine β_2?

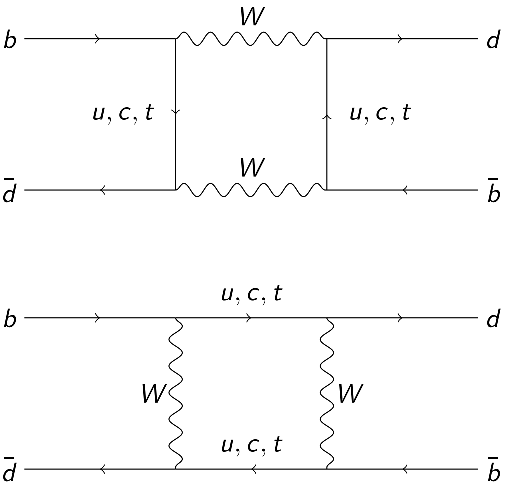
- Consider the 3-point function with $t > t_A > 0$

$$C_3(t, t_A, \vec{q}) = \int_{L^3} \mathrm{d}^3x\, \mathrm{d}^3y\, e^{\mathrm{i}\vec{q}\vec{z}} \left\langle A_4^{ll}(t, \vec{x}) A_k^{hl}(t_A, \vec{z}) \mathcal{B}_k^{*\dagger}(0, \vec{0}) \right\rangle$$

$$= \mathrm{const} \times \frac{e^{-(t-t_A)E_{\pi,\vec{q}}}}{E_{\pi,\vec{q}}} \left(1 - \frac{\beta_2}{\alpha} E_{\pi,\vec{q}}\right) + \mathcal{O}\left(e^{-t_A E_\pi}\right)$$

- Form ratio with 2-point functions and compute it for various $\vec{q}$'s

$B^0 - \bar{B}^0$ Mixing



$B^0 - \bar{B}^0$ Mixing

- Hamiltonian for time-evolution

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

- Hamiltonian is **not diagonal** $\Rightarrow$ non-degenerate mass-eigenstates possible
- Mass Eigenstates

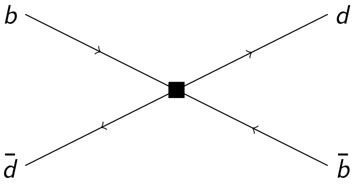
$$|B_L\rangle = C_1 \left|B^0\right\rangle + C_2 \left|\bar{B}^0\right\rangle$$
$$|B_H\rangle = C_1 \left|B^0\right\rangle - C_2 \left|\bar{B}^0\right\rangle$$

- Time-dependent $B^0 - \bar{B}^0$ mixing probability

$$p(t) \sim \frac{e^{-\frac{\Gamma_L + \Gamma_H}{2} t}}{2} \left[\cosh\left(\frac{\Gamma_L - \Gamma_H}{2} t\right) - \cos(\Delta m t)\right]$$

Effective Theory

$$\mathcal{O} = \left[\bar{b}\gamma^\mu(1-\gamma_5)d\right]\left[\bar{b}\gamma_\mu(1-\gamma_5)d\right]$$



$$\left\langle \bar{B}^0 \middle| \mathcal{O} \middle| B^0 \right\rangle = \frac{8}{3} f_B^2 m_B^2 B_{B_d}$$

$$\Delta m \approx -\frac{G_F^2 m_W^2 \eta_B m_B B_{B_d} f_B^2}{6\pi^2} S_0\left(\frac{m_t^2}{m_W^2}\right)\left(V_{td}^* V_{tb}\right)^2$$

Mixing Operators in HQET

- $\mathcal{O}$ matched to two operators in HQET:

$$\mathcal{O}_L = \left[Q^\dagger \gamma_\mu (1-\gamma_5) d\right] \left[\tilde{Q} \gamma_\mu (1-\gamma_5) d\right]$$
$$\mathcal{O}_S = \left[Q^\dagger (1-\gamma_5) d\right] \left[\tilde{Q} (1-\gamma_5) d\right]$$

- Matching:

$$\mathcal{O}(m_b) = -2\Big((1+O(\alpha_s))\mathcal{O}_L(m_b) + O(\alpha_s)\mathcal{O}_S(m_b)\Big)$$

- Transformation under chiral symmetry (flavour index a)

$$\mathcal{O}_{i,aa} \to L_{ac} L_{ab} \mathcal{O}_{i,bc}$$

- $\mathcal{O}_L$ invariant under heavy quark spin transformations $\to$ restricts number of LECs of HMChPT operator, e.g.

$$\mathcal{O}_L^{LO} = \rho(B\bar{B}^\dagger - B_k^* \bar{B}_k^{*\dagger})$$

Mixing Operators in HMChPT

- $\mathcal{O}_L$: one LO and one NLO LEC
- $\mathcal{O}_S$: two LO and two NLO LECs
- Summed Correlator, ratio with 2-point function

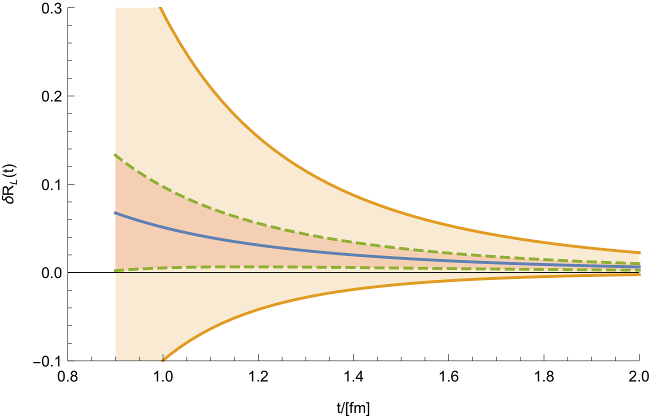
$$C_{3,i}(t, t_\mathrm{I}) = \int \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{y}\,\left\langle \bar{\mathcal{B}}(t,\vec{x})\mathcal{O}_i(t_\mathrm{I},\vec{y})\mathcal{B}^\dagger(0,\vec{0})\right\rangle$$

$$R_i = 2\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}t_\mathrm{I}\,\frac{C_{3,i}(t,t_\mathrm{I})}{C_2(t)} = \left\langle \bar{B}^0\middle|\mathcal{O}_i\middle|B^0\right\rangle(1+\delta R_i(t))$$

- Result for $\mathcal{O}_L$:

$$\delta R_L(t) = \sum_{\vec{p}} \frac{3}{2(fL)^2(E_{\pi,\vec{p}}L)}\frac{\vec{p}^2}{E^2_{\pi,\vec{p}}}\left(g^2 + g\frac{\tilde{\beta}_1 E_{\pi,\vec{p}}}{\tilde{\alpha}} + 2g\frac{\omega_1 E_{\pi,\vec{p}}}{\rho}\right)e^{-tE_{\pi,\vec{p}}}$$

Plot of $\delta\mathcal{O}_L$



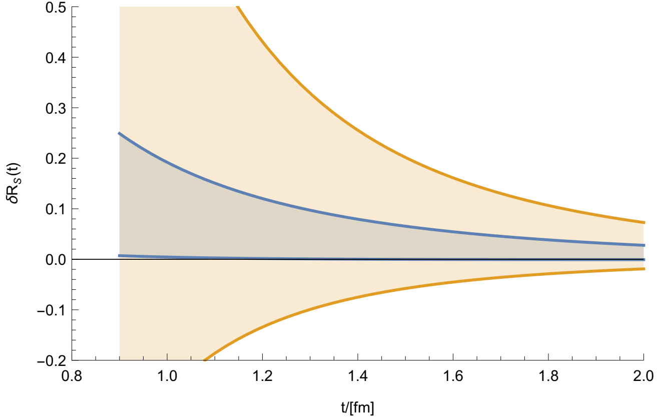
Mixing Operators in HMChPT II

- **Two** LECs at **LO**, parametrise $\langle \bar{B}^0 | \mathcal{O}_S | B^0 \rangle$ and $\langle \bar{B}^{*0} | \mathcal{O}_S | B^{*0} \rangle$
- No cancellation of $\delta C_2(t)$

$$\delta R_S(t) = \sum_{\vec{p}} \frac{3}{8(fL)^2(E_{\pi,\vec{p}}L)} \frac{\vec{p}^2}{E_{\pi,\vec{p}}^2} \left(8e^{-tE_{\pi,\vec{p}}} \left(g^2 \frac{\hat{\eta}}{\eta_1} + g \frac{\tilde{\beta}_1 \hat{\eta} E_{\pi,\vec{p}}}{\tilde{\alpha}\eta_1} + g \frac{\hat{\phi} E_{\pi,\vec{p}}}{\eta_1} \right) - \left(\frac{\eta_2}{\eta_1} + 1 \right) (1 - tE_{\pi,\vec{p}}) \left(g + \frac{\tilde{\beta}_1 E_{\pi,\vec{p}}}{\tilde{\alpha}} \right)^2 e^{-tE_{\pi,\vec{p}}} \right)$$

$$\hat{\eta} = \frac{3\eta_1 + \eta_2}{4}$$

Plot of $\delta\mathcal{O}_S$



$$-2 < \frac{\eta_2}{\eta_1} < 2$$

Conclusions

- Excited states can severely affect lattice calculations
- Chiral Perturbation Theory is an analytic tool to estimate them
- We have derived
  - Interpolating Fields for $B^{(*)}$ and $\bar{B}^{(*)}$ mesons
  - Axial Current
  - Operator for $B^0 - \bar{B}^0$ mixing

  and computed the excited states contamination for
  - $\hat{f}$ and m_{eff}
  - g_π
  - Vector Form Factors $h_\parallel$ and $h_\perp$
  - $B^0 - \bar{B}^0$ mixing amplitudes
- Strategies to determine LECs of interpolating fields