Domain wall dCFTs in N=4 SYM

Charlotte Kristjansen

Niels Bohr Institute

Based on:

- C.K., D.L. Vu & K.Zarembo, ArXiv:2112.10438, JHEP02 (2022) 070
- C.K., D. Müller & K. Zarembo, ArXiv:2106.08116, JHEP09 (2021) 004
- M. de Leeuw, A. Ipsen, C.K, K. Vardinghus, M. Wilhelm, ArXiv:1705.03898, JHEP 08 (2017) 020
- M. de Leeuw, C.K. & K. Zarembo , ArXiv:1506.06958, JHEP 08 (2015) 098

DESY, Hamburg

Jan 16th, 2023

AdS/CFT

$\mathcal{N} = 4$ SYM in 4D $\longleftrightarrow$ IIB strings on $AdS_5 \times S^5$

- Conformal symmetry
- Supersymmetry
- Planar integrability

AdS/dCFT

$\mathcal{N} = 4$ SYM in 4D with 3D domain wall	$\longleftrightarrow$	IIB strings on $AdS_5 \times S^5$ with probe brane

- Conformal symmetry partially broken
- Supersymmetry partially or completely broken

In the planar limit, $N \to \infty$:

Motivation

- Insights on the interplay between conformal symmetry, supersymmetry and integrability
- Tests of AdS/dCFT dictionary for set-ups with and without supersymmetry (so far all positive)
- Exact results for novel types of observables such as one-point functions, bulk-to-boundary correlators etc.
- Interesting connections to statistical physics: matrix product states and quantum quenhes.
- Possible cross-fertilization with the boundary conformal bootstrap program.

Plan of the talk

I. The defect set-up and its parameters (field and string picture)

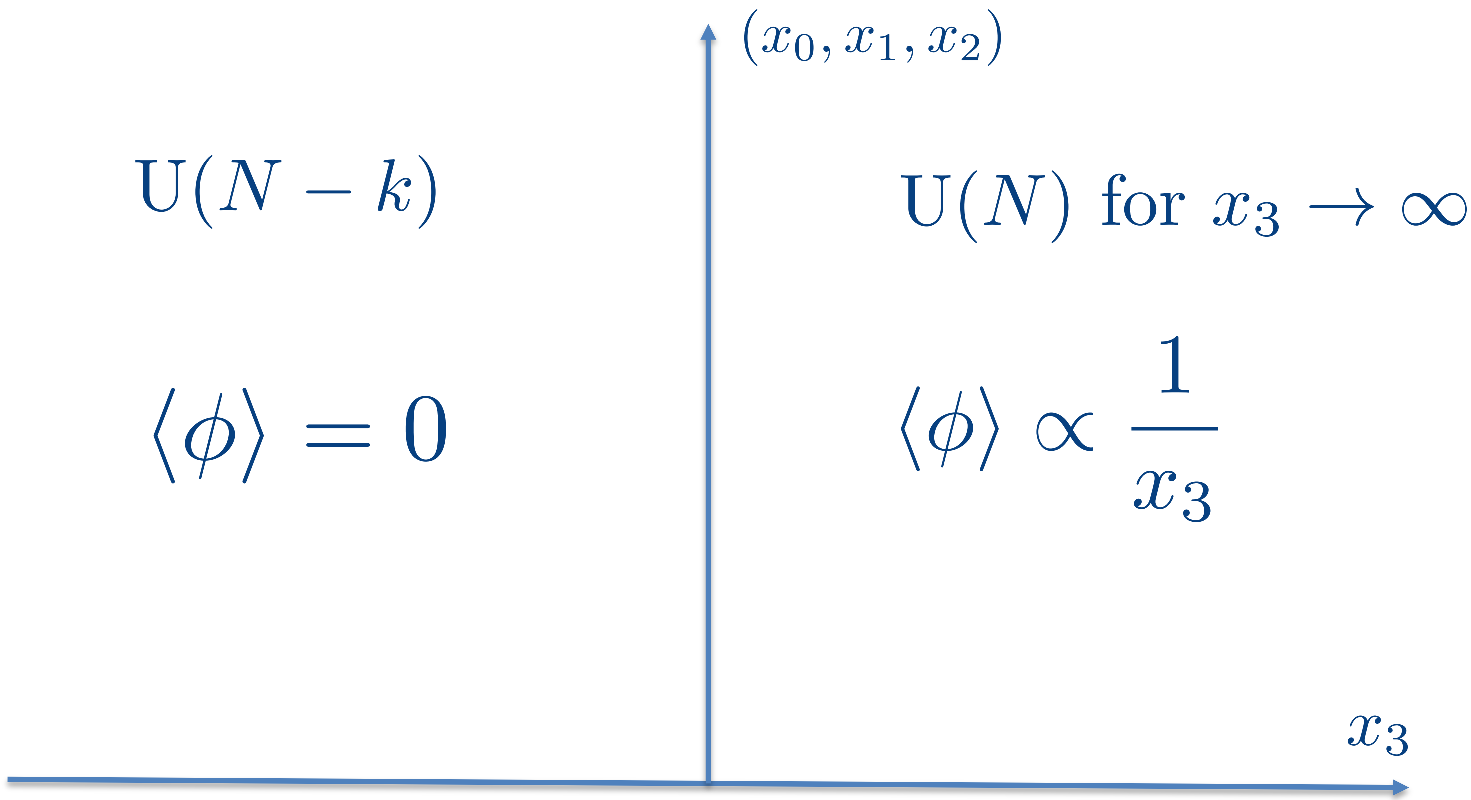
II. One and two-point functions in the bulk

III. The theory on the defect

IV. Summary & Open problems

The defect set-up

$$\mathcal{N} = 4 \quad \mathrm{SYM}$$



Classical Fields

Assume only x_3 -dependence and $x_3 > 0$, $A_\mu^{\rm cl} = 0, \qquad \Psi_A^{\rm cl} = 0$

Classical e.o.m.:

(x_3 is distance to defect)

$$\frac{d^2\phi_i^{\rm cl}}{dx_3^2} = \left[\phi_j^{\rm cl}, \left[\phi_j^{\rm cl}, \phi_i^{\rm cl}\right]\right].$$

Solution:

$$\phi_i^{\rm cl} = \frac{1}{x_3}\begin{pmatrix} (t_i)_{k\times k} & 0 \\ 0 & 0 \end{pmatrix}, \; i = 1,2,3$$

Constable, Myers & Tafjord '99

$$\phi_4^{\rm cl} = \phi_5^{\rm cl} = \phi_6^{\rm cl} = 0$$

$$[t_i, t_j] = i\,\epsilon_{ijl}\,t_l$$

i.e. $t_i,\; i = 1,2,3$ constitute a k-dimensional irreducible representation of SU(2)

Set-up ½ BPS (for appropriate choice b.c. for zero-modes, Gaiotto & Witten '08)

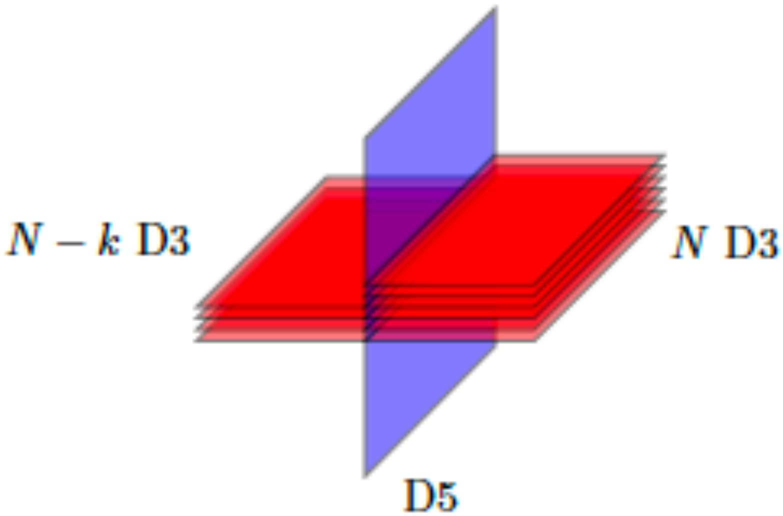
Different cases

$$\phi_i^{\mathrm{cl}} = \frac{1}{x_3} \begin{pmatrix} (t_i)_{k\times k} & 0 \\ 0 & 0 \end{pmatrix}, \; i = 1,2,3$$

- $k > 1$: Nahm pole boundary conditions, $\langle \phi \rangle \propto \frac{1}{x_3}$
- $k = 1$: Limiting case, $\langle \phi \rangle = 0$, analytical continuation
- $k = 0$: Completely different system. Extra fundamental fields on defect. Not the topic of this talk.

AdS/dCFT --- The string theory side

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	$\times$	$\times$	$\times$	$\times$						
D5	$\times$	$\times$	$\times$		$\times$	$\times$	$\times$			



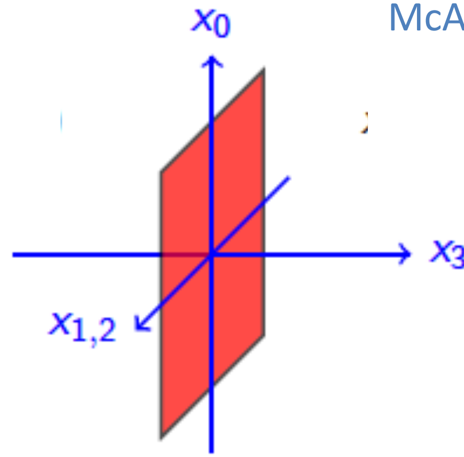
Karch & Randall '01,

Geometry of D5 brane: $AdS_4 \times S^2$

Background gauge field: k units of magnetic flux on S^2

Novel features in dCFTs

Cardy ´84
McAvity & Osborn '95



Defect of co-dimension one

- One-point functions

$$\langle \mathcal{O}^{\rm bulk}_{\Delta}(x)\rangle = \frac{C}{|x_3|^{\Delta}}$$

- Two-point functions between op's with different conf. dims.

$$\langle \mathcal{O}^{\rm bulk}_{\Delta}(x)\mathcal{O}^{\rm bulk}_{\Delta'}(x')\rangle = \frac{1}{|x_3|^{\Delta}|x_3'|^{\Delta'}}\, f(\xi), \qquad \xi = \frac{|x_\mu - x'_\mu|^2}{|x_3||x_3'|}$$

- Mixed correlators involving bulk and defect fields

$$\langle \mathcal{O}^{\rm bulk}_{\Delta}(x)\hat{\mathcal{O}}^{\rm defect}_{\Delta'}(\vec{x}')\rangle = \frac{\mu_{\Delta\Delta'}}{x_3^{\Delta-\Delta'}|x-(\vec{x}',0)|^{2\Delta'}}$$

One-point functions in the dCFT

$$\langle \mathcal{O}^{\text{bulk}}_{\Delta}(x)\rangle = \frac{C}{|x_3|^{\Delta}}$$

Cardy ´84
McAvity & Osborn '95

Normalization given by: $\lim_{x_3\to\infty} \langle \mathcal{O}^{\text{bulk}}_{\Delta}(y+x)\mathcal{O}^{\text{bulk}}_{\Delta'}(z+x)\rangle = \frac{\delta_{\Delta\Delta'}}{|y-z|^{2\Delta}}$

Due to vevs scalar operators can have non-zero 1-pt fcts at tree-level

$$\langle \mathcal{O}_{\Delta}(x)\rangle = \left(\text{Tr}(\phi_{i_1}\ldots\phi_{i_\Delta}) + \ldots\right)\Big|_{\phi_i\to\phi_i^{\text{cl}}=\frac{t_i}{x_3}}$$

Tree level and one-loop 1-pt functions of *conformal* scalar operators can be found in closed form using the tools of integrability

deLeeuw, C.K. & Zarembo '15, Buhl-Mortensen de Leeuw, C.K & Zarembo '16 de Leeuw, C.K & Mori, '17. de Leeuw, C.K & Linardopoulos, '18. C.K, Müller, Zarembo '20

An exact formula for any loop order can be found by bootstrapping arguments

Komatsu & Wang '20 Gombor & Bajnok '20

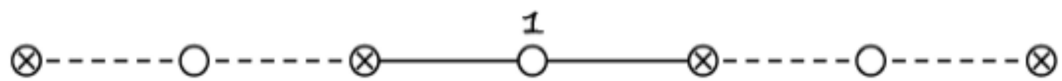
Closed expression for one-point functions

$k = 1$, Leading order result:

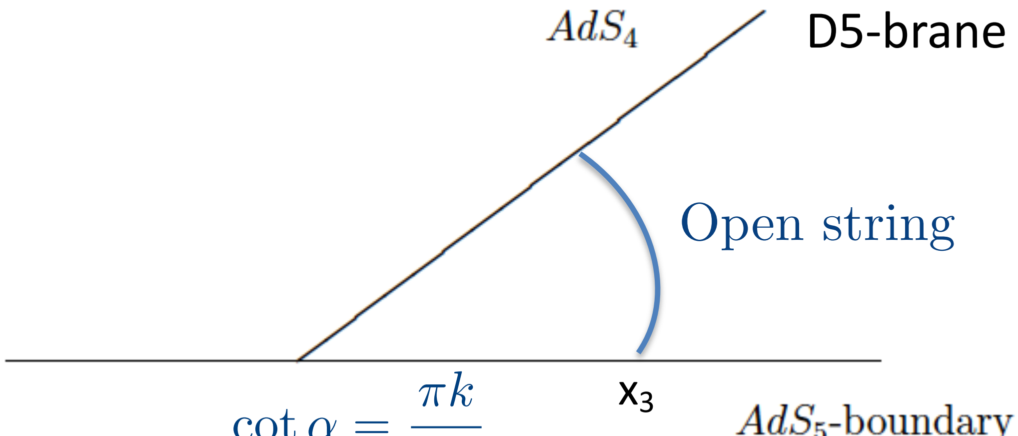
C.K, Müller
Zarembo '21

$$C = \frac{Q_1(0)Q_3(0)Q_4(0)Q_5(0)Q_7(0)}{Q_2(0)Q_2(\frac{i}{2})Q_4(\frac{i}{2})Q_6(0)Q_6(\frac{i}{2})} S \det G$$

$PSU(2,2|4)$:



The string theory side



$$\cot\alpha = \frac{\pi k}{\sqrt{\lambda}}$$

Probe brane system suggests a double scaling limit

$$\lambda \to \infty,\ k \to \infty,\ \frac{\lambda}{k^2}\ \text{finite} \qquad (N \to \infty)$$

One can compare perturbative gauge theory to semi-classical string theory (or sugra).

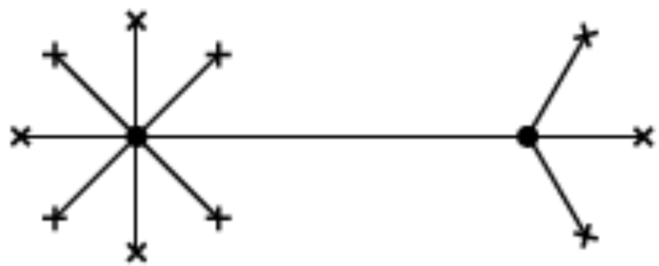
Nagasaki & Yamaguchi '12, C.K., Semenoff & Young '12, Buhl-Mortensen, de Leeuw Ipsen, C.K. & Wilhelm '16

Supersymmetric localization provides exact answer for chiral primary Komatsu & Wang '20

Two-point functions

New feature: Overlap between operators with different conf. dims.

de Leeuw, Ipsen, C.K., Vardinghus, Wilhelm `17



From general arguments

$$\langle \mathcal{O}^{\mathrm{bulk}}_{\Delta}(x)\mathcal{O}^{\mathrm{bulk}}_{\Delta'}(x')\rangle = \frac{1}{|x_3|^{\Delta}|x_3'|^{\Delta'}}\; f(\xi), \qquad \xi = \frac{|x_\mu - x'_\mu|^2}{|x_3||x_3'|}$$

Example

$$\langle \mathrm{tr} Z^{J_1} \mathrm{tr} Z^{J_2}\rangle_{\mathrm{conn}} = \frac{\xi}{\xi+1}\langle \mathrm{tr} Z^{J_1} \mathrm{tr} \bar{Z}^{J_2}\rangle_{\mathrm{conn}}$$

$$= \frac{g^2_{\mathrm{YM}}}{16\pi^2}\frac{J_1}{x_3^{J_1}}\frac{J_2}{x_3'^{J_2}}\sum_{\ell=0}^{\min\{k,J_1,J_2\}}\frac{\alpha_\ell^{J_1-1}\alpha_\ell^{J_2-1}}{\binom{2\ell+1}{\ell+1}}\frac{{}_2F_1(\ell,\ell+1;2\ell+2;-\xi^{-1})}{\xi^\ell(\xi+1)}$$

$\langle \mathcal{O}_L \mathrm{Tr}(\phi_1\phi_2)\rangle$ can be expressed in terms of $\langle \mathcal{O}_L\rangle$ Widén `17

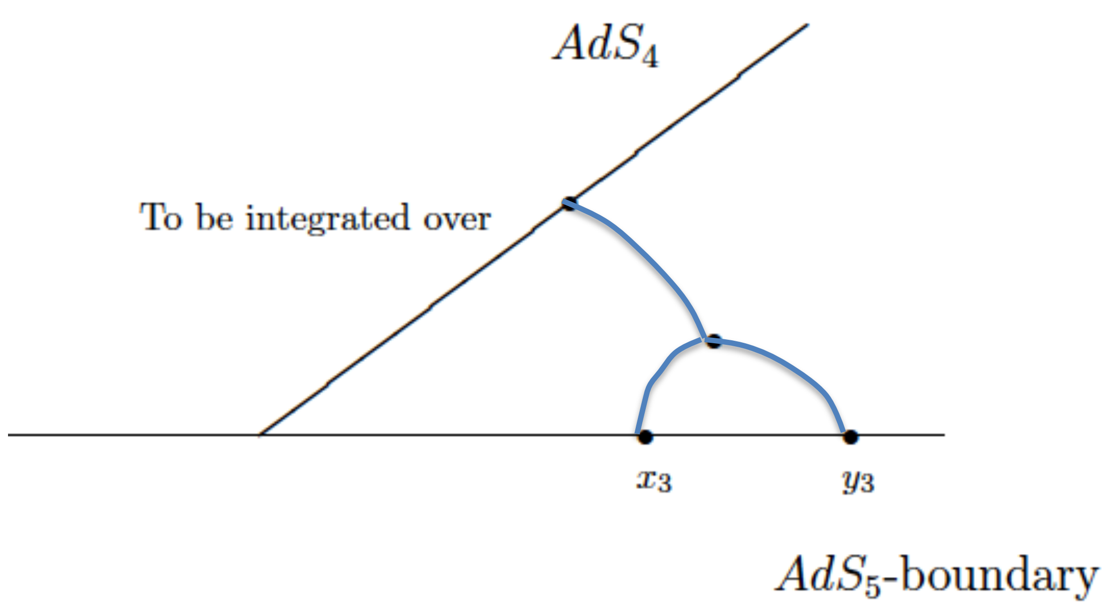
Two-point functions --- string theory side

Prediction for string theory:

$$\lambda = g_{\rm YM}^2 N \to \infty \,, \qquad k \to \infty \,, \qquad \frac{\lambda}{k^2} \quad \text{finite} \,,$$

$$\langle \mathrm{tr} Z^{J_1} \mathrm{tr} \bar{Z}^{J_2} \rangle = \frac{\lambda}{16\pi^2} \frac{1}{N} \left(\frac{k}{2} \right)^{J_1+J_2-1} \frac{1}{x_3^{J_1} y_3^{J_2}} \frac{2\xi+1}{(\xi+1)\xi^2}$$

Relevant string theory computation: (Notice factor of $\frac{1}{N}$)



Open problem

The perturbative program, $k > 1$

Buhl-Mortensen, de Leeuw, Ipsen, C.K., Wilhelm ‘16

Expand the $\mathcal{N} = 4$ action around $\phi^{\rm cl}$

$$\phi_i = \phi_i^{\rm cl} + \tilde{\phi}_i = \frac{t_i}{x_3} + \tilde{\phi}_i \qquad i = 1, 2, 3$$

Complicated mass matrix

-Mixing involving both flavour and colour

-Mass terms involving x_3 dependence

$$A_\mu, \Phi_i, \Psi = \begin{array}{cc} \begin{array}{cc} k \quad & \quad N-k \end{array} & \\ \left[\begin{array}{c|ccc} x & y & y & y \\ \hline y & z & z & z \\ y & z & z & z \\ y & z & z & z \end{array}\right] & \begin{array}{c} k > 1 \\ \\ N-k \\ \\ \end{array} \end{array}$$

x and y fields become massive, $m^2 \propto 1/x_3^2$

Described in terms of AdS_4 propagators

The defect acts as a boundary of an AdS_4 space

The perturbative program, $k=1$

C.K, Müller, Zarembo ‘20

OBS: $\phi^{cl}=0$

Can be understood by analytical continuation of the $k>1$ case

AdS propagators converge to Dirichlet/Neuman propagators

	$\Phi_{4,5,6}, A_{0,1,2}, c$	$\Phi_{1,2,3}, A_3$
x, y	Dirichlet	Neumann
z	no BCs	no BCs

Supersymmetry conserved

Propagators for complex scalars, e.g. $Z=\phi_1+i\phi_4$

$$D_\kappa(x,y)=\frac{1}{4\pi^2}\left(\frac{1}{|x-y|^2}+\frac{\kappa}{|\bar{x}-y|^2}\right), \quad \kappa=\begin{cases}1 & \text{Neumann}\\ -1 & \text{Dirichlet}\\ 0 & \text{no BCs.}\end{cases}$$

$$\bar{x}=(x_0,x_1,x_2,-x_3)$$

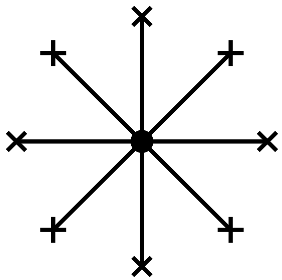
$$\langle X^{1a}(x)X^{b1}(y)\rangle=\frac{g_{\mathrm{YM}}^2\delta^{ab}}{2}\Big(D_1(x,y)-D_{-1}(x,y)\Big)=\frac{g_{\mathrm{YM}}^2\delta^{ab}}{4\pi^2|\bar{x}-y|^2},$$

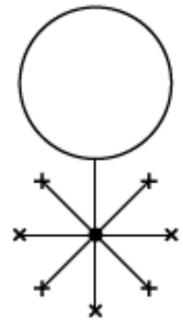
OBS: Finite at coinciding points

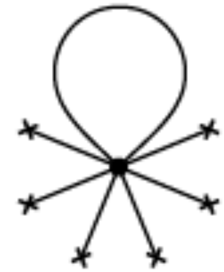
Difference in diagrammatics

One-point functions

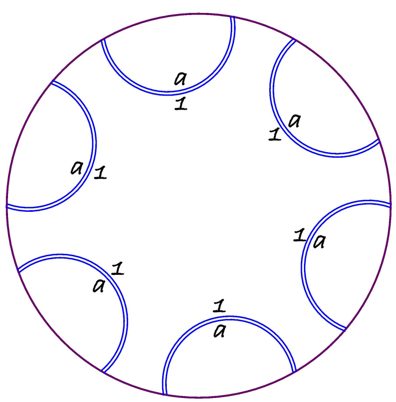
$k > 1$: Tree-level and one-loop order







$k = 1$: Leading order (Large-N)



Boundary fields from bulk fields

De Leeuw, Ipsen, C.K. Wilhelm '17

$$\begin{array}{cc} & \begin{array}{cc} k & N-k \end{array} \\ \Phi = & \left[\begin{array}{c|c} \Phi_{l,m} & \Phi_{n,a} \\ \hline \Phi_{a,n} & \end{array}\right] \end{array} \begin{array}{c} k>1 \\ N-k \end{array}$$

$$l = 0, 1, \ldots k-1,$$
$$m = -l, \ldots, l$$
$$n = 1, \ldots, k$$
$$a = k+1, \ldots, N$$

Extracting boundary fields: Example

$$\langle \phi^1_{lm}(x)\phi^1_{lm}(y)\rangle \sim \frac{{}_2F_1(l-1, l, 2l, -\xi^{-1})}{(1+\xi)(\xi^l)}\frac{1}{x_3 y_3}, \qquad \xi = \frac{|x-y|^2}{x_3 y_3}$$

$$\sim x_3^{2l}\frac{1}{|\vec{x}-\vec{y}|^{2(l+1)}}, \qquad \text{as } x_3 = y_3 \to 0_+$$

Hence,

$$\hat{\phi}^1_{lm}(\vec{x}) = \lim_{x_3\to 0_+} x_3^{-l}\phi^1_{lm}(x), \qquad \hat{\Delta}_{\phi^1_{lm}} = l+1$$

Boundary fields

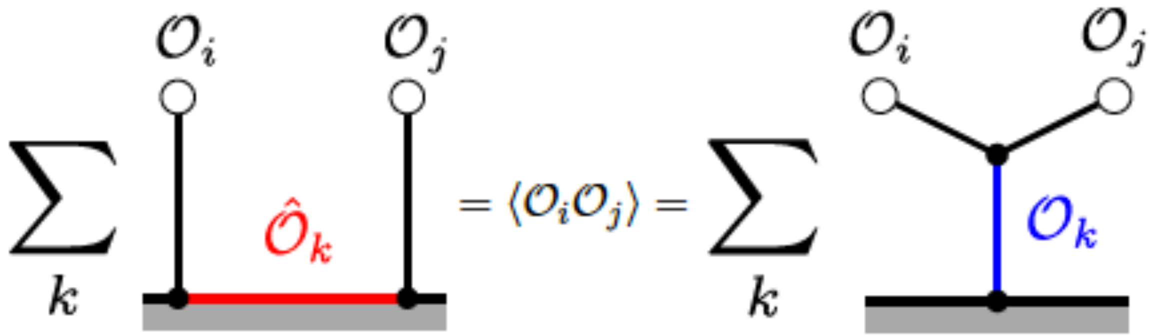
Φ	$\hat{\Delta}$	$\mathrm{U}(N-k)$
$(\hat{\phi}_{1,2,3})_{\ell m}$	$\ell+1$	singlet
$(\hat{\phi}_{4,5,6})_{\ell m}$	$\ell+2$	singlet
$(\hat{A}_{\hat{\mu}})_{\ell m}$	$\ell+2$	singlet
$(\hat{\psi}_{1,2,3,4})_{\ell m}$	$\ell+\frac{3}{2}$	singlet
$[\hat{\phi}_{1,2,3}]_{n,a}$	$\frac{k+1}{2}$	fundamental
$[\hat{\phi}_{4,5,6}]_{n,a}$	$\frac{k+3}{2}$	fundamental
$[\hat{A}_{\hat{\mu}}]_{n,a}$	$\frac{k+3}{2}$	fundamental
$[\hat{\psi}_{1,2,3,4}]_{n,a}$	$\frac{k+2}{2}$	fundamental

$$n = k, k+1, \ldots, N,$$
$$a = 1, 2, \ldots, k$$

NB: $\hat{\mu} = 0, 1, 2,$ A_3 has been used to define gauge covariant fields

Gauge invariant operators can be constructed

Bulk-boundary couplings by BOE



Liendo,Rastelli Van Rees ’12

Liendo & Meneghelli. 16,

OPE: $$\mathcal{O}_i(x)\mathcal{O}_j(y) = \frac{M_{ij}}{|x-y|^{\Delta_i+\Delta_j}} + \sum_k \frac{\lambda_{ij}{}^k}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}} C(x-y,\partial_y)\mathcal{O}_k(y)$$

BOE: $$\mathcal{O}_i(x) = \sum_j \frac{\mu_i{}^j}{(2x_3)^{\Delta_i-\Delta_j}} \hat{C}(x_3,\vec{\partial})\hat{\mathcal{O}}_j(\vec{x})$$

Data mining

de Leeuw, Ipsen
C.K., Vardinghus
& Wilhelm 17,

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \frac{f_{ij}(\xi)}{(2x_3)^{\Delta_i}(2y_3)^{\Delta_j}} \qquad \langle \mathcal{O}_i\rangle = \frac{C_i}{(2x_3)^{\Delta_i}}$$

$$\lim_{z_3\to\infty}\langle \mathcal{O}_i(x+z)\mathcal{O}_j(y+z)\rangle = \frac{M_{ij}}{|x-y|^{\Delta_i+\Delta_j}}$$

From OPE

$$f_{ij}(\xi) = \xi^{-\frac{\Delta_i+\Delta_j}{2}}\left[M_{ij} + \sum_k \lambda_{ij}{}^k C_k \underbrace{F_{\mathrm{bulk}}(\Delta_k, \Delta_i-\Delta_j, \xi)}_{\text{Bulk conformal block}}\right]$$

Structure constants of N=4 SYM from one- and two-point functions (1+2=3)

From BOE

$$f_{ij}(\xi) = C_iC_j + \sum_k \mu_i{}^k \mu_{jk} \underbrace{F_{\mathrm{bdy}}(\Delta_k,\xi)}_{\substack{\text{Boundary}\\ \text{conformal block}}}$$

Bulk-to-boundary couplings from one- and two-point functions

Stress tensor and displacement operator

Improved $\mathcal{N} = 4$ SYM stress tensor

$$T^{\text{scalars}}_{\mu\nu} = \frac{2}{g^2_{YM}} \mathrm{Tr}\left\{ -\frac{2}{3}(\partial_\mu \phi_i)(\partial_\nu \phi_i) + \frac{1}{3}\phi_i(\partial_\mu \partial_\nu \phi_i) + \frac{1}{6} g_{\mu\nu} \left((\partial_\rho \phi_i)^2 + \frac{1}{2}[\phi_i, \phi_j]^2 \right) \right\}$$

Symmetric, traceless and conserved

$$T_{\mu\nu} = T_{\nu\mu}, \qquad g^{\mu\nu} T_{\mu\nu} = 0, \qquad \partial^\mu T_{\mu\nu} = 0$$

$$\langle T_{\mu\nu}(x) \rangle = 0$$

Displacement operator

$$\partial_\mu T^{\mu\perp}(x) = -D(\vec{x})\delta(x_\perp)$$

$$D(\vec{x}) = \lim_{x_3 \to 0} \; -T^{\perp\perp}(\vec{x}, x_3)$$

Displacement operator two-point function

C.K., Linardopoulos
Volk

Leading order

$$\langle D(\vec{x}) D(\vec{y}) \rangle = \lim_{x_3, y_3 \to 0_+} \langle T^{\perp\perp}(x) T^{\perp\perp}(y) \rangle = \frac{5 \cdot 2^6}{9\pi^2 g_{\mathrm{YM}}^2} \, k(k^2 - 1) \frac{1}{|\vec{x} - \vec{y}|^8}$$

Special case of the following result for $x_3, y_3 > 0$

$$\begin{aligned} \langle T_{\mu\nu}(x) T_{\rho\sigma}(y) \rangle = & \frac{1}{|x-y|^8} \left\{ \left(X_\mu X_\nu - \frac{g_{\mu\nu}}{4} \right) \left(Y_\rho Y_\sigma - \frac{g_{\rho\sigma}}{4} \right) A(v) \right. \\ & + \left(X_\mu Y_\rho I_{\nu\sigma} + X_\mu Y_\sigma I_{\nu\rho} + X_\nu Y_\rho I_{\mu\sigma} - g_{\mu\nu} Y_\rho Y_\sigma - g_{\rho\sigma} Y_\mu Y_\nu + \frac{1}{4} g_{\mu\nu} g_{\rho\sigma} \right) B(v) \\ & \left. + I_{\mu\nu\rho\sigma} C(v) \right\} \end{aligned}$$

$$A(v) = 4\alpha(6v^6 + 3v^4 + v^2), \quad B(v) = -\alpha(3v^6 - v^4 - 2v^2), \quad C(v) = \alpha v^2 (v^2 - 1)^2$$

$$v = \frac{\xi}{1+\xi}, \qquad \xi = \frac{|x-y|^2}{4 x_3 y_3}, \quad X_\mu = v \left(\frac{2 x_3 (x^\mu - y^\mu)}{|x-y|^2} - n_\mu \right), \quad Y_\mu = X_\mu |_{x \leftrightarrow y}$$

$$\alpha = \frac{8k\,(k^2 - 1)}{9\pi^2 g_{\mathrm{YM}}^2}$$

Other similar set-ups and results

	D3-D5	D3-D7	D3-D7	D2-D4 (IIA)
Susy	1/2 BPS	None	None	1/2 BPS
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$	$AdS_3 \times CP^1$
$\|MPS\rangle$	Integrable	Non-integrable	Integrable	Integrable
1-pt functions	Exact formula derived at tree level and one-loop. Bootstrapped to all orders. Finite size effects via TBA.	?	Exact formula at tree level. —	Exact formula at tree level. —
Match to strings:	yes	yes	yes	?

Future directions

- Transport properties of the defect
- Other conf. data related to displacement operator two-point function
- Complete classification of boundary operators
- Bulk-boundary couplings from BOE and 2-pt functions
- Bootstrapping defect three-point functions

Thank you