

Equivariant Point Cloud Generation for Particle Jets

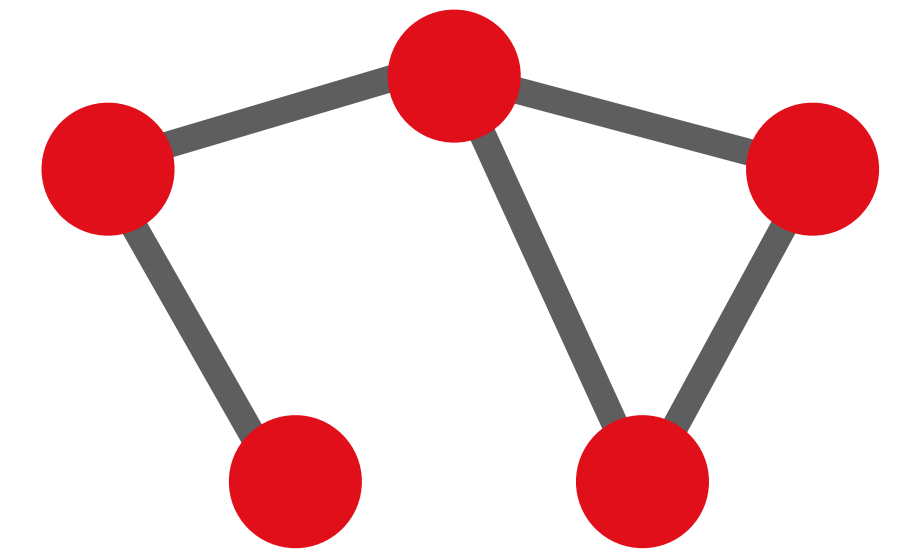
Erik Buhmann*

with Gregor Kasieczka and Jesse Thaler

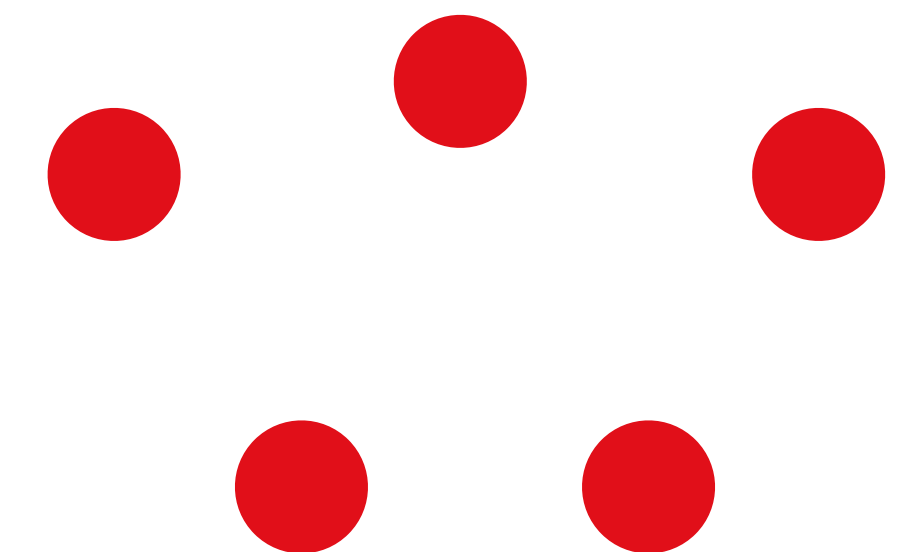
Generative Modelling of Point Clouds

- Particle jets are **variable-length, unordered sets** of particles
 - Natural representation as “point cloud”
- Requirements for **generative model**:
 - A. Variable number of points N
 - B. Permutation equivariance
 - C. Fast generation for large multiplicities (ideally $\mathcal{O}(N)$)
- Current approaches using graph & transformer models
- Our simple Deep Sets-based generative model is **equally performant, yet less complex and significantly faster**

Graph



Point Cloud
(Graph without edges)



Point Cloud: $C = (P, g)$

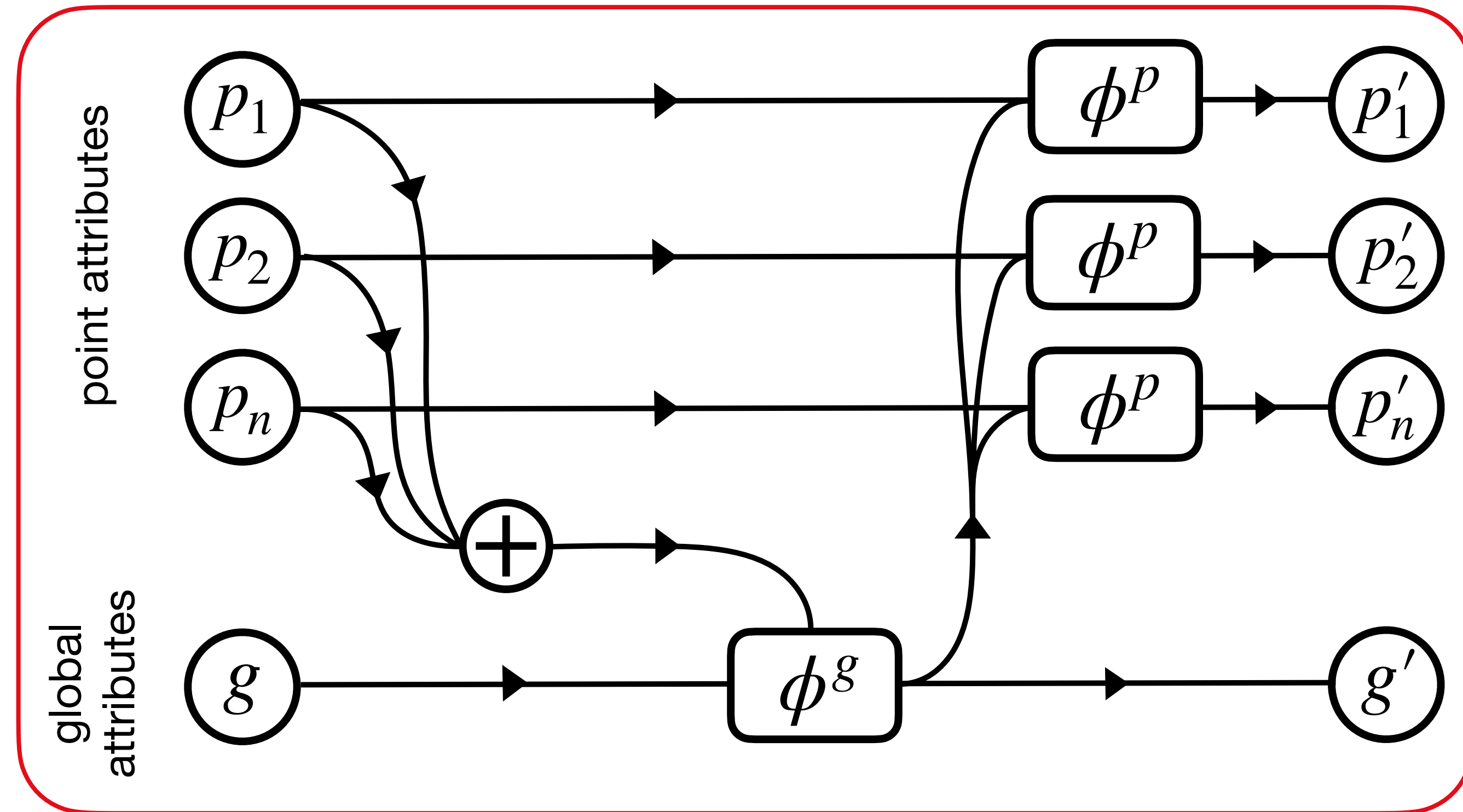
P : Point's attributes p_i

g : Global attributes

Equivariant Point Cloud (EPiC) Layer

- 2 step update process per layer:
 1. Global attributes \mathbf{g} are updated based on particle-wise attributes \mathbf{p}_i
 2. Particle attributes \mathbf{p}_i are updated based on the updated global attributes \mathbf{g}'
- Control of communication between local vectors via:
 - A. Length of global vector $[\mathbf{g}]$
 - B. Number of stacked EPiC layers R
- Optimisation of these two hyperparameters to develop minimal generative model

+ : sum & mean aggregation $\rho^{p \rightarrow g}$
 ϕ : 2-layer MLP



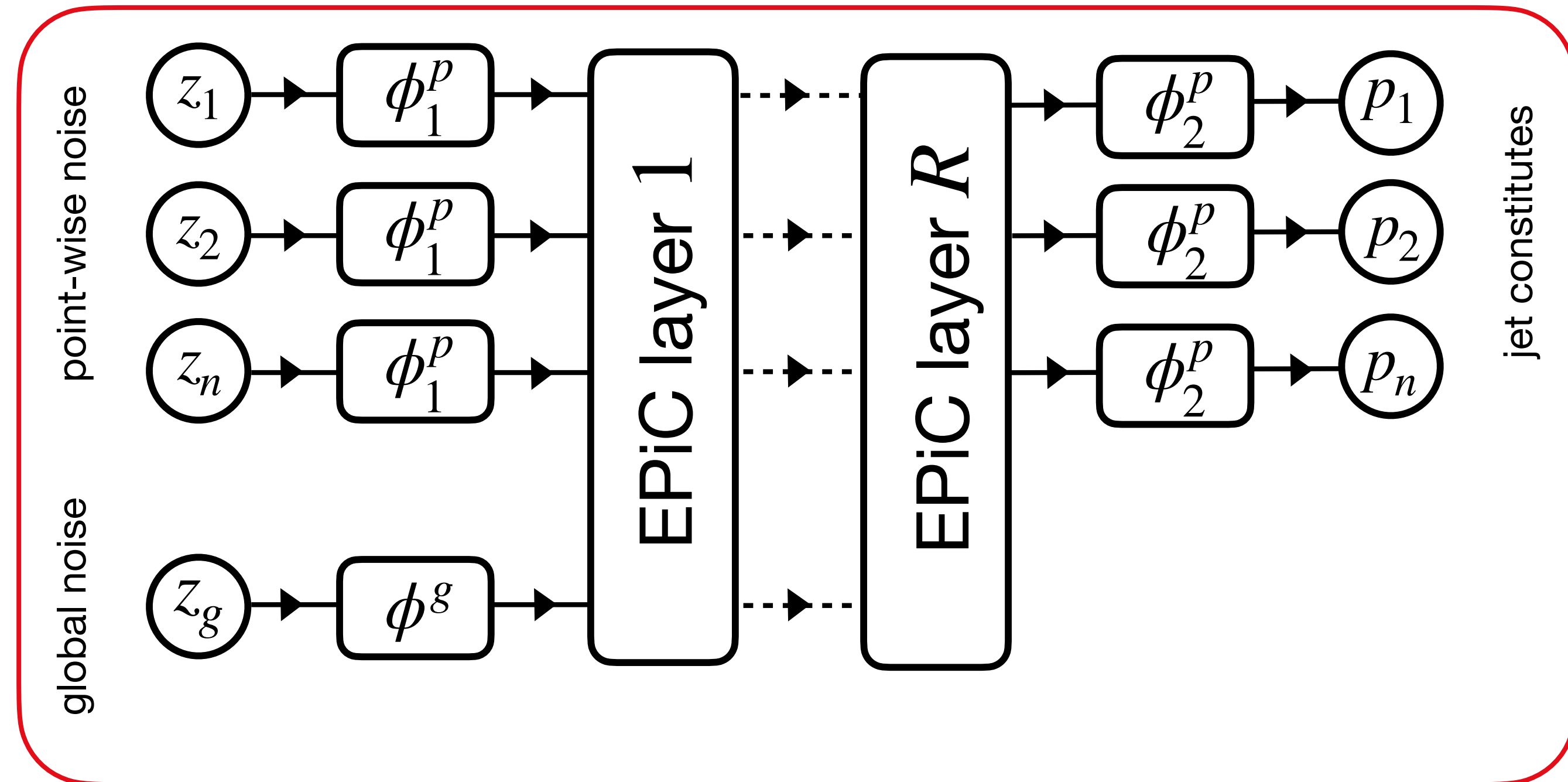
Computations in layer: (1) $\mathbf{g}'_i = \phi^g(\mathbf{g}, \rho^{p \rightarrow g}(P))$

(2) $\mathbf{p}'_i = \phi^p(\mathbf{g}', \mathbf{p}_i)$

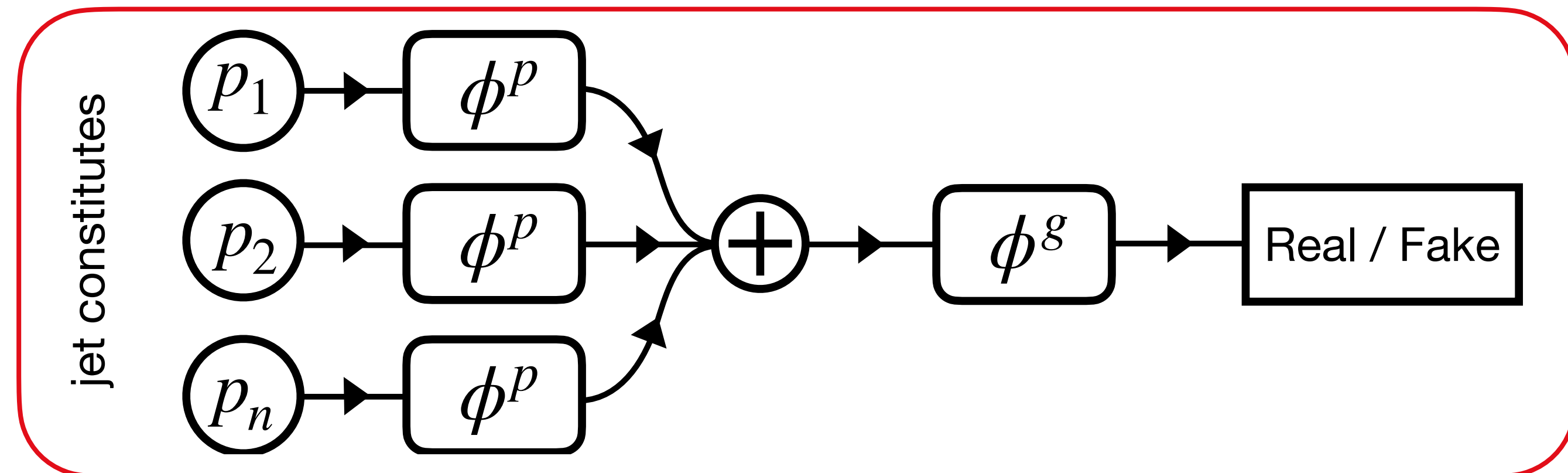
Equivariant Point Cloud (EPIc) GAN

- Generator (470k parameters):
 - Multiple R Equivariant Point Cloud (EPIc) layer stacked
- Discriminator (100k parameters):
 - Invariant Deep Sets structure
- Particle multiplicity defined when sampling (KDE estimation)
- Trained for 1000 epochs, best epoch chosen based on jet mass distribution on validation set

Generator:



Discriminator:



LSGAN loss objective:

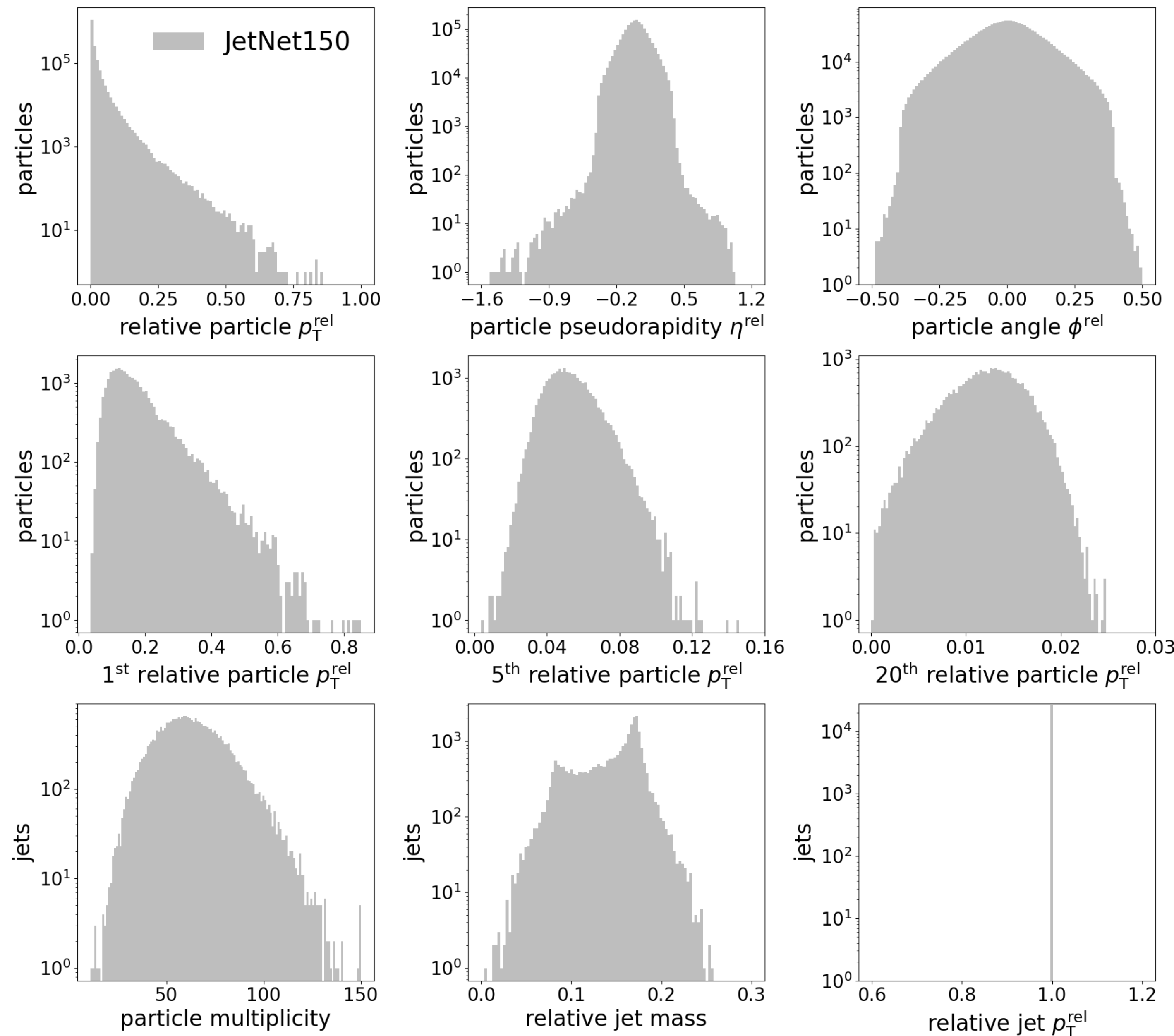
$$\min_D L(D) = \frac{1}{2} \mathbb{E}[(D(x) - 1)^2] + \frac{1}{2} \mathbb{E}[(D(G(z)))^2]$$

$$\min_D L(D) = \frac{1}{2} \mathbb{E}[(D(G(z)) - 1)^2]$$

X Mao et al: Least Squares Generative Adversarial Networks

Point Cloud Data

- Benchmark dataset: JetNet150 [1]
- Pythia simulated jets from proton-proton collisions
- Anti- k_T clustered with $R = 0.8$ and maximum particle multiplicity $N = 150$
- Particle collider coordinates normalised and centred
 - $p_T^{\text{rel}} = p_T^{\text{particle}} / p_T^{\text{jet}}$
 - $\eta^{\text{rel}} = \eta^{\text{particle}} - \eta^{\text{jet}}$
 - $\phi^{\text{rel}} = \phi^{\text{particle}} - \phi^{\text{jet}}$
- Here: Only top jets dataset

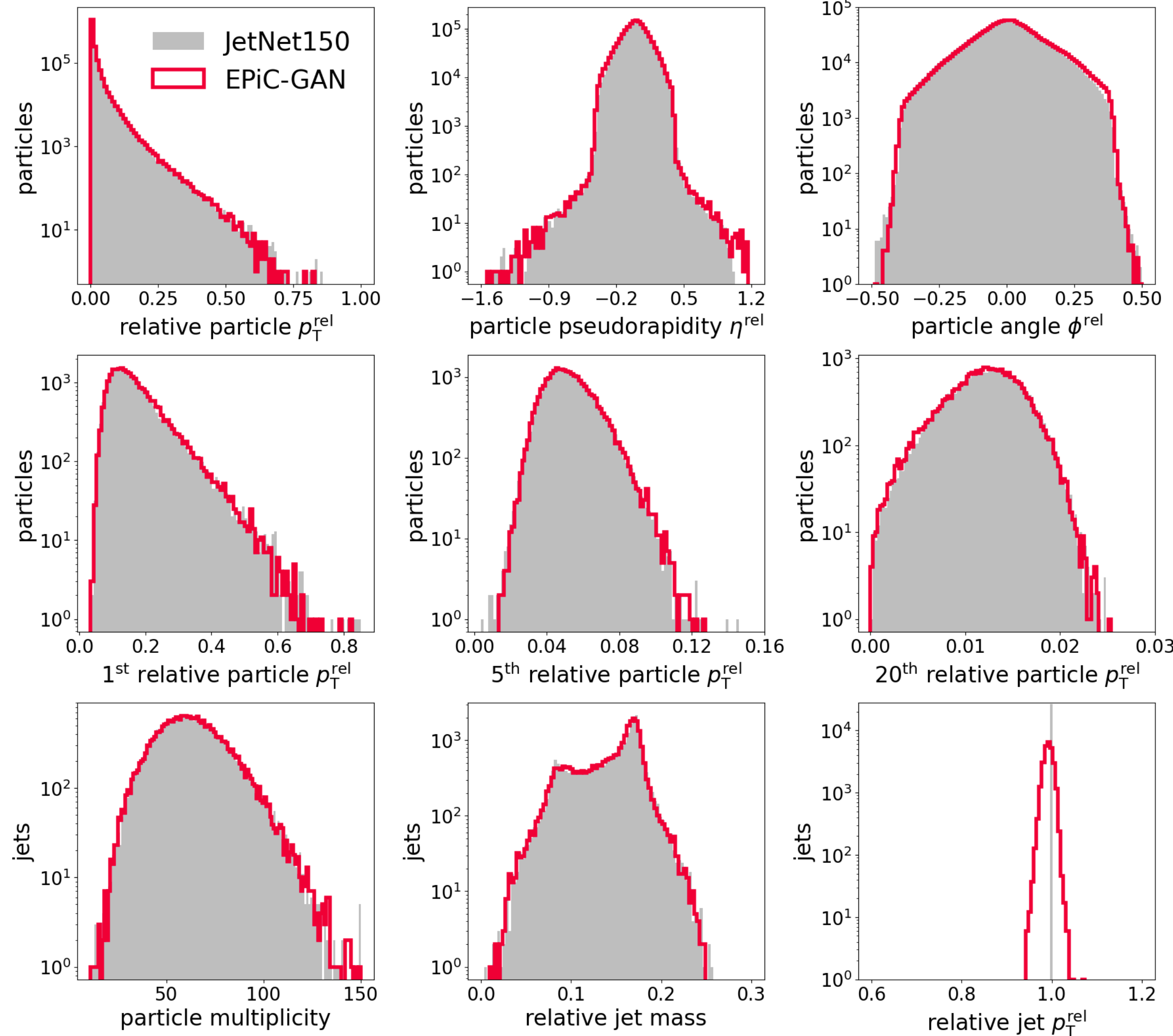


R Kansal, et al: Particle Cloud Generation with Message Passing Generative Adversarial Networks

[1] R Kansal, et al: JetNet150 (2.0.0) [Data set]. Zenodo

Results on JetNet150 tops

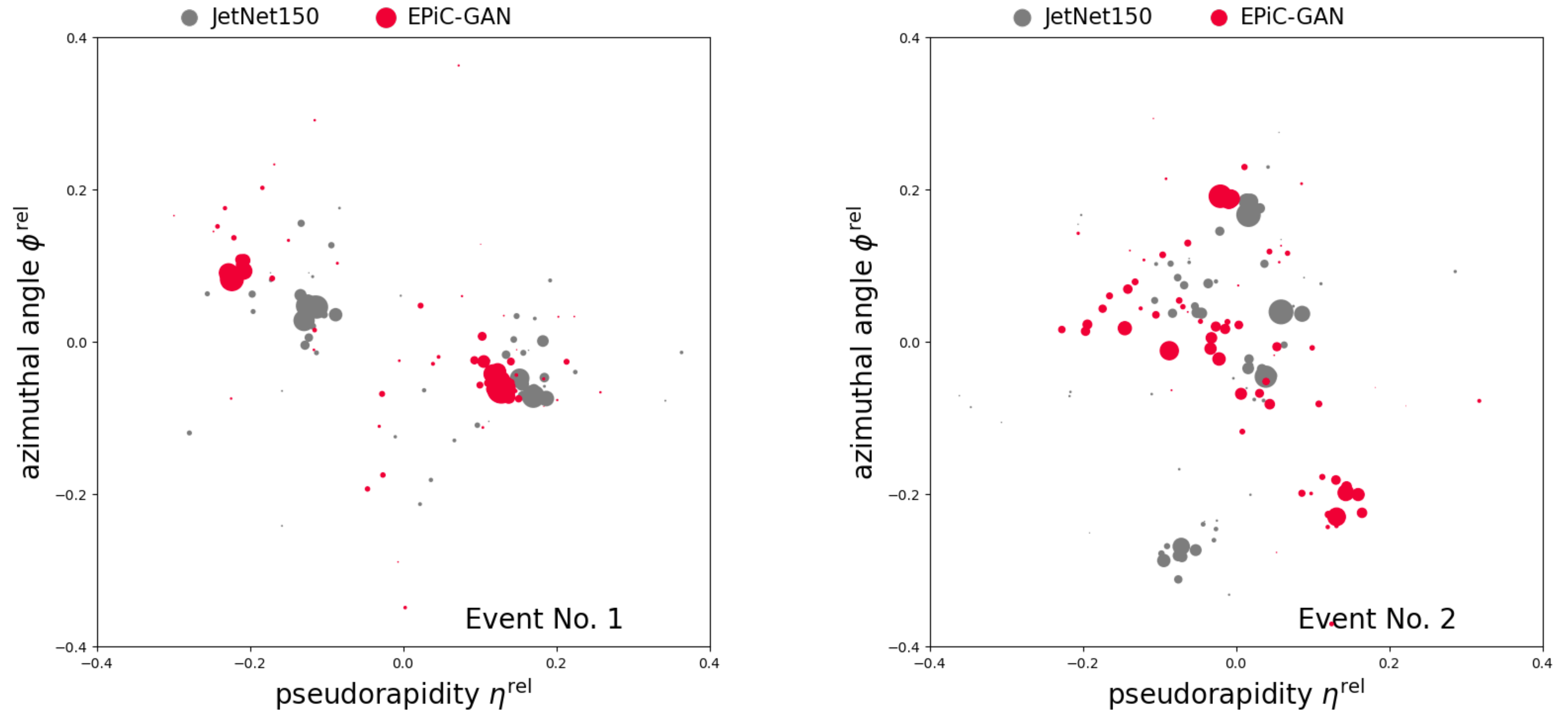
- EPiC-GAN setup:
 - $R = 6$ equivariant layers
 - $[g] = 10$ global attributes
- High generative fidelity after six equivariant update steps
- Distributions well represented by EPiC-GAN, jet mass distributions particularly challenging
- Sharp relative jet p_T distribution challenging; can be resolved by calibration



R Kansal, et al: Particle Cloud Generation with Message Passing Generative Adversarial Networks

[1] R Kansal, et al: JetNet150 (2.0.0) [Data set]. Zenodo

Point Clouds: Top jet visualisation



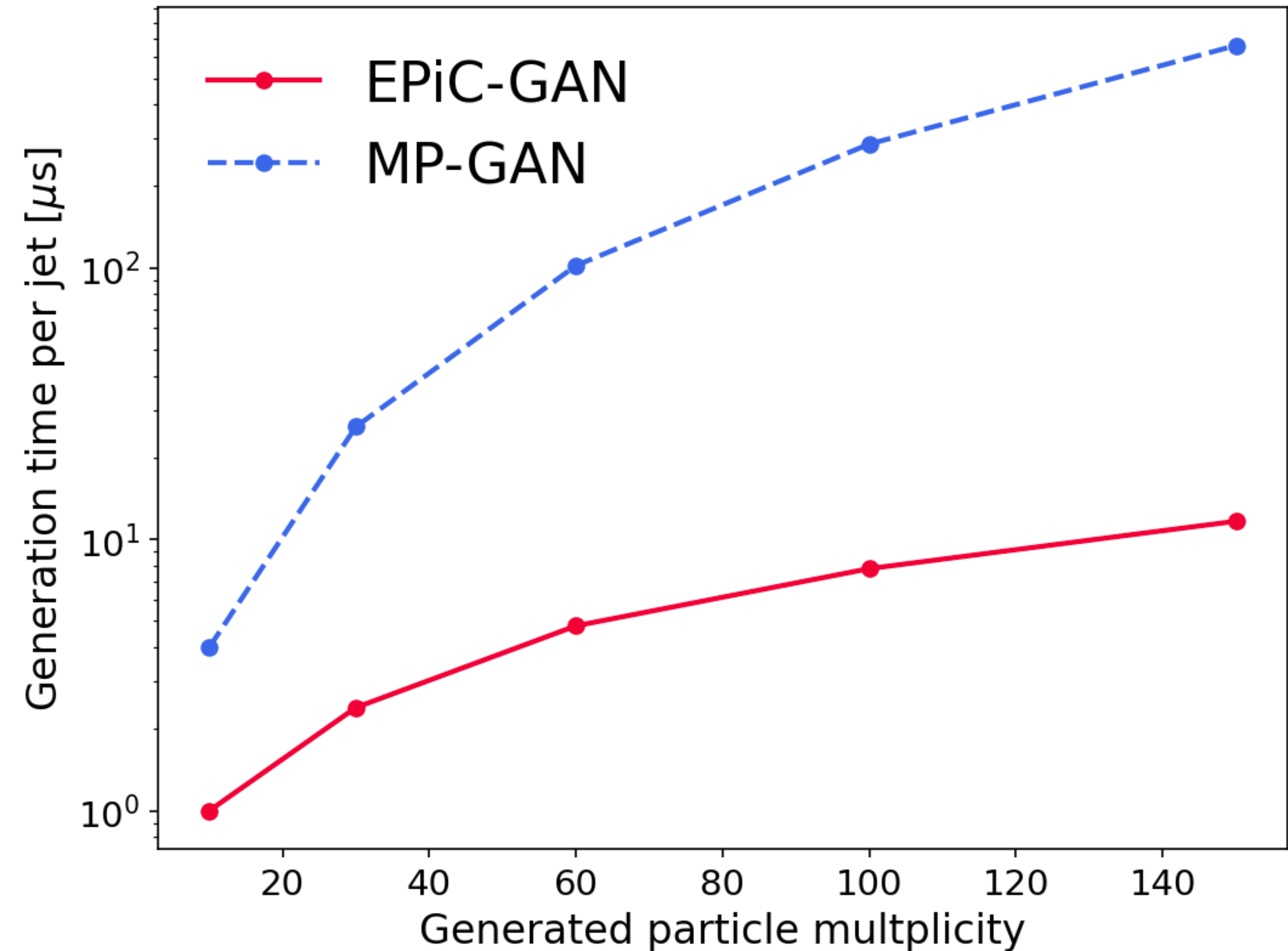
Point size corresponding to particle's transverse momentum p_T^{rel}

Scaling of generation time

- EPiC-GAN scales $\approx \mathcal{O}(N)$
- Message-Passing (MP)-GAN scales $\approx \mathcal{O}(N^2)$
- Batch size was adjusted for optimal generation speed

Generation time per jet:

Particles	Pythia ^[1]	MP-GAN*	EPiC-GAN*
$N = 30$	46 ms	26 μs	2 μs
$N = 150$	46 ms	660 μs	12 μs



* 500k jets generated on hardware:
Intel Gold-5218 (64 CPUs @ 2.3 Ghz), 384GB RAM, Nvidia A100-40GB

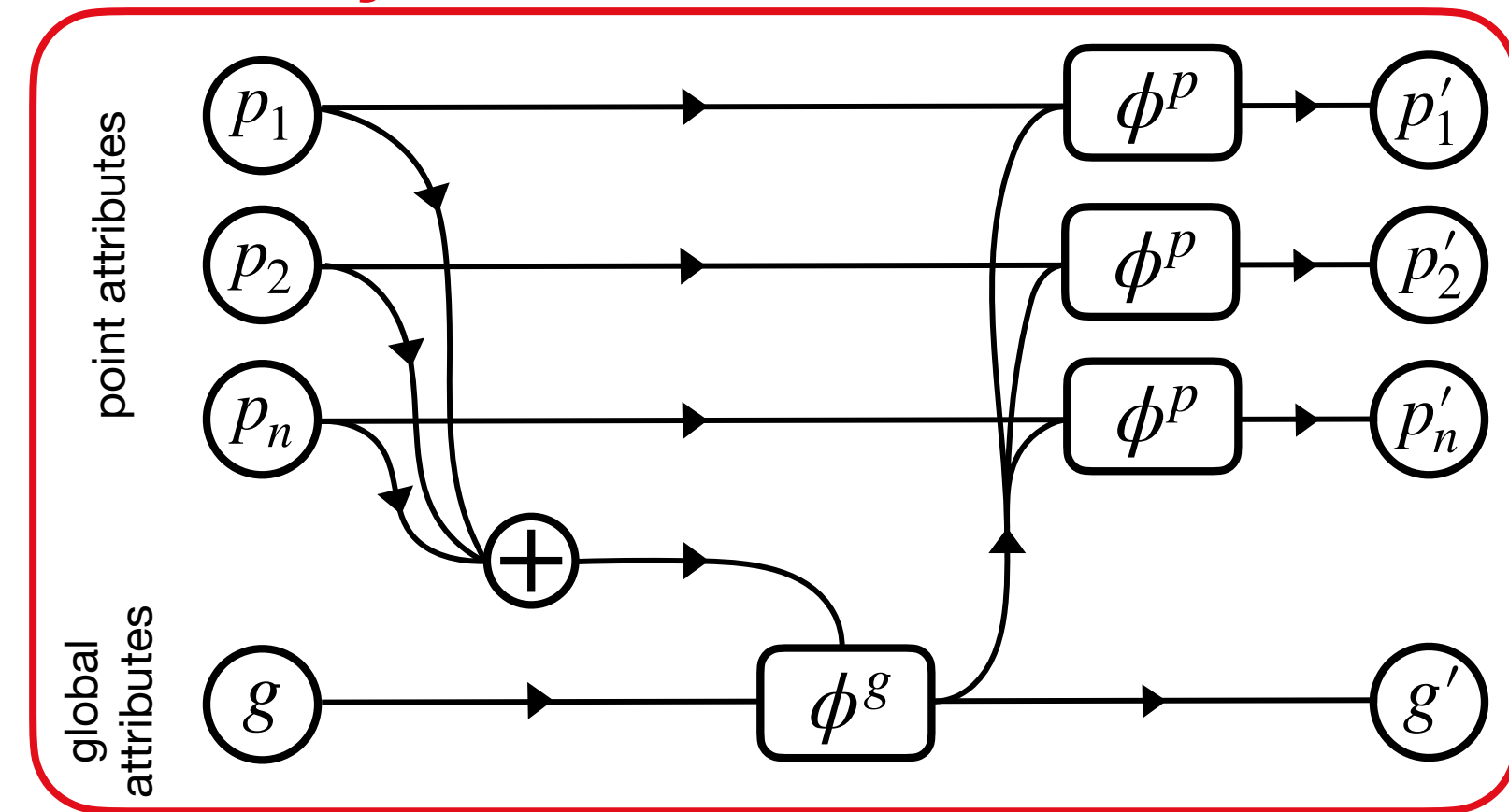
[1] R Kansal, et al: Particle Cloud Generation with Message Passing Generative Adversarial Networks

Summary

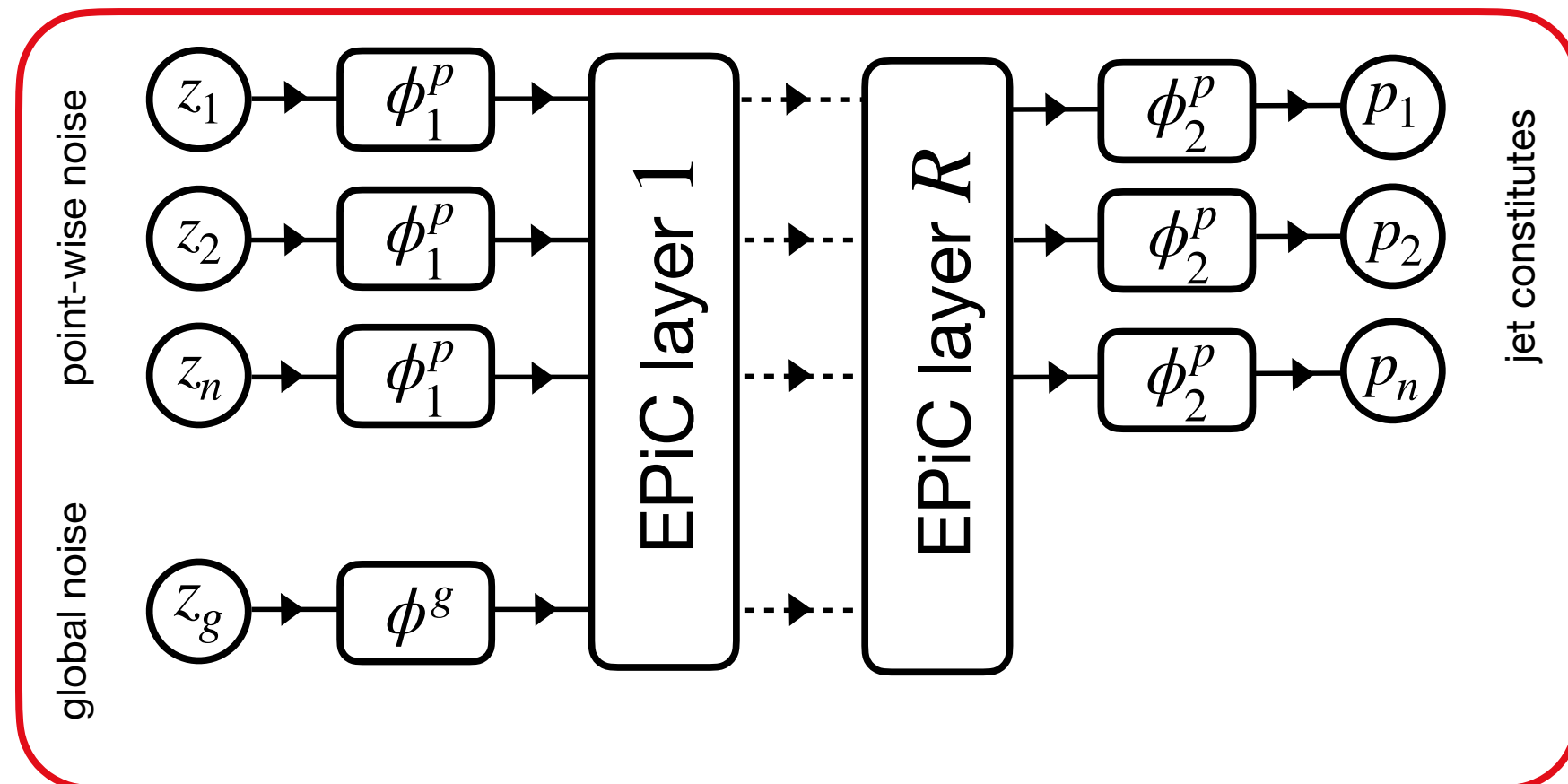
- Generative modelling of point clouds usually done with graph and transformer-based models
- Equivariant Point Cloud (EPIc)-GAN offers a **simple and fast alternative**
- Model generates realistic particle jets with **variable particle multiplicity**
- Fine-tuning possible for **minimal global information & model complexity**

Stay tuned for the paper!

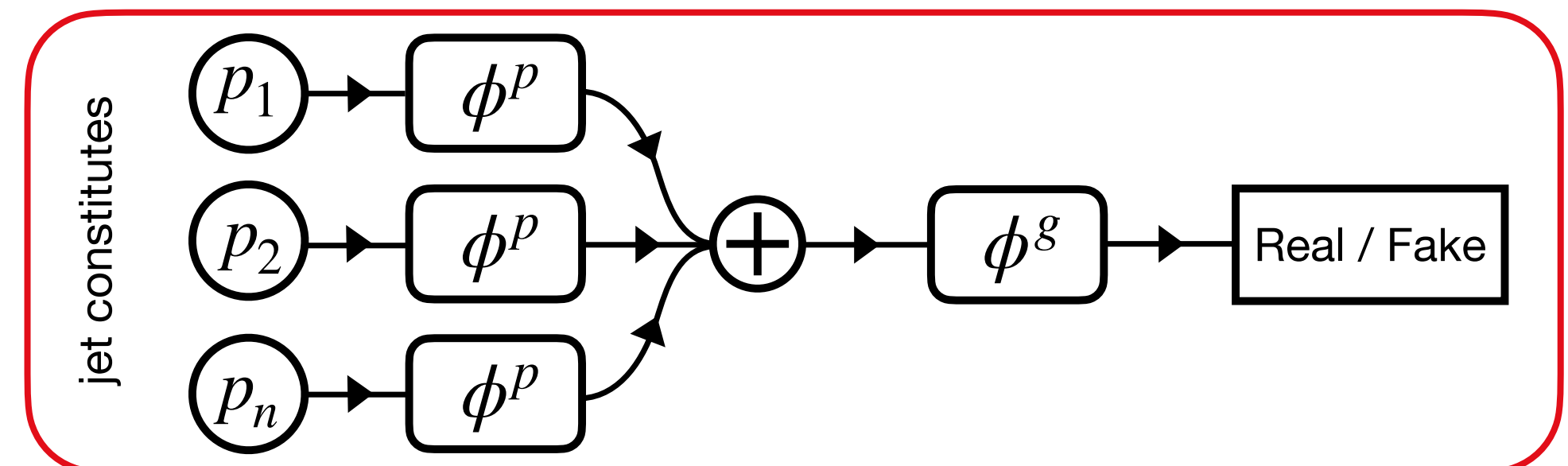
EPIc layer:



Generator:



Discriminator:



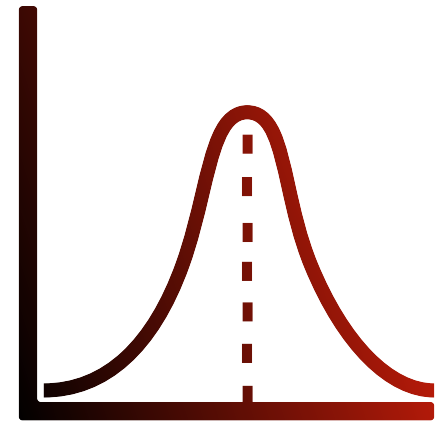
Bonus slides

EPiC-GAN Hyperparameter

- Trained for 500 / 1000 epochs (quarks / tops)
- Learning rate of Adam optimiser: $1e-4$
- Maximum batch size = 128
- Pre-processing: Each feature standardised to $\mathcal{N}(0,5\sigma)$
- Generator: 470 parameters (with $R = 6$ & $[g] = 10$)
- Discriminator: 100k parameters
- All fully connected layer size = 128
- $[p_i]$: For each particle one local noise vector of length 10
- Training / validation / test set: 127k / 26k / 26k
- Shown plots from test set with 26k events

Generative Models: Use cases in HEP research

Amplification of statistics



- Strong inductive bias of architectures help models to learn underlying distribution
- Powerful data augmentation technique

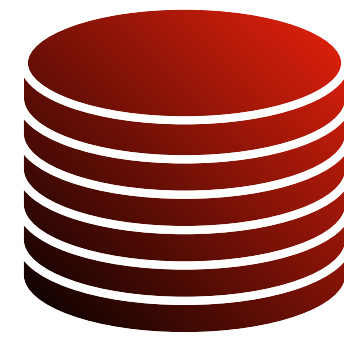
[S Bieringer et al: Calomplification - The Power of Generative Calorimeter Models](#)

[A Butter et al: GANplifying Event Samples](#)

[J Kummer et al: Radio Galaxy Classification with wGAN-Supported Augmentation](#)

[...]

Amortised computation



- Minimisation of local computing resources by upfront central model training
- Storing model weights instead of data

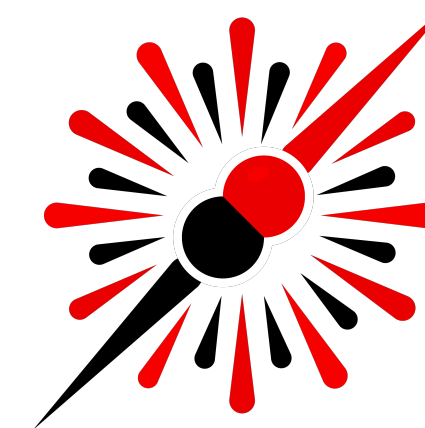
[M Paganini et al: CaloGAN: Simulating 3D High Energy Particle Showers in Multi-Layer Electromagnetic Calorimeters with Generative Adversarial Networks](#)

[EB, Sascha Diefenbacher et al: Getting High: High Fidelity Simulation of High Granularity Calorimeters with High Speeds](#)

[A Butter et al: Machine Learning and LHC Event Generation](#)

[...]

Generation from detector data



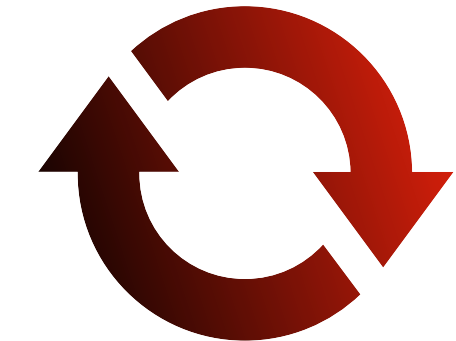
- Unsupervised training on real events instead of tuning Monte Carlo simulations
- I.e. for estimation of background densities

[JN Howad et al: Learning to Simulate High Energy Particle Collisions from Unlabeled Data](#)

[A Hallin et al: Classifying Anomalies THrough Outer Density Estimation \(CATHODE\)](#)

[...]

Differentiable models



- Optimisation of experimental setup based on explicit data likelihood
- Backpropagation through analysis chain

[T Dorigo et al: Toward the End-to-End Optimization of Particle Physics Instruments with Differentiable Programming: a White Paper](#)

[A Adelmann et al: New directions for surrogate models and differentiable programming for High Energy Physics detector simulation](#)

[...]