DIPARTIMENTO DI FISICA E ASTRONOMIA Galileo Galilei

Strings and quantum gravity bounds in 4d N=1 EFTs

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2104.05726, 2006.15154, 2205.04532

Motivations

- In string theory models, plenty of fundamental 4d axions

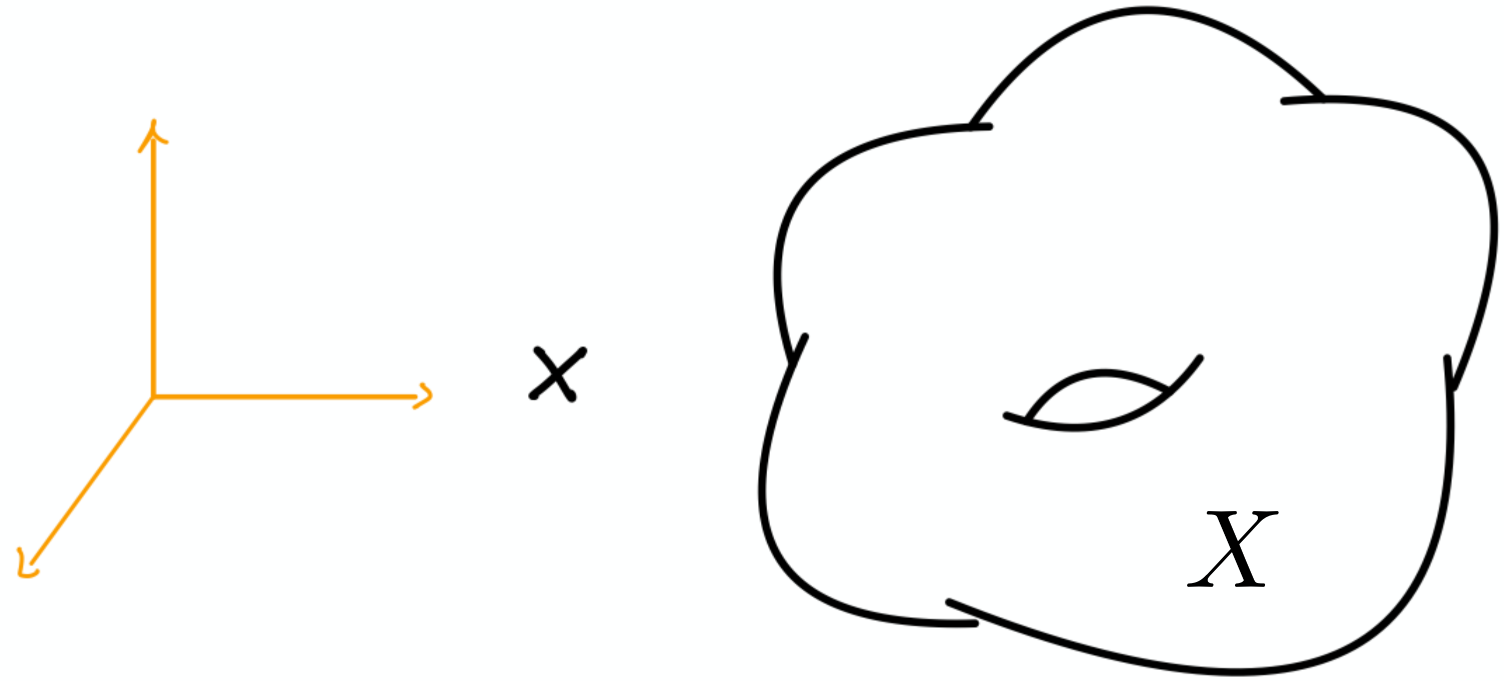
(~~$\phi = v e^{ia}$~~)

see also [Reece '18]

$$A_q = a^i(x)\omega_i$$

$$\omega_i \in H^q(X, \mathbb{Z})$$

$$a^i \simeq a^i + 1$$



- Naturally associated with fundamental axionic strings

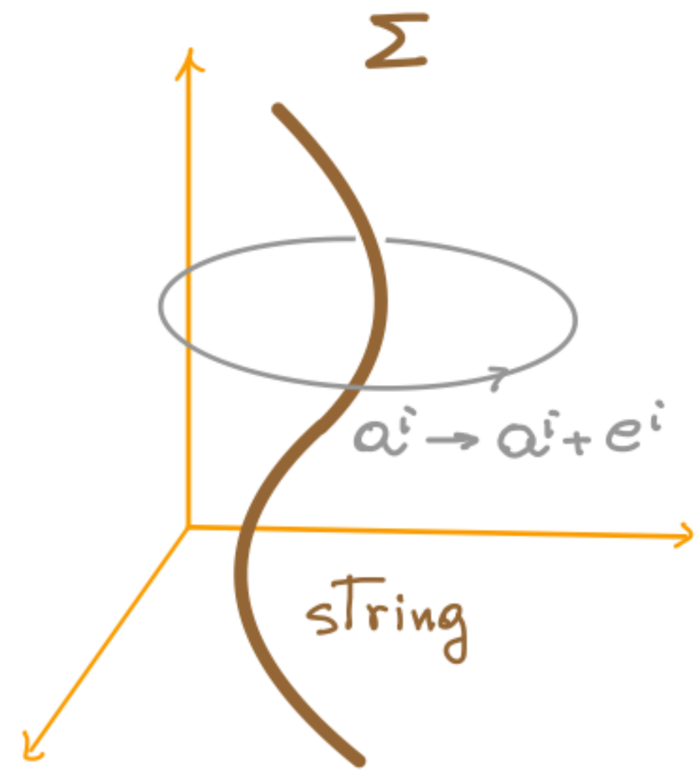
see also [Reece '18]

⇝ natural probes of quantum gravity UV completion

More generically, fundamental strings are required by quantum gravity principles:

* Completeness hypothesis [Polchinski '03, Banks & Seiberg '1 Halow-Ooguri '18,...]

$$*da \sim dB_2 \implies e^{i\int_\Sigma B_{2,i}}$$



* No global symmetries in QG

[Misner-Wheeler '56,...,
Kallosh-Linde-Linde-Susskind '95, ...,
Banks & Seiberg '06,...,
Harlow-Ooguri '19]

$$j_3 \equiv *da \implies d * j_3 = \delta_2(\Sigma)$$ broken by strings!

2-form global symmetry $\mathcal{B}_2 \to \mathcal{B}_2 + \Lambda_2$

[Gaiotto-Kapustin-Seiberg-Willet '15]

- Completeness really justified in presence of axionic shift symmetries

If e.g. $V(a) \neq 0$ $\Longrightarrow$ $\mathcal{B}_2 \to \mathcal{B}_2 + \Lambda_2$, $C_3 \to C_3 + \mathrm{d}\Lambda_2$ — B_2 eaten by massive 3-form

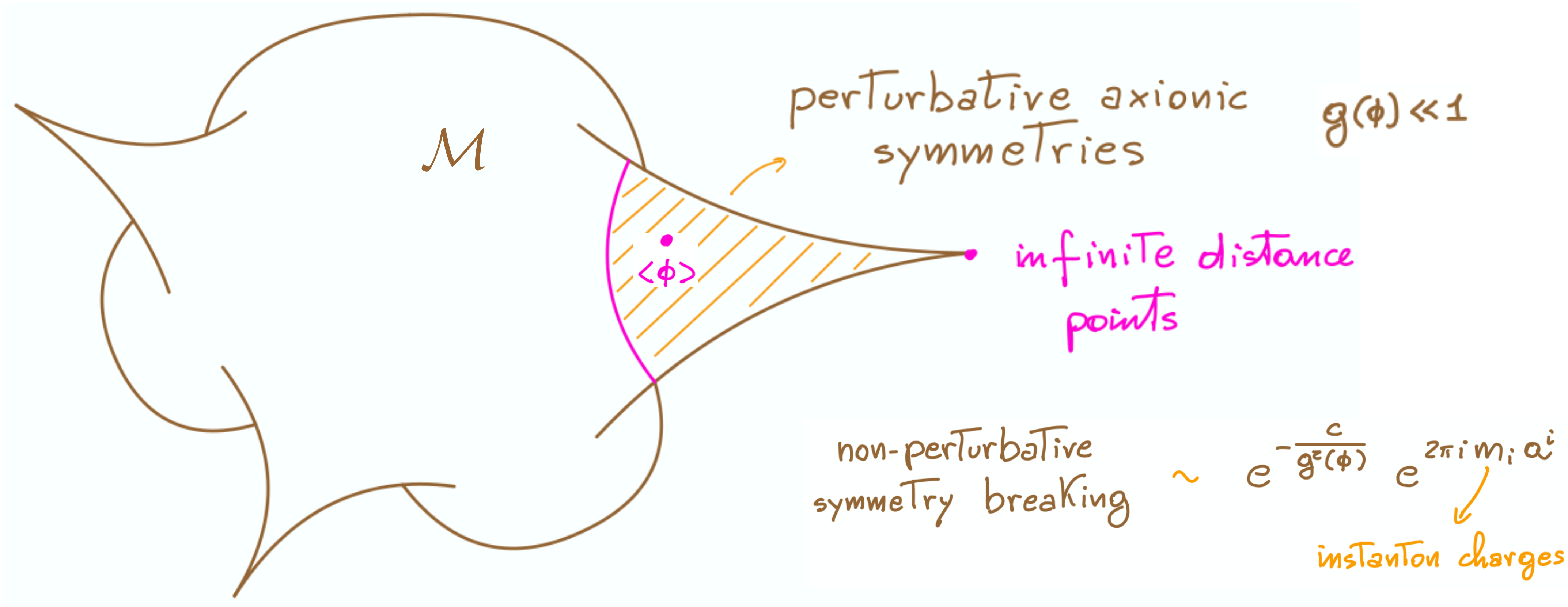
[Dvali '05]

- ✱ no 2-form global symmetry to break
- ✱ no 2-form gauge field to invoke completeness
- ✱ strings must always bound membranes

- ~~global symmetries~~ $\Longrightarrow$ no exact axionic shift symmetries $\Longrightarrow$ at best, perturbative axionic shift symmetries w.r.t. $g \ll 1$

- No adjustable couplings in quantum gravity: $g = g(\phi)$

[Ooguri-Vafa '06]



- Fundamental strings as natural probes of asymptotic field space regions

What can they tell us about the EFT structure?

$\mathcal{N} = 1$ models and EFT strings

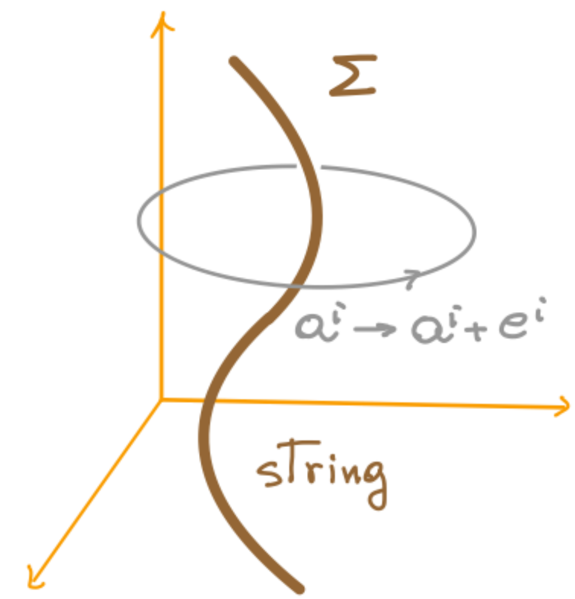
- I'll focus on 'fundamental' BPS strings in d=4 $\mathcal{N}=1$ EFT

see also [Reece '18]

$$S_{4d} = \frac{M_P^2}{2}\int_{\text{bulk}} R + \ldots - \int_\Sigma T_e\sqrt{-g} + e^i\int_\Sigma B_{2,i} + \ldots$$

[Lanza-Marchesano-LM-Sorokin '19]

$$dB_2 \sim * da$$



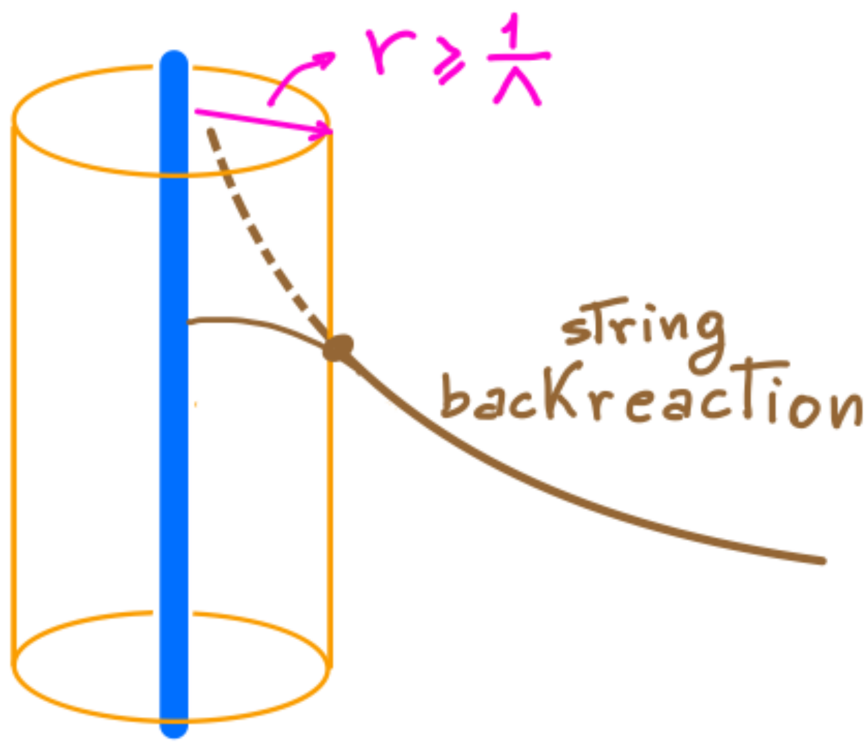
- Warning: strong back-reaction:

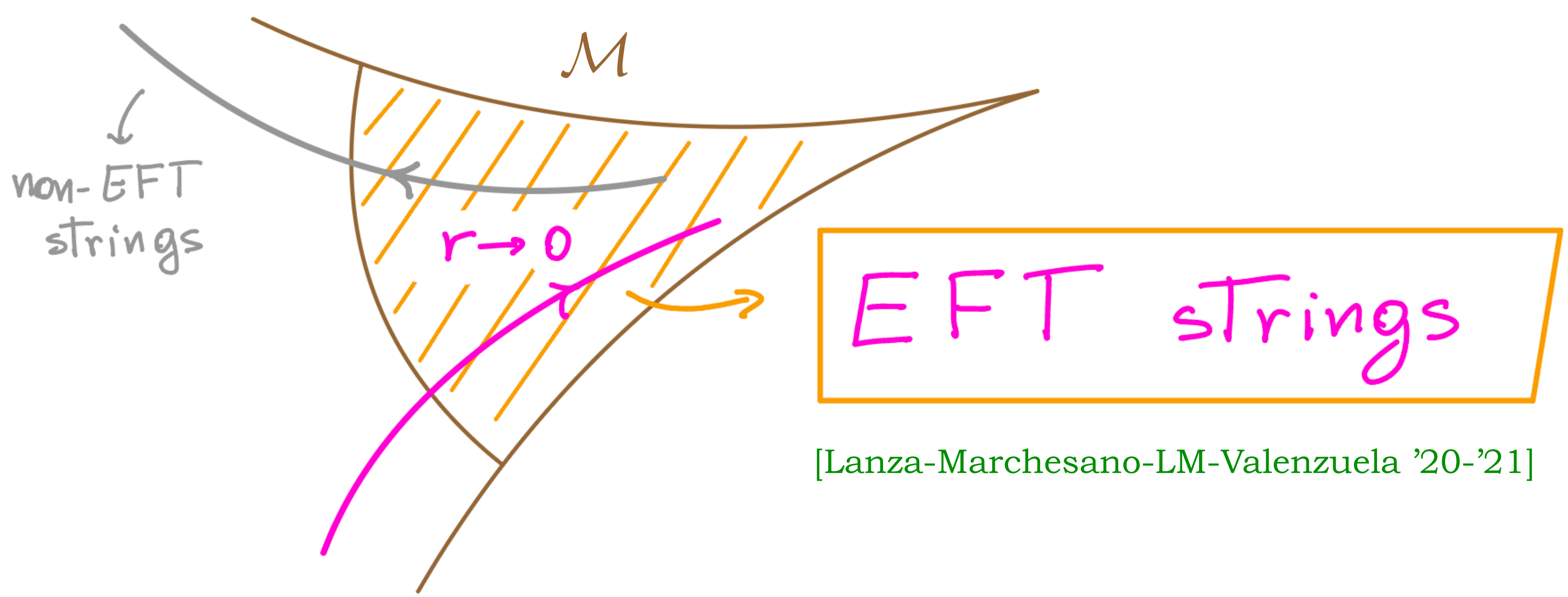
  - bulk vacuum destroyed
  - fields possibly driven to strongly coupled regions
  - IR effects out of control

See [Marchesano-Wiesner '22]

- However, strings can still have a well defined EFT description

[..., Goldberger&Wise '01,
Michel-Mintun-Polchiski-Puhm-Saad '14, Polchinski '15...,]





[Lanza-Marchesano-LM-Valenzuela '20-'21]

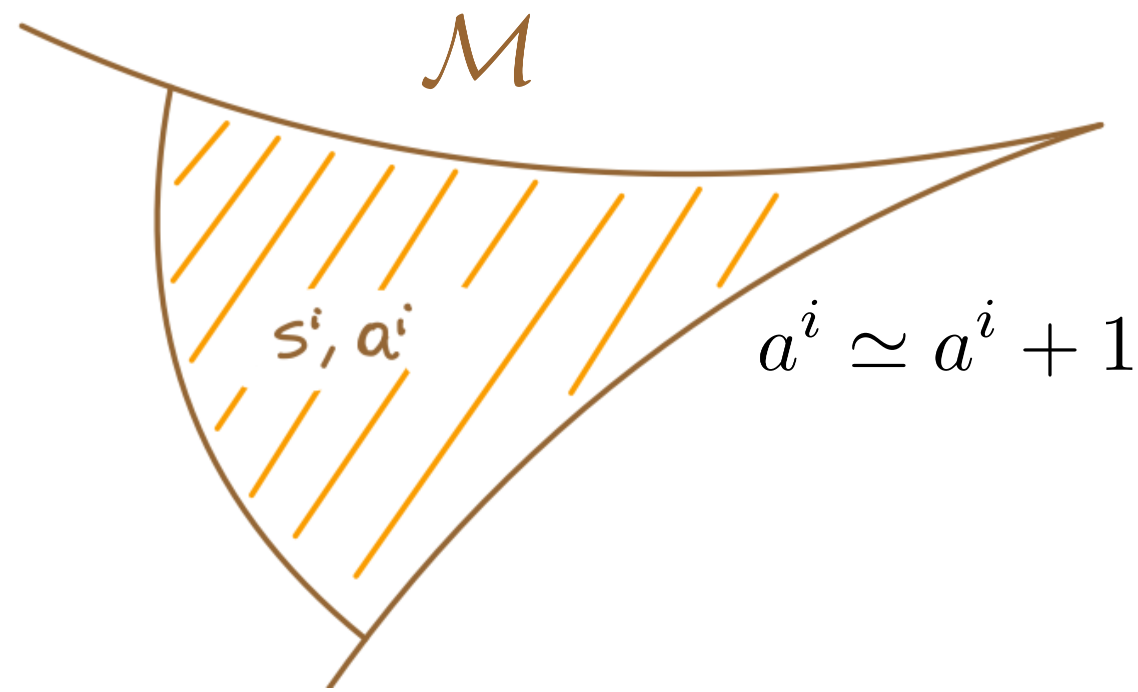
EFT strings

- Perturbative region:

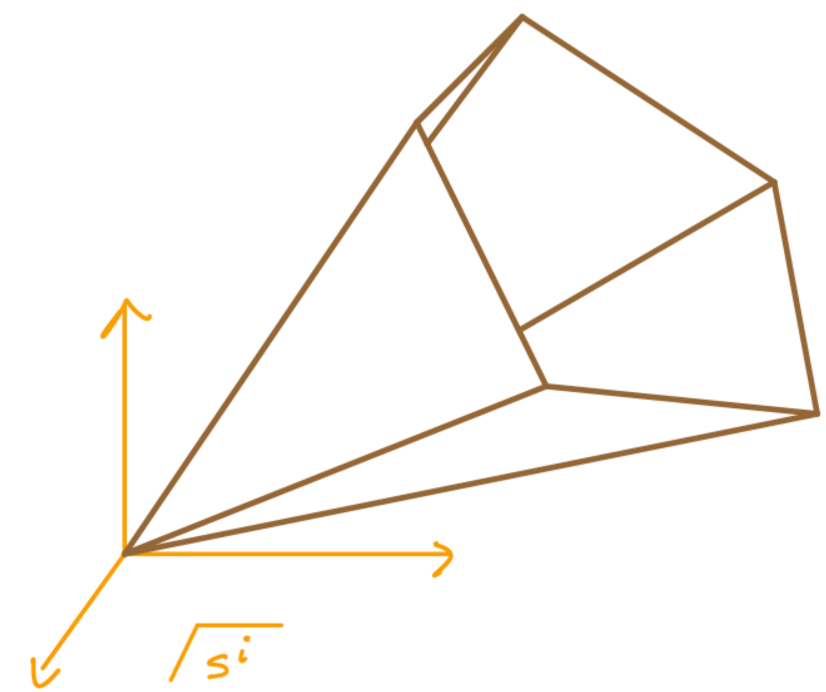
$$t^i \equiv a^i + \mathrm{i}\, s^i$$

axions ↙ ↘ saxions

(chiral mult.)



- $\{s^i\} \in \{\text{saxionic cone}\}$



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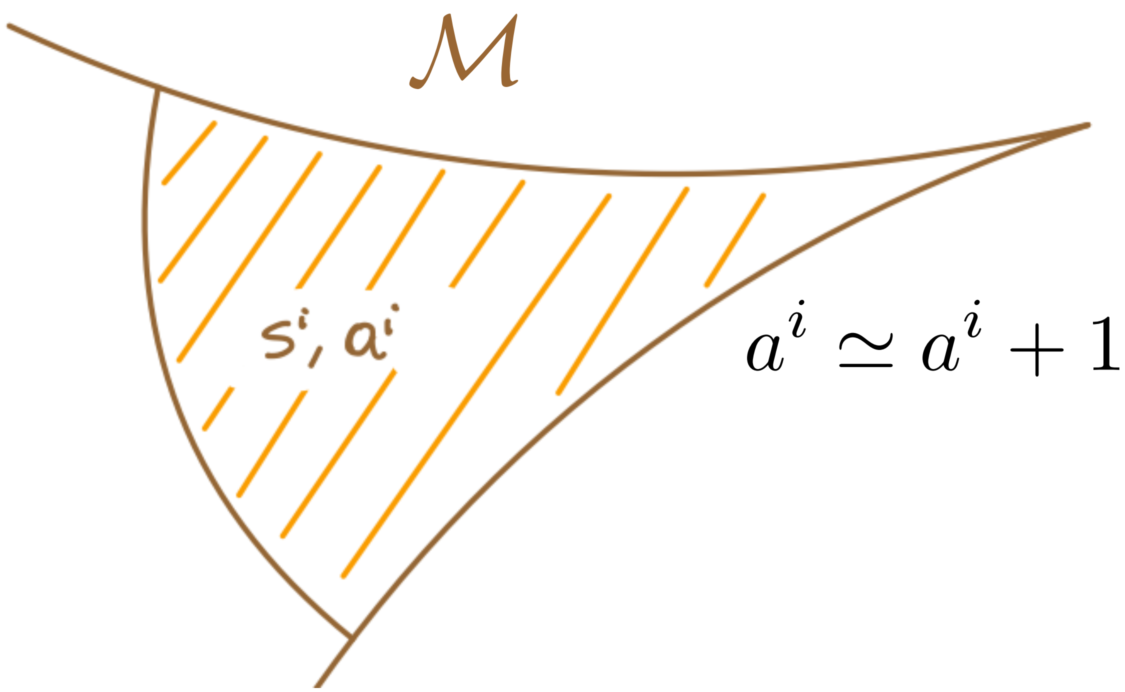
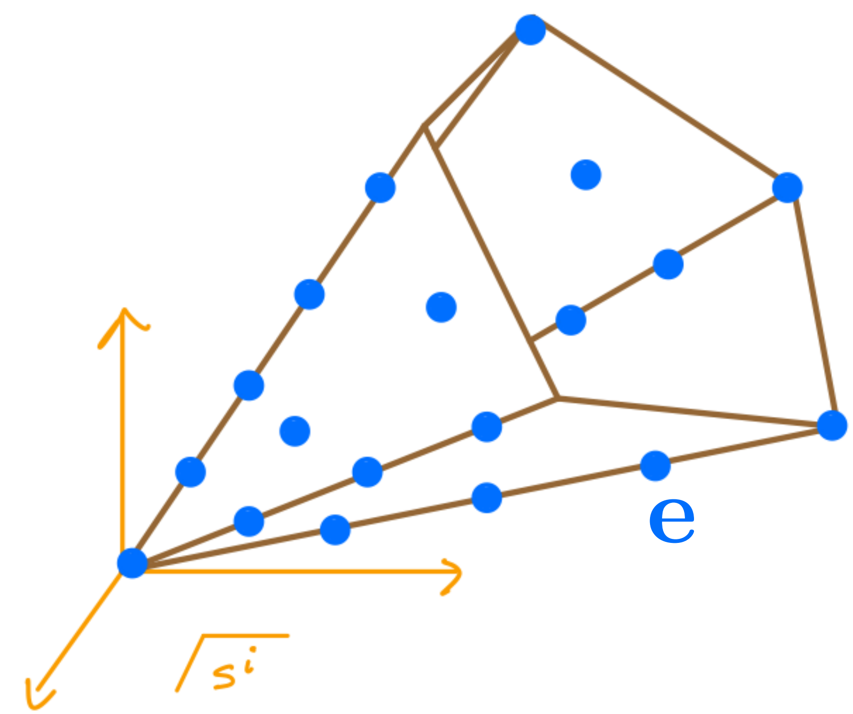


- $\{s^i\} \in \{\text{saxionic cone}\}$

generated by EFT string charges

$$\mathbf{e} \equiv \{e^i\} \in \mathcal{C}_{\mathrm{S}}^{\mathrm{EFT}} \equiv \{\text{saxionic cone}\}_{\mathbb{Z}}$$



EFT strings

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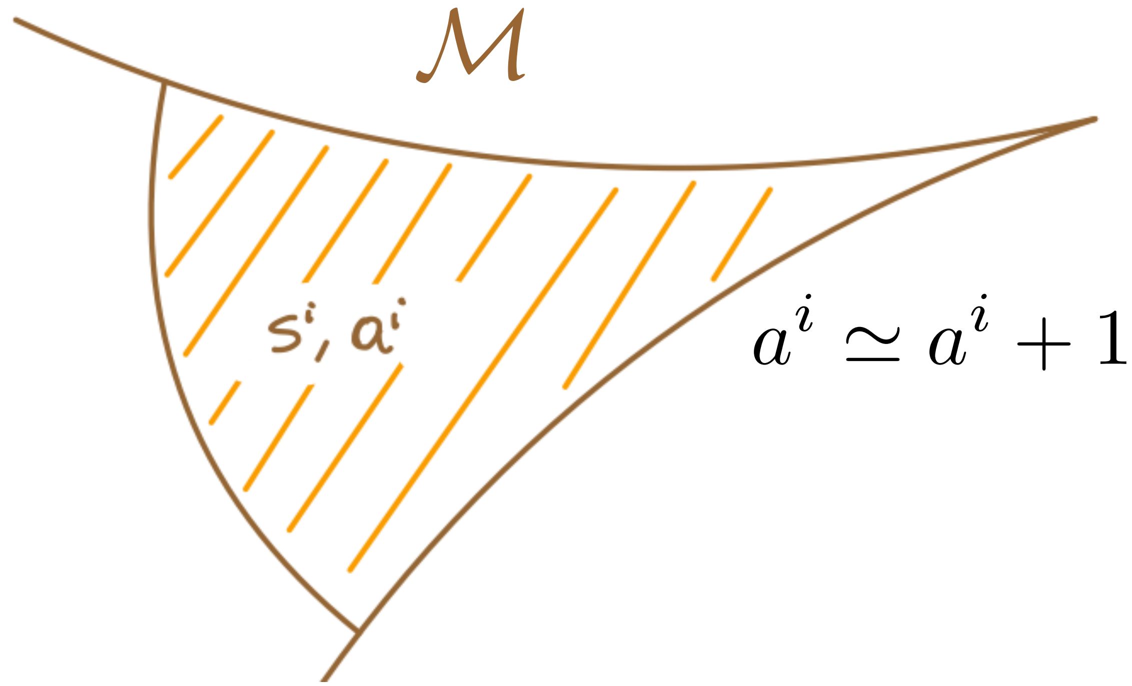
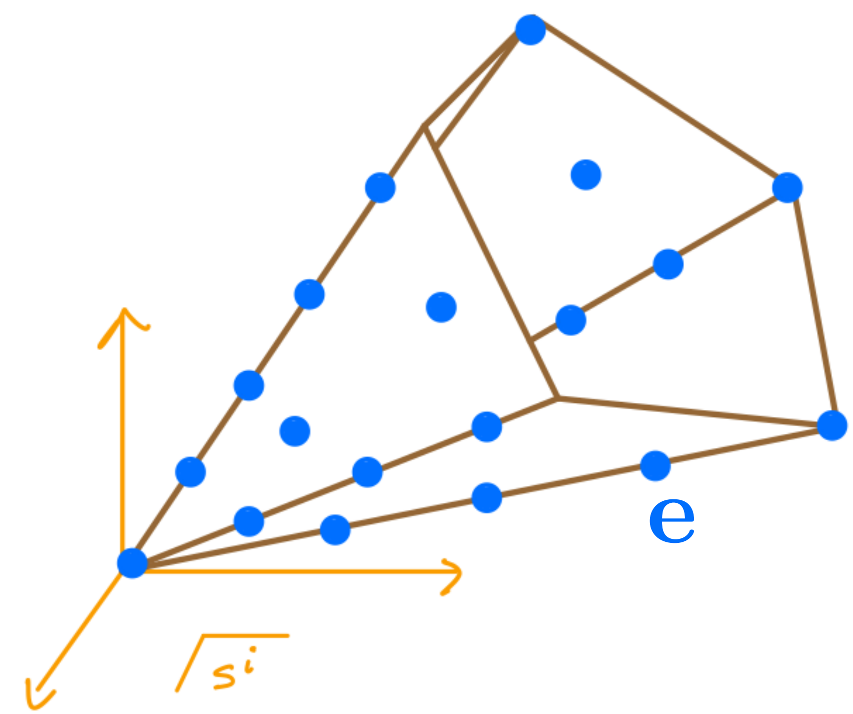


EFT strings strongly characterize asymptotic field space regions!

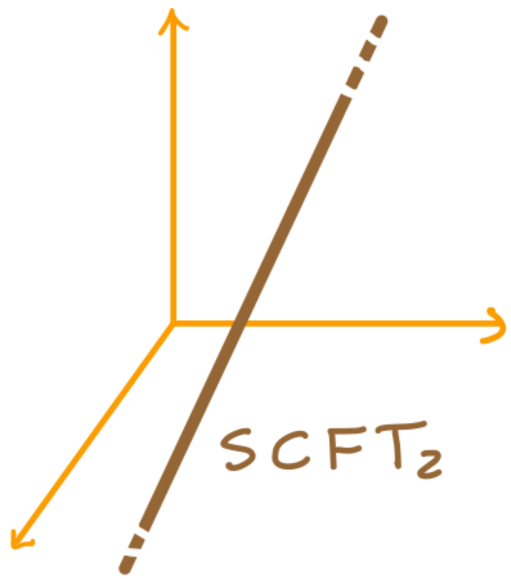
[Lanza-Marchesano-LM-Valenzuela ’20-’21]



see also [Buratti, Calderón-Infante, Delgado, Uranga `21] [Marchesano, Wiesner`21]
[Angius, Calderón-Infante, Delgado, Huertas, Uranga `22] [Grimm, Lanza, Li `22]
[Fierro Cota, Mininno, Weigand, Wiesner`22] [Wiesner`22] [Marchesano, Melotti`22]

Quantum consistency?

- BPS strings as quantum probes of $d \geq 5$ supergravities!



Quantum consistency of IR SCFT$_2$

$\Downarrow$

constraints on bulk EFT

[Kim-Shiu-Vafa `19]

[Lee-Weigand `19]

[Kim-Tarazi-Vafa '20]

[Katz-Kim-Tarazi-Vafa '20]

[Angelantonj-Bonnefoy-Condeescu-Dudas'20]

[Tarazi-Vafa '21]

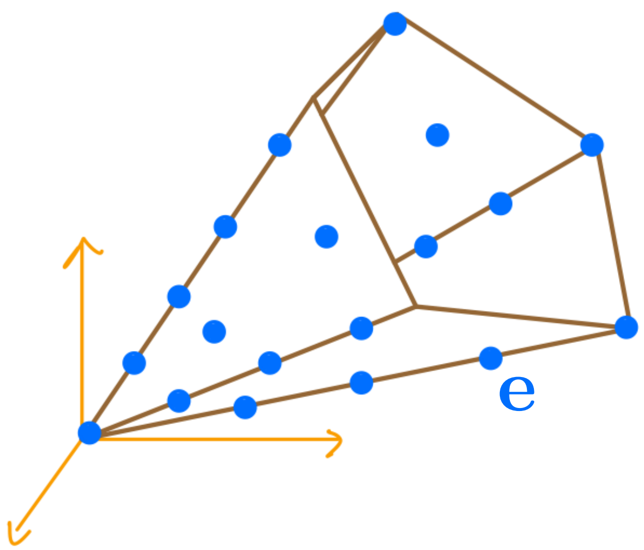
- In $d = 4$ we cannot assume IR SCFT, however...

Quantum consistency?

- … EFT strings support weakly-coupled (0,2) NLSM!

- EFT string completeness + bulk+string EFT quantum consistency

[Lanza-Marchesano-LM-Valenzuela '20-'21]

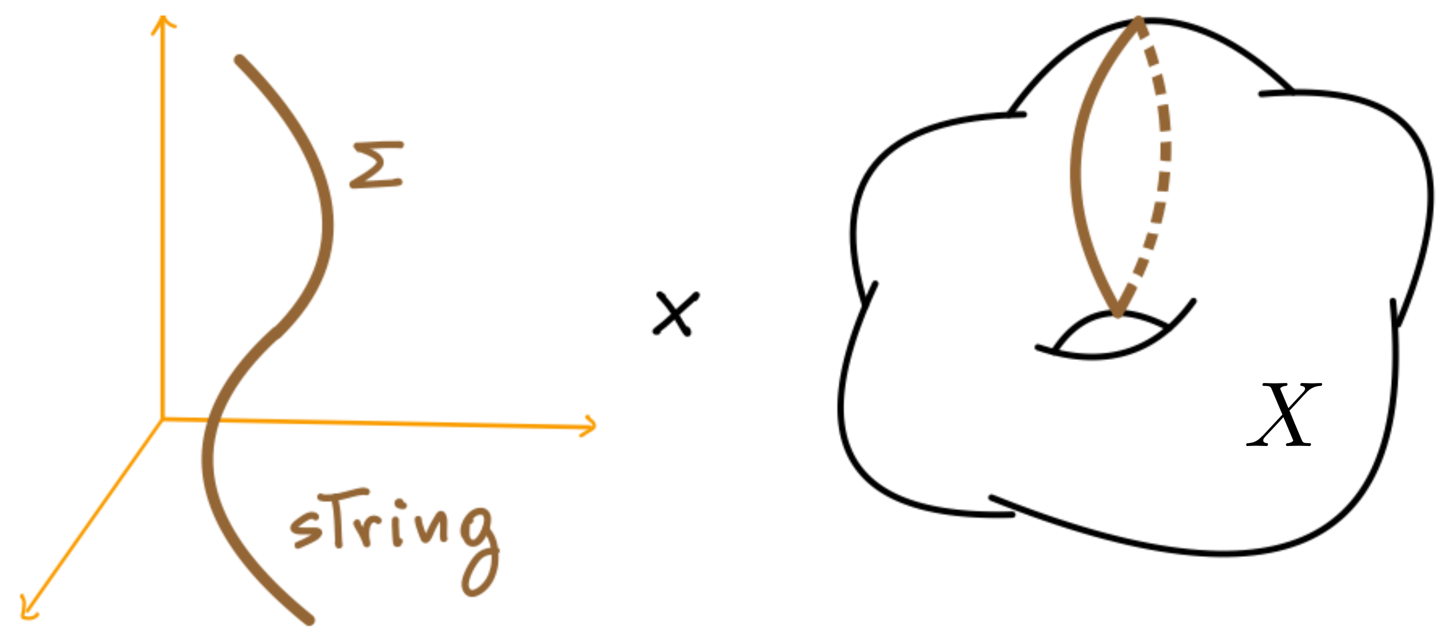


⇓

EFT quantum gravity constraints!

EFT strings in UV-complete models

- ✱ F1 strings in heterotic models
- ✱ D3 on movable curves in F-theory
- ✱ NS5 on nef divisors in IIA and heterotic models
- ✱ …



more subtle:

- ✱ M5/D4 on calibrated cycles dual to stable 3-forms
- ✱ CS strings in IIB & heterotic
- ✱ …

freely move along X

↓

Truly gravitational

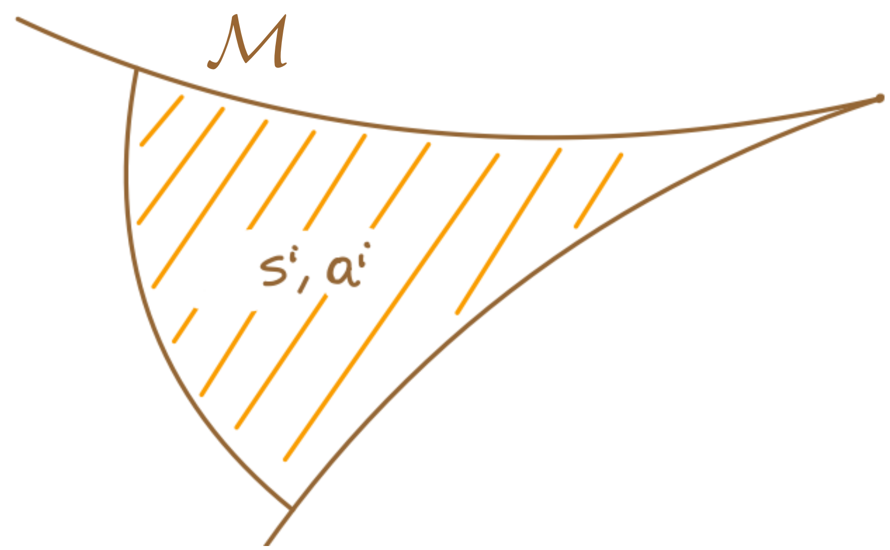
EFT strings probing gauge and (curvature)^2 terms

[LM-Risso-Weigand ’22]

- Bulk perturbative gauge group:

$$-\frac{1}{2}\int (C_i s^i + \ldots)\ \mathrm{Tr}(F\wedge *F) - \frac{1}{2}\int (C_i a^i + \ldots)\ \mathrm{Tr}(F\wedge F)$$

$\frac{1}{g^2} \gg 1$ in



$\Longrightarrow$ $C_i s^i > 0$

- Gauss-Bonnet and Pontryagin terms

$$-\frac{1}{48}\int (\tilde{C}_i s^i + \ldots)\ [\mathrm{Tr}(R\wedge *R) + \ldots] - \frac{1}{48}\int (\tilde{C}_i a^i + \ldots)\ \mathrm{Tr}(R\wedge R)$$

sign ?

positivity suggested by

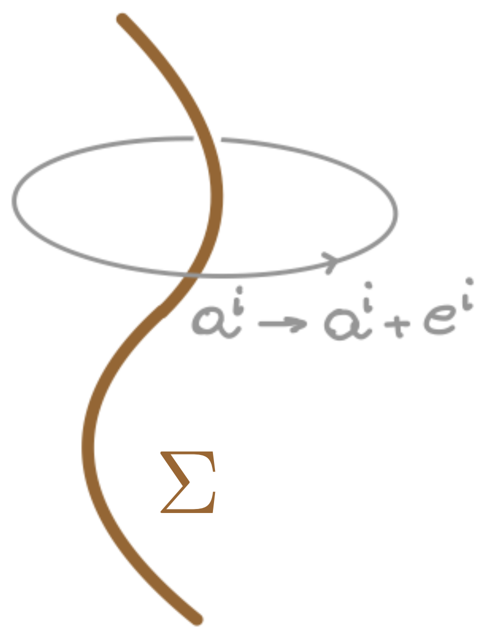
[..., Kallosh-Linde-Linde-Susskind ‘95, Cheung-Remmen ’16, GarcíaEtxebarria-Montero-Sousa-Valenzuela ’20, Aalsma-Shiu ’22, Chin Ong ’22, ...]

The axionic couplings detect the presence of EFT strings:

* $$S_{\rm bulk} \supset \int a^i I_{4,i} = -\int {\rm d}a^i \wedge I_{3,i}$$

with $$I_{4,i} = {\rm d}I_{3,i} = -\frac{1}{2}C_i\,{\rm Tr}(F\wedge F) - \frac{1}{48}\tilde{C}_i\,{\rm Tr}(R\wedge R)$$

* $${\rm d}^2 a^i = e^i \delta_2(\Sigma)$$



$$\delta\, S_{\rm bulk} = -e^i \int_\Sigma \delta I_{2,i} \neq 0$$ $$({\rm d}\delta I_{2,i} = \delta I_{3,i})$$

anomaly inflow [Callan-Harvey '85]

- Anomaly inflow must be cancelled by total world-sheet anomaly [Callan-Harvey '85]

$$I_4^{\rm ws} = -\frac{1}{48}(e^i\tilde{C}_i)\,{\rm Tr}(R_{\rm T}\wedge R_{\rm T}) + \frac{1}{24}(e^i\tilde{C}_i)\,{\rm Tr}(F_{\rm N}\wedge F_{\rm N}) - \frac{1}{2}(e^iC_i^{AB})F_A\wedge F_B \;+(\text{non-Cartan})$$

$SO(1,1)_T$ $U(1)_N$ ≥ 0 Cartan + $U(1)$s

- Weakly-coupled (0,2) NLSM on EFT string:

(0,2) multiplet	#	$U(1)_{\rm N}$ charge	$U(1)_A$ charge	fermion
chiral U	1	1	0	ρ_+
chiral Φ	$n_{\rm C}$	0	$*$	χ_+
Fermi Ψ	$n_{\rm F}$	0	q_A	ψ_-
Fermi Λ	$n_{\rm N}$	$\frac{1}{2}$	0	λ_-

~ higher order obstructions

$$\int d\theta^+\Lambda\, J(\Phi) \Longrightarrow n_{\rm C}^{\rm eff} \equiv n_{\rm C} - n_{\rm N}$$

unobstructed target space directions

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~ higher order obstructions

$$\int {\rm d}\theta^+\Lambda\, J(\Phi) \Longrightarrow n_{\rm C}^{\rm eff} \equiv n_{\rm C} - n_{\rm N}$$

- World-sheet anomaly:

$$I_4^{\rm ws} = -\frac{n_{\rm F} - n_{\rm C}^{\rm eff} - 1}{48}{\rm Tr}(R_T\wedge R_T) + \frac{3n_{\rm C}^{\rm eff}+3}{24}F_{\rm N}\wedge F_{\rm N}$$
$$-\frac{1}{2}\Big[\sum_{\mathbf{q}\in{\rm Fermi}} q^Aq^B + k_{\rm (GS)}^{AB} + (\text{neg. def. terms})\Big]F_A\wedge F_B$$

gauging of target space isometries

[Blaszczyk-Groot Nibbelink-Ruehle, Quigley-Sethi '11]

- completness + anomaly matching $\Longrightarrow$ QG bounds! [LM-Risso-Weigand '22]

$$-\frac{1}{2}\int (C_i^{AB}s^i+\ldots)\, F_A\wedge *F_B - \frac{1}{48}\int (\tilde{C}_i s^i+\ldots)\,[\mathrm{Tr}(R\wedge *R)+\ldots]$$

(1)

$$\boxed{\tilde{C}_i e^i \in 3\mathbb{Z}_{\geq 0}\,, \qquad \forall \mathbf{e}\in \mathcal{C}_{\rm S}^{\rm EFT}} \longrightarrow \boxed{\tilde{C}_i s^i > 0}$$

positive GB coupling!

(2)

$n_{\rm F}+2n_{\rm C}^{\rm eff}$

$$\boxed{r(\mathbf{e}) \leq r(\mathbf{e})_{\rm max} \equiv 2\tilde{C}_i e^i - 2\,, \quad \forall \mathbf{e}\in \mathcal{C}_{\rm S}^{\rm EFT}}$$

bounds on ranks determined by GB!

$\mathrm{rank}\{C_i^{AB}e^i\}$ = total rank of gauge sector 'coupled' to EFT string

- possible stricter bound $r(\mathbf{e})^{\rm strict}_{\rm max} < r(\mathbf{e})_{\rm max}$ from UV information

Examples and UV tests

Simplest example

- Single-field model

$$-\frac{1}{2}\int (Cs+\ldots)\ \mathrm{Tr}(F\wedge *F) - \frac{1}{48}\int (\tilde{C}s+\ldots)\ \mathrm{Tr}(R\wedge *R+\ldots)+\ldots$$

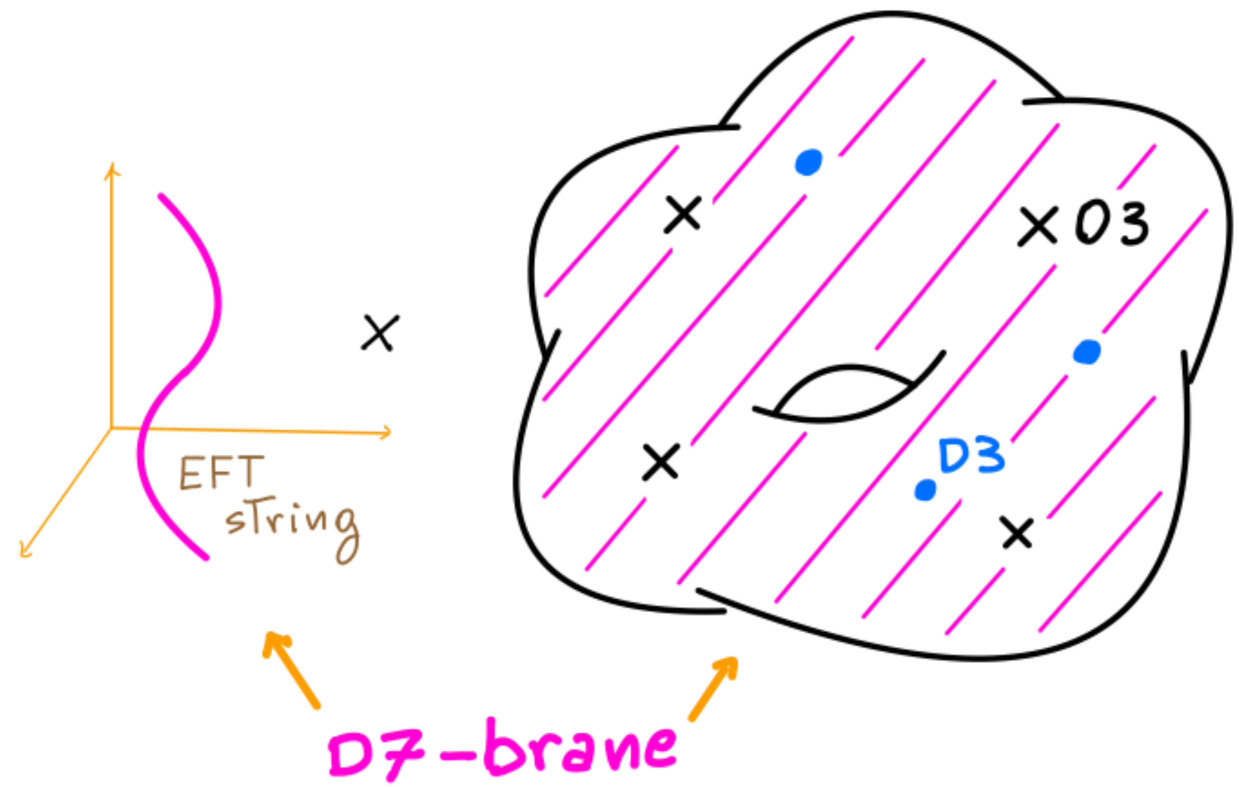
- $\{\text{saxionic cone}\} = \mathbb{R}_{>0}\ ,\quad \mathcal{C}_{\rm S}^{\rm EFT} = \mathbb{Z}_{\geq 0}$

(1) $\tilde{C} = 3k \quad , \quad k \in \mathbb{Z}_{\geq 0}$

(2) $\mathrm{rk}(\mathfrak{g}) \leq 2\tilde{C} - 2 = 6k - 2$

UV test: O3/D3 models

* (s)axion: $a + \mathrm{i}s \equiv C_0 + \mathrm{i}e^{-\phi}$



(1) $\tilde{C} = \frac{3}{16} n_{\mathrm{O3}} \in 3\mathbb{Z}_{\geq 0}$

$\Downarrow$

$$n_{\mathrm{O3}} \in 16\mathbb{N}$$

→ Tested over ~10^6 models by F. Carta [Carta-Moritz-Westphal '20]

agrees with Theorem by [Favale '17]

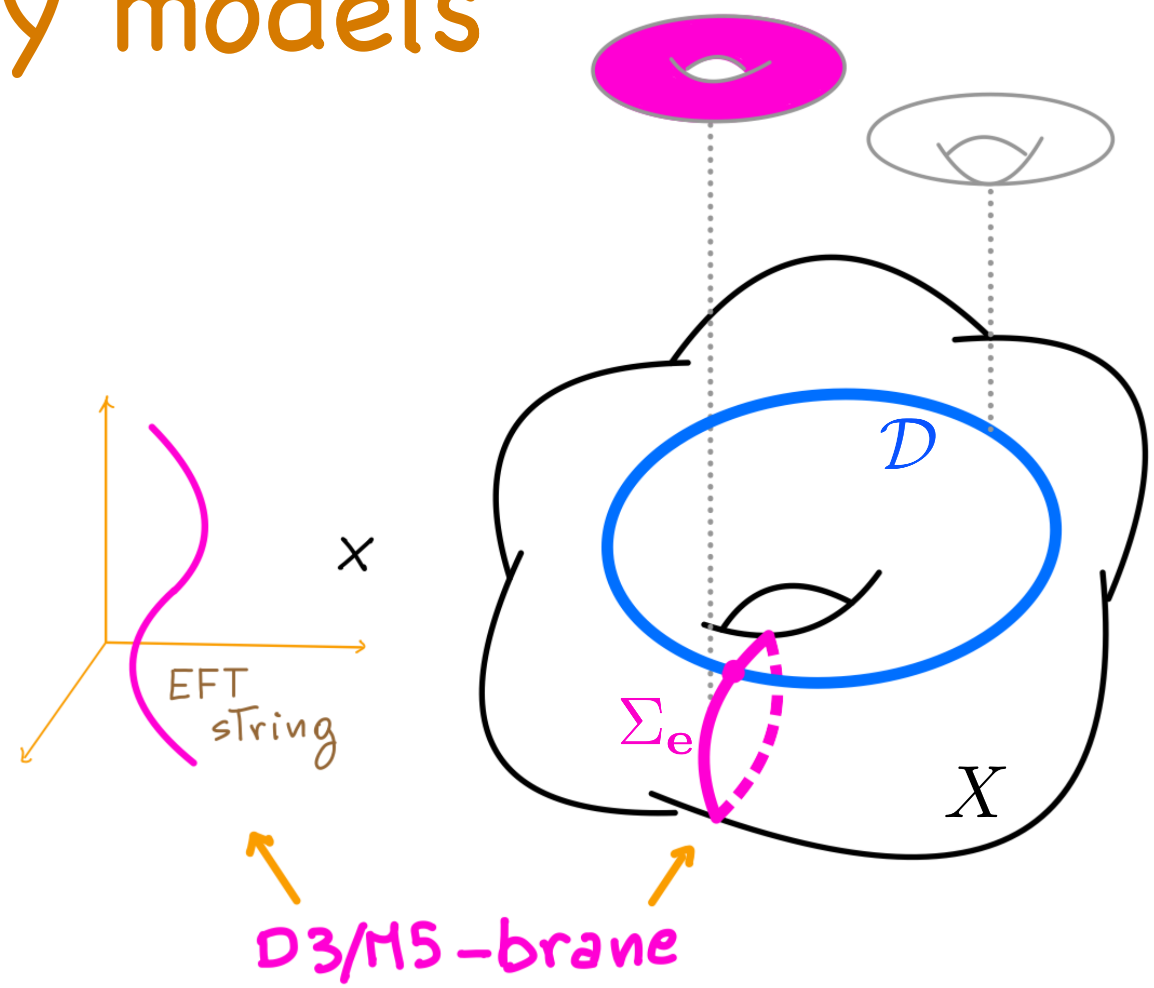
(2) $r(\mathbf{e}) = n_{\mathrm{D3}} \leq r_{\mathrm{F}}(\mathbf{e})_{\max} \leq r(\mathbf{e})_{\max}$

$$\frac{4}{3}\tilde{C}_i e^i = \frac{1}{4} n_{\mathrm{O3}} \quad \longleftrightarrow \quad 4n_{\mathrm{D3}} \leq n_{\mathrm{O3}} \quad \checkmark$$

D3 Tadpole bound!

UV test: F-theory models

- $\mathcal{C}_{\rm S}^{\rm EFT} = \{\text{movable curves } \Sigma_{\mathbf{e}}\}$

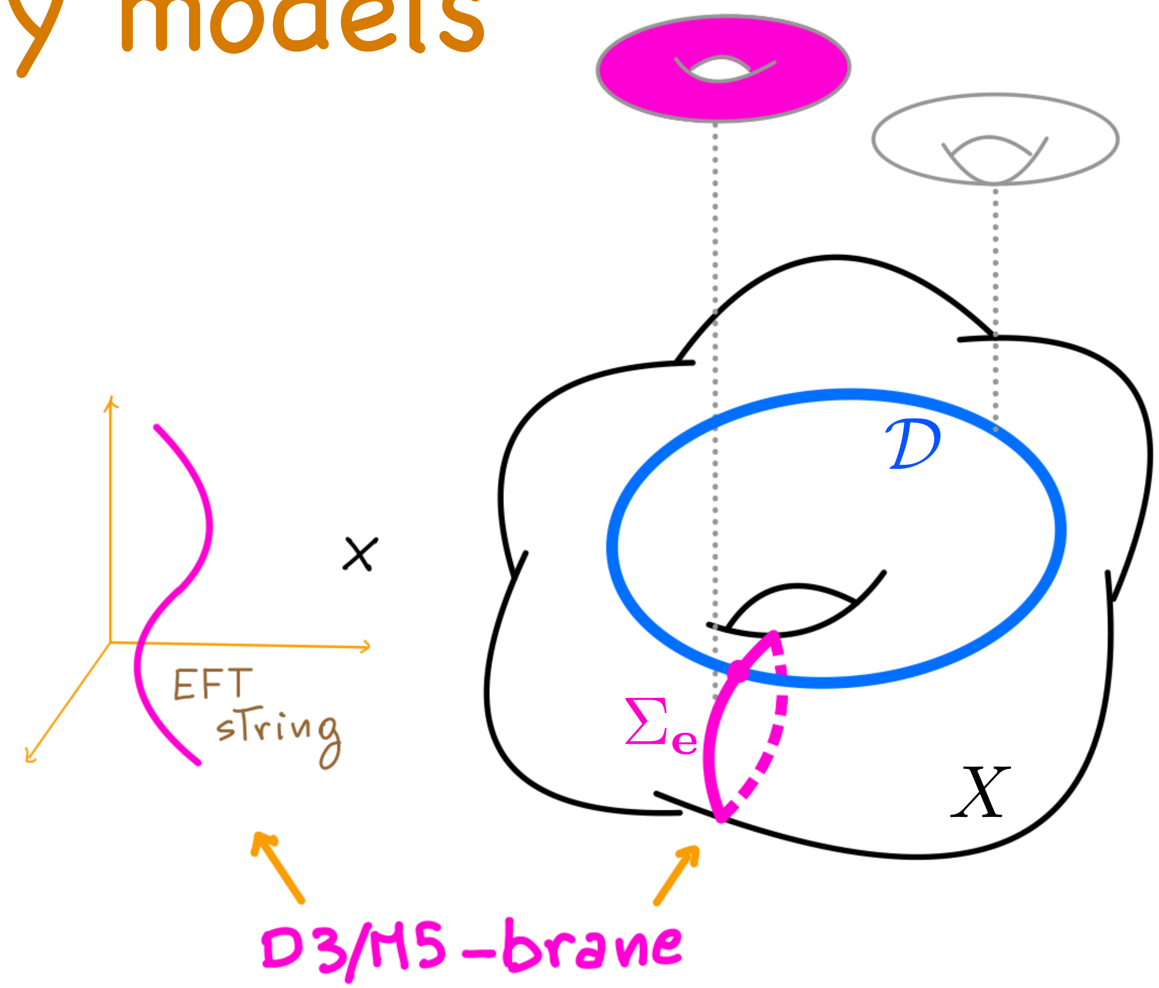


(1) $\tilde{C}_i e^i = 6\Sigma_{\mathbf{e}} \cdot \overline{K}_X \in 3\mathbb{Z}_{\geq 0}$

- Detailed world-sheet spectrum worked out in [Lawrie, Schafer-Nameki, Weigand '16]

(0,2) multiplet	#	D3/M5 modes
chiral U	1	c.o.m.
chiral $\Phi_{(1)}$	$n_{\rm C}^{(1)} = h^0(\Sigma_{\mathbf{e}}, N_{\Sigma_{\mathbf{e}}/X})$	embedding
chiral $\Phi_{(2)}$	$n_{\rm C}^{(2)} = \overline{K}_X \cdot \Sigma_{\mathbf{e}} + g - 1$	self-dual 2-form
Fermi Ψ	$n_{\rm F} = 8\Sigma_{\mathbf{e}} \cdot \overline{K}_X$	3-7 modes
Fermi $\Lambda_{(1)}$	$n_{\rm N}^{(1)} = h^1(\Sigma_{\mathbf{e}}, N_{\Sigma_{\mathbf{e}}/X})$	GS fermions
Fermi $\Lambda_{(2)}$	$n_{\rm N}^{(2)} = g$	GS fermions

UV test: F-theory models



- $\mathcal{C}_{\mathrm{S}}^{\mathrm{EFT}} = \{\text{movable curves } \Sigma_{\mathbf{e}}\}$

(1) $\tilde{C}_i e^i = 6\Sigma_{\mathbf{e}} \cdot \overline{K}_X \in 3\mathbb{Z}_{\geq 0}$

(2)
$$r(\mathbf{e}) = \sum_{I, \mathcal{D}_I \cdot \Sigma_{\mathbf{e}} \neq 0} \text{rank}(G_I)$$
$$\leq r(\mathbf{e})^{\text{strict}}_{\max} = \frac{5}{3}\tilde{C}_i e^i - 2 = 10\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X - 2$$
$$\leq r(\mathbf{e})_{\max} = 2\tilde{C}_i e^i - 2 = 12\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X - 2$$

← ($\mathcal{N}=1$, smooth base, only $\Phi_{(2)}$ contribution

- matches Kodaira's bounds!
- bounds rank of Mordell-Weil group

see also [Lee-Weigand `19]

UV test: F-theory models

- $\mathcal{C}_{\mathrm{S}}^{\mathrm{EFT}} = \{\text{movable curves } \Sigma_{\mathbf{e}}\}$

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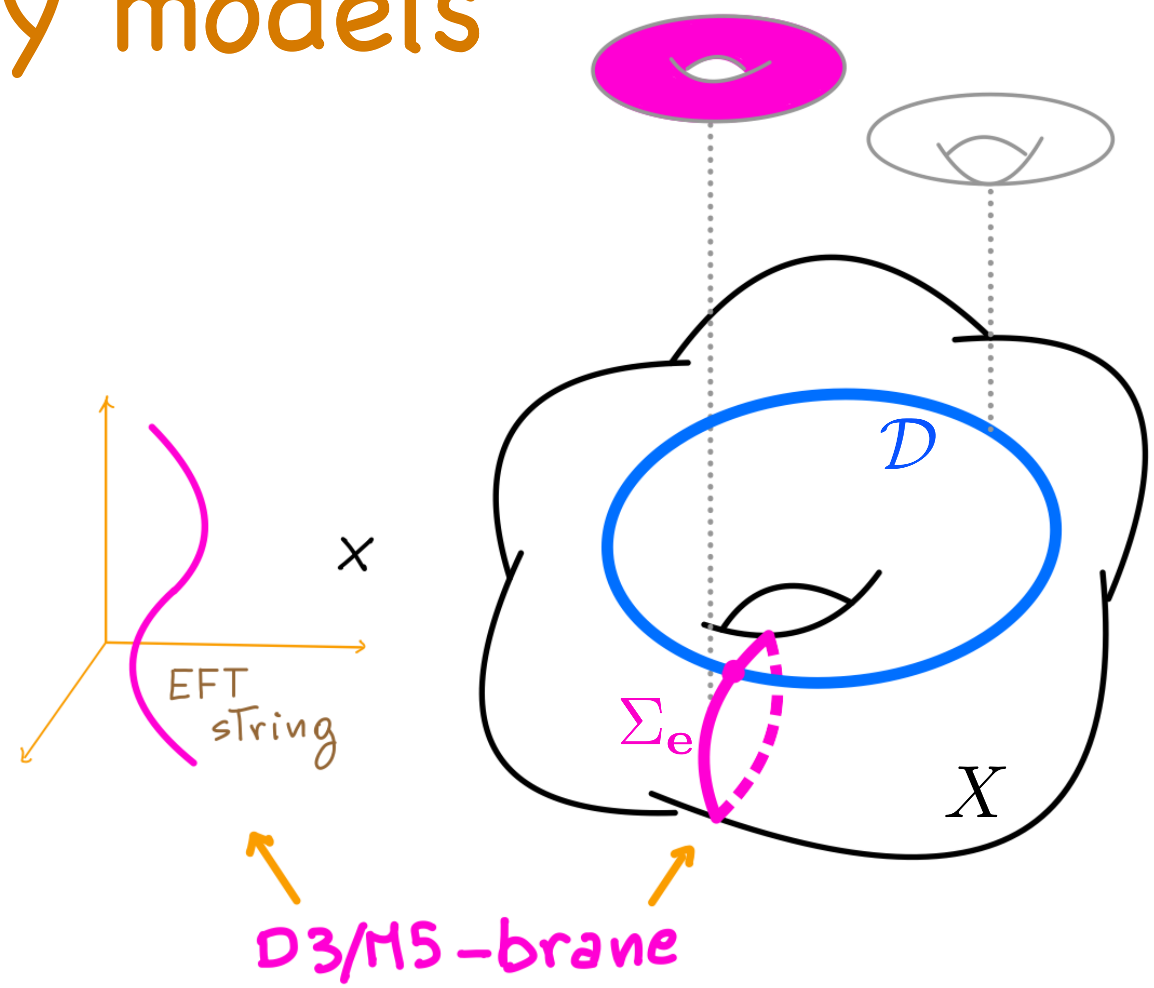


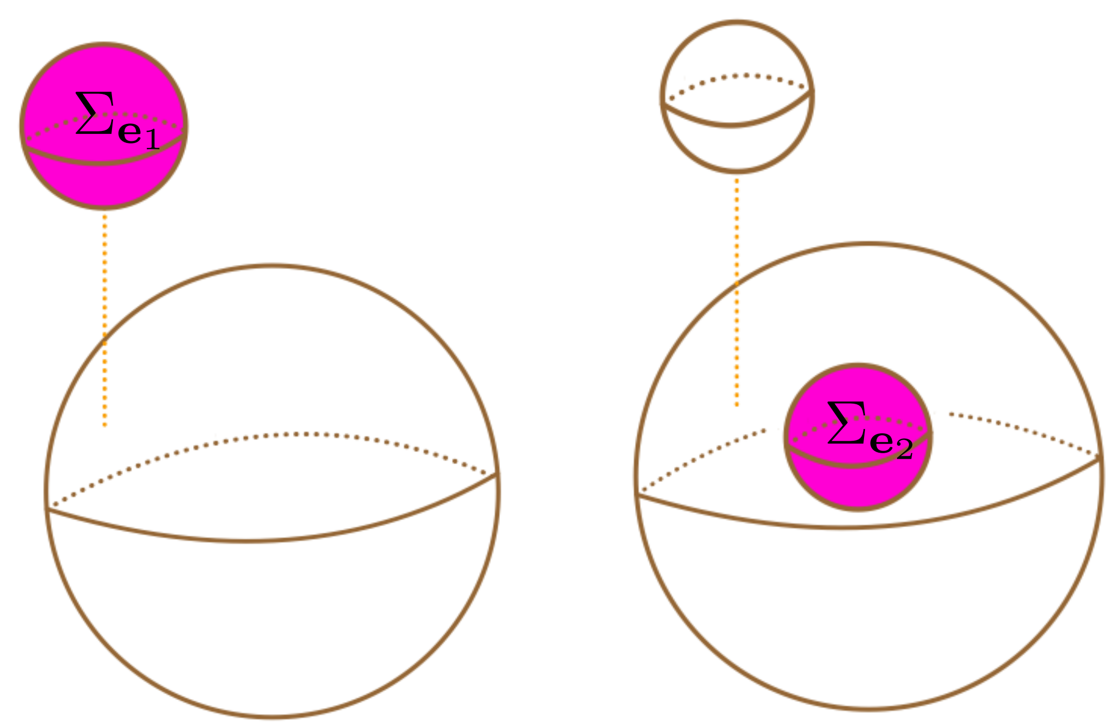
- Example 2: $\mathbb{P}^1 \overset{n \geq 0}{\hookrightarrow} X \to \mathbb{P}^2$,

* $r(\mathbf{e}_1)^{\text{strict}}_{\text{max}} = 18$

e.g. n= 0 : $G_1 = E_6^3 \to$ rank= 18

* $r(\mathbf{e}_2)^{\text{strict}}_{\text{max}} = 28 + 10n$

e.g. n= 0 : $G_2 = E_6^2 \times E_7^2 \to$ rank= 26



UV test: F-theory models

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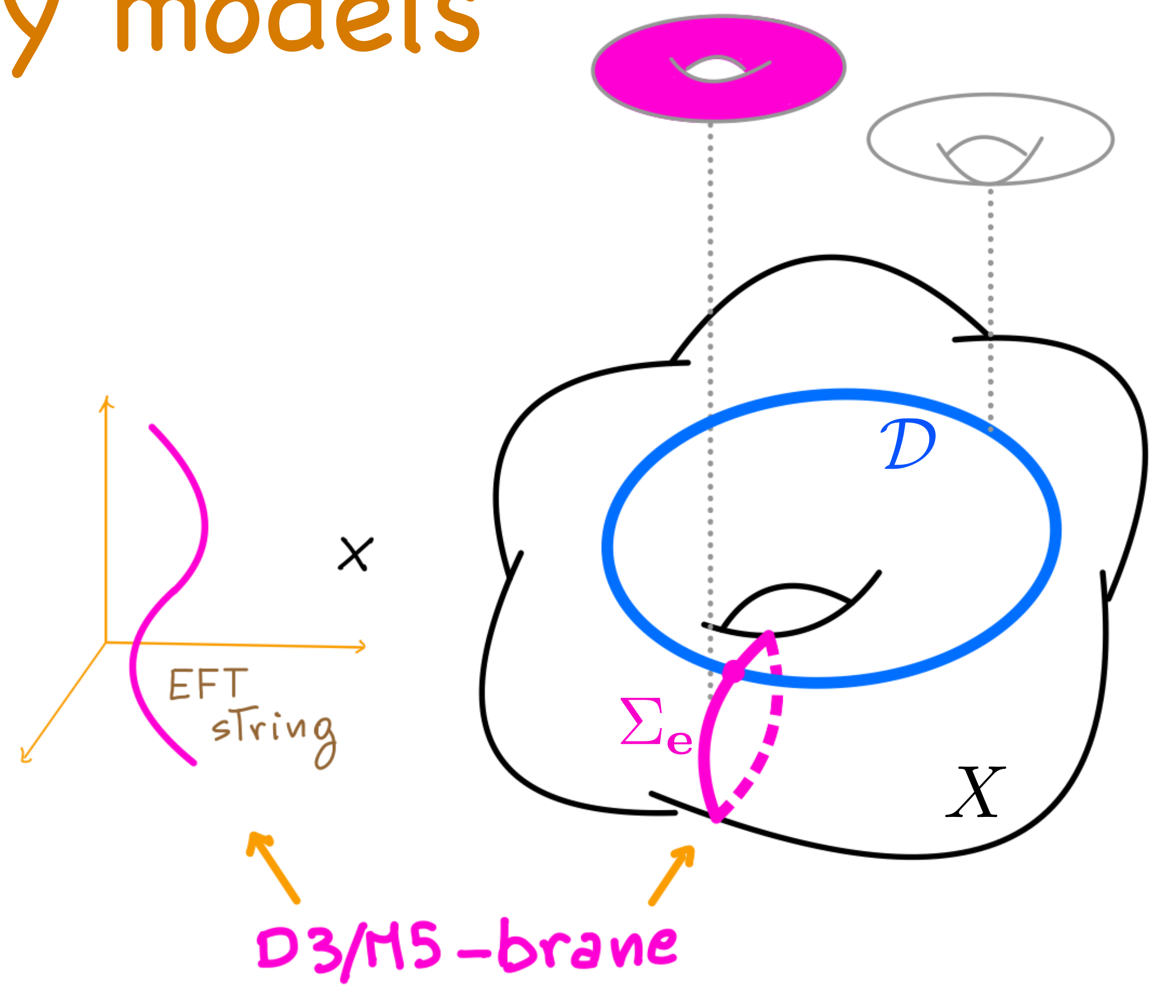

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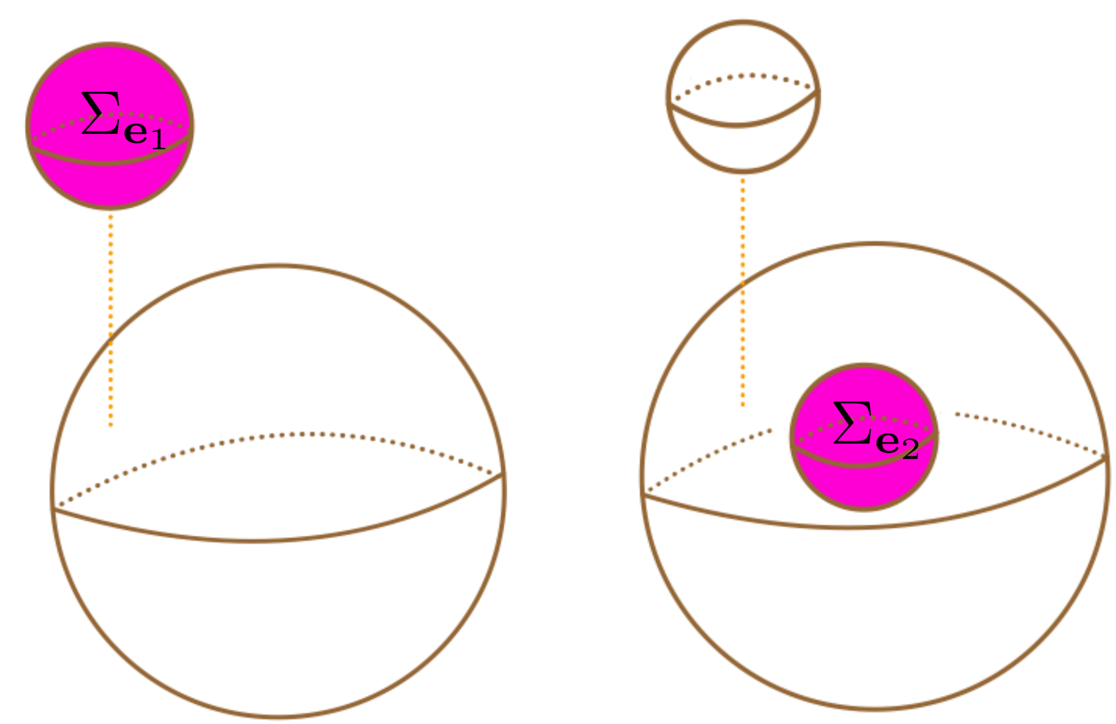
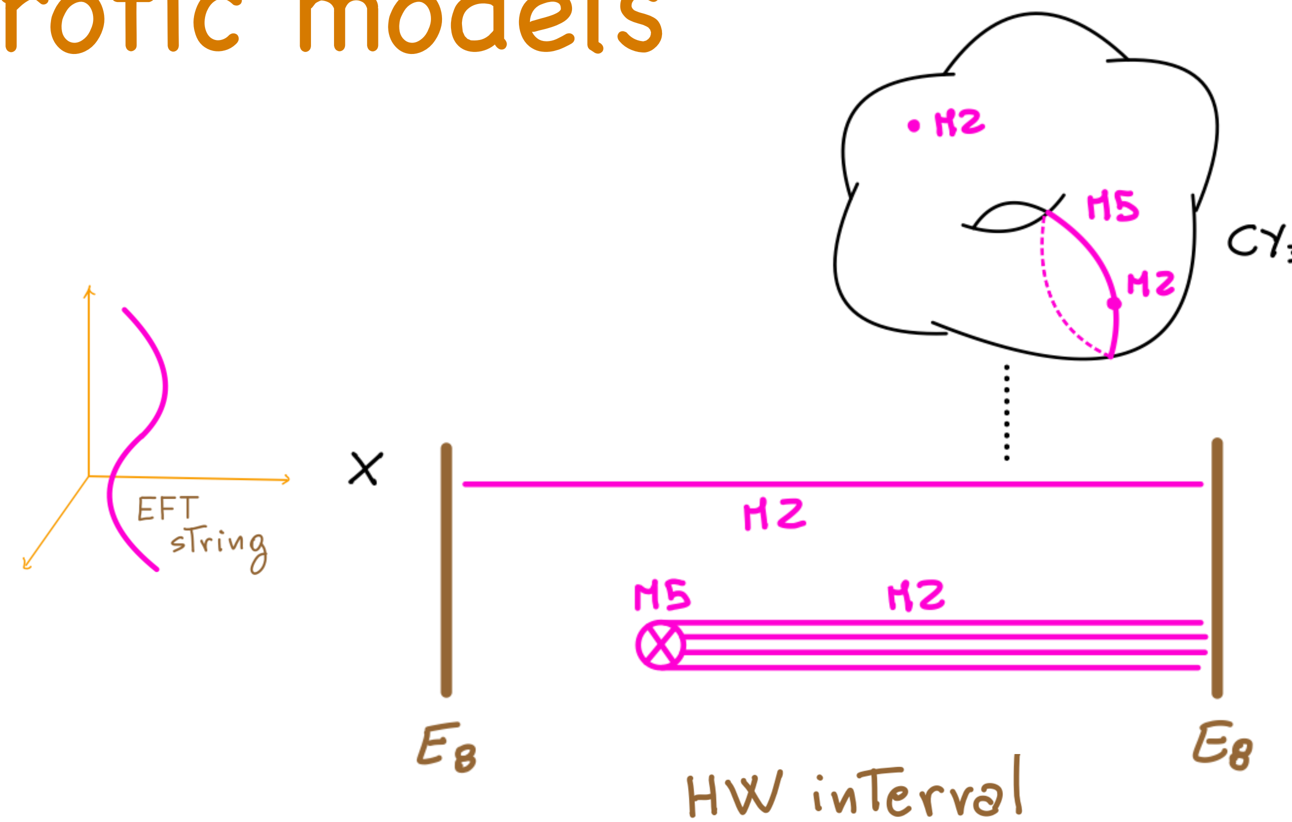


* $r(\mathbf{e}_1)^{\rm strict}_{\rm max} = 18 \quad < \quad r(\mathbf{e}_1)_{\rm max} = 22$

* $r(\mathbf{e}_2)^{\rm strict}_{\rm max} = 28 + 10n \quad < \quad r(\mathbf{e}_2)_{\rm max} = 34 + 12n$

UV test: heterotic models

- EFT strings:
  - ✱ F1/M2
  - ✱ NS5/M5 on nef divisors



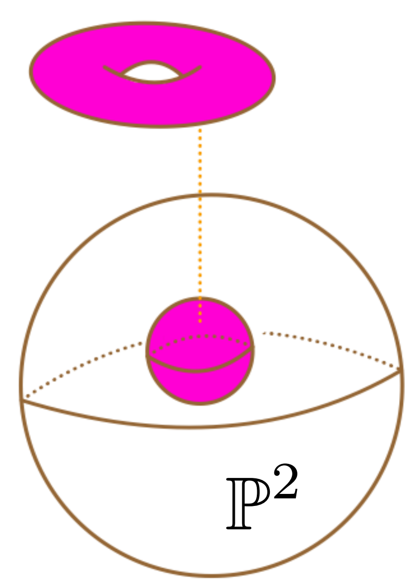
- $c_2(\mathrm{CY}_3)$ + bundle + bulk NS5s $\longrightarrow$ $\tilde{C}_i$

(1) ✓

(2) ✱ $r(\mathbf{e}_{\mathrm{F1/M2}}) \leq 22$ as with 16 superchages [Kim-Tarazi-Vafa '20]

✱ e.g. $r(\mathbf{e}_{\mathrm{NS5/M5}}) \leq 34 + 12n$

as in dual F-Theory model

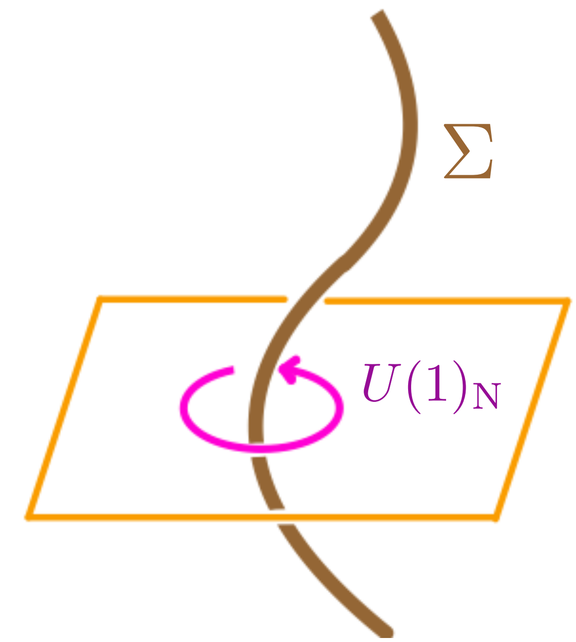


A subtle contribution

- Axionic strings in 4 dimensions can support additional term

[Witten `96]
[Becker-Becker `99]

$$-\frac{1}{24}\hat{C}_{ijk}e^j e^k \int_\Sigma \mathrm{d}a^i \wedge A_{\mathrm{N}}$$

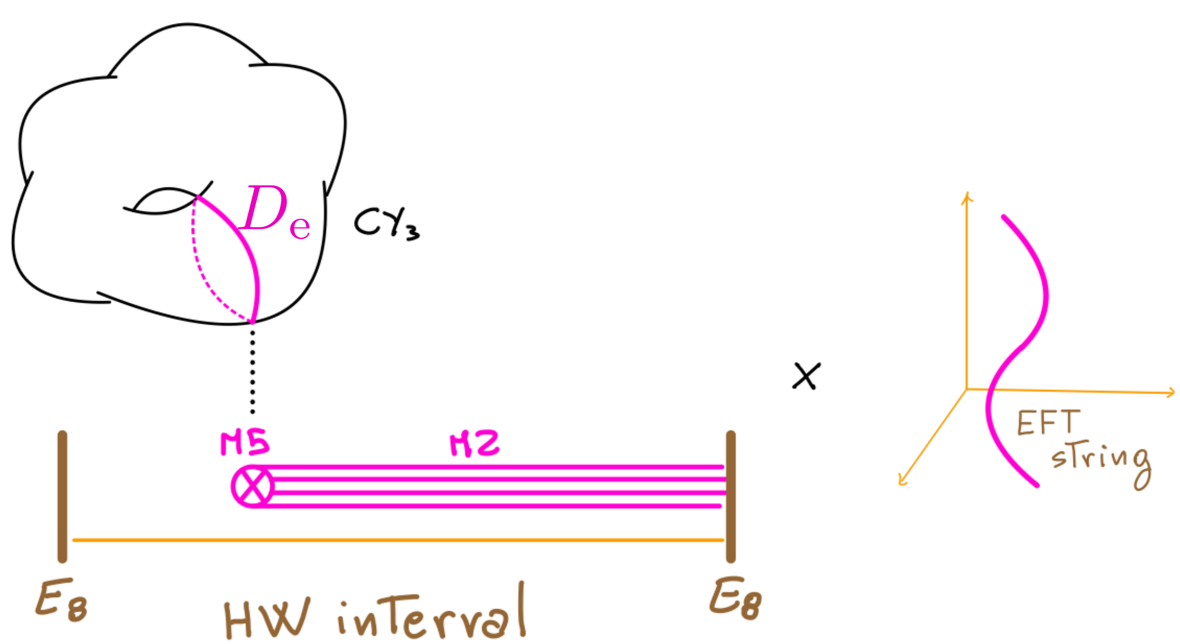


it captures hidden 5d structure

$$\hat{C}_{ijk}\int_{5d} A^i \wedge F^j \wedge F^k$$

- It appears on EFT strings whose flow identifies a preferred 5th dimension

e.g.



NS5/M5 on nef D_e

$$D_e^3 = \hat{C}_{ijk}e^i e^j e^k \neq 0$$

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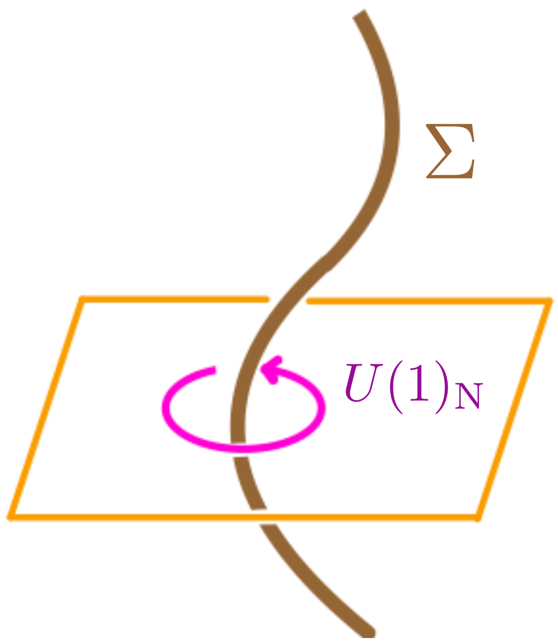
[Witten `96]
[Becker-Becker `99]

$$-\frac{1}{24}\hat{C}_{ijk}e^j e^k \int_\Sigma \mathrm{d}a^i \wedge A_{\mathrm{N}}$$



it captures hidden 5d structure

$$\hat{C}_{ijk} \int_{5d} A^i \wedge F^j \wedge F^k$$

⇓

contributes to anomaly matching

(1) $$\tilde{C}_i e^i + \hat{C}_{ijk} e^i e^j e^k \in 3\mathbb{Z}_{\geq 0}\,, \quad \forall \mathbf{e} \in \mathcal{C}_{\mathrm{S}}^{\mathrm{EFT}}$$

(2) $$r(\mathbf{e}) \leq 2\tilde{C}_i e^i + \hat{C}_{ijk} e^i e^j e^k - 2\,, \quad \forall \mathbf{e} \in \mathcal{C}_{\mathrm{S}}^{\mathrm{EFT}}$$

Conclusions

- EFT strings are physical probes of asymptotic field space regions

- Bounds on gauge and $(\text{curvature})^2$ sectors
  - Positivity of GB terms
  - Upper bounds on gauge group ranks set by GB term
  - All bounds microscopically satisfied (… so far)

- In UV-complete models, these QG bounds may provide non-trivial geometrical information

Future directions

- Bounds on matter representations?
- Phenomenological implications?
- Extension to fundamental membranes?
- Stückelberg gauging of axion and 2-form symmetries?

Thanks!

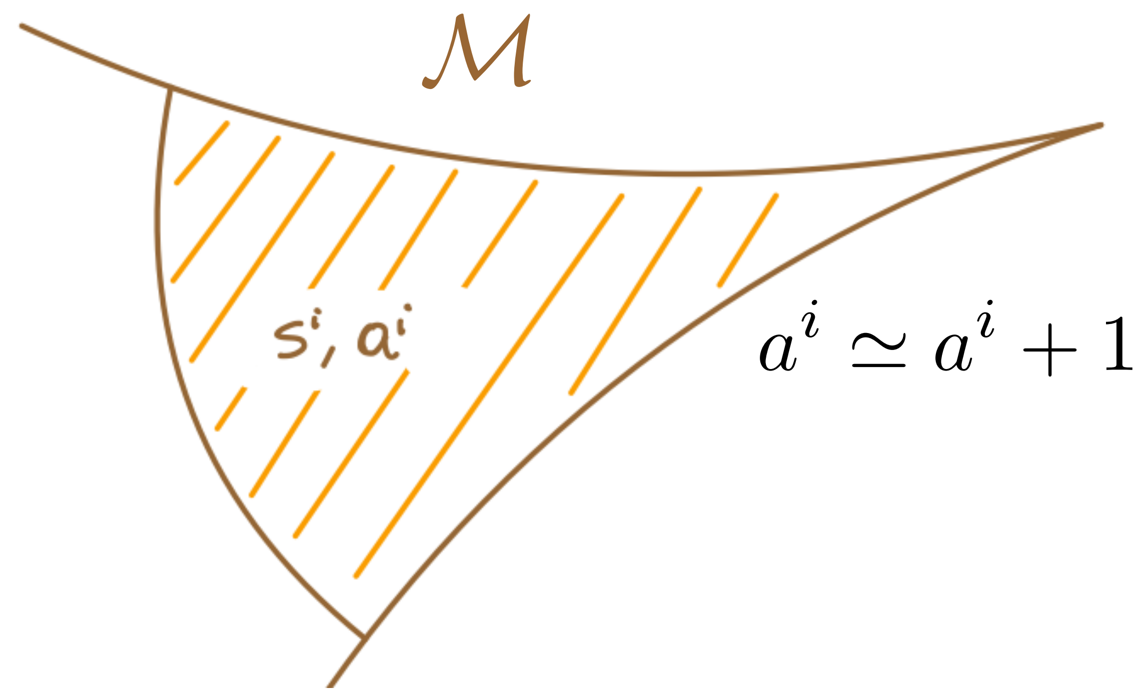
EFT strings

- Perturbative region:

$$t^i \equiv a^i + \mathrm{i}\, s^i$$

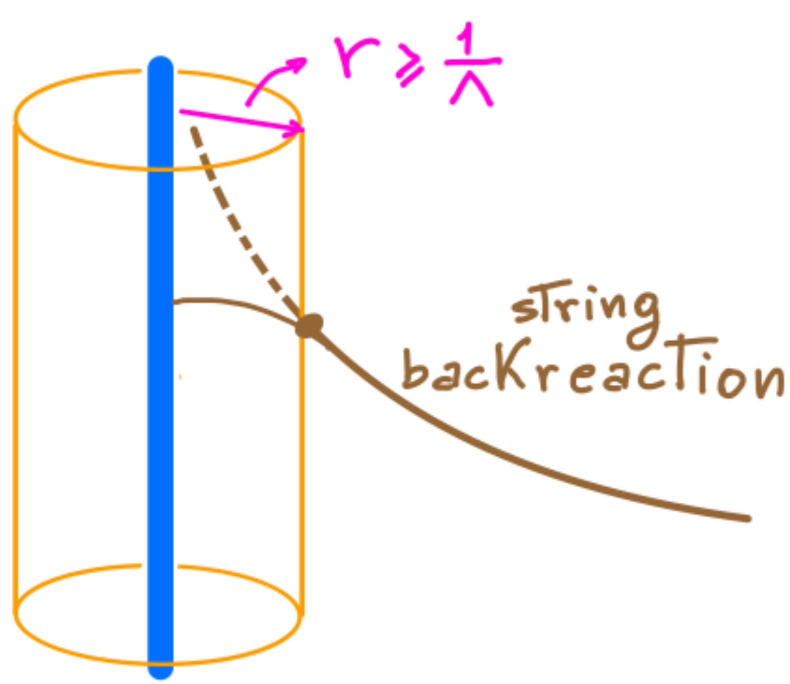
axions ↙ ↘ saxions

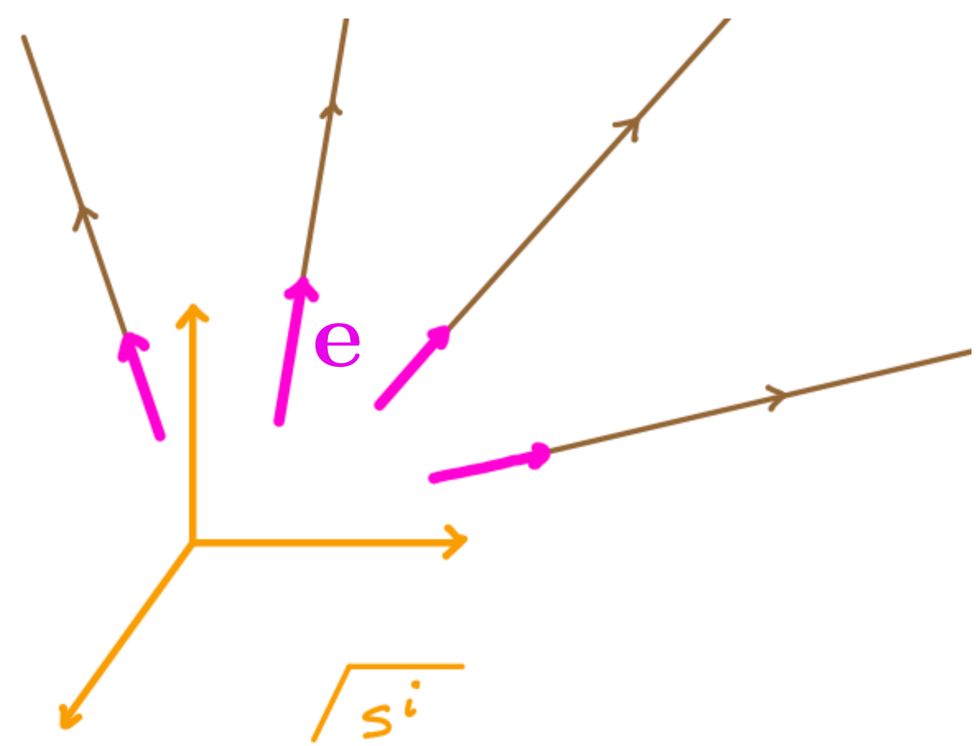
(chiral mult.)



- BPS string flows: straight saxionic lines generated by $\mathbf{e} = \{e^i\}$

[… Greene-Shapere-Vafa-Yau ’90,
Dabholkar-Gibbons-Harvey-Ruiz Ruiz ’90, …]

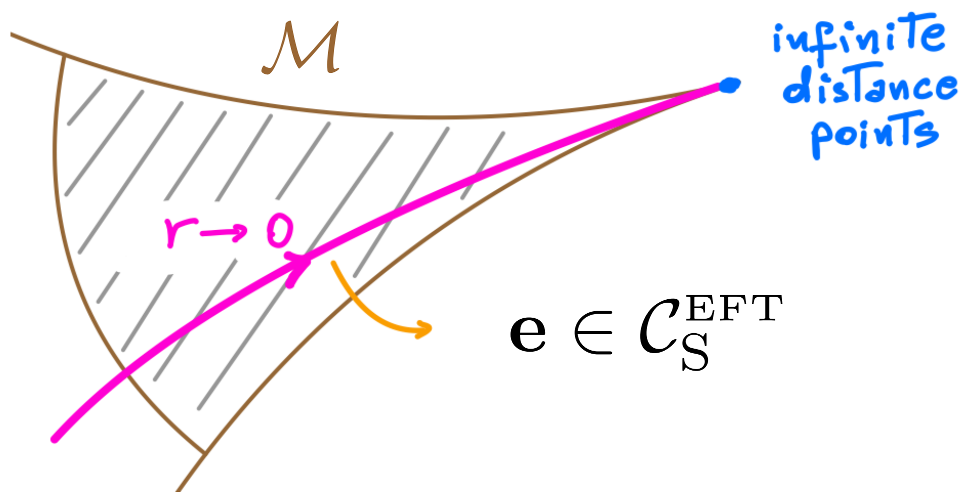




EFT strings and infinite distances

[Lanza-Marchesano-LM-Valenzuela '20-'21]

*



Distant Axionic String Conjecture

(see also [Grimm, Lanza, Li '22])

* $\mathcal{T}_{\mathbf{e}} \to 0$ along EFT string flows

EFT realization of Distance Conjecture

[Ooguri-Vafa '06]

* $$m^2_{\text{UV-tower}} \sim M_{\rm P}^2 \left(\frac{\mathcal{T}_{\rm e}}{M_{\rm P}^2}\right)^{w_{\rm e}} \longrightarrow 0$$

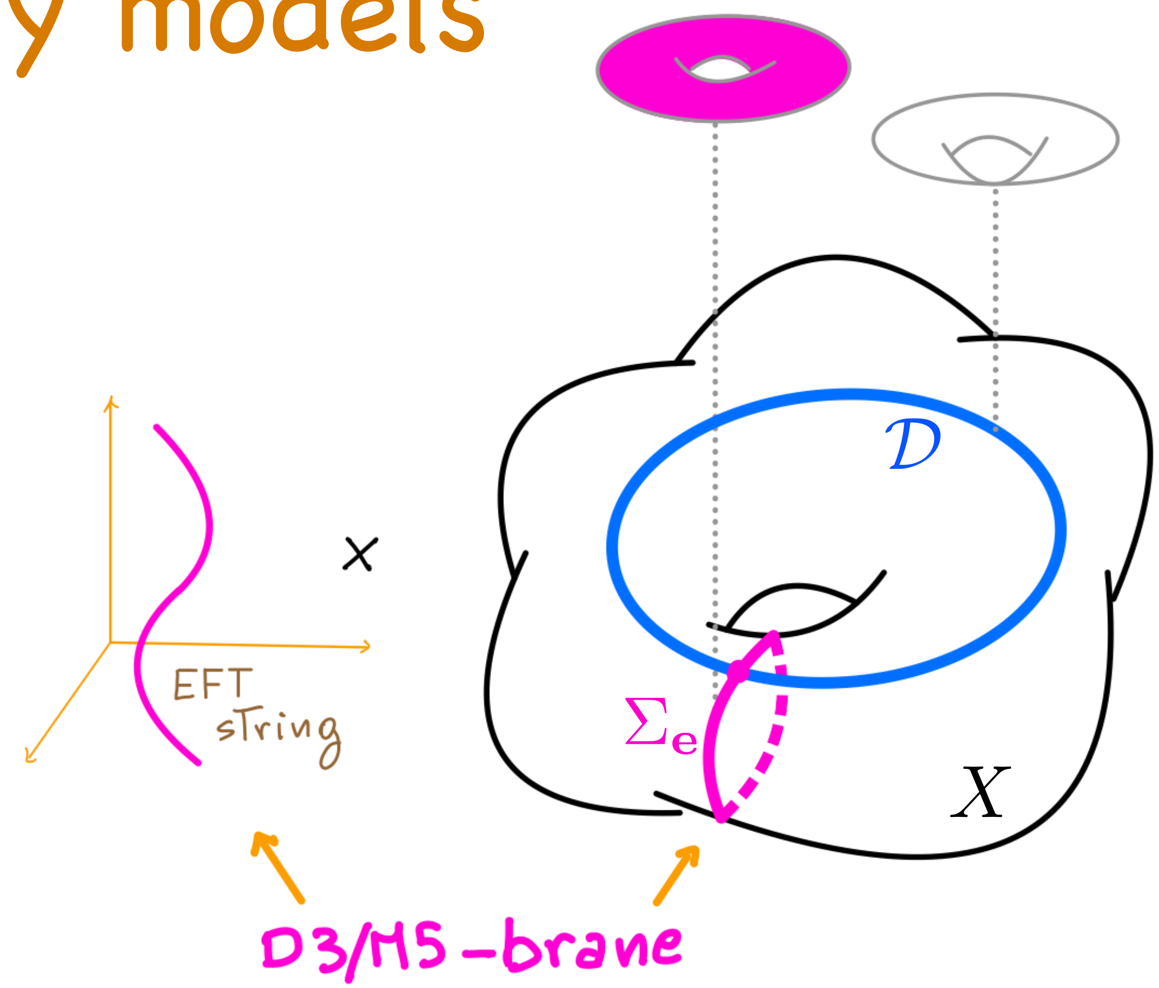
$w_{\mathbf{e}} = 1, 2, 3$
scaling weight

Integral Scaling Weight Conjecture

UV test: F-theory models

- $\mathcal{C}_{\rm S}^{\rm EFT} = \{\text{movable curves } S_{\mathbf{e}}\}$

(1) $\tilde{C}_i e^i = 6\Sigma_{\mathbf{e}} \cdot \overline{K}_X \in 3\mathbb{Z}_{\geq 0}$



- Example 1: $X = \mathbb{P}^3\,, \quad \Sigma_{\mathbf{e}} = \mathbb{P}^1 \subset \mathbb{P}^3$

$\Longrightarrow \quad r(\mathbf{e})^{\rm strict}_{\rm max} = 10\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X - 2 = 38$

e.g. $E_6 \times E_7^4 \rightarrow$ rank= 34