

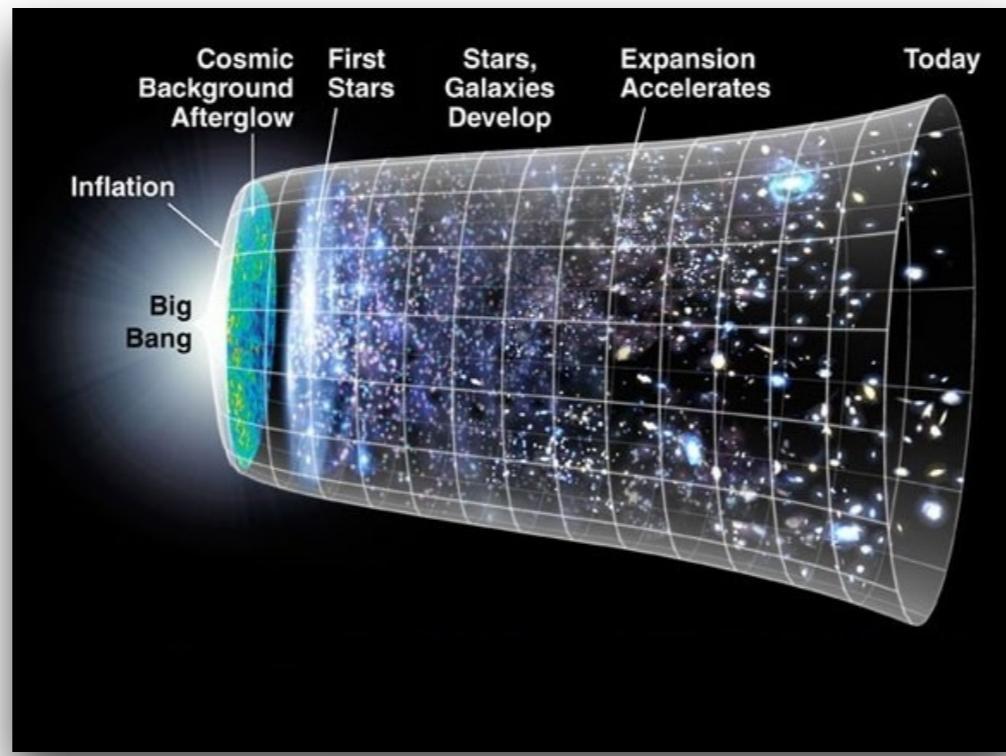
# Gravitational waves from inflation

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*The University of Groningen*

Theory Colloquium - Hamburg - October 26th 2022

*Talk based on papers in collaboration with:*

Peter Adshead, Niayesh Afshordi, Matteo Fasiello, Tomohiro Fujita,  
Marc Kamionkowski, Eugene Lim, Ameek Malhotra,  
Daan Meerburg, Giorgio Orlando, Maresuke Shiraishi,  
Gianmassimo Tasinato



## Inflation predicts a stochastic gravitational wave background

- How does it look like?
- What info does it provide on inflation?
- How do we **characterise** it (and distinguish it from other SGWBs)?

— Frequency profile  
— Chirality  
— Non-Gaussianity  
— Anisotropies

# Stochastic background of gravitational waves

## Cosmological sources:

- \* **Inflation**
- \* Reheating
- \* Phase transitions
- \* Cosmic strings
- ...

## Astrophysical sources:

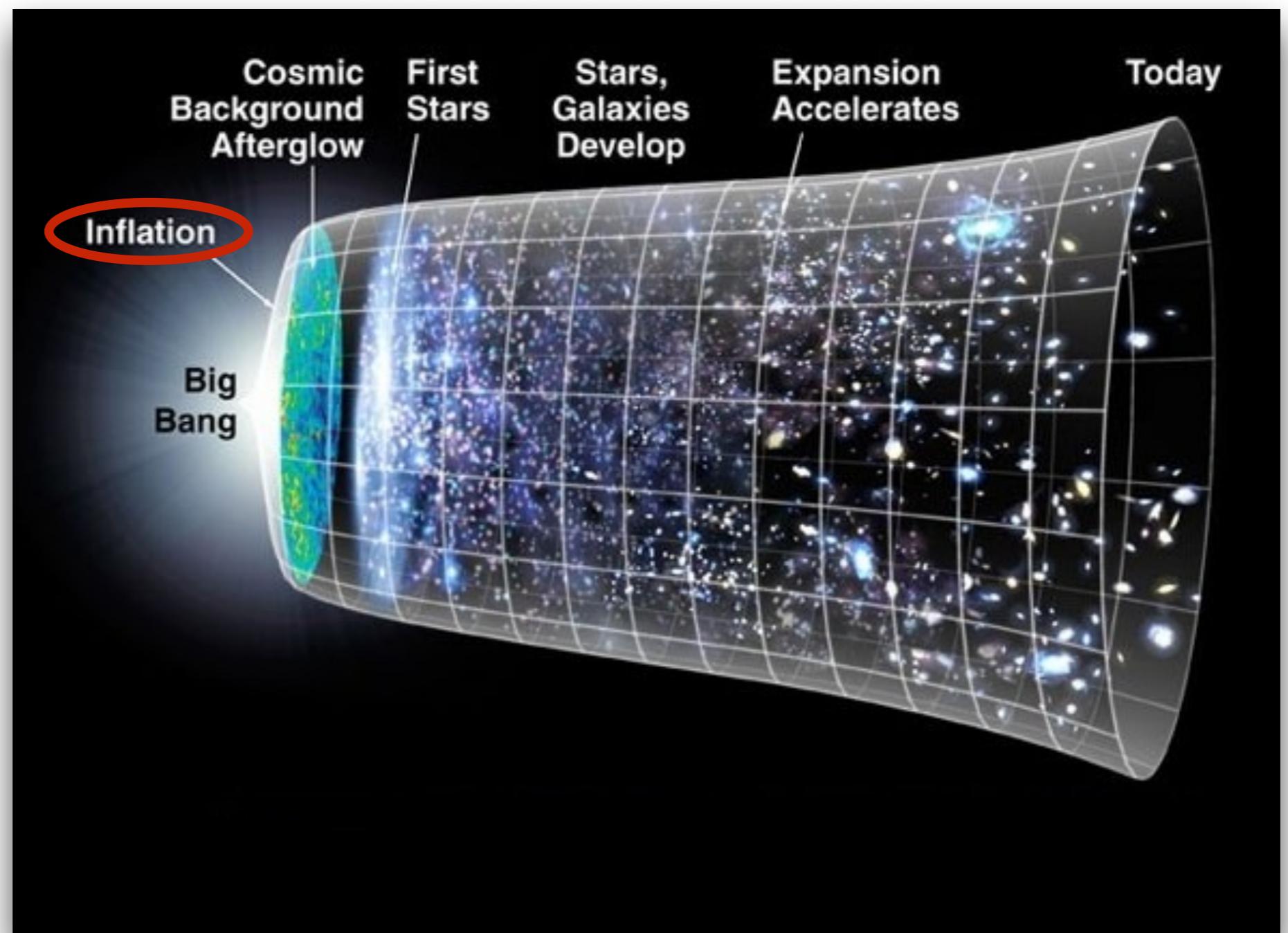
a stochastic gravitational wave background is expected, due e.g. to the superposition of signals from a large number of astrophysical sources (e.g. mergers of black holes, neutron stars,...)

# Introduction and motivations: cosmic inflation and primordial gravitational waves

# Inflation

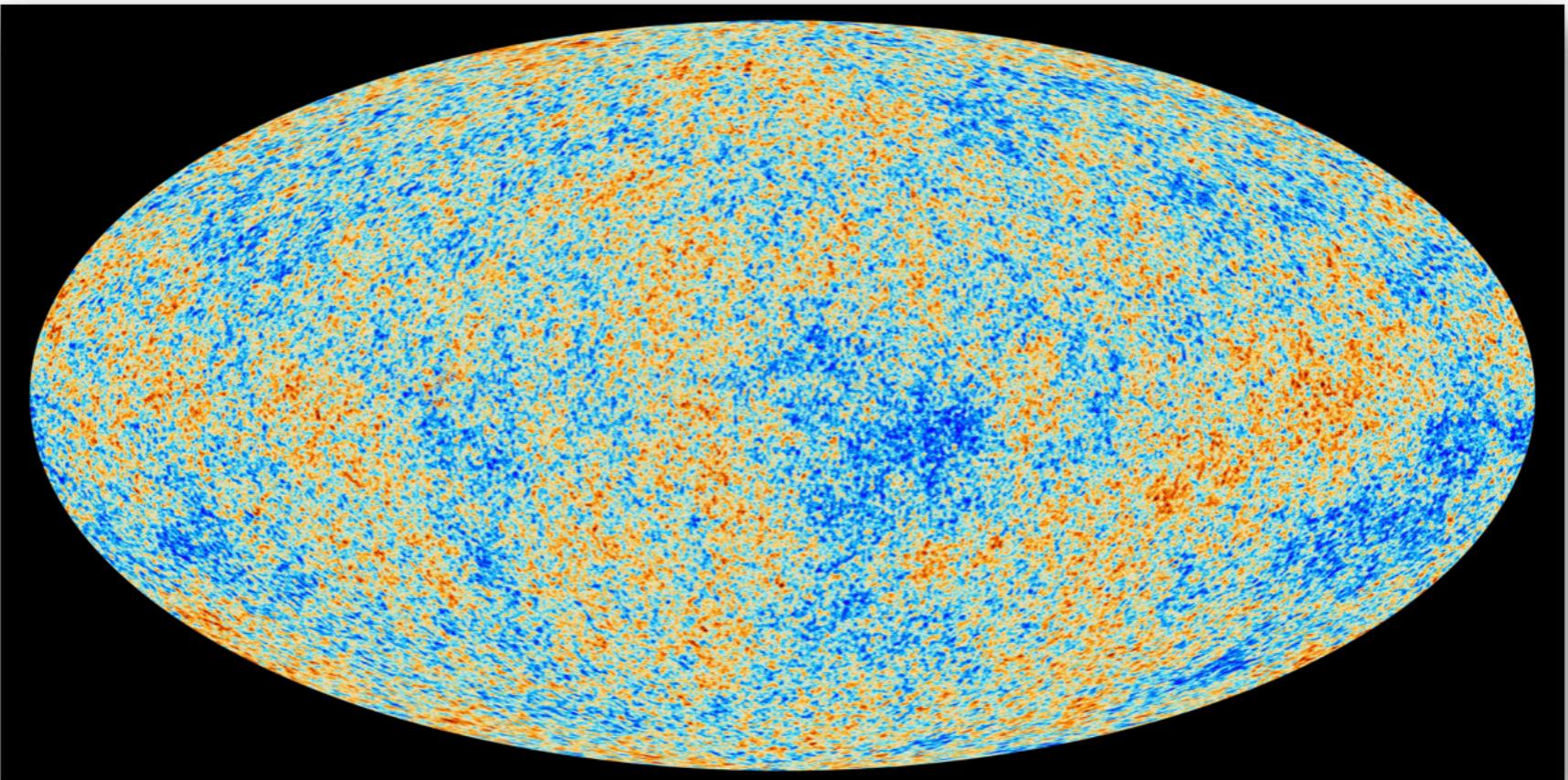
- era of accelerated (exponential) expansion

The universe over time



# Inflation

- era of accelerated (exponential) expansion
- explains why CMB is nearly uniform



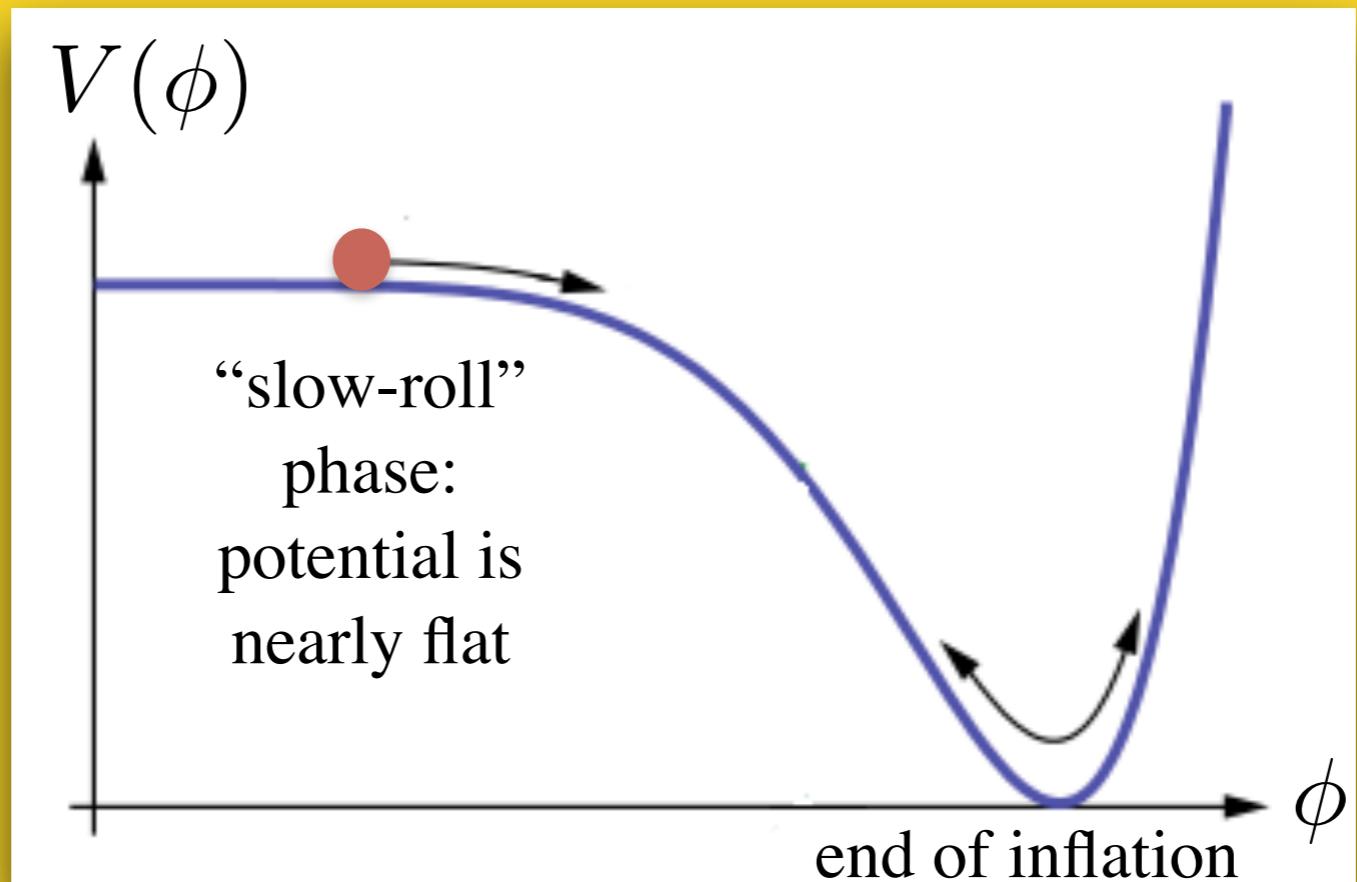
blackbody spectrum:  $\bar{T}_{CMB} \sim 2.7 K$

nearly isotropic:  $\Delta T \sim 10^{-5}$

# Inflation

- era of accelerated (exponential) expansion
- explains why CMB is nearly uniform
- explains how those fluctuations are generated

Simplest realization: single-scalar field in slow-roll (SFSR)



# Inflation

- era of accelerated exponential expansion
- explains why CMB is nearly uniform
- explains how those fluctuations are generated

$$\varphi(\vec{x}, t) = \phi(t) + \delta\phi(\vec{x}, t)$$

classical  
homogeneous  
background



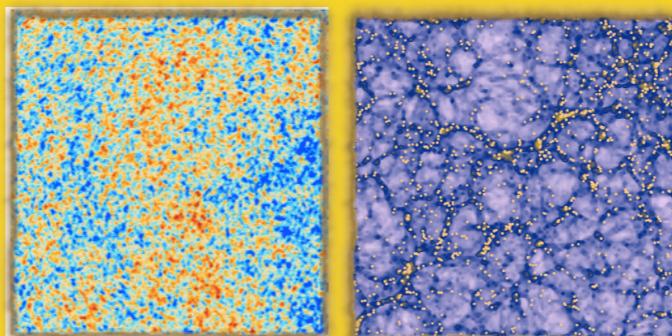
quantum  
fluctuations

perturbation modes are stretched by the expansion,  
become super horizon and freeze out to their value at horizon exit

$$\lambda = a(t)\lambda_c$$

$$\Delta T \quad \delta\rho$$

cosmological  
perturbations

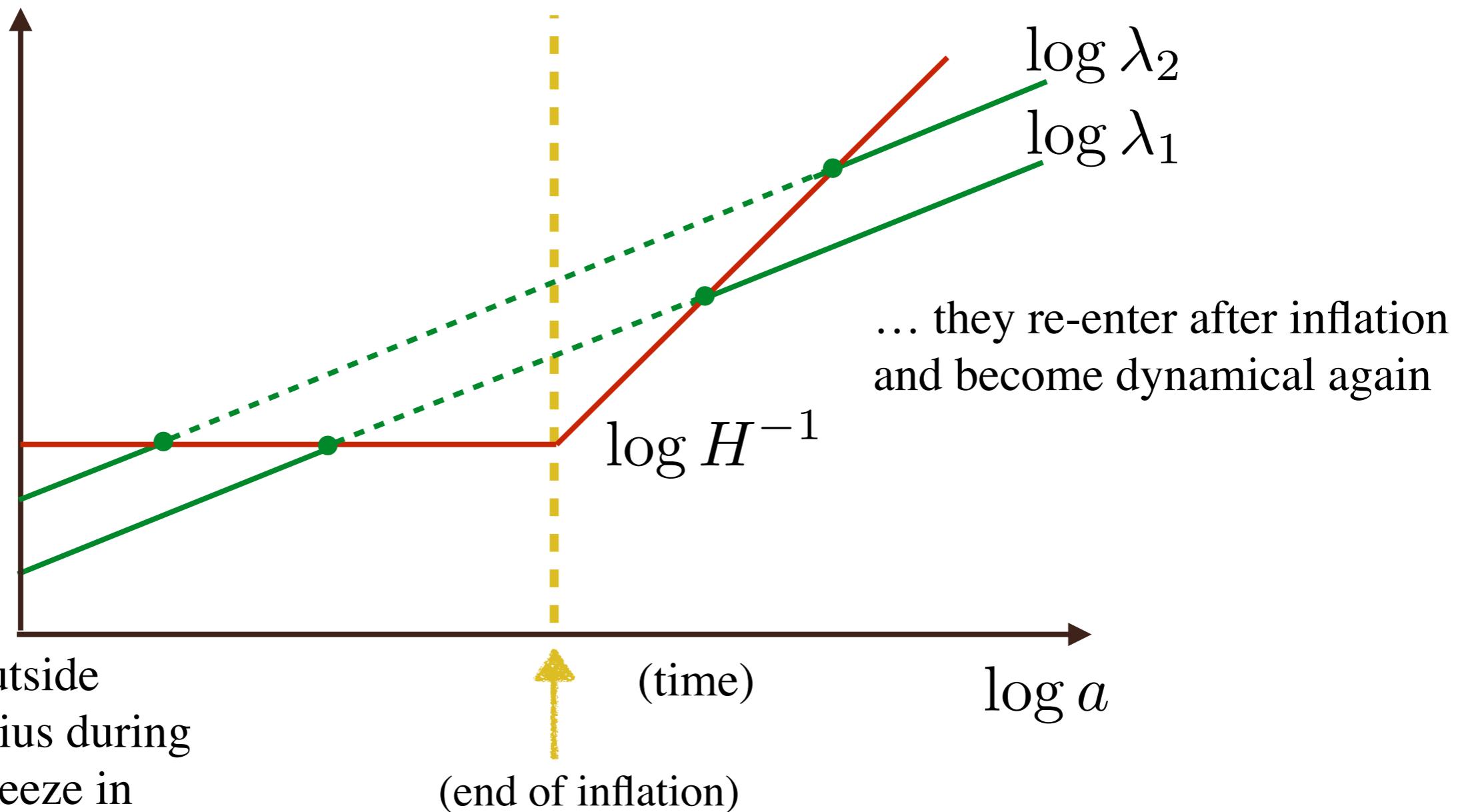


# Scales

wavenumber → e-folding → time of re-entry

$k$

$N_k$



# Gravitational waves

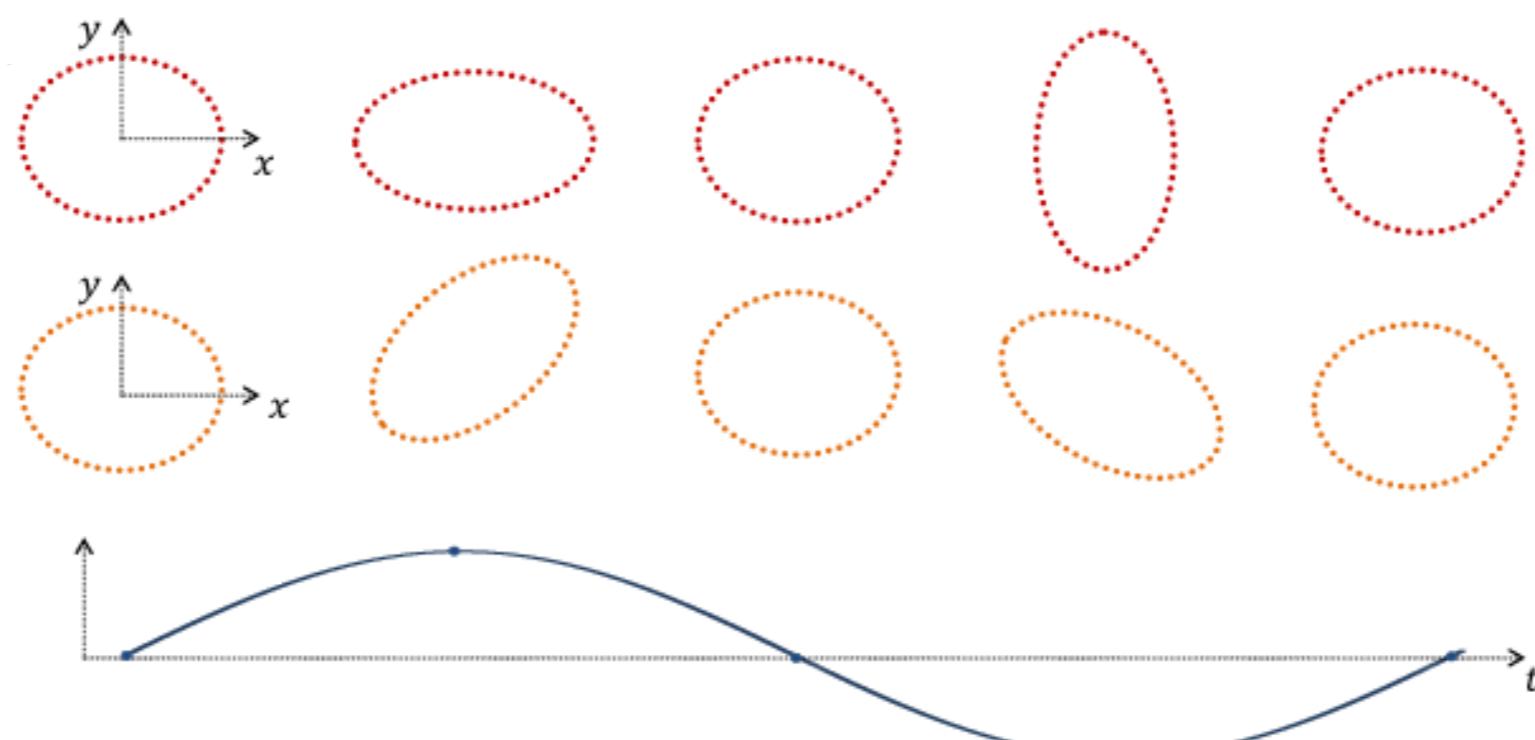
## Einstein equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

# Perturbation around FRLW (homogenous&isotropic) background

$$ds^2 = -dt^2 + a^2(t) (\delta_{ij} + \gamma_{ij}) dx^i dx^j$$

$\gamma_i^i = \partial_i \gamma_{ij} = 0$   **two polarization states of the graviton: +, x**



# Gravitational waves

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G \Pi_{ij}^{TT}$$

- **homogeneous** solution: GWs from **vacuum fluctuations**

Production of gravitons out of the vacuum  
in an expanding universe!

- **inhomogeneous** solution: GWs from **sources**

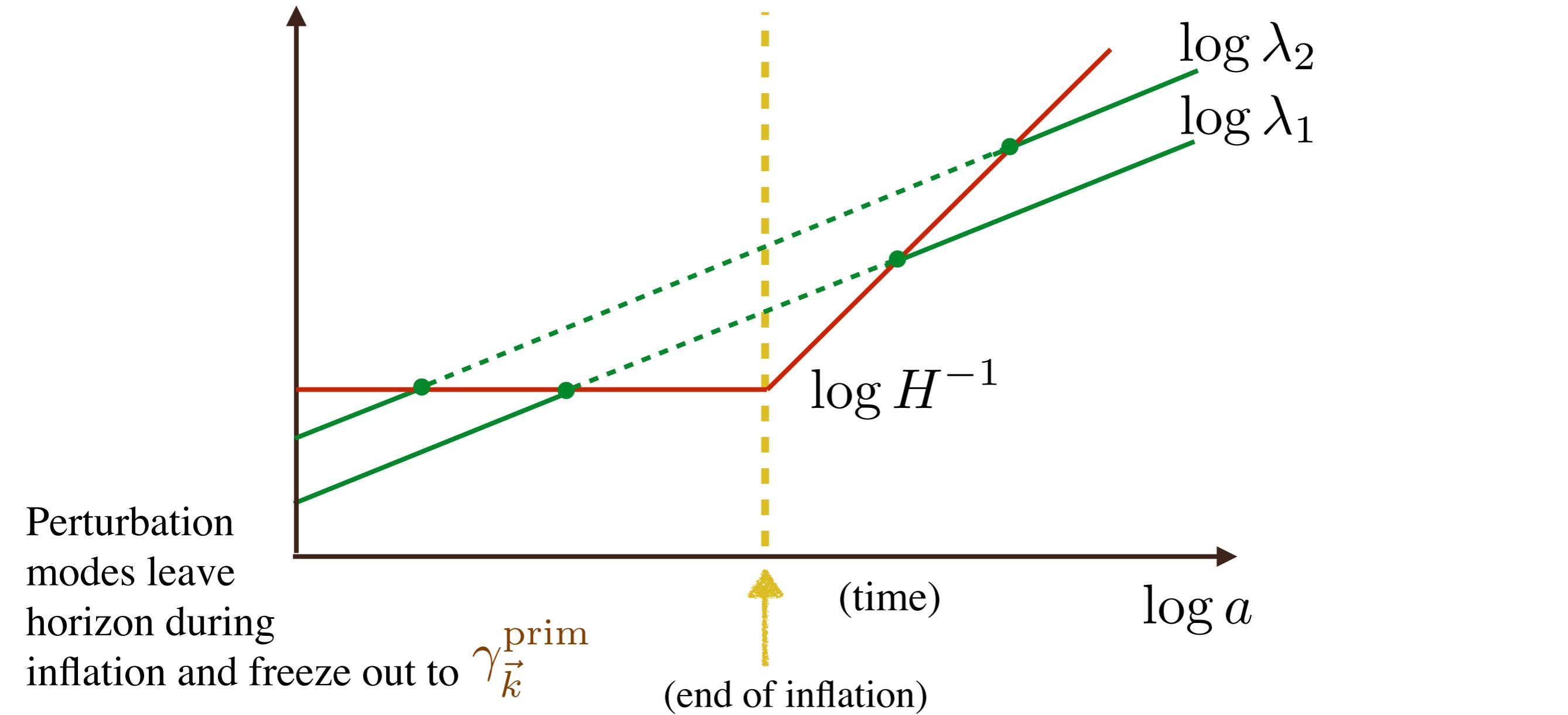
$$\Pi_{ij}^{TT} \propto \{\text{scalar fields, vector fields, fermions, tensors ...}\}$$

# Scales

... they re-enter after inflation  
and become dynamical again

$$\gamma_{\vec{k}}(\tau) = T(\tau, k) \gamma_{\vec{k}}^{\text{prim}}$$

transfer function  
(subhorizon evolution)



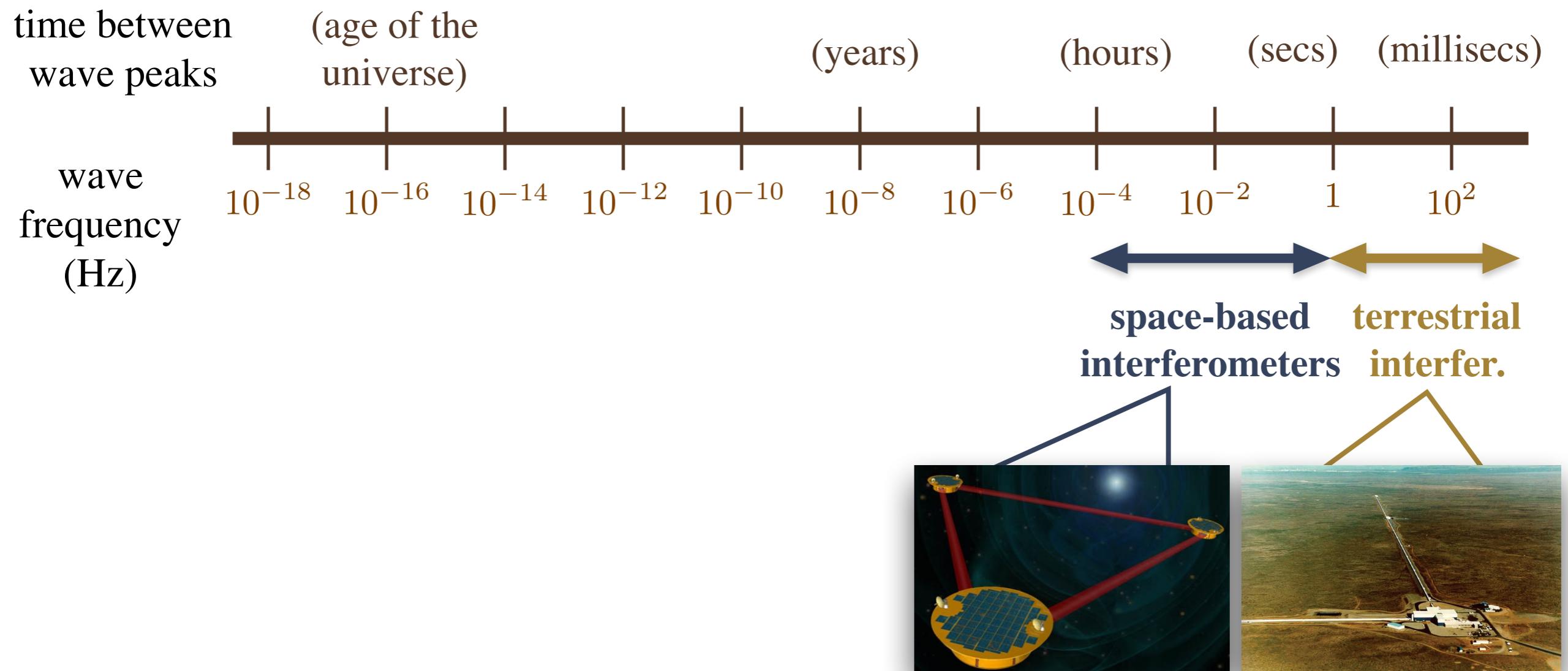
# Current bounds and detectability

# Direct detection

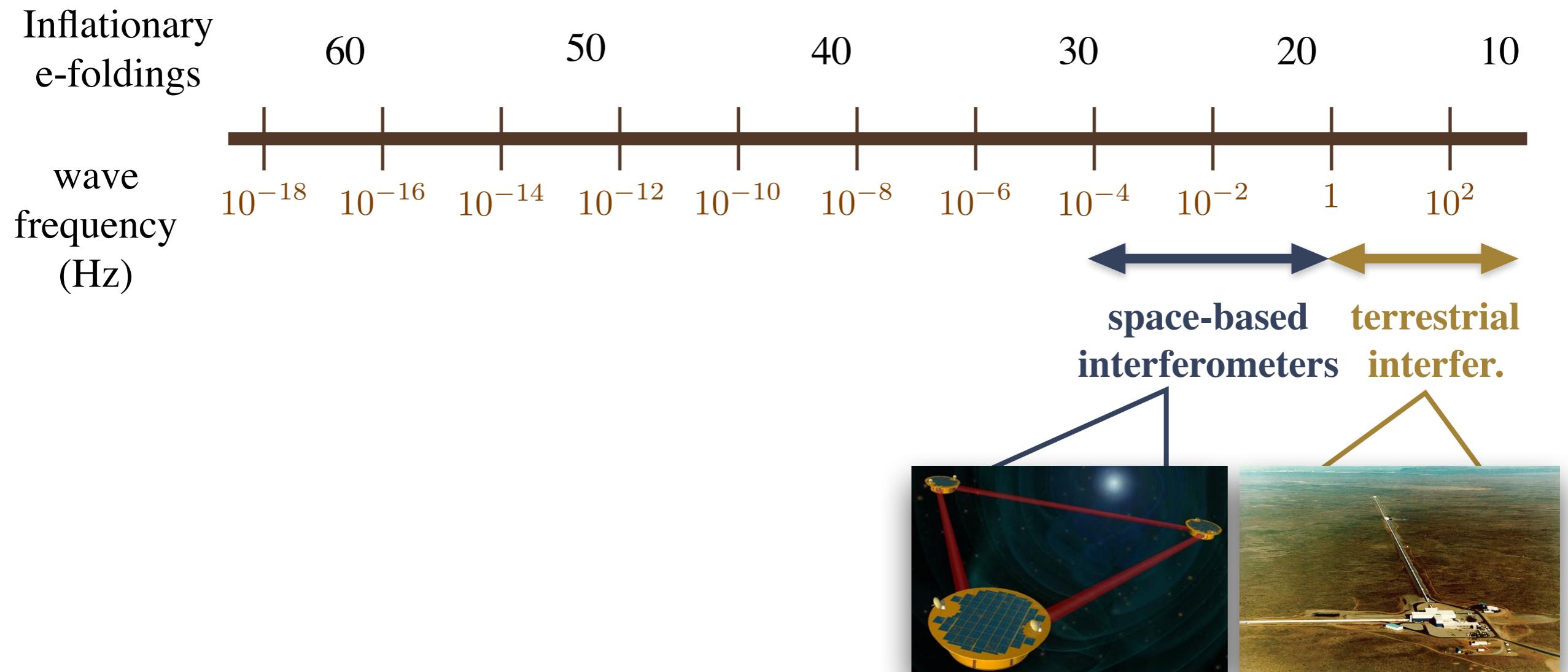
Gravitational waves travel freely until today

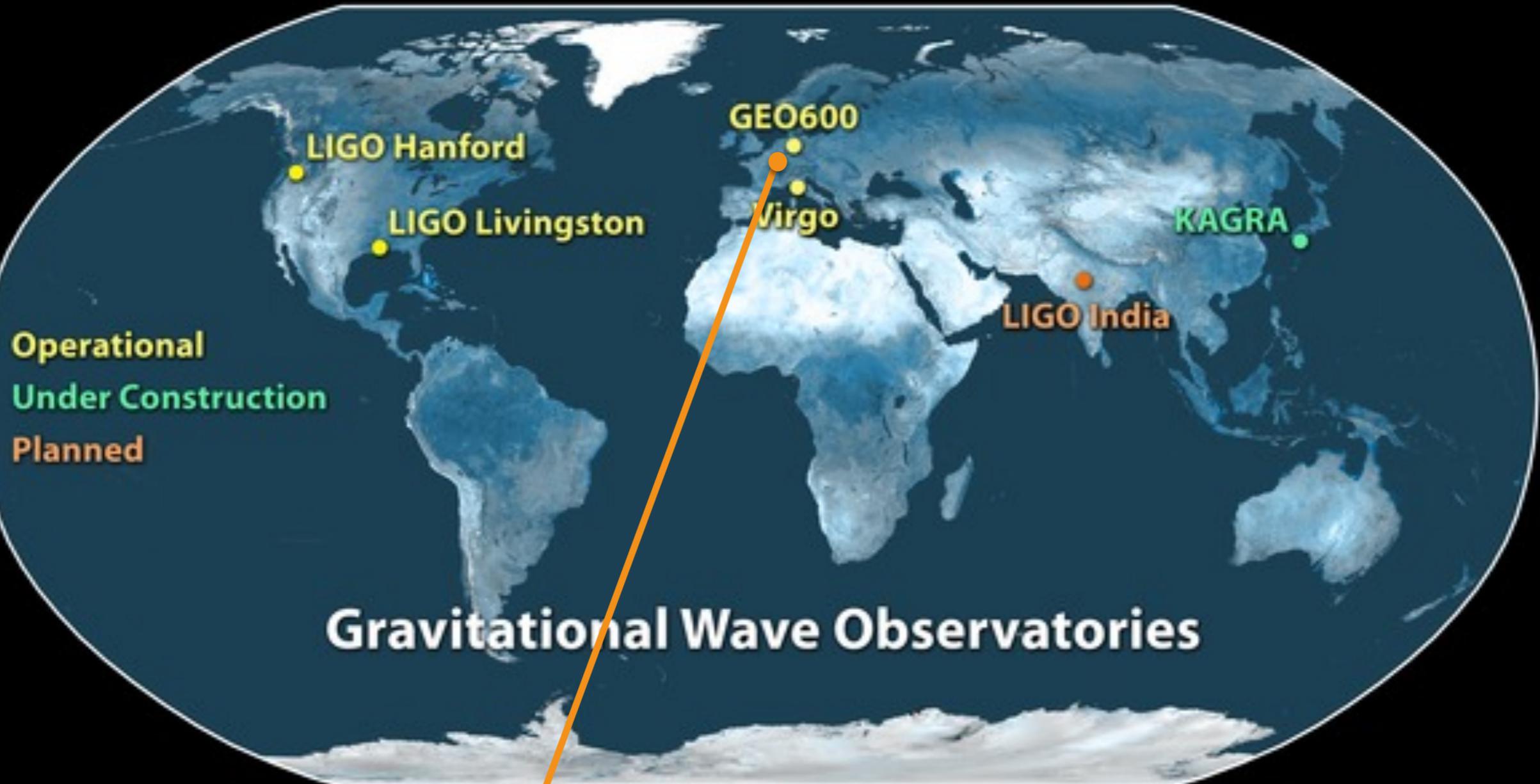
Space get distorted as gravitational wave passes by a detector

# Scales – Experiments



# Scales – Experiments

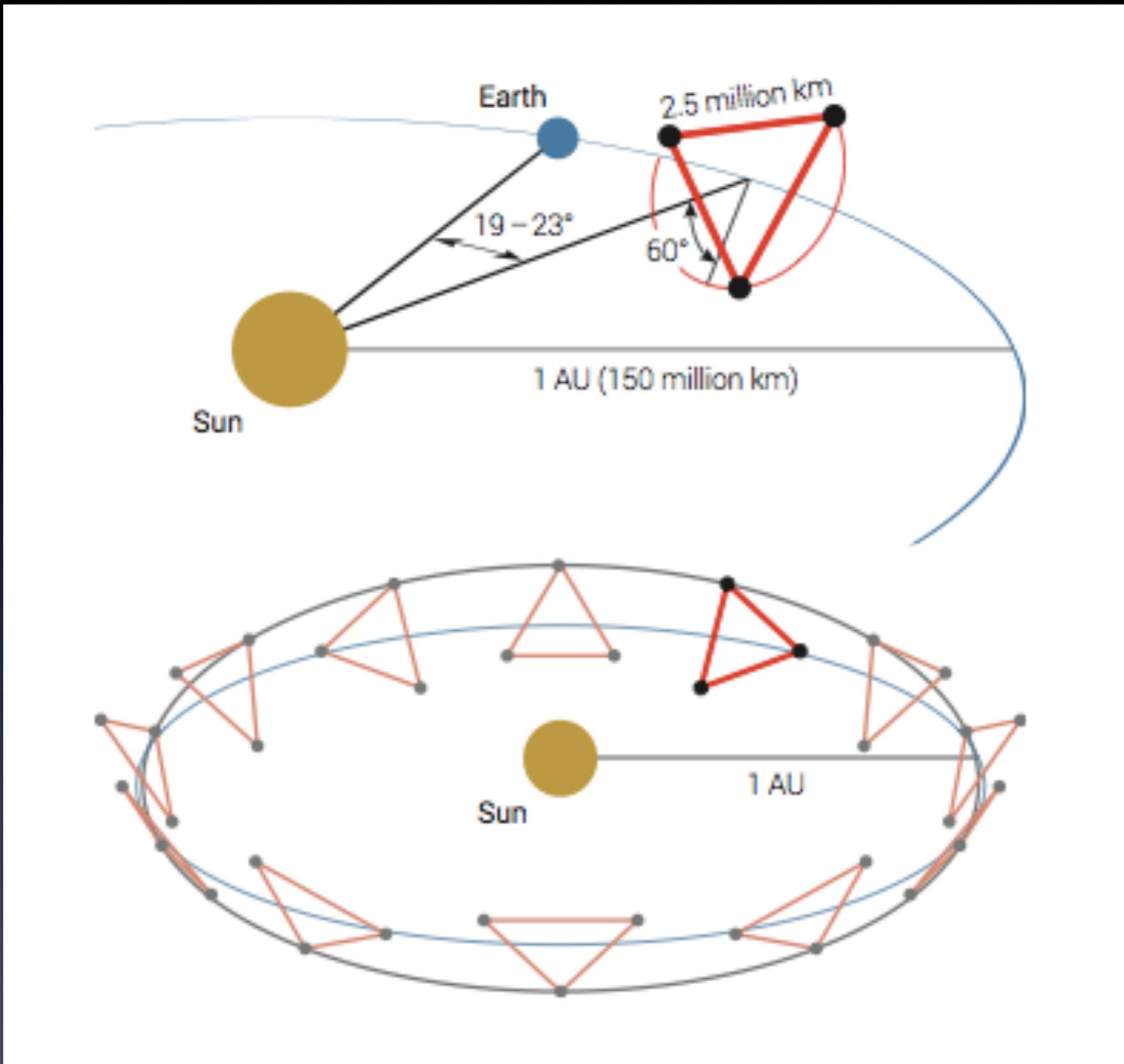




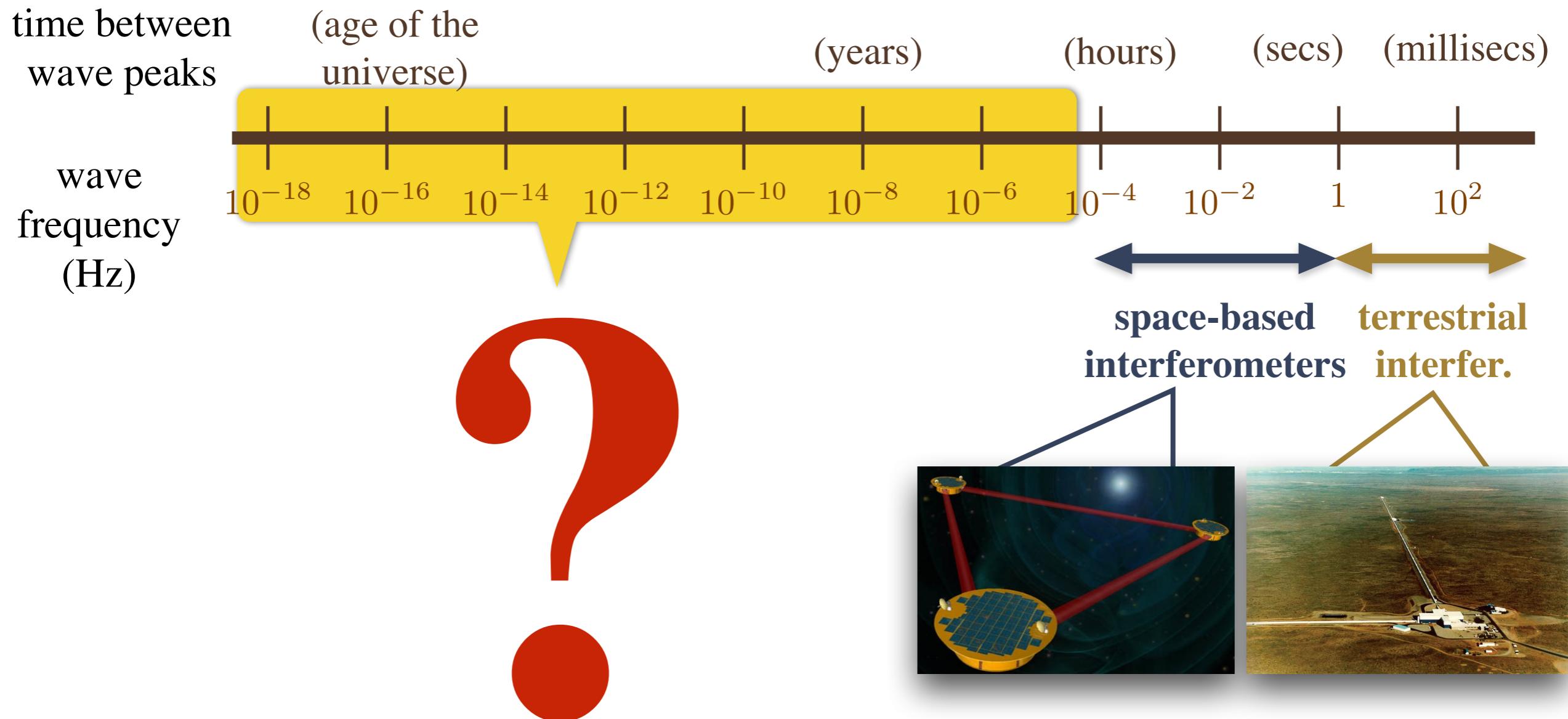
Einstein Telescope (ET)

LIGO website

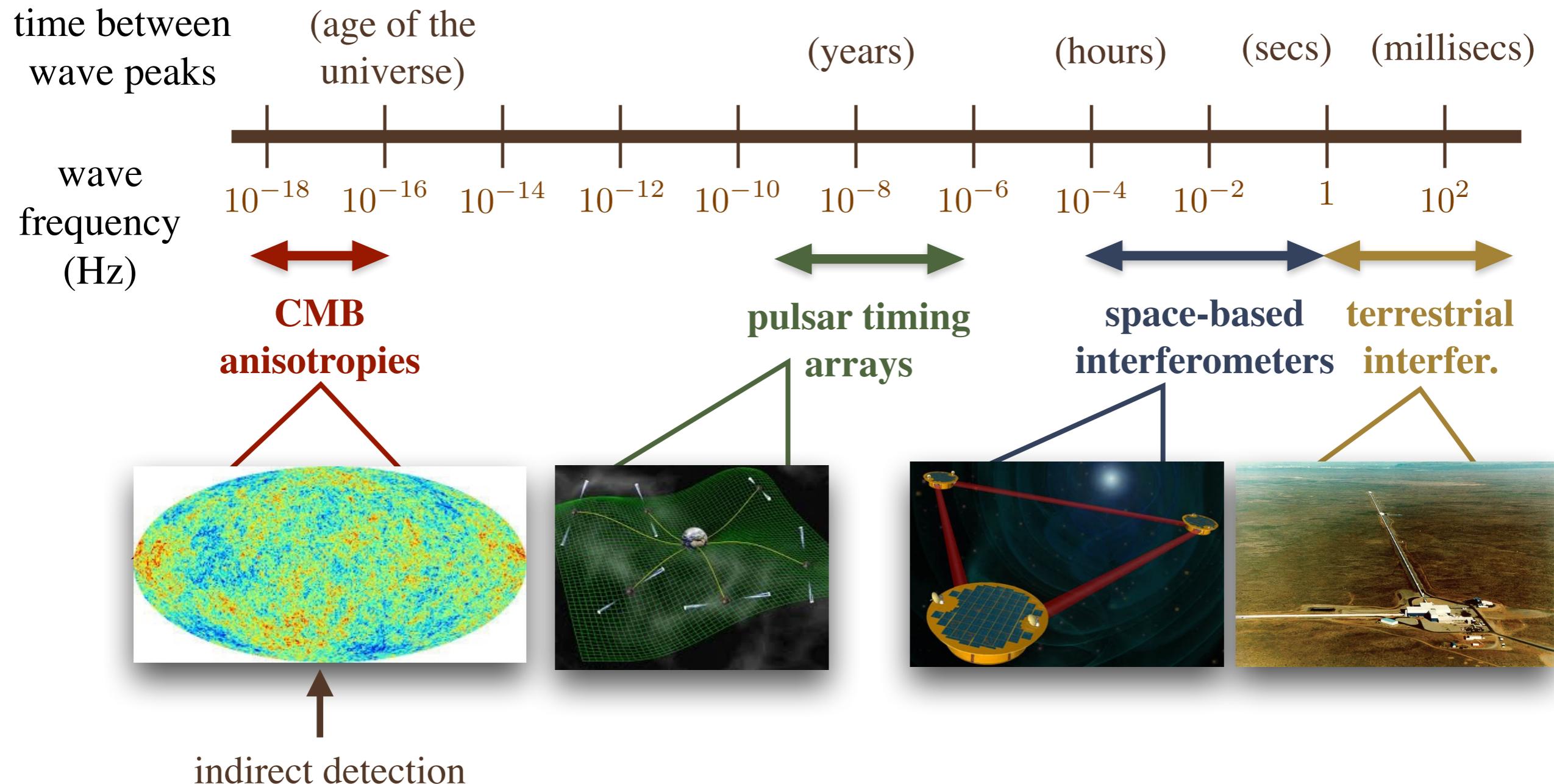
# LISA: laser interferometer space antenna



# Scales – Experiments



# Scales – Experiments



# Primordial GW: **indirect** detection

In the presence of GW photon propagation occurs along the perturbed geodesics



**temperature anisotropies**

Thomson scattering of radiation with quadrupole anisotropy by free electrons

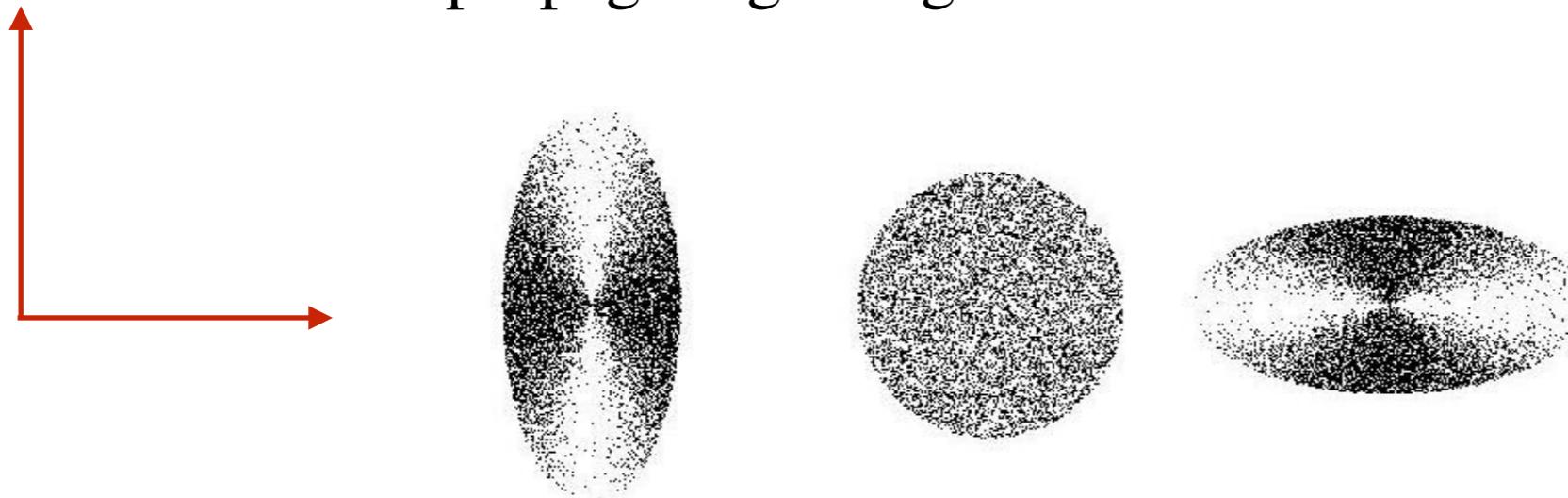


**net linear polarization** for the scattered radiation

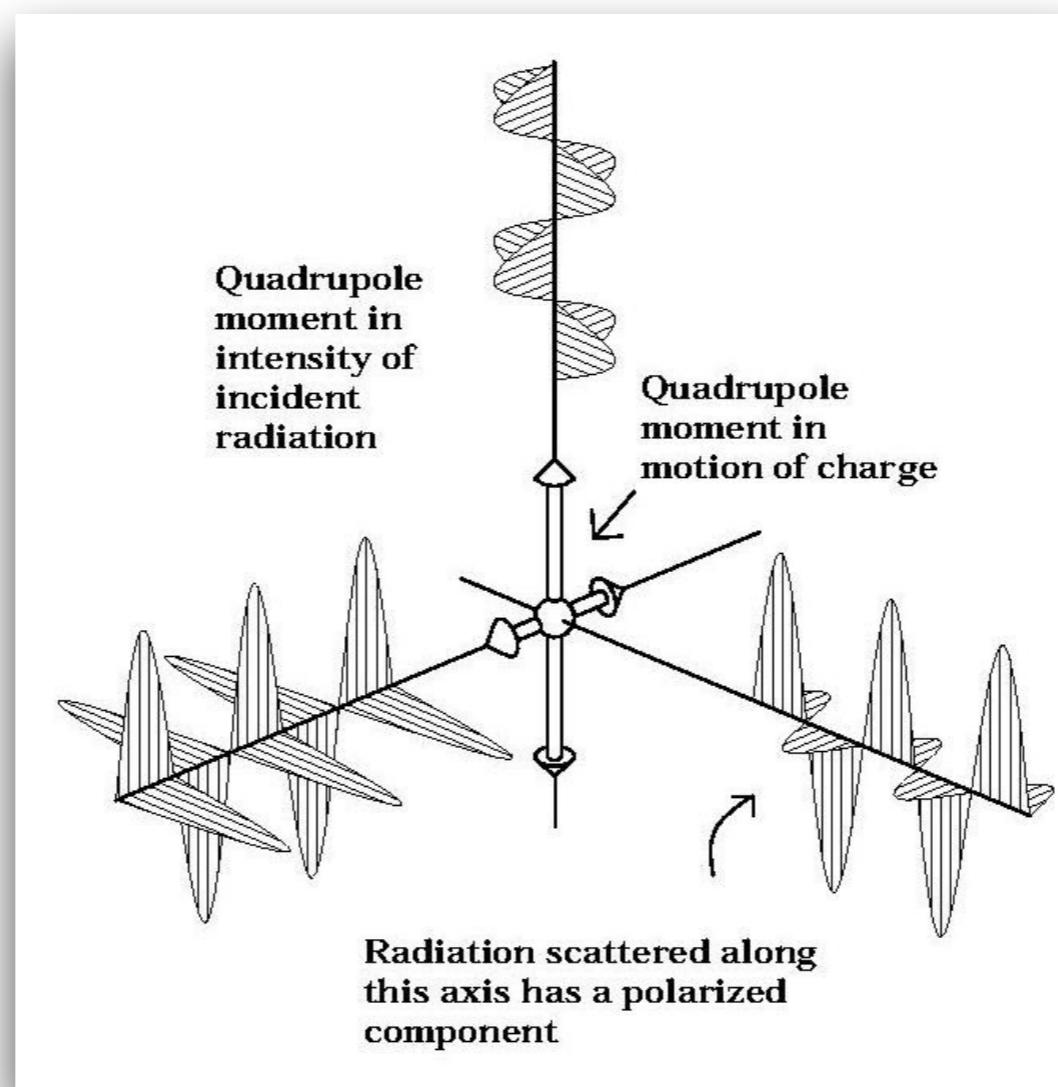


**B modes**

Distortion of a circular patch of homogenous plasma due to the passing of a gravitational wave: wavelength of photons propagating along the two axis are also distorted!



Thomson scattering



# Observational bounds/sensitivities

$r_{0.002 \text{ Mpc}^{-1}} < 0.056$   
(Planck+BICEP2/KECK)



$$V^{1/4} \lesssim 1.6 \times 10^{16} \text{ GeV}$$

Upper bound on the energy scale  
of inflation

More to come:  
BICEP Array, SPT-3G,  
Simons Observatory, CMB-S4,  
LiteBIRD, PICO, ...

What info do gravitational waves  
provide on inflation?

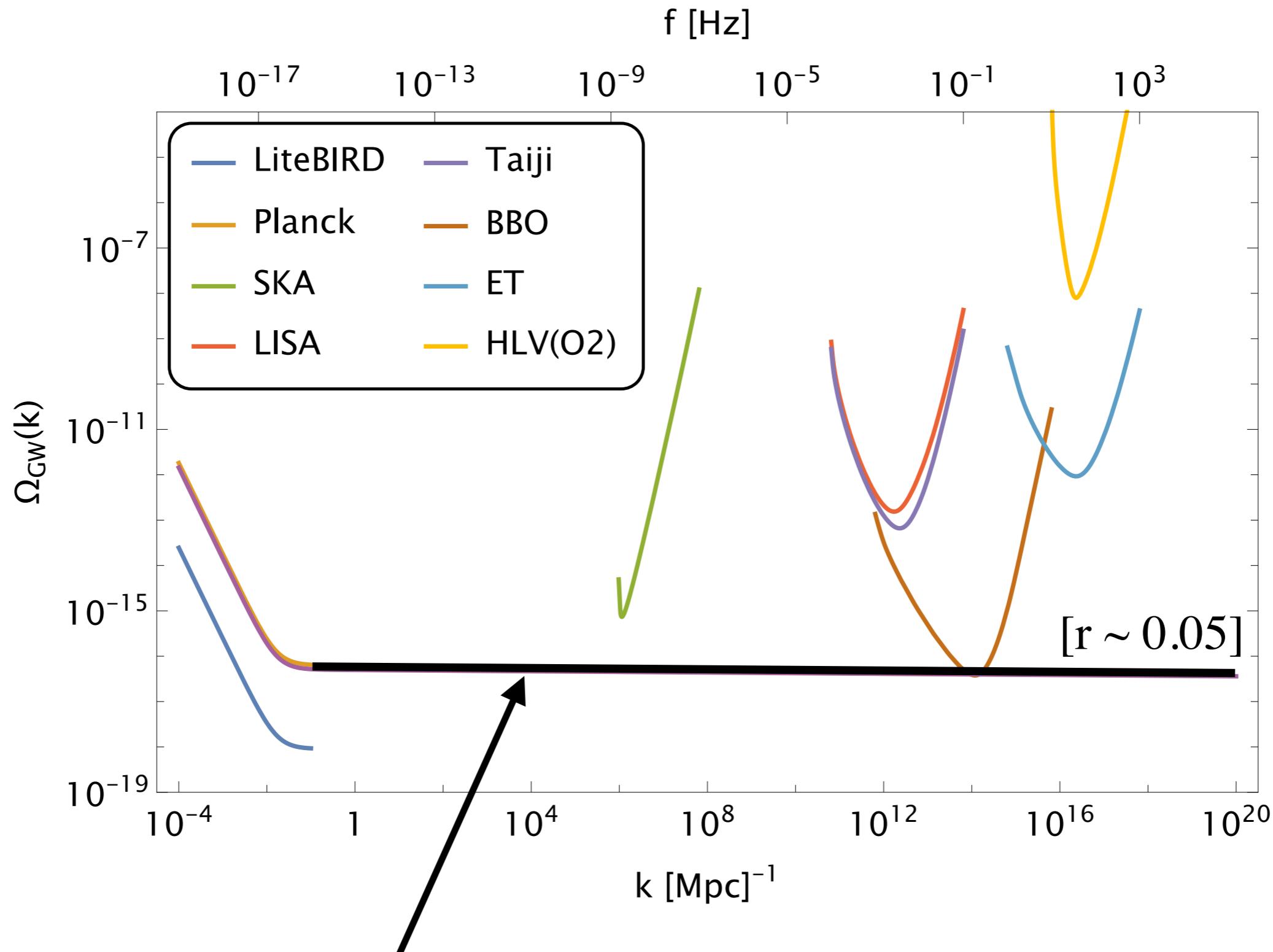
# GW can tell us a whole lot about inflation: examples

- GW from the amplification of vacuum fluctuations

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 0$$

Production of gravitons out of the vacuum  
in an expanding universe!

# Prediction and sensitivity limits



Standard SFSR would go undetected at small scales (**red tilt**)

# Inflationary GW from vacuum fluctuations (SFSR)

- Energy scale of inflation:
$$V_{\text{inf}}^{1/4} \simeq 10^{16} \text{GeV}(r/0.01)^{1/4}$$
$$H \simeq 2 \times 10^{13} \text{GeV}(r/0.01)^{1/2}$$
- Red **tilt**:
$$n_T \simeq -2\epsilon = -r/8$$
- Non-**chiral**:
$$P_L = P_R$$
- Nearly **Gaussian**:
$$f_{\text{NL}} \ll 1$$

## **GW can tell us a whole lot about inflation:**

- GW from the amplification of vacuum fluctuations
- Generation of GW from additional fields during inflation

# GW can tell us a whole lot about inflation: examples

- GW from the amplification of vacuum fluctuations
- Generation of GW from additional fields during inflation

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G \Pi_{ij}^{TT}$$

anisotropic  
stress-energy tensor

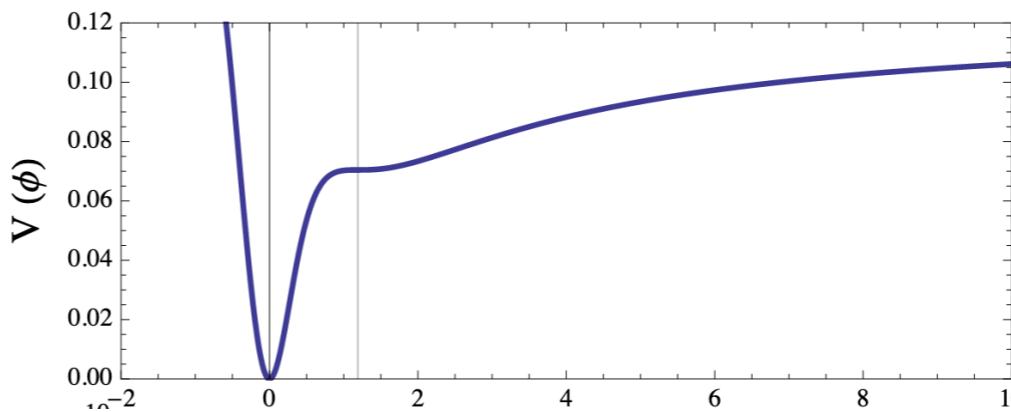
- Axion-gauge field models  $\frac{\lambda\chi}{4f}F\tilde{F}$   
**[Anber - Sorbo 2009, Cook - Sorbo 2011, Barnaby - Peloso 2011, Adshead - Wyman 2011, Maleknejad - Sheikh-Jabbari, 2011, ED - Fasiello - Tolley 2012, ED - Peloso 2012, Namba - ED - Peloso 2013, Adshead - Martinec - Wyman 2013, ED - Fasiello - Fujita 2016 Agrawal - Fujita - Komatsu 2017, Caldwell - Devulder 2017, Domcke et al. 2018, ... ]**
- GW from extra non-minimally coupled spin-2 field (EFT formulation)  
**[Bordin et al, 2018; ...]**
- Spectator fields with small sound speed  
$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G \Pi_{ij}^{TT} \propto \partial_i\sigma\partial_j\sigma$$
  
**[Biagetti, Fasiello, Riotto 2012, Biagetti, ED, Fasiello, Peloso 2014, ...]**
- ...

# GW can tell us a whole lot about inflation:

- GW from the amplification of vacuum fluctuations
- Generation of GW from additional fields during inflation
- Second order GW from peaks in the scalar power spectrum

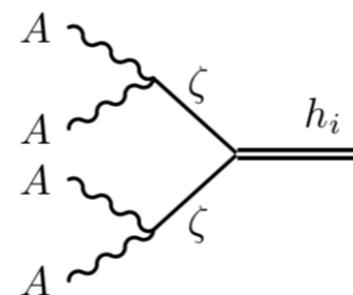
[Ananda et al - 2007,  
Baumann et al - 2007, ...]

- Potentials with inflection points



[Garcia-Bellido, Morales 2017]

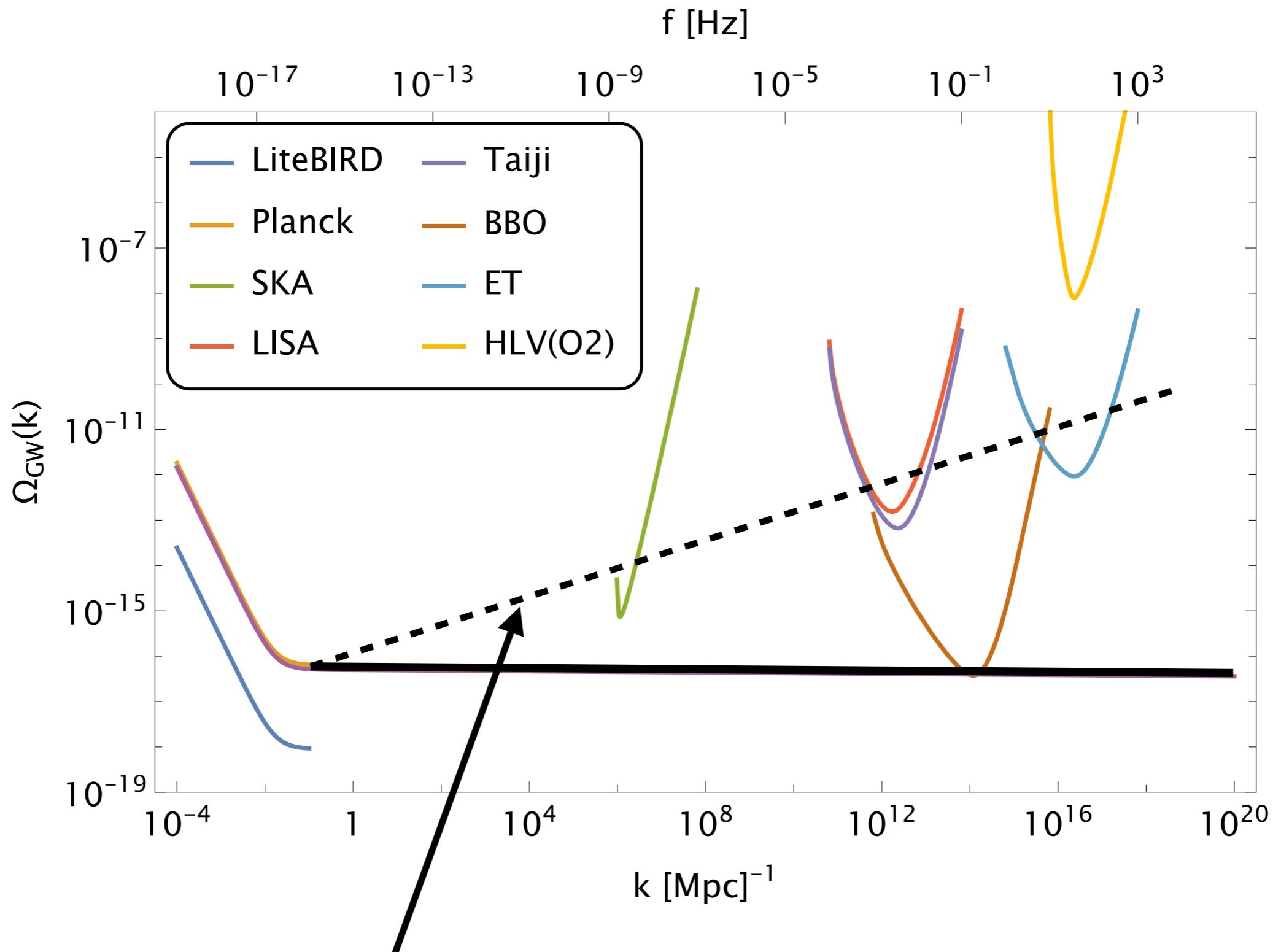
- Inflation with axion and gauge fields



[Garcia-Bellido, Peloso, Unal 2017]

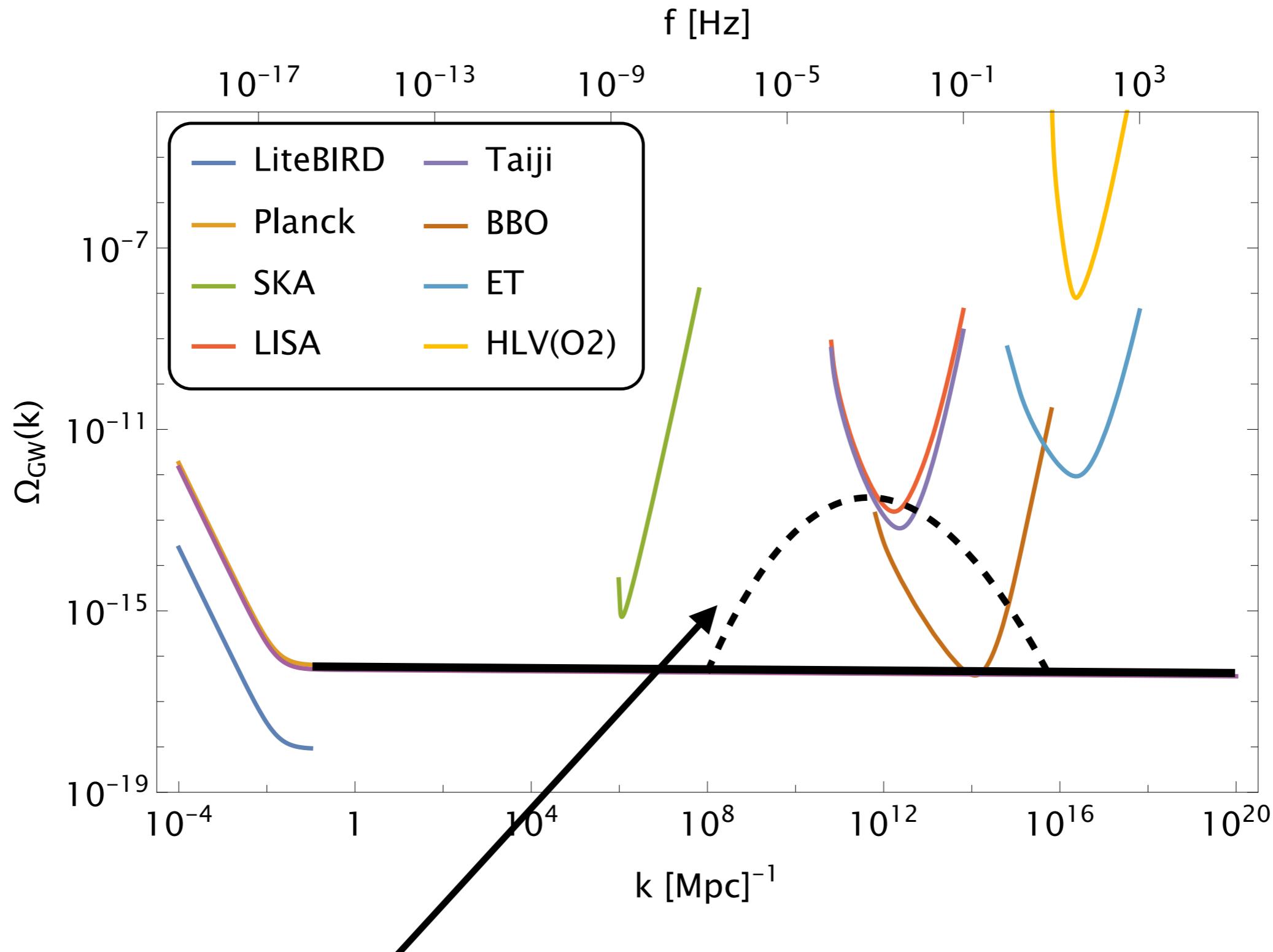
- ...

# Prediction and sensitivity limits



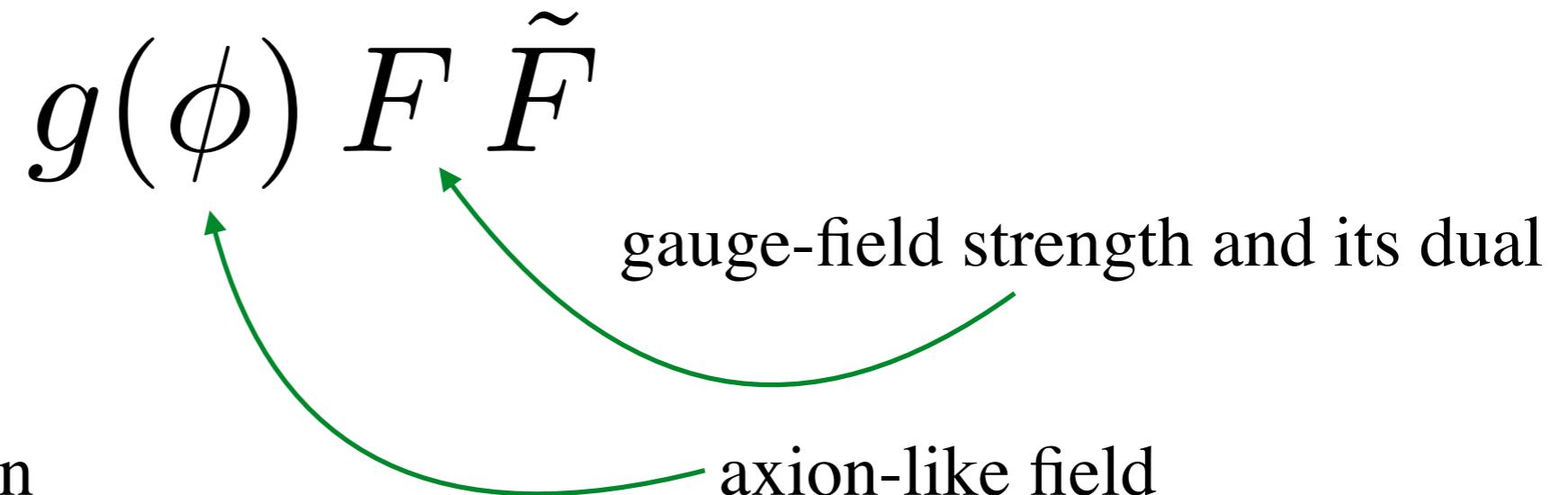
Power spectrum larger at small scales: e.g. **blue tilt**

# Prediction and sensitivity limits



Power spectrum larger at small scales: e.g. **bump**

# Axion-Gauge fields models: Chern-Simons coupling



- naturally light inflaton
- support reheating
- mechanism for baryogenesis
- primordial black holes formation
- **sourced chiral gravitational waves**

[Freese - Frieman - Olinto 1990, Anber - Sorbo 2009, Cook - Sorbo 2011, Barnaby - Peloso 2011, Adshead - Wyman 2011, Maleknejad - Sheikh-Jabbari, 2011, ED - Fasiello - Tolley 2012, ED - Peloso 2012, Namba - ED - Peloso 2013, Adshead - Martinec - Wyman 2013, ED - Fasiello - Fujita 2016, Garcia-Bellido - Peloso - Unal 2016, Agrawal - Fujita - Komatsu 2017, Fujita - Namba - Obata 2018, Domcke - Mukaida 2018, Kaloper-Westphal 2021, Iarygina - Sfakianakis 2021, ...]

# Axion-Gauge fields models: SU(2)



[Adshead - Wyman 2011]

$$\mathcal{L} = \mathcal{L}_{\text{inflaton}} - \frac{1}{2} (\partial\chi)^2 - U(\chi) - \frac{1}{4} FF + \frac{\lambda\chi}{4f} F\tilde{F}$$

$\downarrow$

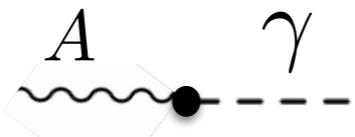
$$P_{\gamma, \text{vacuum}} \quad \mathcal{L}_{\text{spectator}} \rightarrow P_{\gamma, \text{sourced}}$$

- Inflaton field dominates energy density of the universe
- Spectator sector contribution to curvature fluctuations negligible

$$A_0^a = 0$$
$$A_i^a = aQ\delta_i^a$$

slow-roll background attractor solution

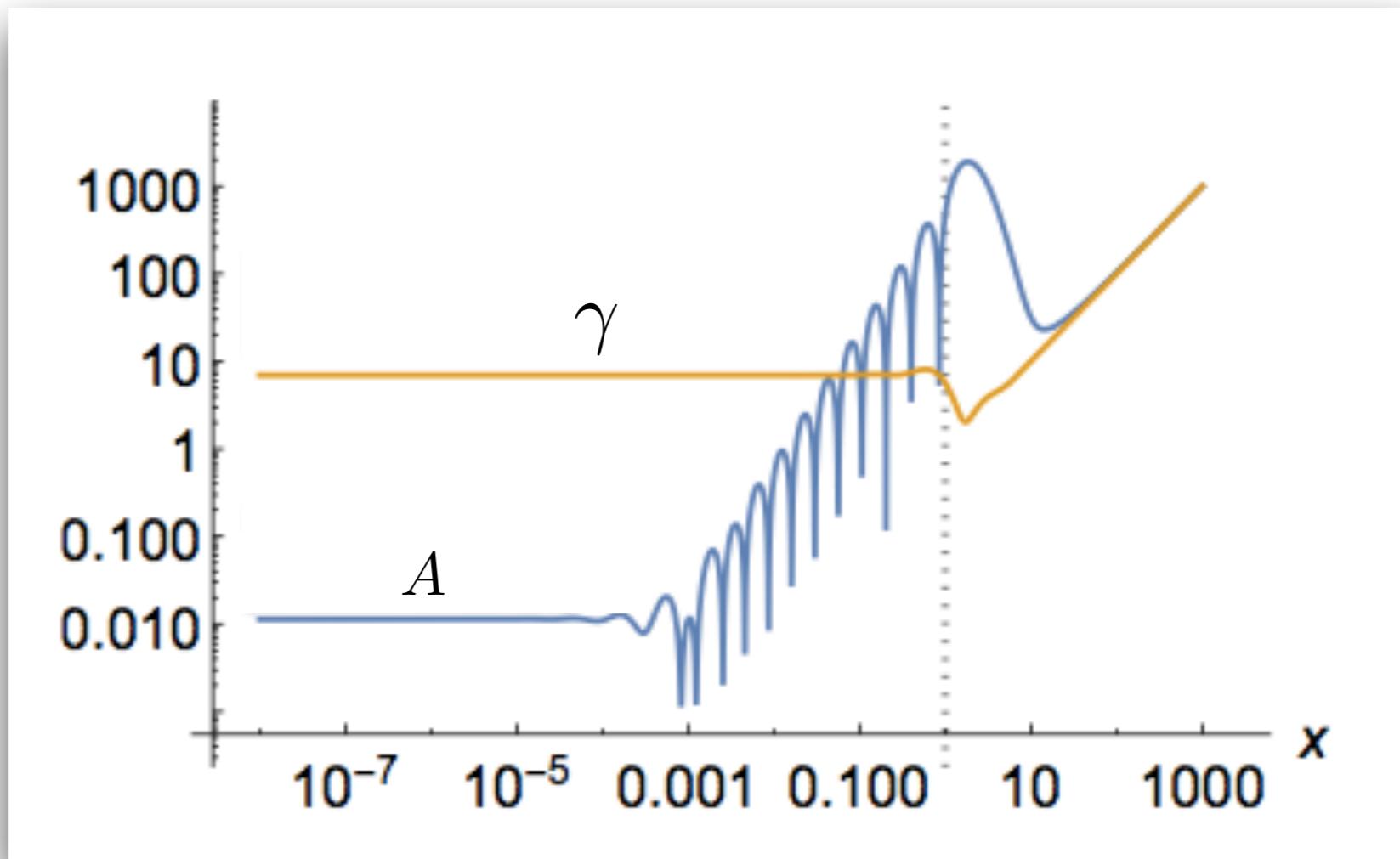
$$\delta A_i^a = t_{ai} + \dots \quad \text{TT-component}$$



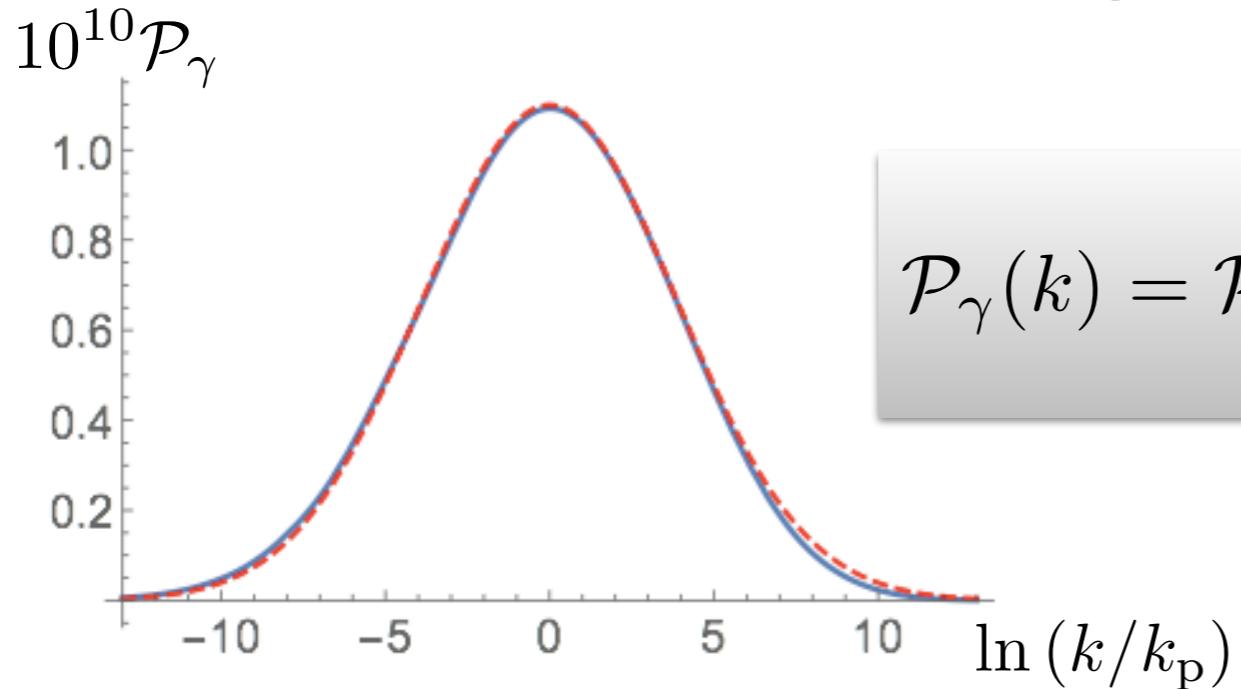
[ED-Fasiello-Fujita 2016]

# Axion-Gauge fields models: SU(2)

One helicity of the gauge field fluctuations is amplified from coupling with axion → the same helicity of the tensor mode is amplified

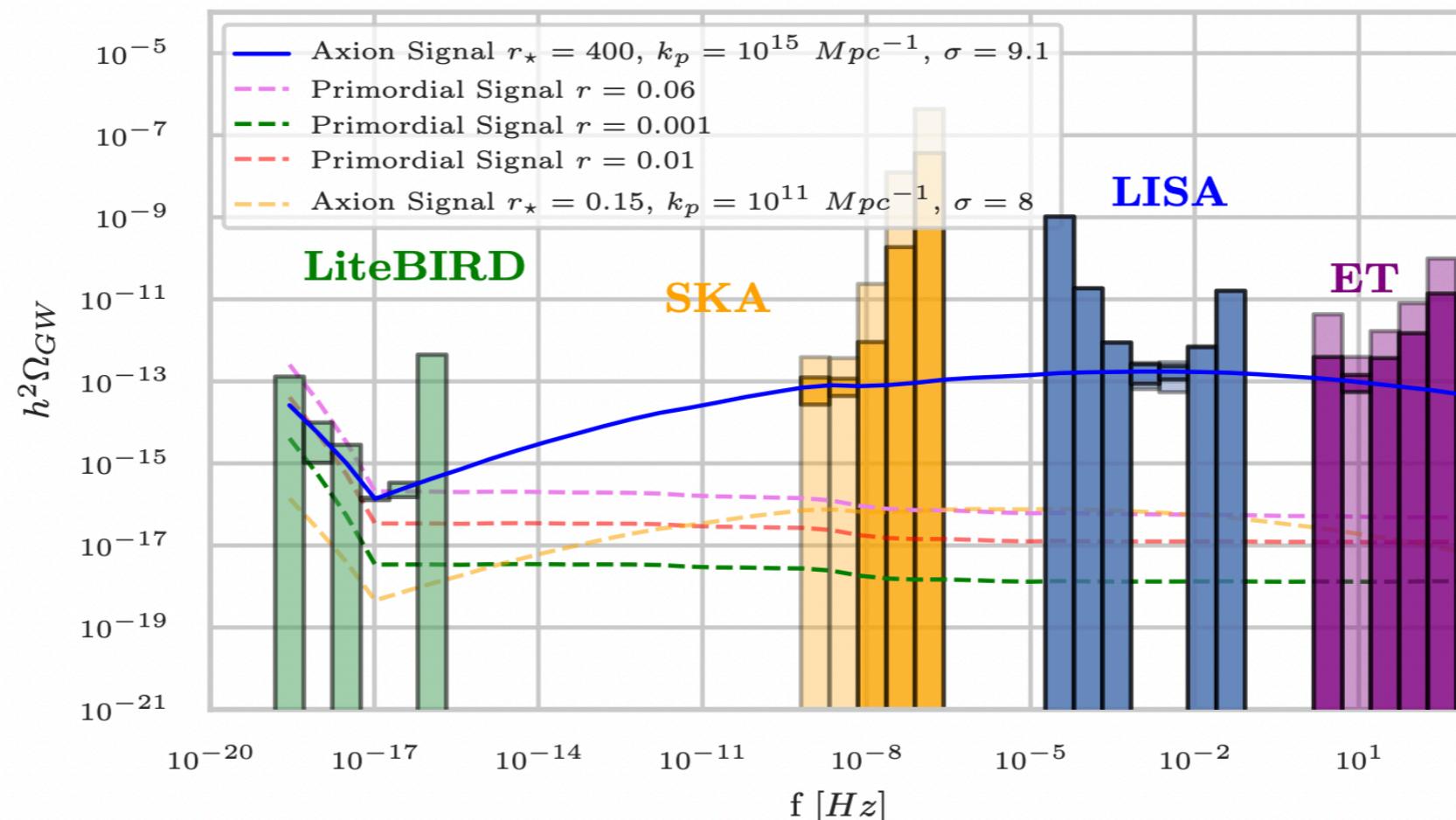


# Axion-Gauge fields models: SU(2)



$$\mathcal{P}_\gamma(k) = \mathcal{P}_{\gamma, L}^{(\text{sourced})}(k) = r_* \mathcal{P}_\zeta(k) e^{-\frac{1}{2\sigma^2} \ln^2(k/k_p)}$$

[ED-Fasiello-Fujita, 2016 – Thorne et al, 2017]



[Campeti et al, 2020]

# Inflationary GW from vacuum fluctuations (SFSR)

- ~~Energy scale~~ of inflation:  $V_{\text{inf}}^{1/4} \simeq 10^{16} \text{GeV}(r/0.01)^{1/4}$   
 $H \simeq 2 \times 10^{13} \text{GeV}(r/0.01)^{1/2}$
- ~~Red tilt~~:  $n_T \simeq -2\epsilon = -r/8$
- ~~Non-circular~~:  $P_L = P_R$
- Nearly ~~Gaussian~~:  $f_{\text{NL}} \ll 1$

How do we detect chirality?

# CMB angular power spectra & chirality

$$\mathcal{C}_\ell^{XY} = \int dk \Delta_\ell^X(k, \eta_0) \Delta_\ell^Y(k, \eta_0) [\mathcal{P}_\gamma^R(k) + \epsilon \cdot \mathcal{P}_\gamma^L(k)]$$

$X, Y = T, E, B$

$$\epsilon = \begin{cases} 1 & \text{for TT, EE, BB, TE} \\ -1 & \text{for TB, EB} \end{cases}$$

For parity-conserving theories

$$\langle TB \rangle, \langle EB \rangle = 0$$

For parity-violating theories

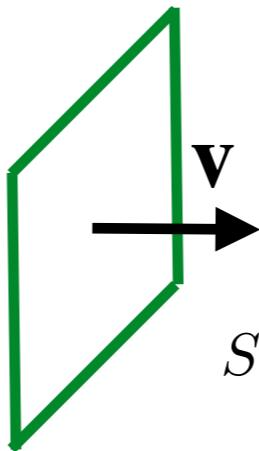
$$\langle TB \rangle, \langle EB \rangle \neq 0$$

# Constraining chirality at high frequencies (interferometers)

A **planar** detector cannot distinguish L from R  
for an isotropic SGWB

An ‘effective’ **non-planar** geometry can be realised by:

- using different (non co-planar) detectors at once → monopole
- exploiting the motion of a detector → higher multiples



use of kinematically induced dipole:

$$SNR \simeq \frac{v}{10^{-3}} \frac{\Omega_{\text{GW},R} - \Omega_{\text{GW},L}}{1.4 \cdot 10^{-11}} \sqrt{\frac{T}{3 \text{ years}}} \\ (\text{LISA, ET})$$

\* For (networks of) space-based interferometers see: Domcke et al, 2020 - Orlando et al., 2021

\* For ground-based networks see, e.g.: Seto-Taruya, 2007 – Smith-Caldwell, 2017

\* For PTA: Belgacem-Kamionkowski, 2020

[See also Seto 2006-2007]

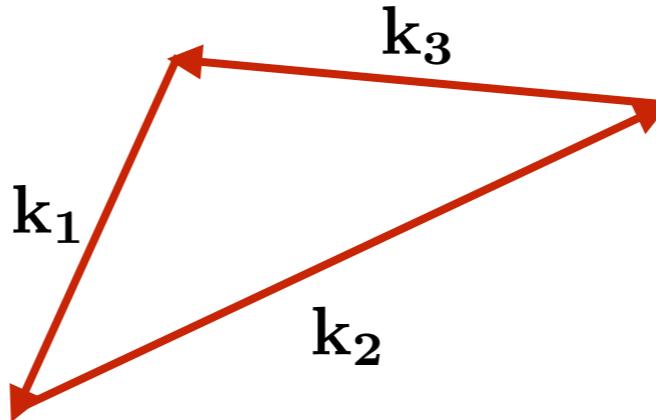
[See Komatsu et al 2017 for forecasts on chirality from our model with interferometers]

# Inflationary GW from vacuum fluctuations

- Energy scale of inflation:  $V_{\text{inf}}^{1/4} \simeq 10^{16} \text{GeV}(r/0.01)^{1/4}$   
 $H \simeq 2 \times 10^{13} \text{GeV}(r/0.01)^{1/2}$
- Red tilt:  $n_T \simeq -2\epsilon = -r/8$
- Non-chiral:  $P_L = P_R$
- Nearly Gaussian:  $f_{\text{NL}} \ll 1$

# Primordial non-Gaussianity and anisotropies in the GW energy density

# Non-Gaussianity: beyond the power spectrum



$$\langle \gamma_{\mathbf{k}_1}^{\lambda_1} \gamma_{\mathbf{k}_2}^{\lambda_2} \gamma_{\mathbf{k}_3}^{\lambda_3} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_{\gamma}^{\lambda_1 \lambda_2 \lambda_3}(k_1, k_2, k_3)$$

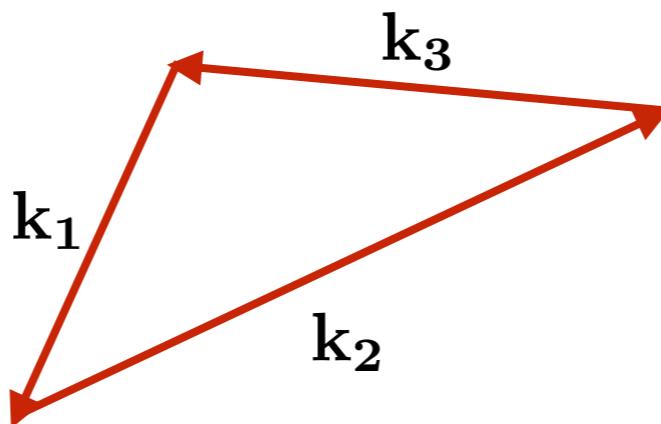
tensor bispectrum

amplitude:  $f_{NL} = \frac{B}{P_{\zeta}^2}$

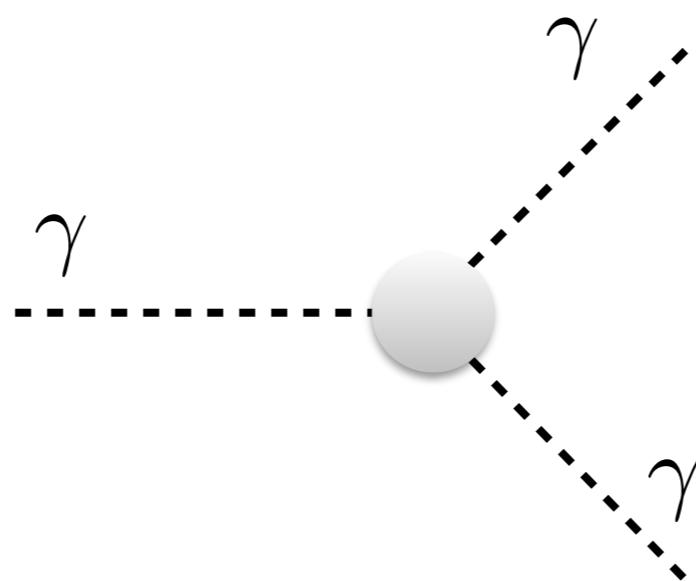
shape:



# Tensor non-Gaussianity

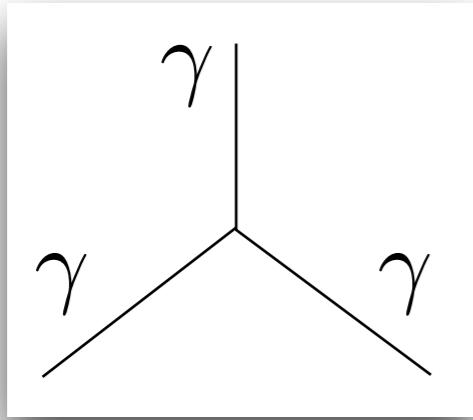


from interactions of the tensors with other fields or from self-interactions



# Tensor non-Gaussianity

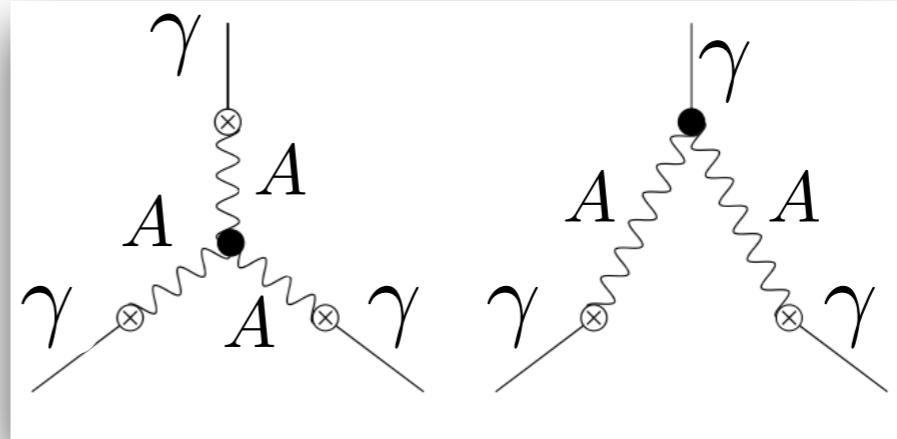
basic single-field inflation



$$f_{NL} = \mathcal{O}(r^2)$$

too small for detection

axion-gauge fields models



$$f_{NL} = r^2 \cdot \frac{25}{\Omega_A} \gtrsim \mathcal{O}(r^2 \cdot 10^6)$$

- detectable by upcoming CMB space missions  
**[Agrawal - Fujita - Komatsu 2017]**

# Non-Gaussianity (tensor / mixed): CMB constraints

- We do have constraints from CMB anisotropies and future B mode observations are expected to bring important improvements

Example: LiteBIRD-like experiment  
could detect an  $O(1)$  signal for

$$f_{\text{NL}}^{tss,\text{sq}} \quad f_{\text{NL}}^{ttt,\text{sq}} \quad f_{\text{NL}}^{ttt,\text{eq}} \quad [\text{Shiraishi, 2019}]$$

- The formalism for constraining non-Gaussianity with CMB anisotropies is by now well developed

# Non-Gaussianity at interferometers

Shapiro time delay:

$$\gamma'' + 2\mathcal{H}\gamma' - [1 + (12/5)\zeta] \gamma_{,kk} = 0$$

GW propagating in FRW background  
+ long-wavelength perturbations

$$\gamma_{ij} = A_{ij} e^{ik\tau + ik \cdot 2 \int^\tau d\tau' \zeta[\tau', (\tau' - \tau_0) \hat{k}]}$$

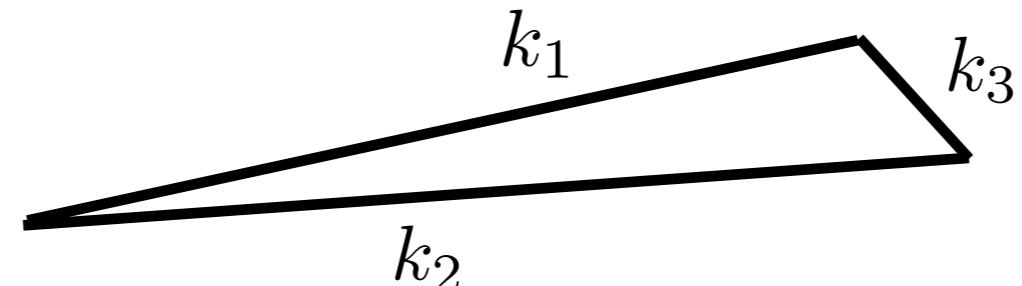
GW from different directions  
undergo different phase shift  
due to intervening structure

→ decorrelation → cannot measure bispectrum directly with interferometers

[Bartolo, De Luca, Franciolini, Lewis, Peloso, Riotto 2018]

Note: signal measured by an interferometer arises from the superposition  
of signals from a large number of Hubble patches (CLT)

# Ultra squeezed non-Gaussianity

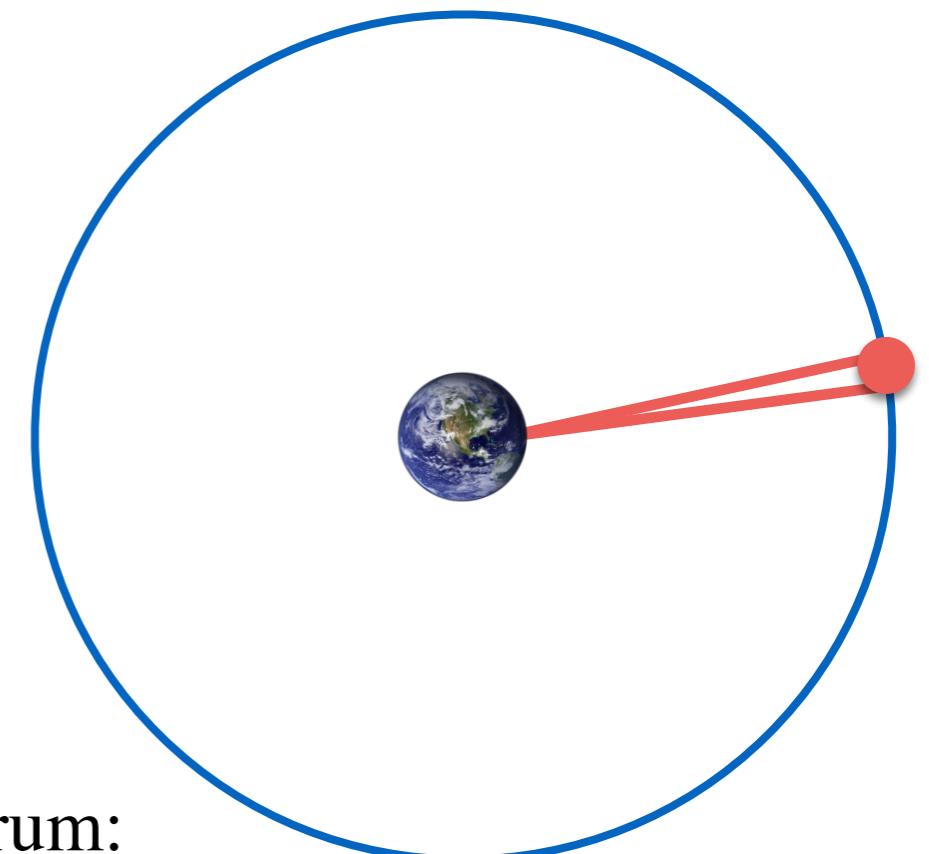


Correlation among two short-wavelength modes (e.g. interferometer scale) and 1 very long-wavelength mode:  
the latter has not undergone propagation!

Signals originate from the same patch!

How do we constrain this ultra-squeezed bispectrum:

Look for anisotropies in the SGWB!



$$\Omega_{\text{GW}}(k) = \bar{\Omega}_{\text{GW}}(k) \left[ 1 + \frac{1}{4\pi} \int d^2 \hat{n} \delta_{\text{GW}}(k, \hat{n}) \right]$$

# SGWB anisotropies from primordial non-Gaussianity

$$\Omega_{\text{GW}}(k) = \bar{\Omega}_{\text{GW}}(k) \left[ 1 + \frac{1}{4\pi} \int d^2 \hat{n} \delta_{\text{GW}}(k, \hat{n}) \right]$$

isotropic component

anisotropic component

$$\Omega_{\text{GW}}(k) \equiv \frac{1}{\rho_{\text{cr}}} \frac{d\rho_{\text{GW}}}{d \ln k}$$

energy density spectrum  
for the stochastic GW background

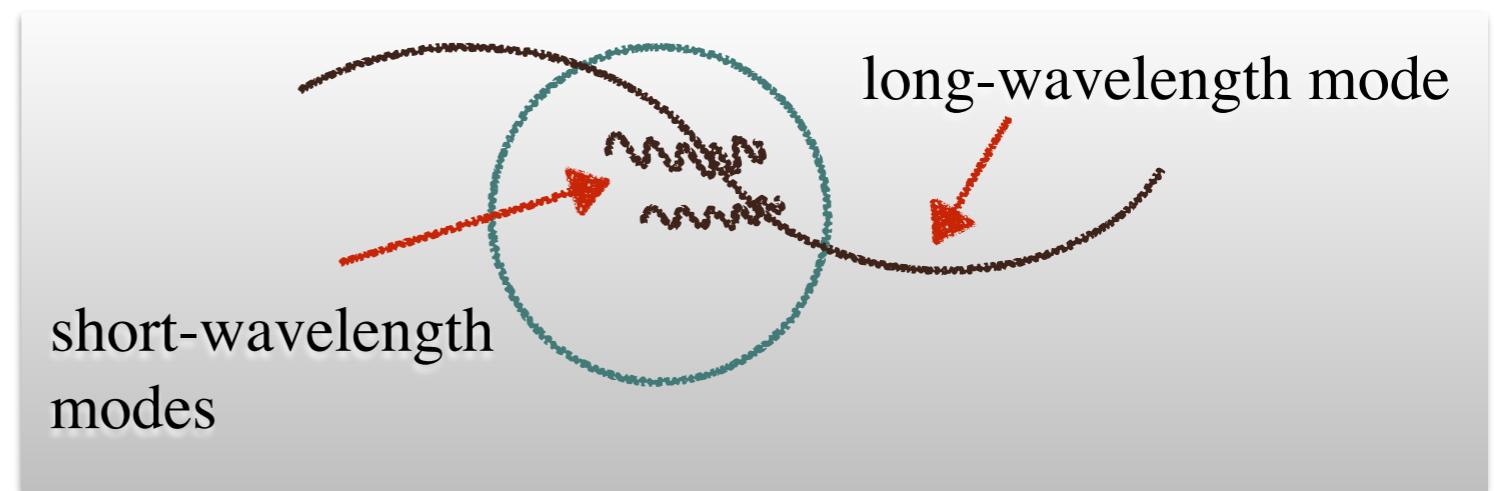
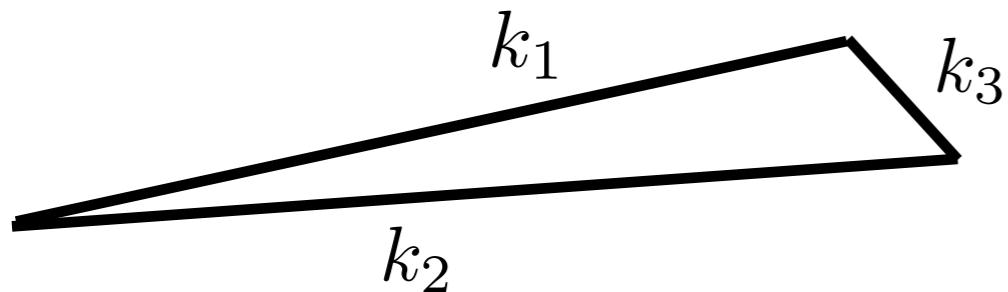
$k$ = comoving wavenumber (proportional to the observed frequency)

$\hat{n}$ = direction of incoming graviton

**How do SGWB anisotropies relate to non-Gaussianity?**

# Soft limits and ‘fossils’

$$k_1 \simeq k_2 \gg k_3$$



long wavelength modes introduces a modulation  
in the primordial power spectrum of the short wavelength modes

$$B^{F\gamma\gamma} \equiv \langle F_L \gamma_S \gamma_S \rangle' \sim F_L \cdot \langle \gamma_S \gamma_S \rangle'_{F_L} f_{\text{NL}}^{F\gamma\gamma}$$

$$\delta \langle \gamma_S \gamma_S \rangle \equiv \langle \gamma_S \gamma_S \rangle_{F_L} \sim \frac{B^{F\gamma\gamma}}{P_F(k_3)} \cdot F_L^* = P_\gamma(k_1) \cdot \frac{B^{F\gamma\gamma}}{P_F(k_3) P_\gamma(k_1)} \cdot F_L^*$$

$$\langle \gamma_S \gamma_S \rangle'_{\text{total}} = P_\gamma(k_1) \left( 1 + f_{\text{NL}}^{F\gamma\gamma} \cdot F_L^* \right)$$

# Soft limits and fossils



$$\delta_{\text{GW}}(k, \hat{n}) = \int \frac{d^3 q}{(2\pi)^3} e^{-i d \hat{n} \cdot \mathbf{q}} \zeta(\mathbf{q}) F_{\text{NL}}^{\text{stt}}(\mathbf{k}, \mathbf{q})$$

large scale variation large scale variations in the energy density of GW

$$\mathbf{d} = -(\eta_0 - \eta_{\text{in}}) \hat{n}$$

$$\Omega_{\text{GW}}(k) = \bar{\Omega}_{\text{GW}}(k) \left[ 1 + \frac{1}{4\pi} \int d^2 \hat{n} \delta_{\text{GW}}(k, \hat{n}) \right]$$

# Soft limits and fossils



[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]



for derivation with in-in formalism and applications:  
see: [ED, Fasiello, Pinol, 2022]

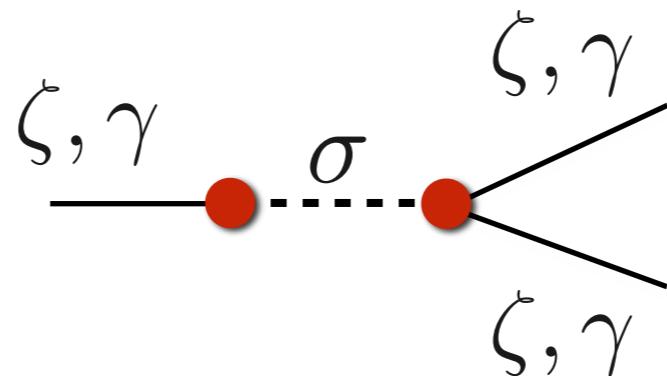
# Soft limits in inflation

- *Extra fields / superhorizon evolution*

[Chen - Wang 2009, Baumann - Green 2011, Chen et al 2013,  
ED - Fasiello - Kamionkowski 2015, ...]

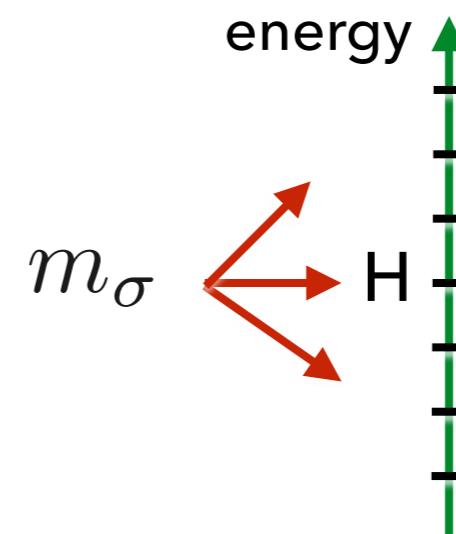
# Soft limits in inflation

- *Extra fields*



Soft limits reveal  
(extra) fields mediating  
inflaton or graviton  
interactions

squeezed bispectrum delivers  
info on mass spectrum!!!



# Soft limits in inflation

- *Extra fields / superhorizon evolution*

[Chen - Wang 2009, Baumann - Green 2011, Chen et al 2013,  
ED - Fasiello - Kamionkowski 2015, ...]

- *Non-Bunch Davies* initial states

[Holman - Tolley 2007, Ganc - Komatsu 2012, Brahma - Nelson - Shandera 2013, ...]

- *Broken space diffs*

(e.g. space-dependent background)

[Endlich et al. 2013, ED - Fasiello - Jeong - Kamionkowski 2014, Celoria - Comelli - Pilo - Rollo 2021...]

Ideal probe for (extra) fields, pre-inflationary dynamics, (non-standard) symmetry patterns

# SGWB anisotropies from primordial non-Gaussianity

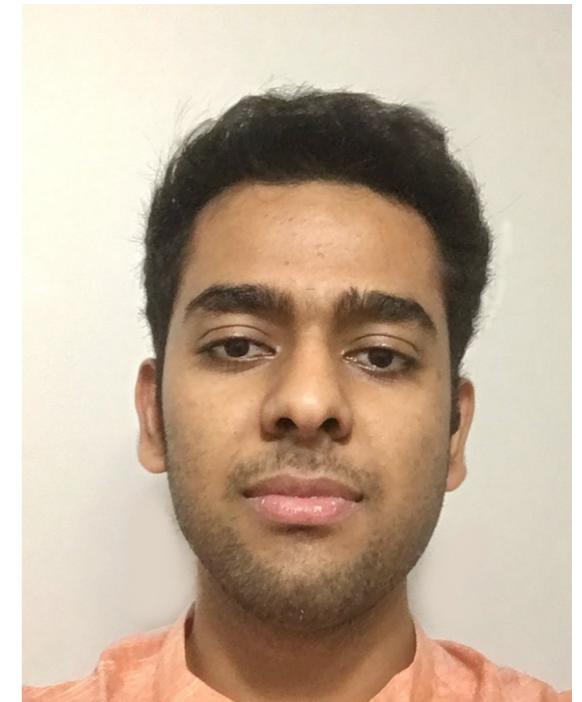
- Typical amplitude of these anisotropies:

$$\delta_{\text{GW}}^{\text{tss}} \sim F_{\text{NL}}^{\text{tss}} \sqrt{A_S}$$

$$\delta_{\text{GW}}^{\text{ttt}} \sim F_{\text{NL}}^{\text{ttt}} \sqrt{r A_S}$$

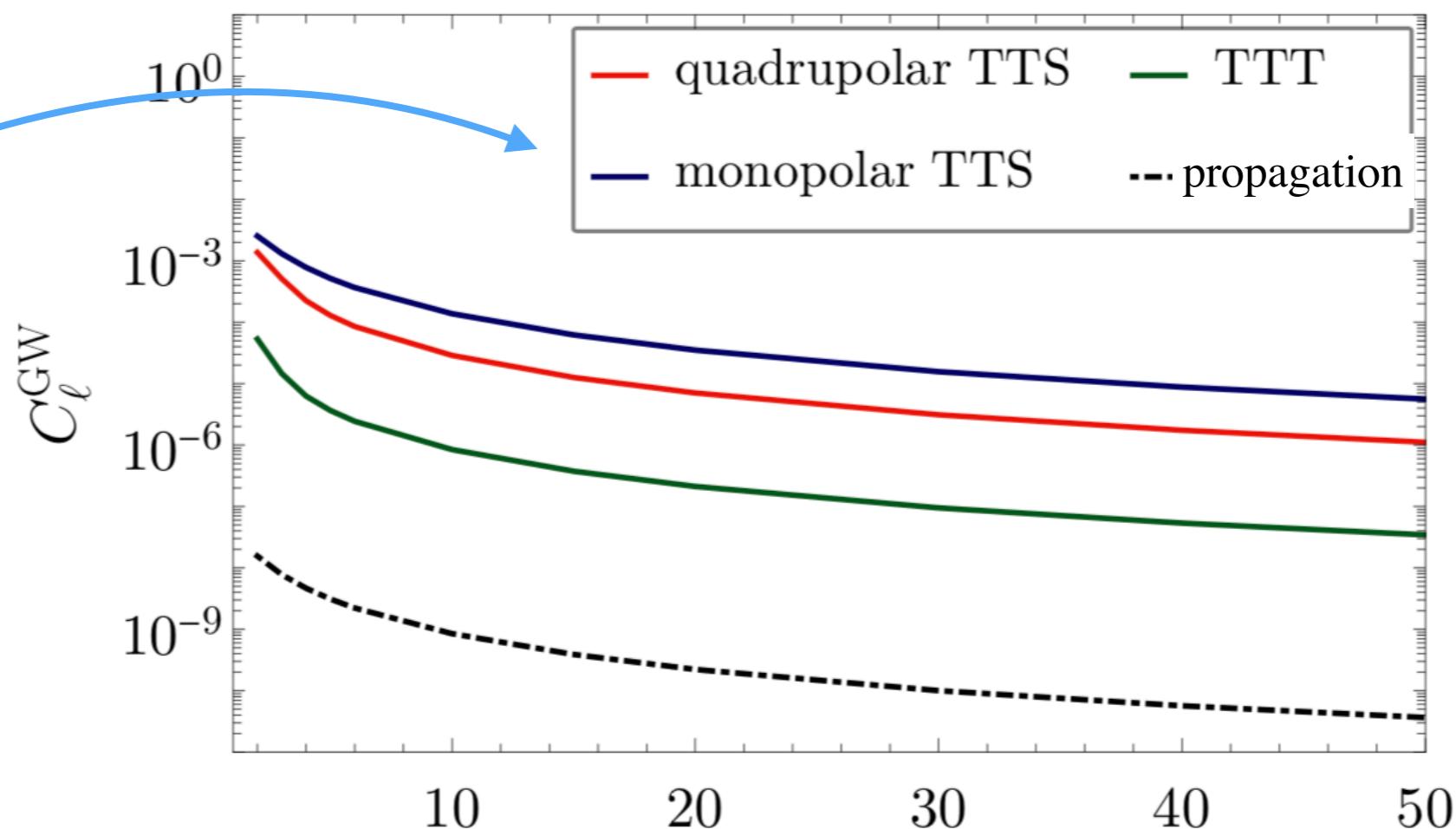
scalar power spectrum  
amplitude at CMB scales

tensor-to-scalar ratio



angular  
dependence of  
 $F_{\text{NL}}(\mathbf{k}, \mathbf{q})$

(scale-invariant case)

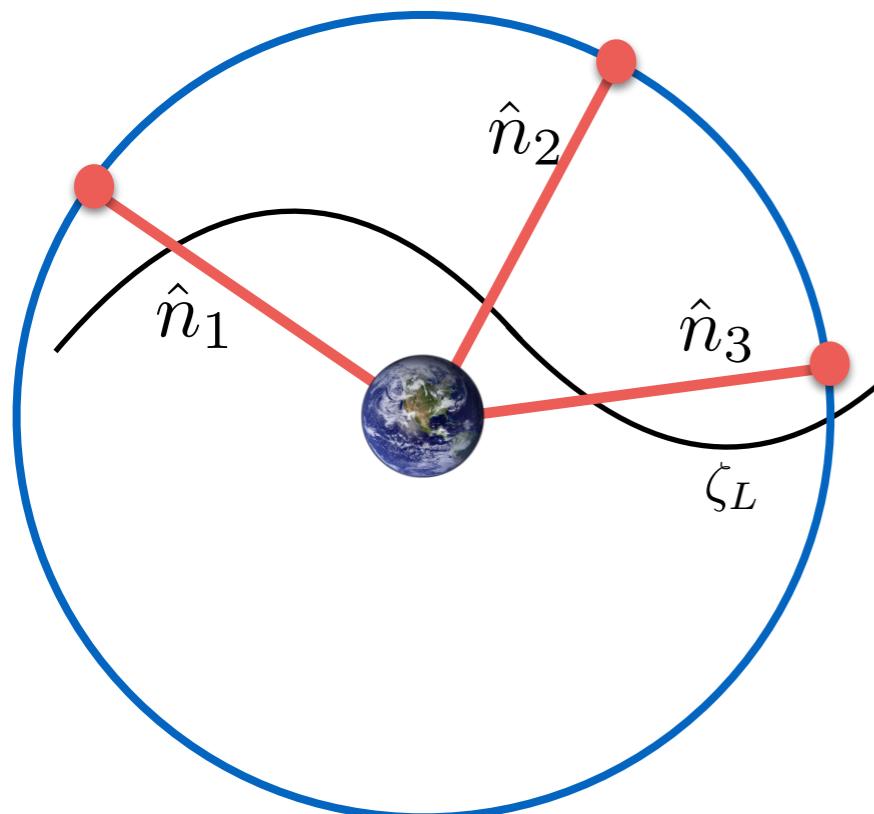


$$\langle \delta_{\text{GW}, \ell_1 m_1} \delta_{\text{GW}, \ell_2 m_2} \rangle$$

[Malhotra, ED, Fasiello, Shiraishi 2020 -  
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

# Anisotropies from propagation

GW propagate through the perturbed universe → subject to Sachs-Wolfe / integrated Sachs-Wolfe ..., just like CMB photons



Large-scales: SW dominates

$$\frac{\delta f(\hat{n})}{f} = \frac{1}{5} [\zeta_{L(\text{today})} - \zeta_L(\hat{n} \cdot \eta_0)]$$

$$\zeta_L(\hat{n}_1 \cdot \eta_0) \neq \zeta_L(\hat{n}_2 \cdot \eta_0)$$

Gravitational  
redshift/blueshift  
of gravitons

[Alba - Maldacena,  
2015]



Direction-dependent frequency shift



Anisotropy in the GW energy density

$$\delta_{\text{GW}} \sim \left( \frac{\partial \ln \Omega_{\text{GW}}}{\partial \ln k} \right) \zeta_L$$



$$\delta_{\text{GW}} \simeq \mathcal{O}(1) \zeta_L \simeq 10^{-5} \quad (\text{for SFSR Inflation})$$

# Boltzmann treatment for gravitational waves

FRLW in Poisson gauge:  $ds^2 = a^2(\eta) [-e^{2\Phi} d\eta^2 + (e^{-2\Psi} \delta_{ij} + \chi_{ij}) dx^i dx^j]$



Boltzmann equation for  
the distribution function of gravitons  
 $(\delta_{\text{GW}} \propto \Gamma)$

$$\Gamma(\eta_0, \vec{x}_0, \hat{n}, q) = \underbrace{\Gamma(\eta_i, \vec{x}_i, q)}_{\Gamma_I} + \underbrace{\Phi(\eta_i, \vec{x}_i)}_{\text{Initial perturbations}} + \underbrace{\int_{\eta_i}^{\eta_0} d\eta (\Phi' + \Psi')}_{\Gamma_S}.$$

$$\vec{x}_i = \vec{x}_0 - (\eta_0 - \eta_i) \hat{n}$$

Time and position  
of the observer

Time of emission

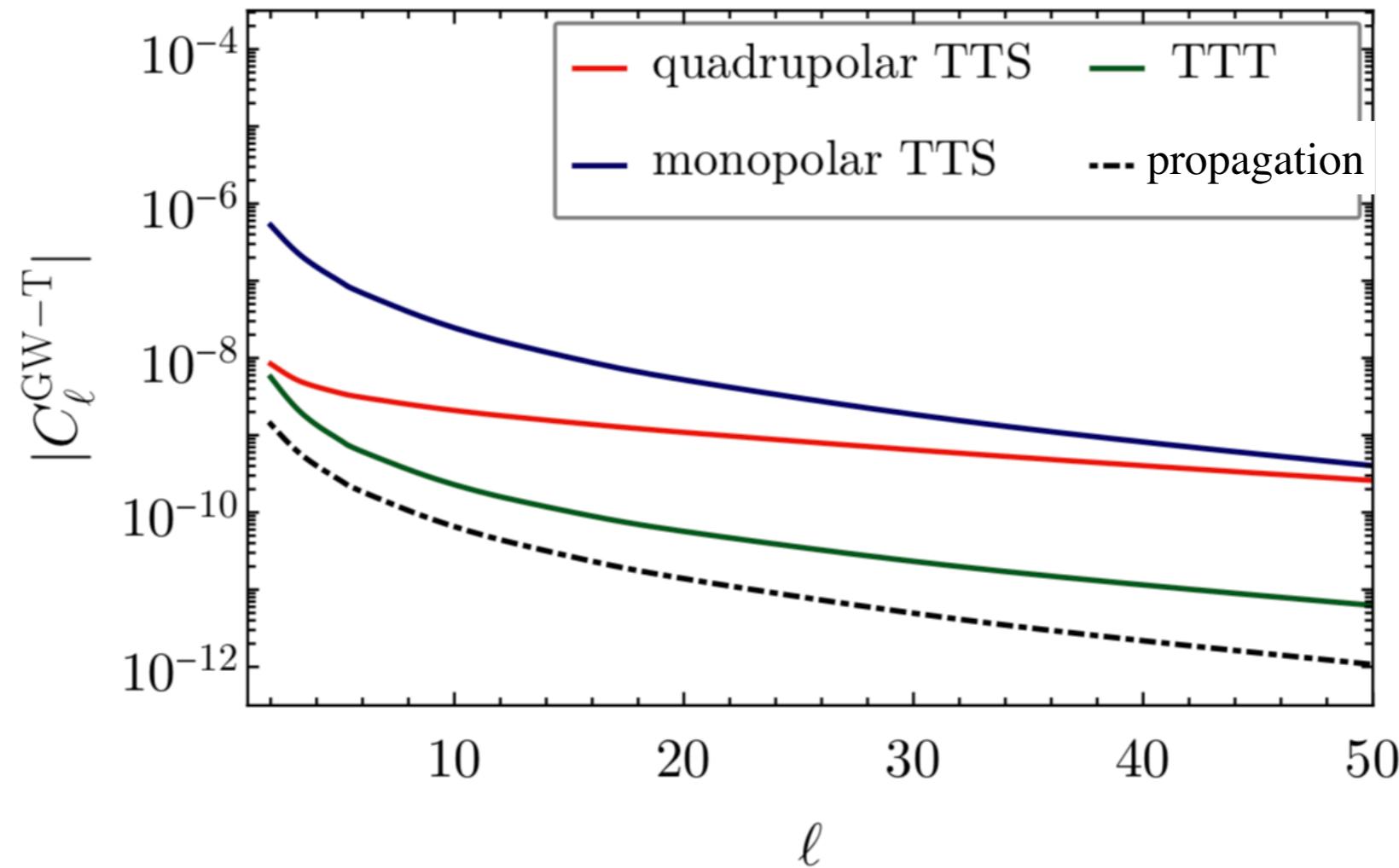
From propagation in the  
inhomogeneous universe  
(SW + ISW)

# Cross-correlations of GW and CMB anisotropies

$$\delta_{\text{GW}}^{\text{propagation}} \sim \zeta_L$$

$$\delta_{\text{GW}}^{\text{sst}} \sim F_{\text{NL}}^{\text{sst}} \cdot \zeta_L$$

$$\left. \begin{aligned} \delta_{\text{GW}}^{\text{sst}} &\sim F_{\text{NL}}^{\text{sst}} \cdot \zeta_L \\ \frac{\Delta T}{T} &\sim \zeta_L \end{aligned} \right\} C_{\ell}^{\text{GW-T}} \sim F_{\text{NL}}^{\text{sst}} \cdot C_{\ell}^{TT}$$



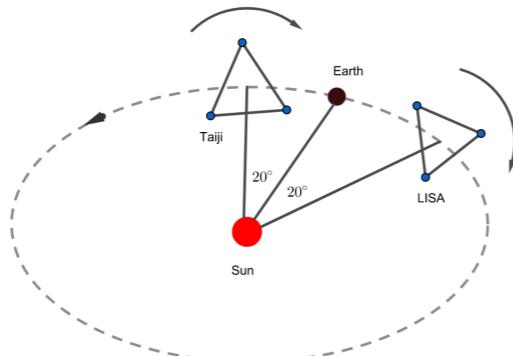
[Adshead, Afshordi, ED, Fasiello, Lim, Tasinato 2020  
Malhotra, ED, Fasiello, Shiraishi 2020  
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

# Projected constraints on $F_{\text{NL}}^{\text{tss}}$

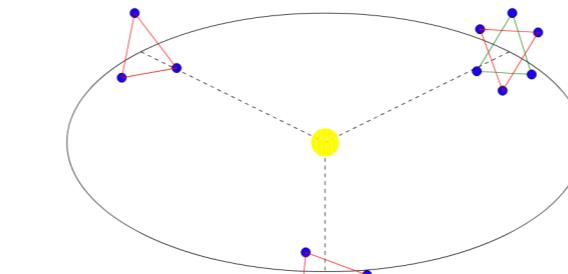
$$F_{ij} = \sum_{XY} \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \frac{\partial C_{\ell}^X}{\partial \theta_i} (C_{\ell}^{XY})^{-1} \frac{\partial C_{\ell}^Y}{\partial \theta_j} \quad X, Y = \{\text{TT}, \text{GW}, \text{GW-T}\}$$

$$\mathcal{C}_{\ell} = \frac{2}{2\ell+1} \begin{bmatrix} (C_{\ell}^{\text{TT}})^2 & (C_{\ell}^{\text{GW-T}})^2 & C_{\ell}^{\text{TT}} C_{\ell}^{\text{GW-T}} \\ (C_{\ell}^{\text{GW-T}})^2 & (C_{\ell}^{\text{GW}})^2 & C_{\ell}^{\text{GW}} C_{\ell}^{\text{GW-T}} \\ C_{\ell}^{\text{TT}} C_{\ell}^{\text{GW-T}} & C_{\ell}^{\text{GW}} C_{\ell}^{\text{GW-T}} & \frac{1}{2}(C_{\ell}^{\text{GW-T}})^2 + \frac{1}{2}C_{\ell}^{\text{TT}} C_{\ell}^{\text{GW}} \end{bmatrix}$$

- BBO: 4 LISA-like constellations



- LISA+Taiji



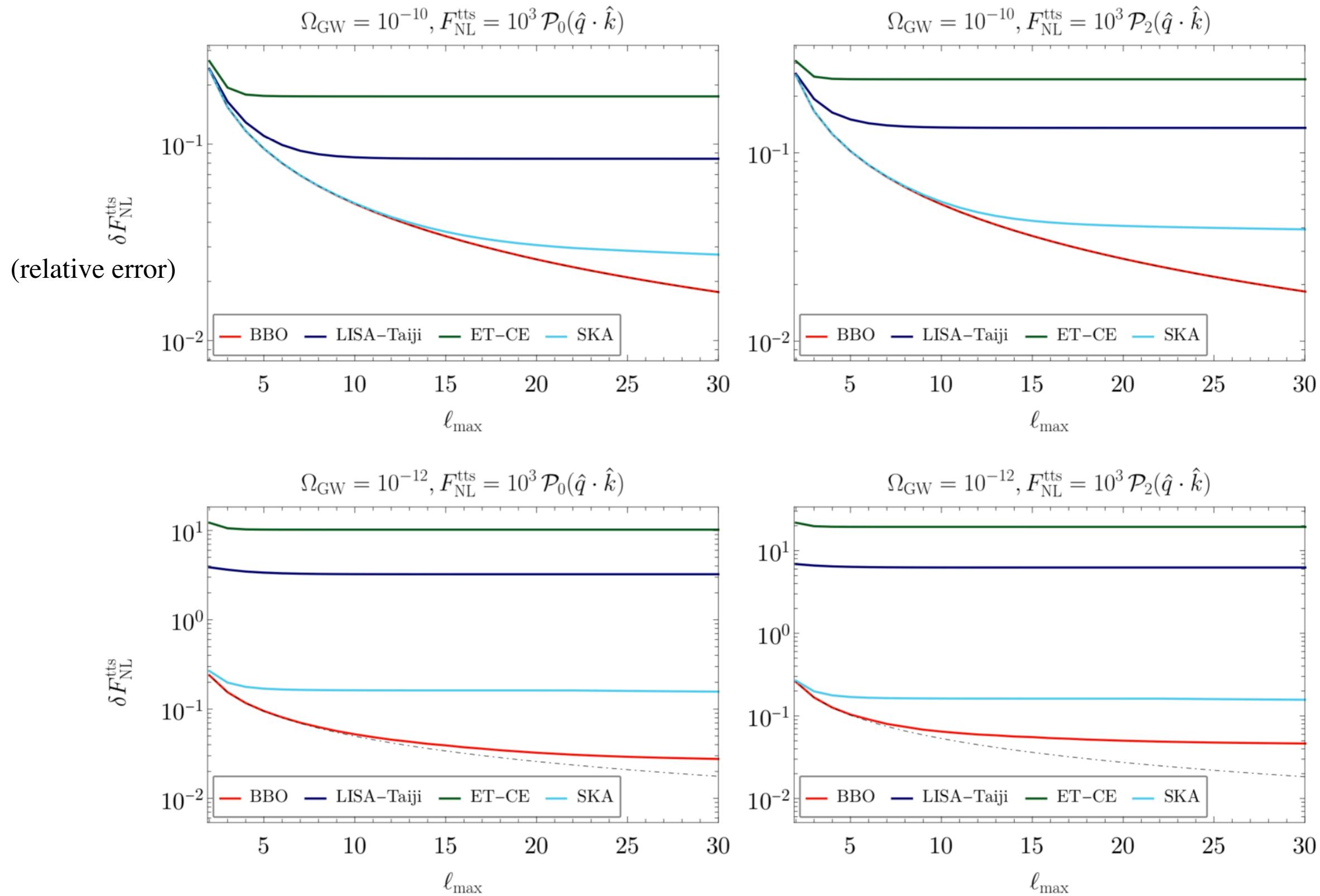
[Ruan et al, 2020]

- ET + CE

- SKA (assumed 50 identical pulsars)

[Crowder - Cornish, 2005]

# Projected constraints on $F_{\text{NL}}^{\text{tss}}$



# SGWB anisotropies: astrophysical sources

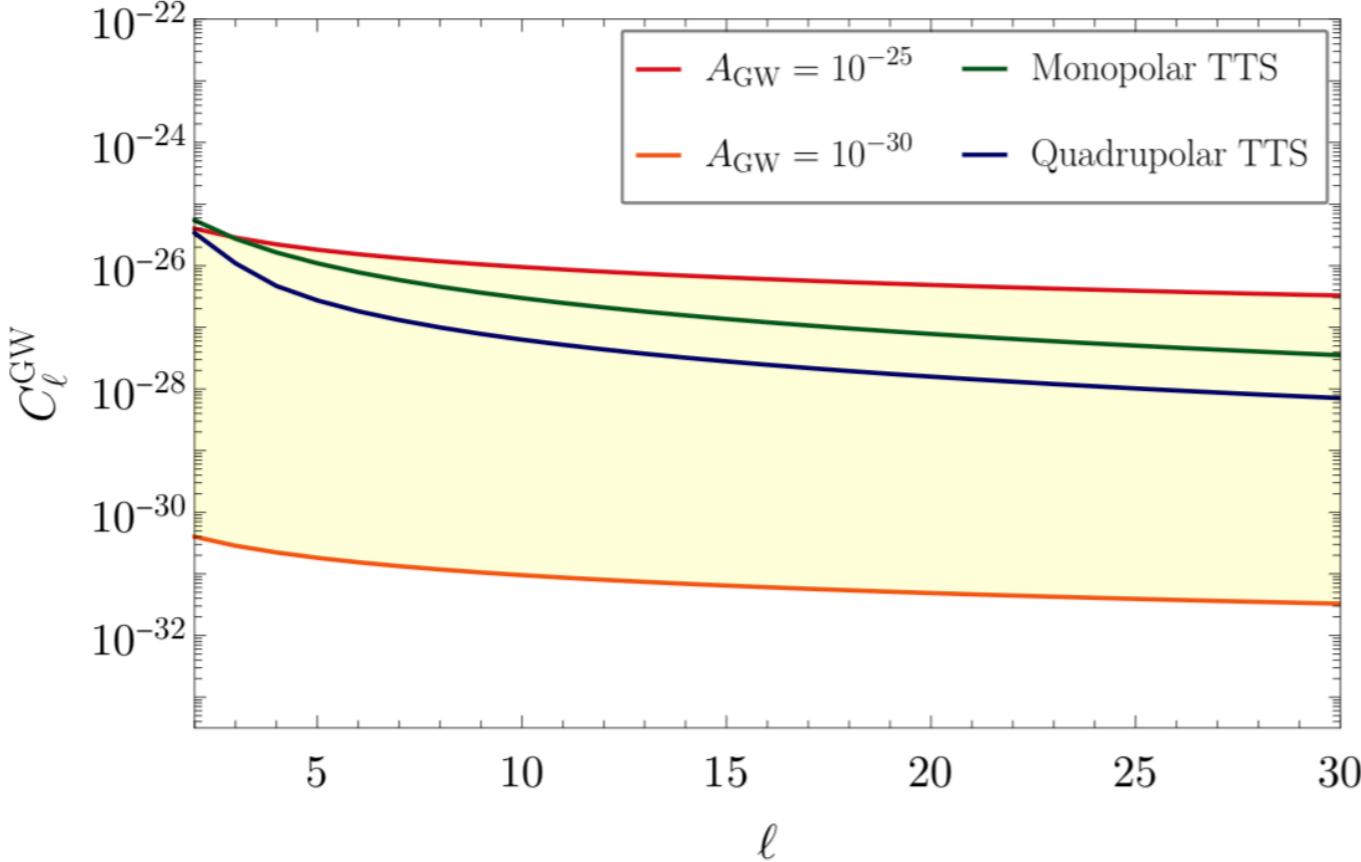
- SGWB from superposition of signals from black holes, neutron star binaries
- ASGWB also expected to be anisotropic due to the distribution of sources
- Anisotropies in the ASGWB can inform us about many things (e.g. start formation model, mass distribution, etc) **[see e.g. Cusin et al, 2018-19-20]**
- On large scales anisotropies in the ASGWB do not correlate strongly with CMB, (cross-correlations with LSS observables much more effective) **[Ricciardone et al, 2021]**



GW-CMB correlation excellent probe of cosmological SGWB!

# Astrophysical foregrounds

$$\Omega_{\text{GW}} = 10^{-12}, |\tilde{F}_{\text{NL}}| = 5 \times 10^3, k_{\text{ref}} = k_{\text{BBO}}$$



SNR for GW-CMB cross-correlations:

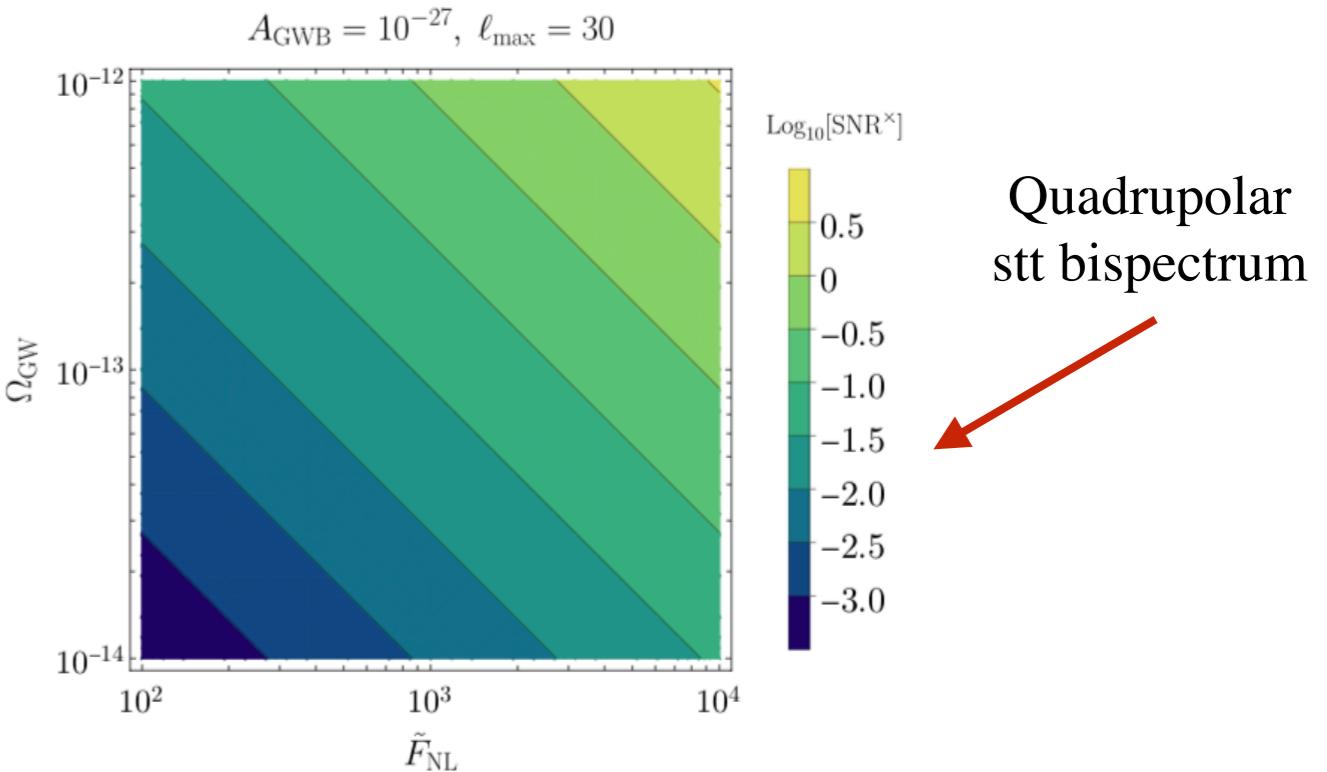
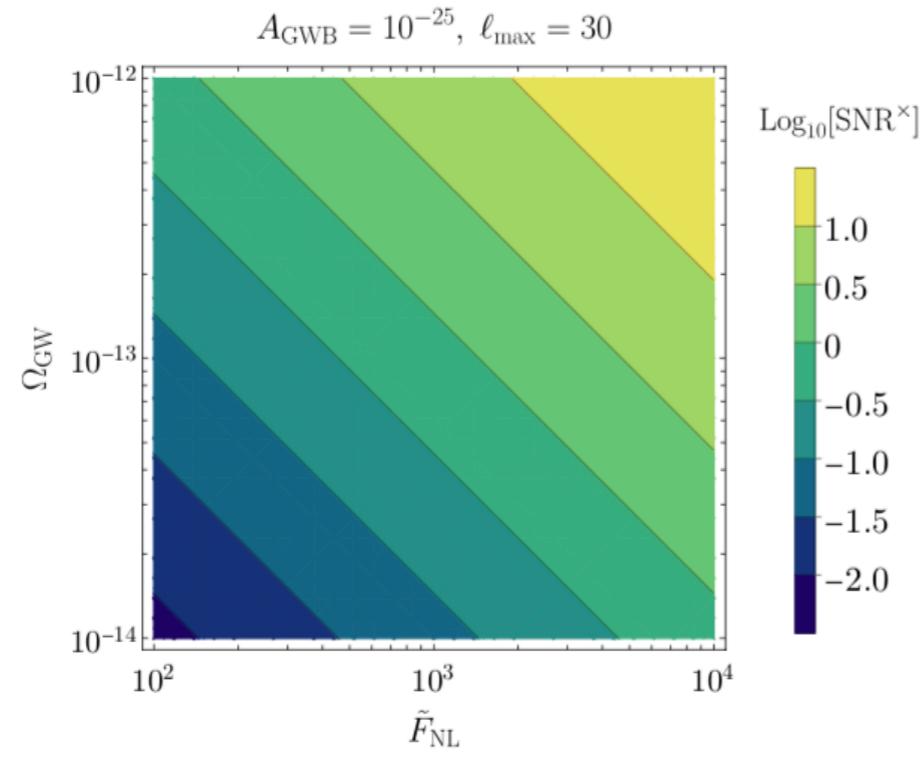
$$\text{SNR}^\times = \left[ \sum_{\ell_{\min}}^{\ell_{\max}} (2\ell + 1) \frac{\left( C_\ell^{\text{GW-T,signal}} \right)^2}{\left( C_\ell^{\text{GW-T,total}} \right)^2 + C_\ell^{\text{GW,total}} C_\ell^{\text{TT}}} \right]^{1/2}$$

$$C_\ell^{\text{GW-T,signal}} = C_\ell^{\text{GW-T,tts}}$$

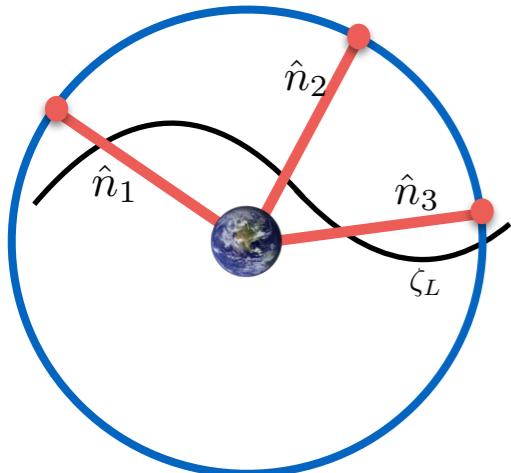
$$C_\ell^{\text{GW-T,total}} = C_\ell^{\text{GW-T,signal}} + C_\ell^{\text{GW-T,induced}},$$

$$C_\ell^{\text{GW,total}} = C_\ell^{\text{GW,tts}} + C_\ell^{\text{GW,induced}} + C_\ell^{\text{GW,astro}} + N_\ell^{\text{GW}}$$

Monopolar  
stt bispectrum

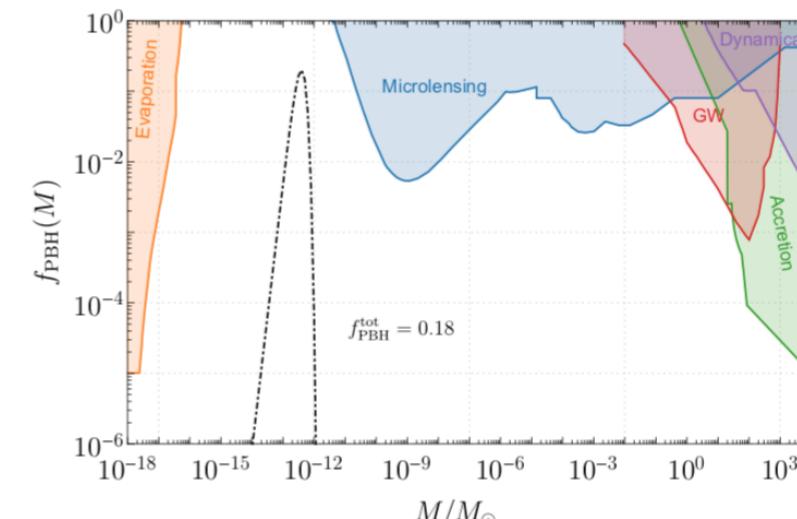
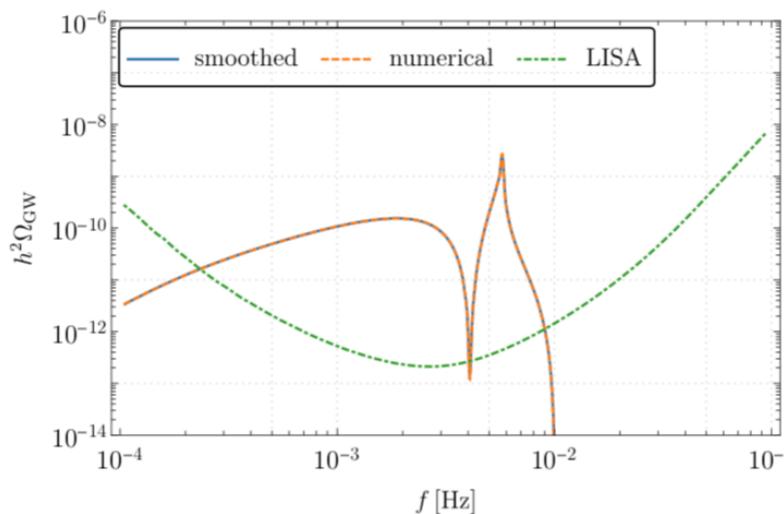


# Anisotropies from propagation



Models with sharp peaks in the scalar power spectrum (e.g. PBH production)

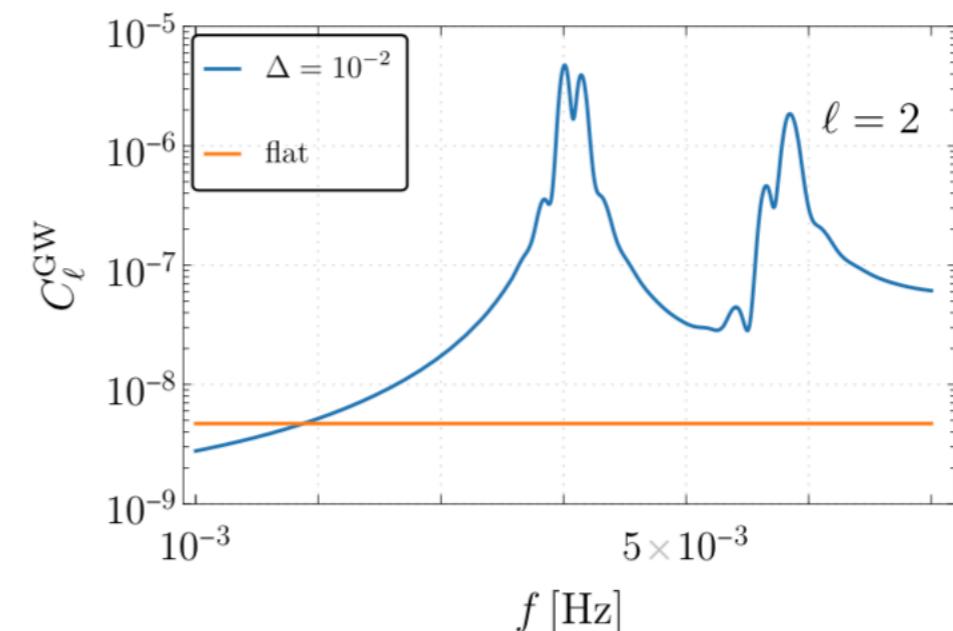
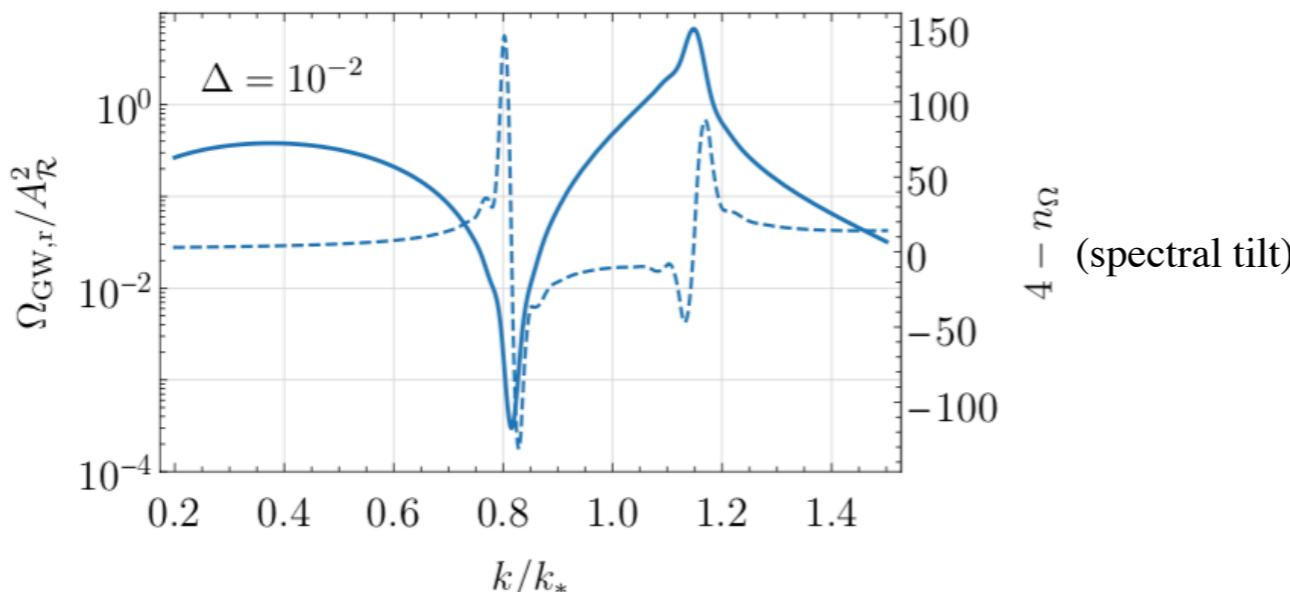
- a large GW background with sharp peaks induced at second order from scalar perturbations



- the anisotropies can be typically enhanced by O(10-100)

$$\delta_{\text{GW}} \sim \left( \frac{\partial \ln \Omega_{\text{GW}}}{\partial \ln k} \right) \zeta_L$$

- the angular power spectrum of the SGWB anisotropies inherits the frequency dependence



# Primordial gravitational waves

- Different production mechanisms during inflation lead to a variety of signals
- We can characterise the various GW sources from inflation using:
  - spectral shape
  - chirality
  - non-Gaussianity
  - SGWB anisotropies
- Powerful observables with the potential to disentangle inflationary GW from those generated in the post-inflationary universe