The weak gravity conjecture with scalar fields in AdS

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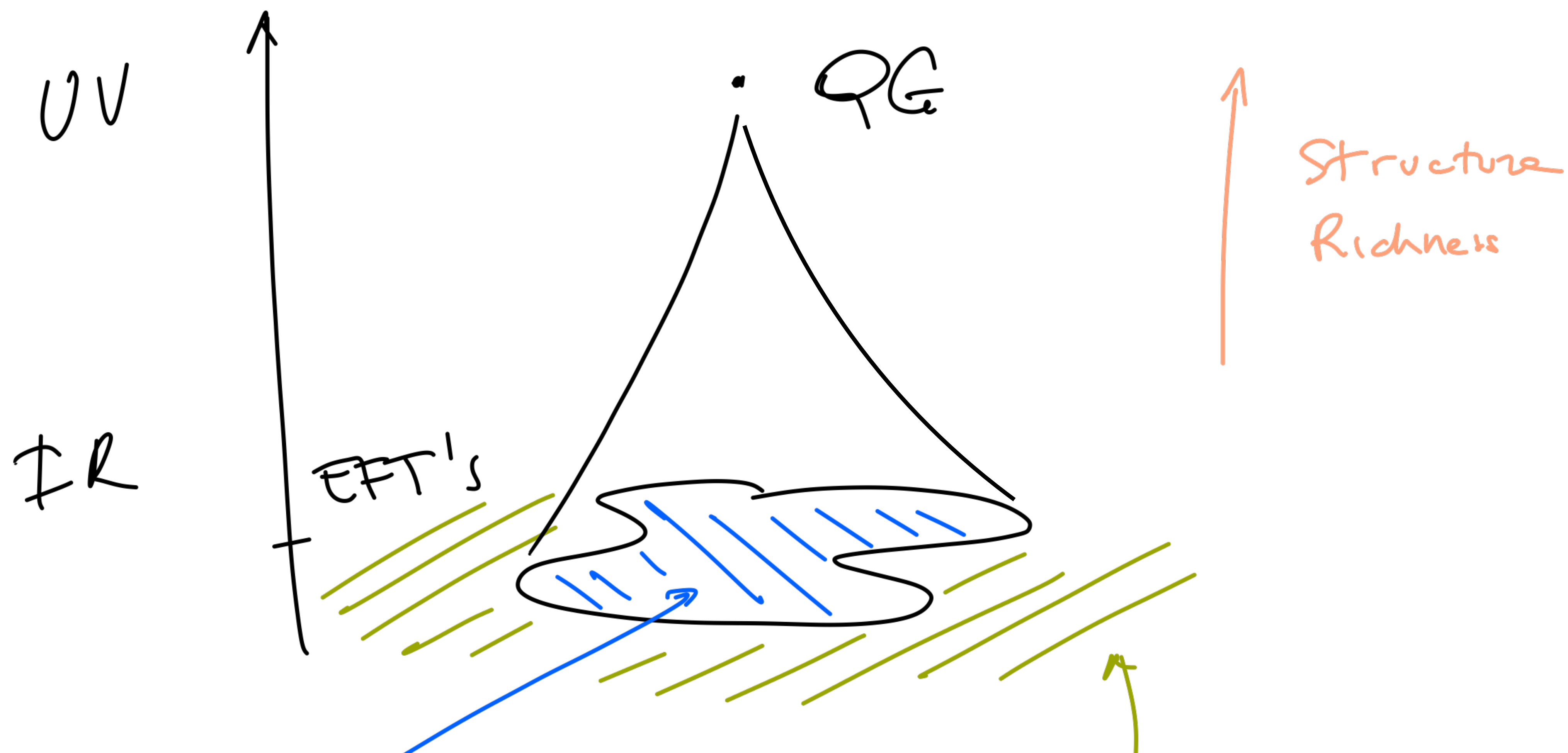
SWAMPLAND

[Vafa '05, Ooguri, Vafa '06]



Landscape:
EFT's with QG completion

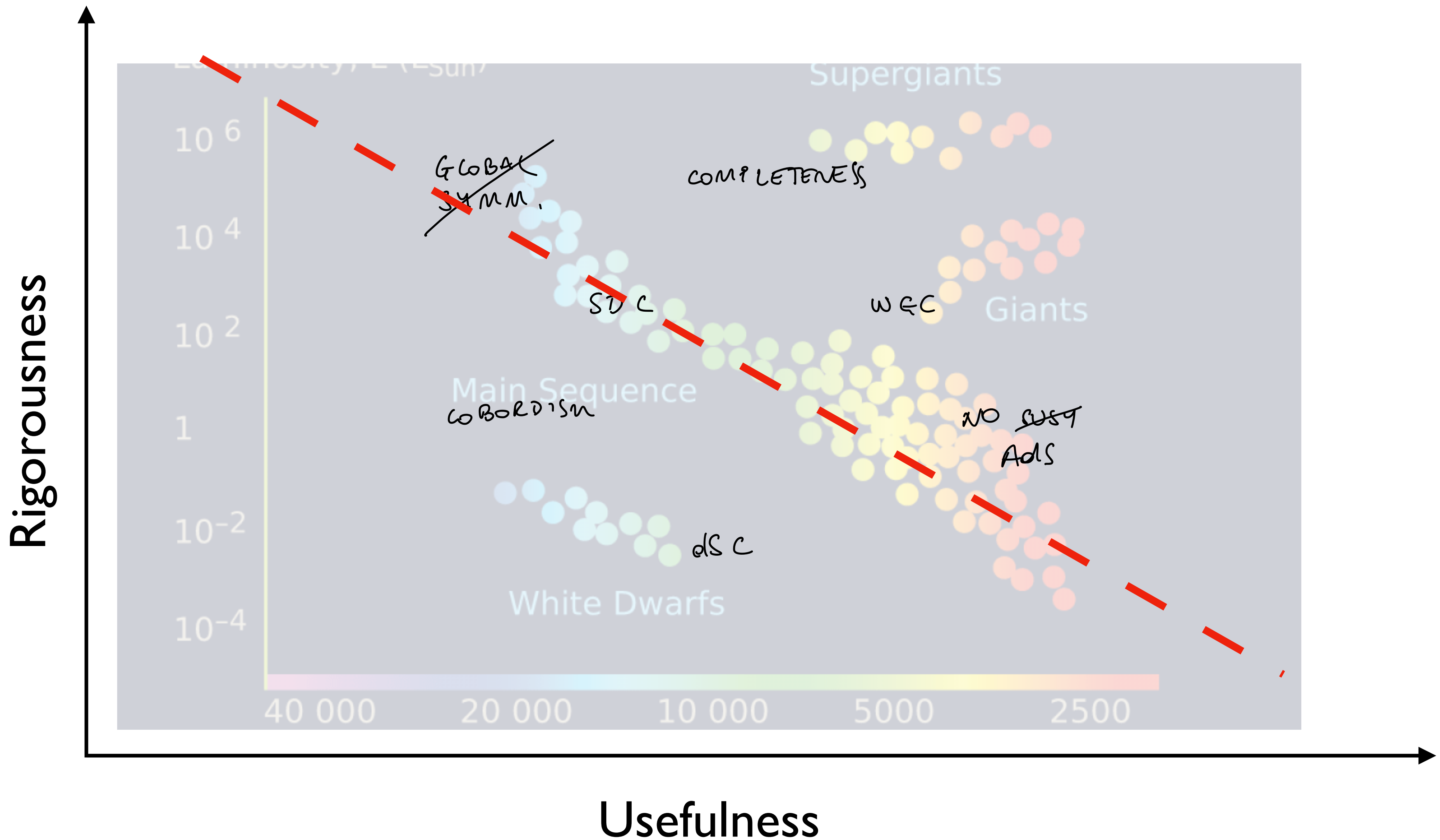
Swampland:
EFT's that do **not** have QG completion

Boundary defined by ***Swampland criteria***

WEB OF CONJECTURES

[Brennan, Carta, Vafa ’17 — Palti ’19
van Beest, Calderón-Infante, Mirfendereski, Valenzuela ’21]

Ooguri-Hertzsprung-Russell diagram



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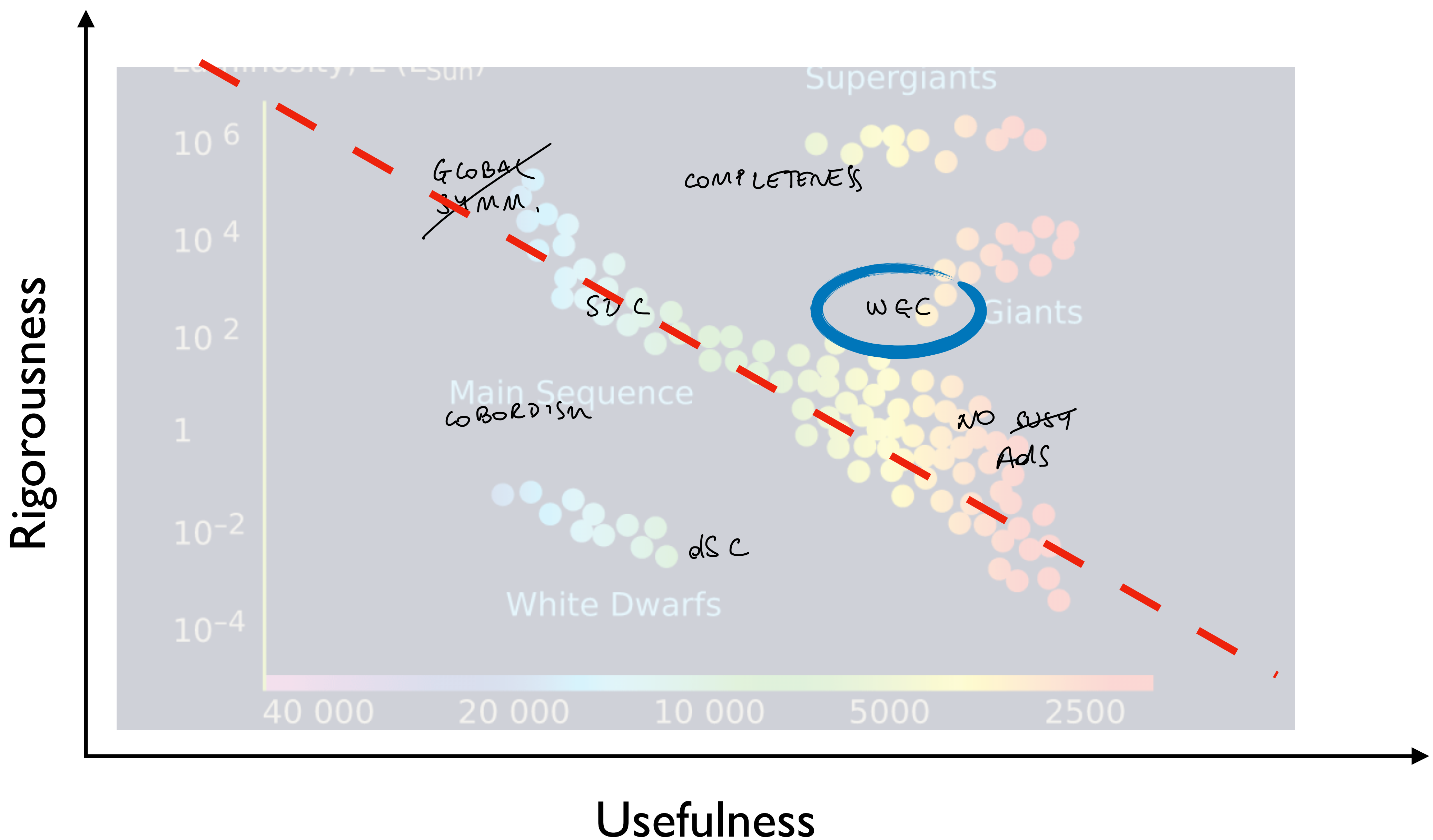

THE WEAK GRAVITY CONJECTURE

- (electric) WGC in **flat** spacetime in d dimensions

In any consistent U(1) gauge theory coupled to gravity, there must exist (at least) a state

$$q^2 g^2 \geq \frac{d-3}{d-2} \frac{m^2}{M_d^{d-2}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

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- Applying this to the magnetic monopole with mass $m_{mag} \sim \frac{\Lambda}{g_{el}^2}$ UV CUT-OFF

$m_{\rm mag} \lesssim g_{\rm mag} M_p \sim \frac{M_p}{g}$ implies a new EFT cut-off $\Lambda \leq g M_p$

- Stronger version: an infinite tower of particles exists at $\Lambda \sim g M_p$ T/sL-WGC

[Heidenreich, Reece, Rudelius '15,…,SA, Junghans, Noumi, Shiu]

e.g. compactifications $\Lambda \sim 1/R \sim g_{KK}$

above which KK modes appear $m \sim n/R$

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Consider it as a statement about particles of charge $q \sim \mathcal{O}(1)$

- In case of (canonically normalized) U(1)^N

$$\sum_i q_i^2 g_i^2 \geq \frac{d-3}{d-2} \frac{m^2}{M_d^{d-2}}$$

- In presence of a massless Yukawa-coupled scalar: Scalar WGC (SWGC)

$$\sum_i q_i^2 g_i^2 \geq \frac{d-3}{d-2} \frac{m^2}{M_d^{d-2}} + \mu^2$$

[Palti '17]

MOTIVATIONS (subtleties modded out!)

Starting from: No exact global symmetries in QG (e.g. global U(1))

1) Perturbative ST: WS continuous global symmetries are gauged in spacetime

[Banks, Dixon '88]

2) AdS/CFT:

- global symmetries in the bdy associated to gauged symmetries in the bulk
- global symmetries in the bulk lead to a contradiction in the CFT

[Ooguri, Harlow '18]

3) Path Integral: global symmetries are violated by Euclidean wormholes

[Giddings, Strominger '88][Coleman Lee '90][…]

4) Black hole heuristics for absence of global symmetries in QG:

- “No-hair” theorem in tension with finiteness of BH entropy:

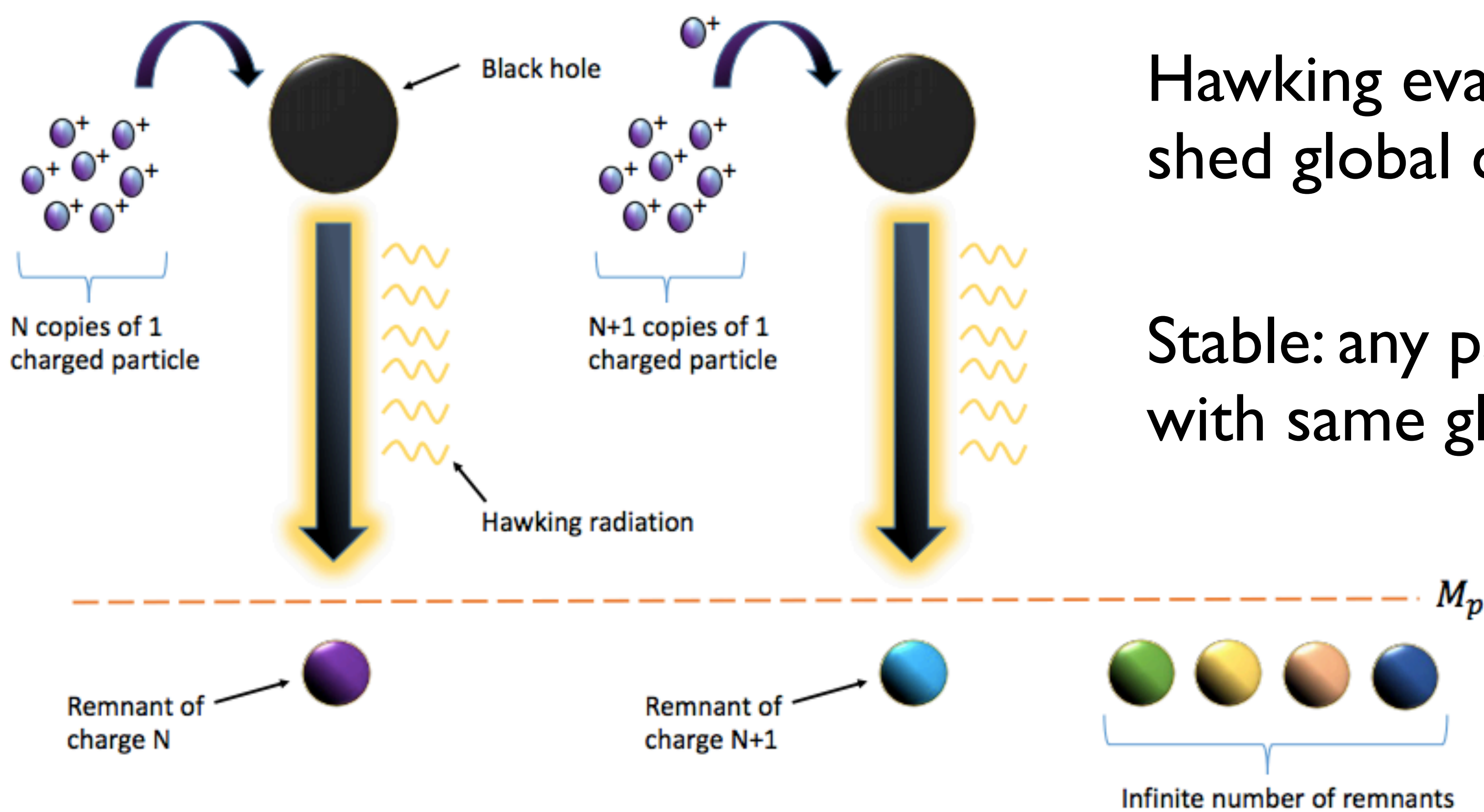
A black hole of fixed mass (other gauge charge, angular momentum) can have any global charge, not reflected in the horizon

Thus, there is an infinite uncertainty (infinite number of microstates) associated to the black hole for an observer, therefore infinite entropy

4) Black hole heuristics for absence of global symmetries in QG:

- single particle charged under global U(1) gives infinite # of stable remnants

$\mathcal{O}(M_p)$



Hawking evaporation does not shed global charge

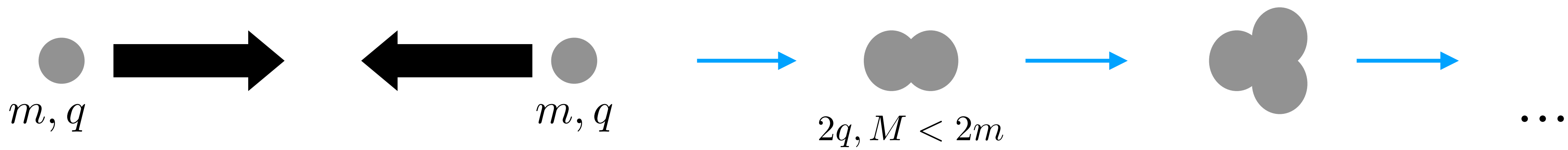
Stable: any particle combination with same global charge is heavier

[taken from Palti '19]

- Issues in deep IR for renormalisation of G [Susskind '95]
- infinite number of states for same black hole geometry, in contradiction with CEB (finite remnant entropy) [Banks, Seiberg '10]

5) Bottom-up rephrasing in terms of infinite number of stable **bound states** using m/q ratio

Take 2 copy of particle with global charge q, m and smallest m/q



Gravity pulls them together into a stable bound state

$$\left(\frac{m}{q}\right)_{\text{bound}} < \left(\frac{m}{q}\right)_{\text{particle}} < \left(\frac{m}{q}\right)_{\text{other particles}}$$

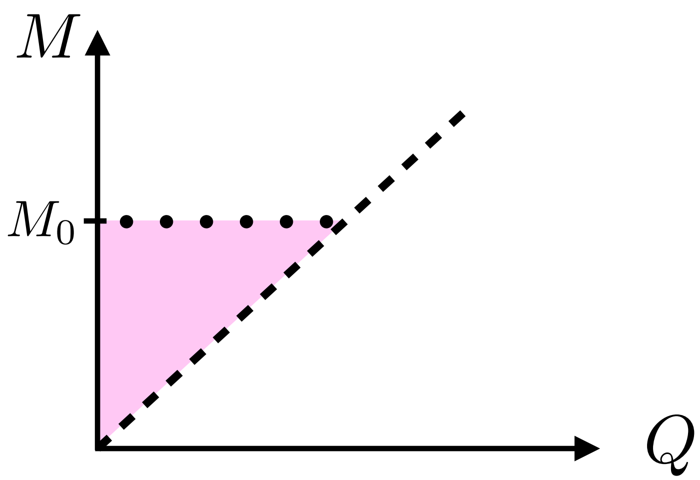
Remember that decay is possible if it exists a by-product with smaller m/q than the decaying object.

Here, for any bound state, all possible by-products have $\left(\frac{m}{q}\right) > \left(\frac{m}{q}\right)_{\text{bound}}$

We end up with an infinite number of stable bound states…

Gauging the U(1) we get rid of this infinite number of remnants/bound states:

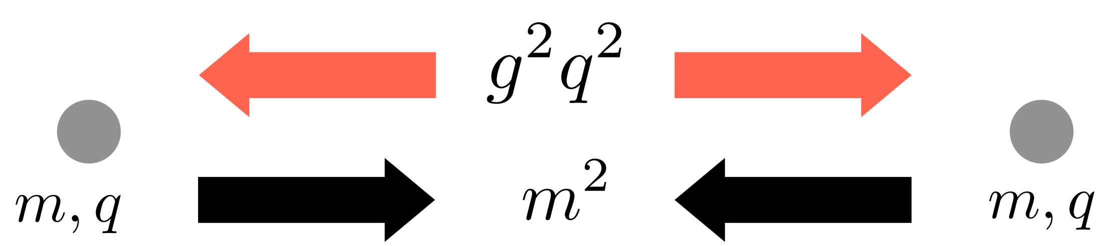
- Charged black hole solutions $M \geq \sqrt{2} g Q M_P$



Finite # black holes below any given mass

$$N_{BH} \sim \frac{M_0}{g M_P}$$

- Additional long range force: gauge VS gravity battle



But for $g \to 0$ it is like we restore the global symmetry with its issues!

We better gauge the U(1) **strongly enough**

s.t. there exists a particle for which gravity is weaker than gauge force

- This is precisely the condition allowing extremal black holes to decay:

$$\left(\frac{m}{gqM_P}\right)_{\text{WGC particle}} < \left(\frac{M}{gQM_P}\right)_{\text{EBH}} = \sqrt{2}$$

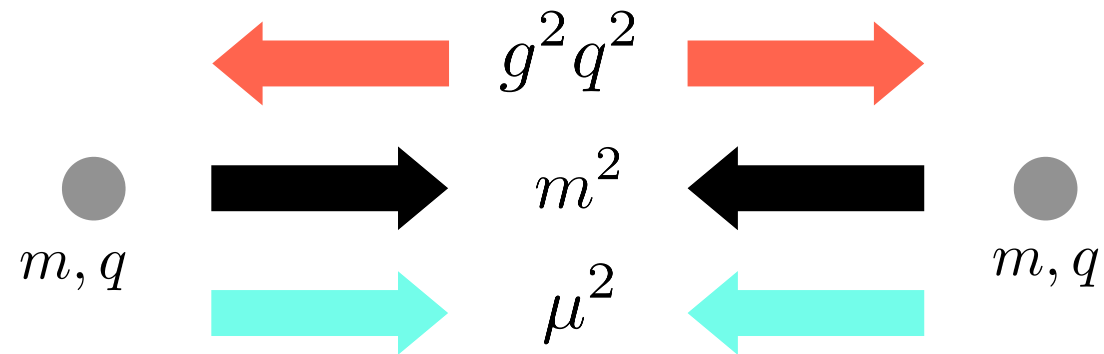
getting rid of infinite # EBHs [Arkani-Hamed, Motl, Nicolis, Vafa '06]

- Also equivalent to Repulsive Force conjecture (RFC) requiring the existence of a self-repulsive particle [Heidenreich, Reece, Rudelius '19]

and so we get rid of infinite # stable bound states

What happens with **massless scalars** - additional long range forces ?

Logic demands the existence of a particle which is self-repulsive under all forces



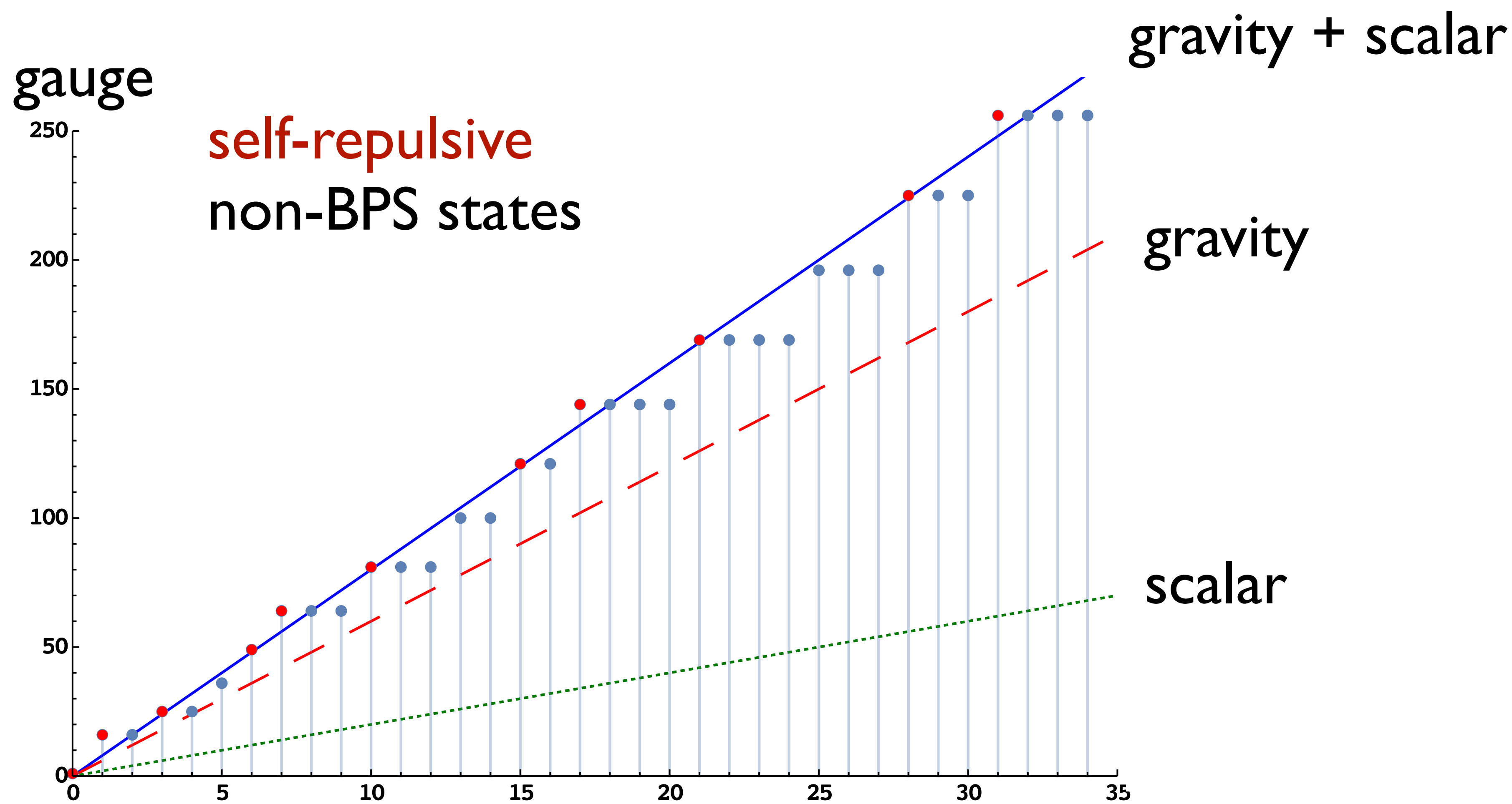
$$\mathcal{L} \supset m^2|\phi|^2 + 2|m|\mu\varphi|\phi|^2$$

WGC bound (4d):

$$q^2g^2 \geq \frac{1}{2}\frac{m^2}{M_P^2} + \mu^2$$

[Palti '19]

Support from string theory (F-theory construction)



[Lee, Lerche, Weigand '18]

* Evidence from non-BPS states in susy setups

Would be nice to check non susy theories/vacua…

- What is the appropriate formulation of WGC in **AdS** ? [Nakayama, Nomura '15] [Aharony, Palti '21]

- It must reduce to the flat spacetime version in the flat limit $L \to \infty$

- It must be saturated by BPS states, as in flat spacetime

- It can teach us about CFT properties $\longrightarrow$ Testable framework (also in non-susy setups!)

$$\Delta(2q) \geq 2\Delta(q)$$

Abelian Convex Charge Conj. $\Delta(n_1 q_0 + n_2 q_0) \geq \Delta(n_1 q_0) + \Delta(n_2 q_0)$

maybe more universal convexity ? [Aharony, Palti '21]

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WGC in AdS

Let us follow the logic of demanding a self-repulsive particle.

In flat space, only long range forces were relevant to this statement.

But AdS is like a box: we cannot separate two particles arbitrarily far away, thus we cannot neglect "short-range" forces.

A more accurate formulation taking this into account is the proposed

Positive Binding Conjecture:
For a (weakly coupled) gravitational theory with a U(1) gauge field, there should exist at least one charged particle with charge of order one, that has a ***non-negative self-binding energy.***

$$\gamma \geq 0$$

[Aharony, Palti '21]

By definition, BPS states saturate the bound

Given an EFT around a AdS vacuum, the self-binding energy can be computed in the Hamiltonian formalism as the difference between the energy of the 2-particle state and twice the single particle state:

$$\gamma \equiv E_{\phi\phi} - 2E_{\phi} = \langle \phi\phi | H | \phi\phi \rangle - 2\langle \phi | H | \phi \rangle$$

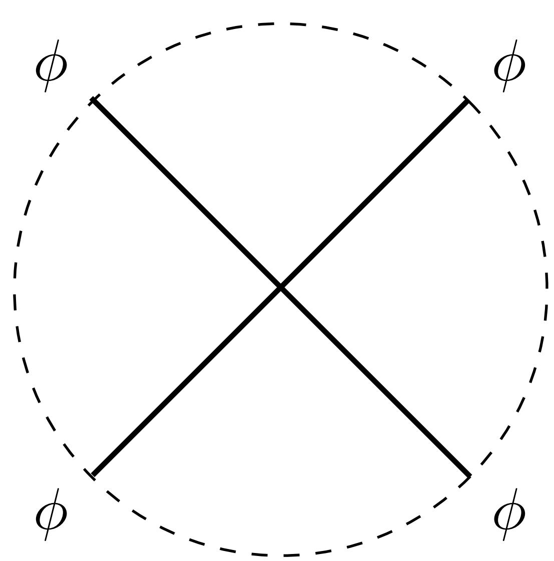
$\gamma > 0$ Self-repulsive

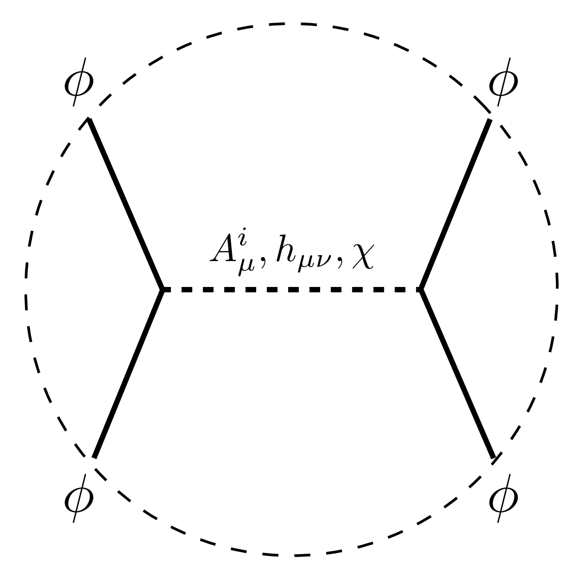
$\gamma < 0$ Self-attractive (bound state)

$\gamma = 0$ No-force condition (BPS) — or free theory

We want to compute γ at tree level, determined by quartic interactions

contact terms





field exchanges

Setup for such an EFT:

$$S[\phi, A^i, h, \chi] = \int d^dx\sqrt{-g}\Big[\frac{1}{\kappa^2}\left(\frac{R}{2} - \Lambda\right) - \sum_{i=1}^{N}\frac{F_i^2}{4} - |D\phi|^2 - m^2|\phi|^2 - V(\phi)$$
$$-\frac{1}{2}(\partial\chi)^2 - \frac{M^2}{2}\chi^2 - Y\chi|\phi|^2 - \alpha\chi|D\phi|^2\Big]$$

$Y = 2|m|\mu$

with a contact term potential

$$V(\phi) = a|\phi|^4 + b|\phi|^2|D\phi|^2 + c\left(\phi^2(D\phi^\dagger)^2 + (\phi^\dagger)^2(D\phi)^2\right)$$

In general, very hard to compute

$$\gamma \equiv E_{\phi\phi} - 2E_\phi = \langle\phi\phi|H|\phi\phi\rangle - 2\langle\phi|H|\phi\rangle$$

Situation simplifies under the assumption of **small interactions** treated as perturbations of the free theory for ϕ

BINDING ENERGY via PERTURBATION THEORY

Operator ϕ and states $|\phi\rangle$, $|\phi\phi\rangle$ are those of free theory, well known

$$S_{\rm free} = \int d^d x \sqrt{-g}\left[-|\partial\phi|^2 - m^2|\phi|^2\right]$$

$$\phi = \sum_{nlJ}\left(a_{nlJ}\psi_{nlJ}(x) + b^\dagger_{nlJ}\psi^*_{nlJ}(x)\right) \qquad\qquad |\phi\phi\rangle \sim b^\dagger_{000} b^\dagger_{000}|0\rangle$$

Hamiltonian of system

$$H = H_{\rm free} + \delta H$$

focus on quartic interactions

Standard Hamiltonian perturbation theory: at leading order in couplings

$$\gamma = \langle\phi\phi|\delta H|\phi\phi\rangle = \langle\phi\phi|\delta H_{\rm contact}|\phi\phi\rangle + \sum_{\alpha\neq\phi\phi}\frac{|\langle\phi\phi|\delta H_{\rm exchange}|\alpha\rangle|^2}{(E_{\phi\phi} - E_\alpha)_{\rm free}}$$

[Fitzpatrick, Katz, Poland, Simmons-Duffin '10]

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[Fitzpatrick, Katz, Poland, Simmons-Duffin '10]

SHORTCUT

[Fitzpatrick, Shih '11]

Classically **integrate out all exchanged fields** to obtain a TL effective action in terms of ϕ

$$S_{\rm eff} = S_{\rm free} - \int d^d x \sqrt{-g}\; V_{\rm eff}[\phi, \phi^\dagger]$$

$$V_{\rm eff}[\phi,\phi^\dagger] = V[\phi,\phi^\dagger] + V_{\rm eff,phot}[\phi,\phi^\dagger] + V_{\rm eff,grav}[\phi,\phi^\dagger] + V_{\rm eff,scal}[\phi,\phi^\dagger]$$

$$\Rightarrow \quad H = H_{\rm free} + \delta H_{\rm eff}\;, \quad \delta H_{\rm eff} = \int d^{d-1}x \sqrt{-g}\; V_{\rm eff} = \delta H_{\rm contact} + \delta \tilde{H}_{\rm contact}$$

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$$\Rightarrow \quad H = H_{\text{free}} + \delta H_{\text{eff}}\,, \quad \delta H_{\text{eff}} = \int d^{d-1} x \sqrt{-g}\; V_{\text{eff}} = \delta H_{\text{contact}} + \boxed{\delta \tilde{H}_{\text{contact}}}$$

Binding energy all within first order PT:

$$\gamma = \int d^{d-1} x \sqrt{-g} \langle \phi\phi | V_{\text{eff}} | \phi\phi \rangle$$

More explicitly, the eoms of graviton, photon, and neutral scalar are

$$\Delta^{\rho\sigma}_{\mu\nu} h_{\rho\sigma}[\phi,\phi^\dagger] = \frac{\kappa}{2} T_{\mu\nu}[\phi,\phi^\dagger] \qquad T_{\mu\nu} = g_{\mu\nu}\left(-|\partial\phi|^2) - m^2|\phi|^2\right) + (\partial_\mu\phi^\dagger\partial_\nu\phi + h.c.)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}F_i^{\mu\nu}[\phi,\phi^\dagger]) = g_i q_i J^\nu[\phi,\phi^\dagger] \qquad J_\mu = i\phi^\dagger\partial_\mu\phi + h.c.$$

$$\Box\chi[\phi,\phi^\dagger] - M^2\chi = Y|\phi|^2 + \alpha|\partial\phi|^2$$

Classically integrating out graviton, photon, and neutral scalar:

$$V_{\rm eff}^{\rm grav} = -\frac{\kappa^2}{4} h^{\mu\nu} T_{\mu\nu}$$

$$V_{\rm eff}^{\rm phot} = \frac{1}{2} A_\mu J^\mu \sum_i g_i^2 q_i^2$$

$$V_{\rm eff}^{\rm scal} = \frac{1}{2}\chi\left(Y|\phi|^2 + \alpha|\partial\phi|^2\right)$$

$h_{\mu\nu}, A_\mu, \chi$

solve eoms

BINDING ENERGY in d=5 $\Delta > 1$

- Contact terms in the scalar potential V

$$\gamma^V = \frac{\pi^2 N_\Delta^4 \left(aL^2 - b(\Delta-2)\Delta + 2c\Delta^2\right)}{(\Delta-1)(2\Delta-1)L^4} \qquad N_\Delta = \sqrt{\frac{\Delta-1}{2\pi^2}}$$

- Photon exchange

$$\gamma^{\text{phot}} = \frac{\pi^2 N_\Delta^4}{L^2(2\Delta-1)} \sum_i g_i^2 q_i^2$$

- Graviton exchange

$$m^2 L^2 = \Delta(\Delta-4)$$

$$\gamma^{\text{grav}} = -\frac{2\pi^2(\Delta-2)\Delta^2\kappa^2 N_\Delta^4}{3(\Delta-1)(2\Delta-1)L^4}$$

BINDING ENERGY in d=5

- Scalar exchange

  - $M^2 = 0 \quad \Delta \geq 2$

$$\gamma^{\rm scal} = \frac{\pi^2 N_\Delta^4}{8(\Delta-2)^2(\Delta-1)^2}\Bigg[Y^2\left(1-\Delta+\frac{1}{\Delta}+\frac{4}{\Delta-1}+\frac{2}{2\Delta-1}+4H_{\Delta-2}-2H_{2\Delta}\right)$$
$$+\frac{2Y\alpha}{L^2(2\Delta-1)}\left(4+7\Delta(\Delta-1)-9\Delta^3+2\Delta^4+2\Delta(2\Delta^2-9\Delta+4)\left(H_{2\Delta}-2H_\Delta\right)\right)$$
$$+\frac{\alpha^2\Delta^2}{L^4}\left(-6+\frac{2}{2\Delta-1}-\Delta(\Delta(\Delta-7)+11)-2\Delta(\Delta-4)^2\left(H_{2\Delta}-2H_\Delta\right)\right)\Bigg]$$

  - $M^2L^2 = -2 \qquad \alpha = 0$

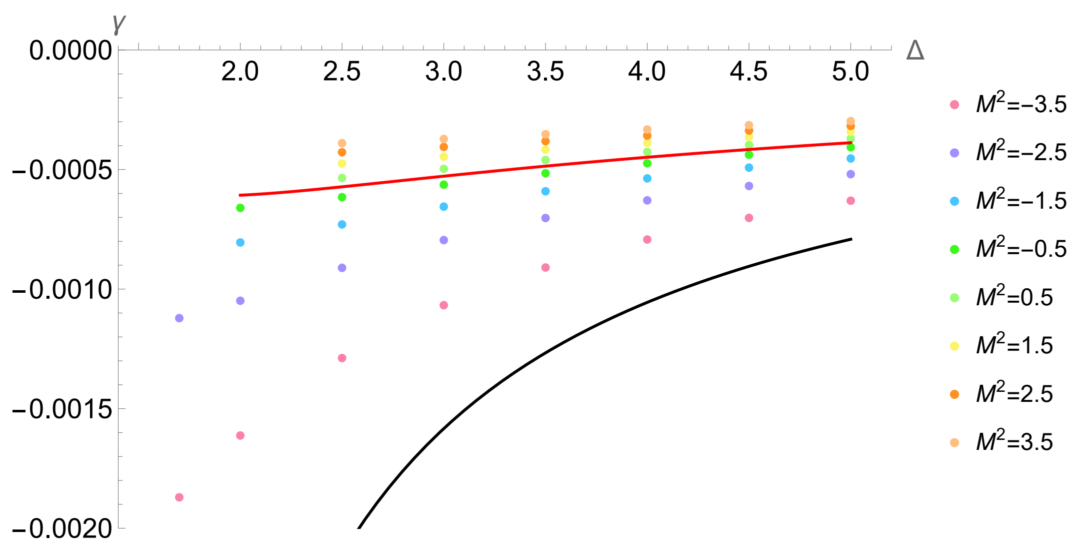
    (BF saturation)

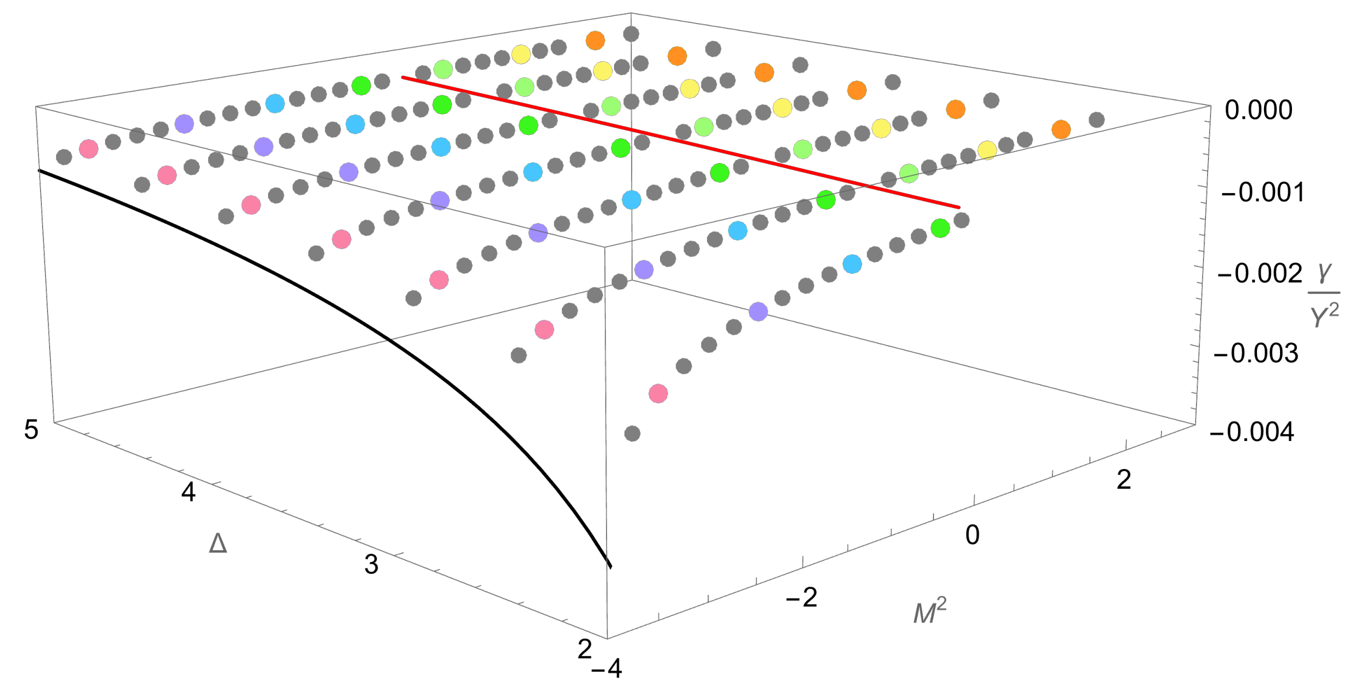
$$\gamma^{\rm scal} = -\frac{Y^2\pi^2 N_\Delta^4}{8(\Delta-1)^3}$$

BINDING ENERGY in d=5

- Scalar exchange

  - numerics for $\alpha = 0$





* Caveat: Perturbative analysis covers parameter range $\Delta \geq 1 + \sqrt{1 + \frac{M^2}{4}}$

For ML>>1: $M \lesssim 2m$

Exchanges of heavy particles can be thought of as being included in HO effective operators in V

TESTING #1 — FLAT LIMIT

$$m^2 L^2 = \Delta(\Delta - 4)$$

Obtained taking $\quad L \to \infty \quad \Delta \to \infty \quad m = \text{fixed}$

The positive binding conjecture is…

$$\gamma = \gamma_{\text{phot}} + \gamma_{\text{grav}} + \gamma^{M=0}_{\text{scal}} + \gamma^{V} \geq 0$$

leading term

$$\sum_i q_i^2 g_i^2 \geq \frac{2}{3} m^2 \kappa^2 + \left(\mu - \frac{\alpha m}{2}\right)^2 \boxed{- \frac{1}{L}\left(\frac{a}{m} - m(b - 2c)\right)}$$

$$\alpha = \frac{\tilde{\alpha}}{M_d}$$

…precisely the SWGC if $\alpha = 0$

…matches the SWGC due to relative suppression $\quad \left(\mu - \frac{m}{M_d}\frac{\tilde{\alpha}}{2}\right)$

[same in d=4]

TESTING #2 — BPS STATES

N=2 d=5: [Ceresole, Dall'Agata, Kallosh, Van Proeyen '01]

one hypermultiplet $q^X = (V, \sigma, \theta, \tau)$ $ds^2 = \frac{dV^2}{2V^2} + \frac{1}{2V^2}(d\sigma + 2\theta d\tau - 2\tau d\theta)^2 + \frac{2}{V}(d\tau^2 + d\theta^2)$

one vector multiplet ρ, A_1

gravity multiplet g, A_0

$$\frac{SU(2,1)}{SU(2)\times U(1)} \times O(1,1)$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{1}{4\rho^8}F_0^2 - \frac{\rho^4}{4}F_1^2 - \frac{1}{2}g_{XY}D_\mu q^X D^\mu q^Y - \frac{6}{\rho^2}\partial_\mu\rho\partial^\mu\rho - V(q,\rho) + CS$$

$$V = g^2\left(-6W^2 + \frac{9}{2}g^{\rho\rho}\partial_\rho W\partial_\rho W + \frac{9}{2}g^{XY}\partial_X W\partial_Y W\right)$$

we gauge $U(1)\times U(1) \subset SU(2)\times U(1)$ via $K_0(\beta,\gamma;q), K_1(\beta,\gamma;q),$

$$D_\mu q^X = \partial_\mu q^X + gA_\mu^0 K_0^X(q) + gA_\mu^1 K_1^X(q)$$

Useful field redefinitions: $V = \frac{1 - |\phi_1|^2 - |\phi_2|^2}{(1+\phi_1)(1+\phi_1^*)}\,, \sigma = \ldots\,, \theta = \ldots\,, \tau = \ldots\,, \rho = e^{\chi}$

AdS SUSY vacuum: $\phi_1 = \phi_2 = \chi = 0$ $\qquad \partial_\rho W = \partial_X W = 0$

$W \neq 0$

yields EFT of type studied:

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{F_0^2}{4} - \frac{F_1^2}{4} - |D\phi_1|^2 - \frac{(\partial\chi)^2}{2} + \frac{6}{\kappa^2 L^2} - m_1^2|\phi_1|^2 - \frac{M^2}{2}\chi^2 - Y_1\chi|\phi_1|^2 - V + \ldots$$

$$V = a_1|\phi_1|^4 + b_1|\phi_1|^2|\partial\phi_1|^2$$

$$D_\mu\phi_i = \partial_\mu\phi_i - i\big(g_0 q_{i0} A_{0\mu} + g_1 q_{i1} A_{1\mu}\big)\phi_i$$

with

$$g_0 q_{10} = \frac{\kappa}{\sqrt{2}L}(2\gamma+1)\,, \quad g_1 q_{11} = \frac{\kappa}{L}(\beta+1) \qquad m_1^2 = \frac{1}{4L^2}\left(-5 + 2(\beta+\gamma)\right)\left(3 + 2(\beta+\gamma)\right)$$

$M^2 = -\frac{4}{L^2}$ **(BF saturation)** $\qquad Y_1 = -\frac{\kappa}{\sqrt{3}L^2}(\beta - 2\gamma)(1 + 2\beta + 2\gamma) \qquad \alpha_1 = 0$

$$a_1 = \frac{\kappa^2}{2L^2}(-6 + 3\beta^2 + 4\beta\gamma + 4\gamma^2)\,, \qquad b_1 = 2\kappa^2 \qquad c_1 = 0$$

The lightest field ϕ_1 has $\Delta = \frac{3}{2} + \beta + \gamma$ $m^2 L^2 = \Delta(\Delta - 4)$

(chiral primary in SCFT,
other is dual with $\Delta_d = \Delta + 1$)

Therefore, in terms of β, Δ its binding energy is

$$
\begin{aligned}
\gamma^V &= \frac{\kappa^2}{L^4} \frac{\pi^2(\beta+1)(3\beta-4\Delta+3)N_\Delta^4}{2\left(2\Delta^2-3\Delta+1\right)}, \\
\gamma^{A_0} &= \frac{\kappa^2}{L^4} \frac{2\pi^2(\beta-\Delta+1)^2 N_\Delta^4}{(2\Delta-1)}, \\
\gamma^{A_1} &= \frac{\kappa^2}{L^4} \frac{\pi^2(\beta+1)^2 N_\Delta^4}{(2\Delta-1)}, \\
\gamma^{\mathrm{grav}} &= -\frac{\kappa^2}{L^4} \frac{2\pi^2(\Delta-2)\Delta^2\kappa^2 N_\Delta^4}{3(\Delta-1)(2\Delta-1)}, \\
\gamma^{\chi} &= -\frac{\kappa^2}{L^4} \frac{\pi^2(3\beta-2\Delta+3)^2 N_\Delta^4}{6(\Delta-1)}
\end{aligned}
$$

The lightest field ϕ_1 has $\Delta = \dfrac{3}{2} + \beta + \gamma$ $\qquad m^2L^2 = \Delta(\Delta - 4)$

(chiral primary in SCFT,
other is dual with $\Delta_d = \Delta + 1$)

Therefore, in terms of β, Δ its binding energy is

$$\begin{aligned}
\gamma^V &= \frac{\kappa^2}{L^4}\,\frac{\pi^2(\beta+1)(3\beta-4\Delta+3)N_\Delta^4}{2\left(2\Delta^2-3\Delta+1\right)}\,,\\
\gamma^{A_0} &= \frac{\kappa^2}{L^4}\,\frac{2\pi^2(\beta-\Delta+1)^2N_\Delta^4}{(2\Delta-1)}\,,\\
\gamma^{A_1} &= \frac{\kappa^2}{L^4}\,\frac{\pi^2(\beta+1)^2N_\Delta^4}{(2\Delta-1)}\,,\\
\gamma^{\mathrm{grav}} &= -\frac{\kappa^2}{L^4}\,\frac{2\pi^2(\Delta-2)\Delta^2\kappa^2N_\Delta^4}{3(\Delta-1)(2\Delta-1)}\,,\\
\gamma^\chi &= -\frac{\kappa^2}{L^4}\,\frac{\pi^2(3\beta-2\Delta+3)^2N_\Delta^4}{6(\Delta-1)}
\end{aligned}$$

$$\gamma^V + \gamma^{A_0} + \gamma^{A_1} + \gamma^{\mathrm{grav}} + \gamma^\chi = 0$$

$\forall\beta, \Delta$
(trust me)

N=2 d=4: [Hristov, Looyestijn, Vandoren '09]

one hypermultiplet $q^X = (\rho, \sigma, \xi_1, \xi_2)$

gravity multiplet g, A

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - g_{XY} D_\mu q^X D^\mu q^Y - \frac{1}{8} F^2 - V(q)$$

we gauge $U(1)$ via $K = \beta(0, 0, -\xi_2, \xi_1)$

$$D_\mu q^X = \partial_\mu q^X - g A_\mu K^X(q)$$

scalar potential $V = g^2 \left(4 K^X K^Y g_{XY} - 3 \vec{P} \cdot \vec{P}\right)$

$$\vec{P} = \beta \left(\frac{2\xi_1}{\rho^{1/2}}, -\frac{2\xi_2}{\rho^{1/2}}, 1 - \frac{\xi_1^2 + \xi_2^2}{\rho} \right)$$

AdS SUSY vacuum: $\xi_1 = \xi_2 = 0$

$$K^X = 0$$
$$\epsilon^{ijk} P^j P^k = 0$$
$$P \cdot \vec{P} > 0$$

yields EFT of type studied: $\rho = e^{\chi}$ $\quad \phi = \xi_2 + i\xi_2$

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{F^2}{4} - |D\phi|^2 - \frac{(\partial\chi)^2}{2} - \frac{(\partial\sigma)^2}{2} + \frac{3}{\kappa^2 L^2} + \frac{2}{L^2}|\phi|^2 + \sqrt{2}\kappa\chi|\partial\phi|^2 - \frac{2\sqrt{2}\kappa}{L^2}\chi|\phi|^2 - V + \dots$$

$$V = -\frac{\kappa^2}{4}(\phi^\dagger\partial\phi - \phi\partial\phi^\dagger)^2 + \frac{\kappa^2}{L^2}|\phi|^4 \qquad D_\mu\phi = \partial_\mu\phi - i\sqrt{2}\frac{\kappa}{L}A_\mu\phi$$

and so non-vanishing parameters are:

$$m^2L^2 = -2\,, \quad g^2q^2 = 2\frac{\kappa^2}{L^2}\,, \quad a = \frac{\kappa^2}{L^2}\,, \quad b = -2c = \frac{\kappa^2}{2}\,, \quad Y = \frac{2\sqrt{2}\kappa}{L^2}\,, \quad \alpha = -\sqrt{2}\kappa$$

Binding energies $\quad m^2L^2 = \Delta(\Delta-3) \qquad \Delta = 2 \qquad$ ($\Delta = 1$ breaks PT)

$$\gamma^V = \gamma^\chi = -\frac{3}{8}\frac{\kappa^2}{L^3}\pi^2 N_2^4\,, \qquad \gamma^{\rm phot} = \frac{5}{4}\frac{\kappa^2}{L^3}\pi^2 N_2^4\,, \qquad \gamma^{\rm grav} = -\frac{1}{2}\frac{\kappa^2}{L^3}\pi^2 N_2^4$$

Total binding energy vanishes !

Summary & Future directions

Positive Binding Energy proposal as formulation of WGC in AdS

Survives tests

- flat spacetime limit (d=4,5)
- BPS states in N=2 (d=4,5) have $\gamma = 0$

Do it in AdS3 *[WIP]*

Extend it to fermions and more charged particles (vs TWGC/sLWGC)

Check with super-radiance instability

Test for non-BPS states in susy and non-susy theories $\gamma > 0$

(eg: string/F-theory , compactification of SM to AdS3 , …)

CFT side of the story (and back): Convexity properties universal ? [Aharony, Palti '21]

Thank you

A tale about Wormholes

Euclidean Wormholes and axionic WGC

[2004.13721 SA, Huang, Noumi, Ooguri, Shiu]

[2205.01119 SA, Shiu, Soler, Van Riet]

AWGC is the generalisation to **p=0** form potential (***axion***):

(d=4)	WGC	AWGC
form field potential	photon A_μ	axion/2-form dual $\theta / B_{\mu\nu}$
charged states	particles & black holes	instantons & grav. instantons
coupling	gauge coupling g	$\frac{1}{f}$ f=axion decay constant
relevant quantities	mass, charge (m, q)	action, charge (S, q)
WGC bound Exists a state s.t.	$\frac{m}{qgM_P} < 1$	$\frac{Sf}{qM_P} < \mathcal{O}(1)$
Extremal obj's	EBH's	**regular** solutions **[Eucl. wormholes]**
Interpretation	Instability of EBH's	tunneling via collection small instantons favoured over single instanton same tot q $S(q_1+q_2) > S(q_1) + S(q_2)$

If really present in the QG path integral, EWHs give rise to series of puzzles: Coleman's baby universes, factorisation problem in AdS/CFT,…

We would like to interpret the AWGC as instability of (axionic) Euclidean WHs in flat spacetime [Giddings, Strominger '88]

Pertubatively unstable? [Hertog, Truijen Van Riet '18]
No, no negative modes that lower their action [Loges, Shiu, Sudhir '22]

Non-perturbative unstable?
NP effects that lower their action: *(EFT regime + UV properties)*

- higher derivative corrections (via positivity and duality) [SA, Huang, Noumi, Ooguri, Shiu '20]
- adding massive dilaton [SA, Shiu, Soler, Van Riet '22]

Perturbative/NP stability of asymptotically AdS addressed in [Marolf, Santos '21]
Finding NP instability (brane nucleation) for EWHs in <u>UV complete setups</u>

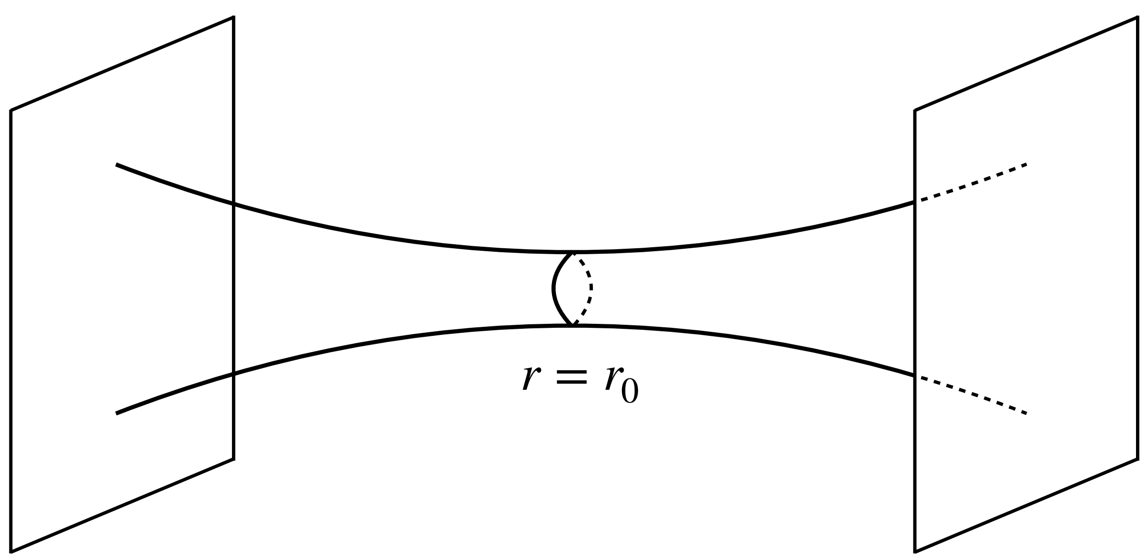
HIGHER DERIVATIVE CORRECTIONS

Classical Axio-dilaton-gravity (ADG) $\xrightarrow{\beta = 0}$ Axion-gravity (AG)

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{f^2}{2} e^{\beta \phi} (\partial_\mu \theta)^2 \right]$$

[Giddings, Strominger '88]

[reviews: Hebecker, Mangat, Theisen, Witkowski '16, Hebecker-Mikhail-Soler '18, Van Riet '20]



$$ds^2 = \frac{dr^2}{1 - \frac{r_0^4}{r^4}} + r^2 d\Omega_3^2$$

$$r_0^4 = \frac{q^2}{6f^2} \cos^2 \left(\frac{\pi}{4} \frac{\sqrt{3}\beta}{\sqrt{2}} \right)$$

HIGHER DERIVATIVE CORRECTIONS

Classical Axio-dilaton-gravity (ADG) $\xrightarrow{\beta=0}$ Axion-gravity (AG)

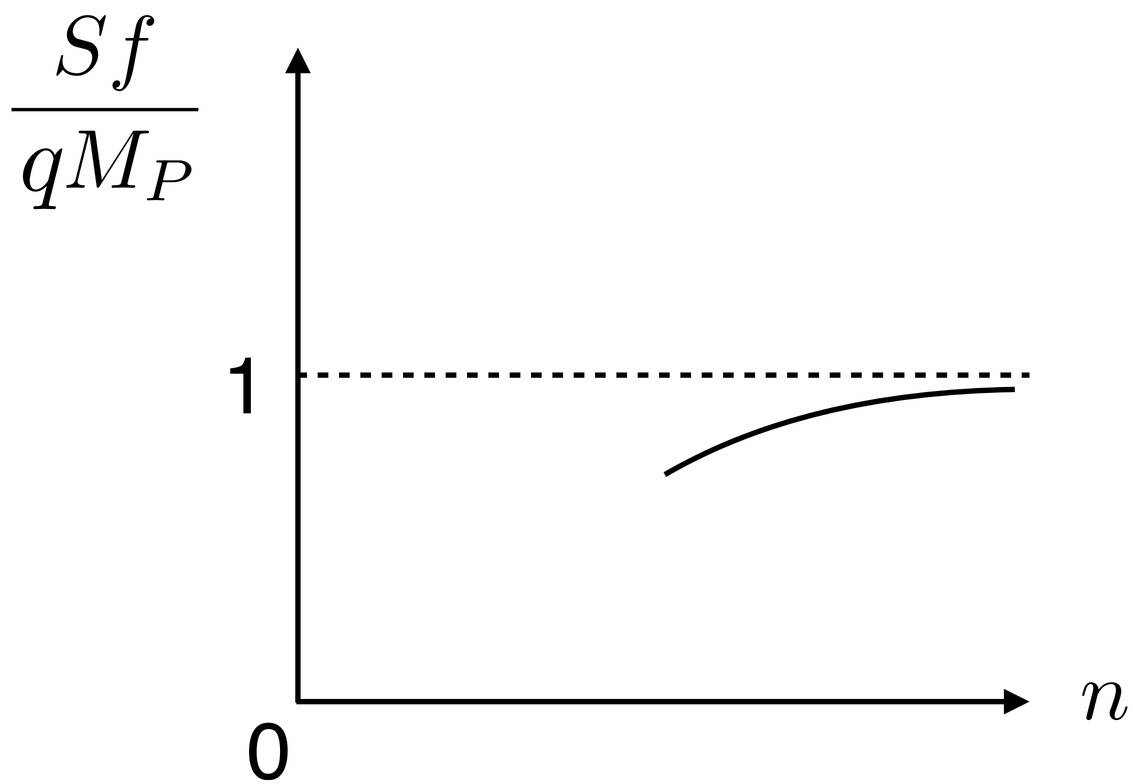
$$S=\int d^4x\sqrt{-g}\Big[\frac{R}{2}-\frac{1}{2}(\partial_\mu\phi)^2-\frac{f^2}{2}e^{\beta\phi}(\partial_\mu\theta)^2\Big]$$

+ **HO** *(4-derivative)* **corrections**, generic

$$\Delta S=\int d^4x\sqrt{-g}\Big[a_1(\phi)(\partial_\mu\phi\partial^\mu\phi)^2+a_2(\phi)f^4(\partial_\mu\theta\partial^\mu\theta)^2$$
$$+a_3(\phi)f^2(\partial_\mu\phi\partial^\mu\phi)(\partial_\mu\theta\partial^\mu\theta)+a_4(\phi)f^2(\partial_\mu\phi\partial^\mu\theta)^2+a_5(\phi)W^2+a_6\theta W\tilde{W}\Big]$$

Wick-rotate and evaluate the action on EWH solutions

ADG $\quad\dfrac{Sf}{qM_P}=\dfrac{8\pi^2}{\beta}\sin\left(\dfrac{\pi}{4}\dfrac{\sqrt{3}\beta}{\sqrt{2}}\right)+\Delta S$ $\qquad$ AG $\quad\dfrac{Sf}{qM_P}=\sqrt{6}\pi^3+\Delta S$



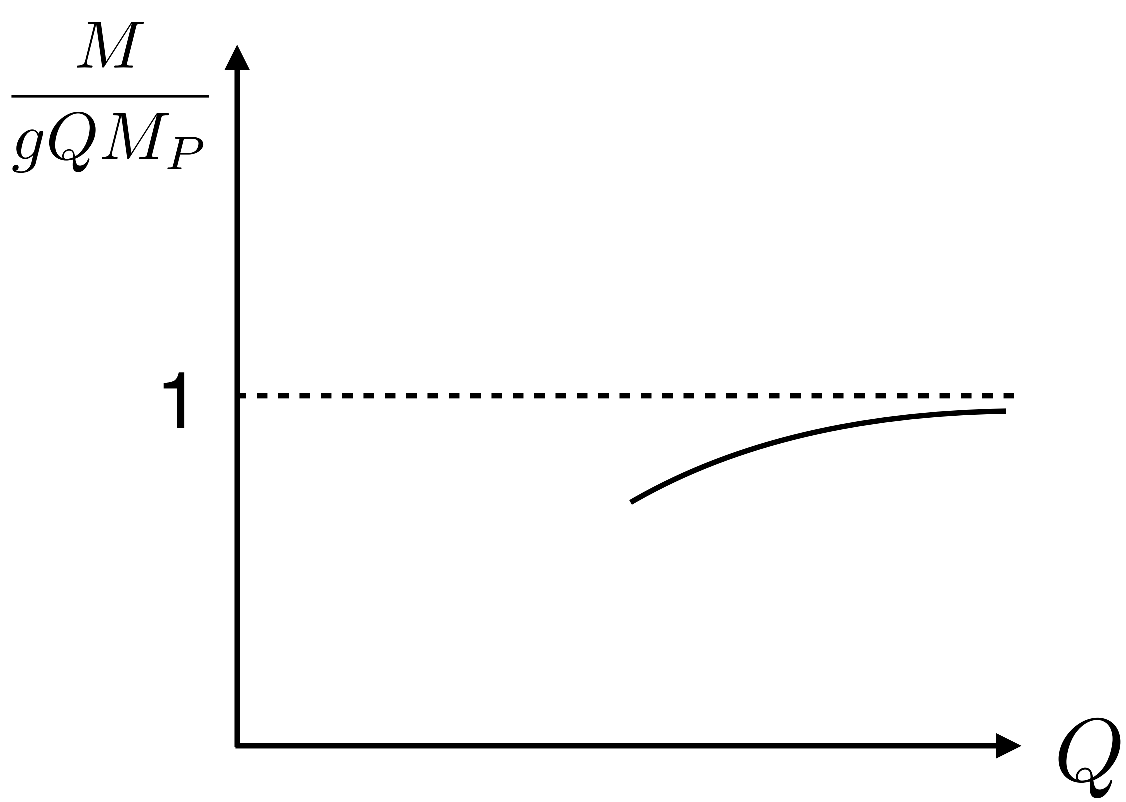
$$\Delta S < 0$$

- AG: from positivity conditions
- ADG: from positivity
  + SL(2,R) on HO terms

$$\left.\frac{Sf}{qM_P}\right|_{HO} = 1 + \frac{\Delta Sf}{qM_P} < 1$$

Reminiscent of how HO (string) corrections modify the classical BH extremality bound in a way that the EBH's (Q,M) can satisfy the WGC

[Kats, Motl, Padi '06]



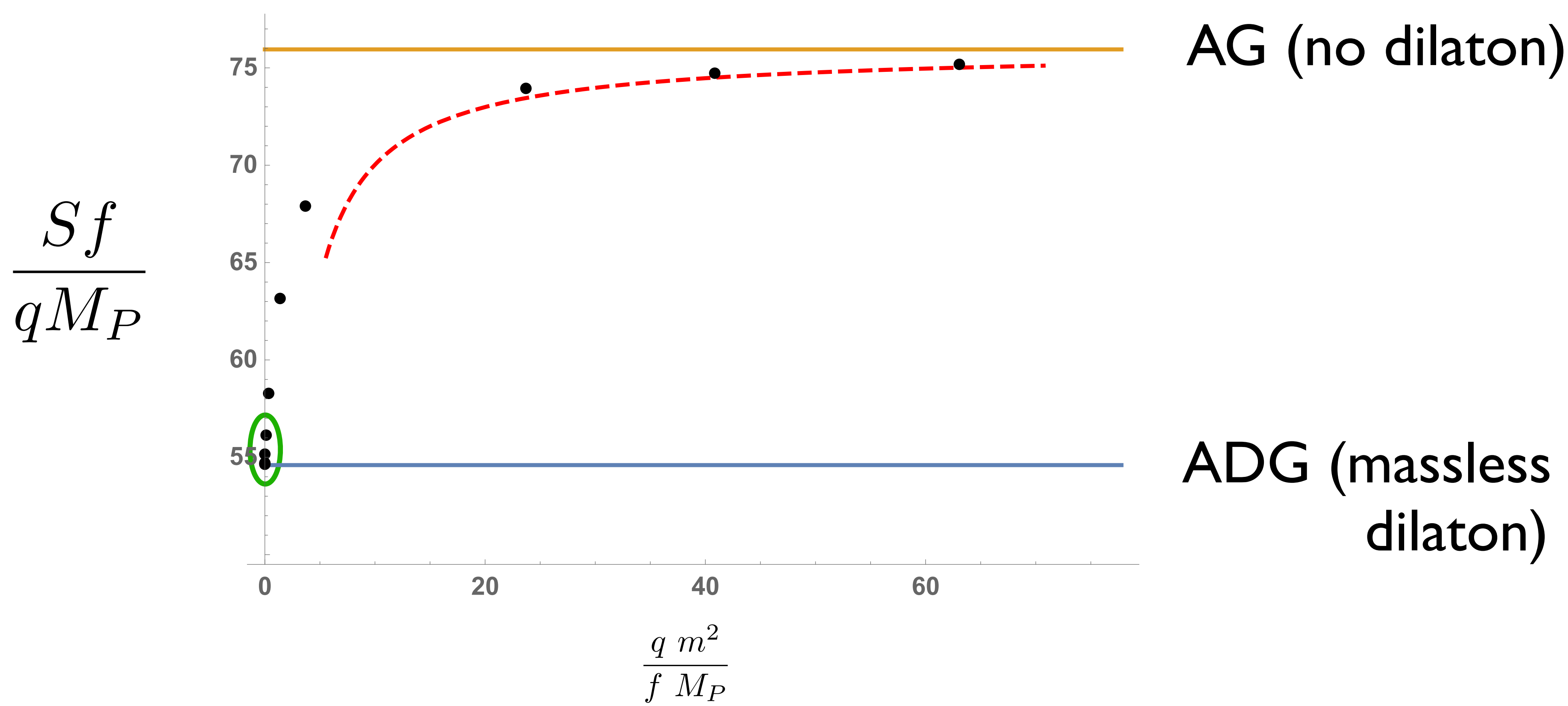
$$\Delta M < 0$$

$$\left.\frac{M}{gQM_P}\right|_{HO} = 1 + \frac{\Delta M}{gQM_P} < 1$$

MASSIVE DILATON

$$S_E = \int d^4x\sqrt{g}\Big[-\frac{M_P^2}{2}R + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{12f^2}e^{-\frac{\beta}{M_P}\phi}F^2 + m^2\phi^2\Big]$$

- $\frac{1}{r_0} \ll m$ analytically $\quad \frac{Sf}{qM_P} = \sqrt{6}\pi^3\left(1 - \frac{\sqrt{6}}{2\pi}\frac{\beta^2 f M_P}{q\,m^2}\right)$
- $\frac{1}{r_0} \gtrsim m$ numerically, for any m value we have the same behaviour



Confirming that (for any dilaton mass), $\quad S(q_1+q_2) > S(q_1) + S(q_2)$

Summary & Future directions

Possible interpretation of the AWGC as "instability"
of Euclidean axionic wormholes (with flat asymptotics, d=4)

$$S(q_1 + q_2) > S(q_1) + S(q_2)$$

Via NP analysis of

- HO derivative corrections

- explicit setup (massive dilaton)

Extend it to more reliable setups,
eg EFT from UV-complete settings a'la Marolf-Santos

However, still in EFT regime. It would be nice to have a better grasp of microscopics…

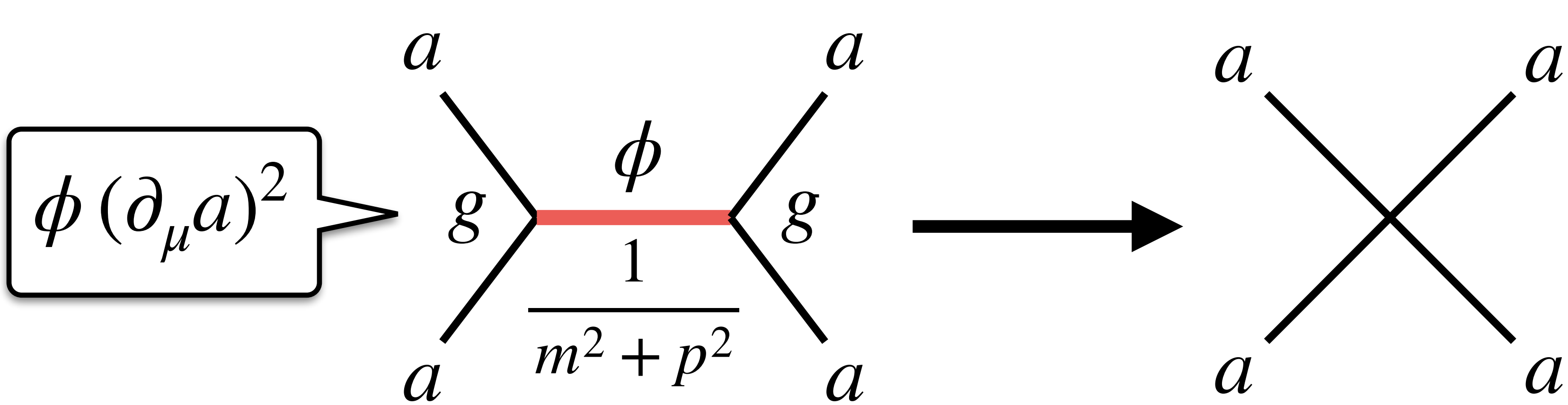
Explore possible UV properties that obstruct such solutions

Thank you

POSITIVITY

Axion-gravity EFT $\qquad \mathscr{L} = -\frac{1}{2}(\partial_\mu a)^2 + \alpha\,(\partial_\mu a \partial^\mu a)^2 + \cdots$

- where, for instance, α arises after **integrating out** massive scalar ϕ



$$\alpha = \frac{g^2}{2m^2} \geq 0$$

and the sing of α is related to the sign of propagator (unitarity)

- generically, $\alpha > 0$ follows from unitarity, analyticity, locality of UV scattering amplitudes [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]

  - **Caveat, assumption:** gravitational Regge states are sub-dominant

$$|\alpha| > 1/\left(M_s^2 M_{\rm Pl}^2\right)$$ [Hamada-Noumi-Shiu '18]