FeynArts and FormCalc

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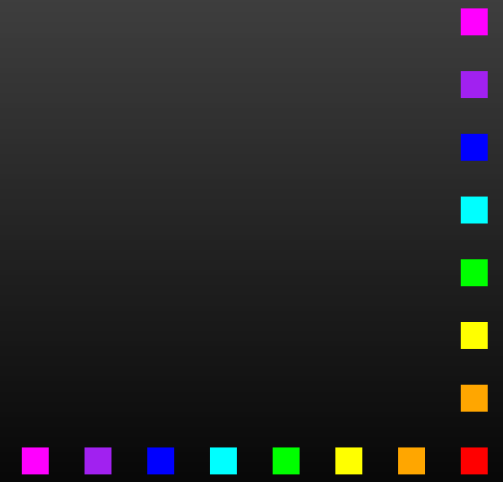


The Diagrammatic Challenge

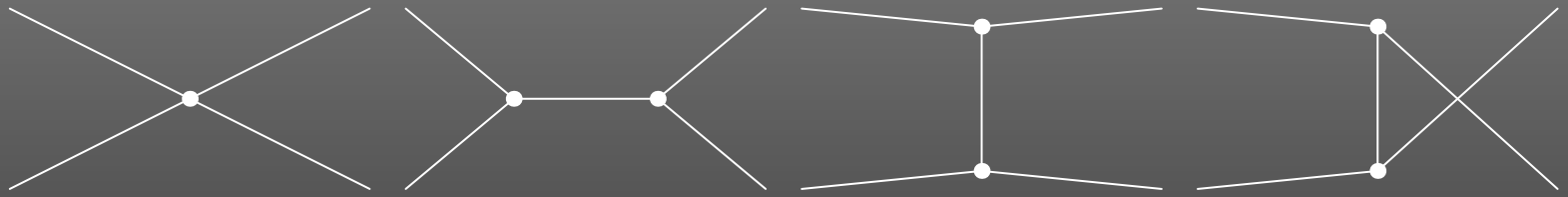
# loops	0	1	2	3+
# $2 \to 2$ topologies	4	99	2214	50051
typical accuracy	10%	1%	.1%	.01%
general procedure known	yes	yes	$1 \to 1$	no
current limits	$2 \to 8$	$2 \to 6$	$2 \to 2$	$1 \to 1$

Plus:

- Phase-space integration,
- Treatment of unstable particles,
- Numerical difficulties,
- ...

Feynman Diagram Cookbook

1. Draw all possible types of diagrams with the given number of loops and external legs

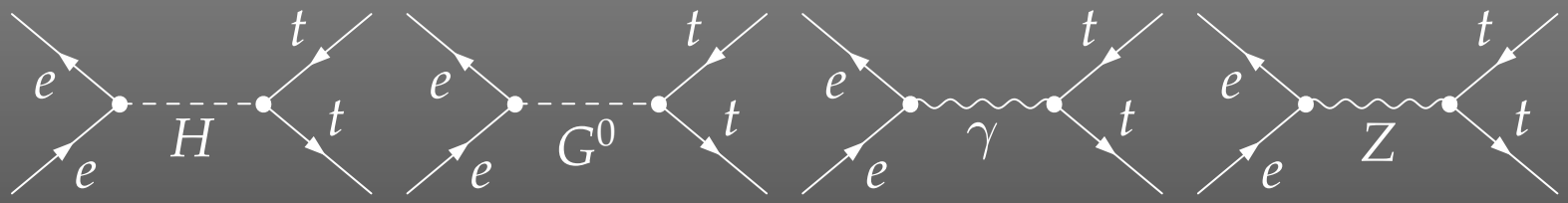


Topological task, no physics input needed*

* Well, almost: need to know allowed adjacencies in physics model, e.g. renormalizable theories have at most 3- and 4-point vertices.

Feynman Diagram Cookbook

2. Figure out what particles can run on each type of diagram



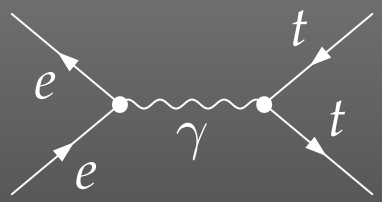
Combinatorial task, requires physics input (model)

In this case, in the SM, three of the topologies were not realized though one was realized multiply.

Note further that the e-e-scalar couplings are suppressed by m_e^2/M_W^2 and thus usually neglected.
These are selections one would typically make at this stage, i.e. diagrammatically.

Feynman Diagram Cookbook

3. Translate the diagrams into formulas by applying the Feynman rules

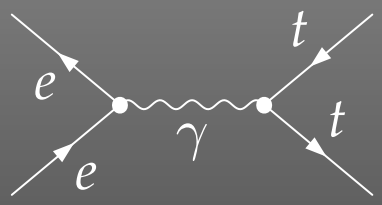


$$= \underbrace{\langle v_1 | \, \mathrm{i}e\gamma^\mu \, | u_2 \rangle}_{\text{left vertex}} \; \underbrace{\frac{g_{\mu\nu}}{(k_1+k_2)^2}}_{\text{propagator}} \; \underbrace{\langle u_4 | \left(-\tfrac{2}{3}\mathrm{i}e\gamma^\nu\right) | v_3 \rangle}_{\text{right vertex}}$$

Database look-up

Feynman Diagram Cookbook

4. Contract the indices, take the traces, etc.



$$= \frac{8\pi\alpha}{3s} F_1\,, \qquad F_1 = \langle v_1|\,\gamma_\mu\,|u_2\rangle\,\langle u_4|\,\gamma^\mu\,|u_3\rangle$$

Also, compute the fermionic matrix elements, e.g. by squaring and taking the trace:

$$\begin{aligned} |F_1|^2 &= \mathrm{Tr}\,\{(\not{k}_1 - m_e)\gamma_\mu(\not{k}_2 + m_e)\gamma_\nu\}\,\mathrm{Tr}\,\{(\not{k}_4 + m_t)\gamma^\mu(\not{k}_3 - m_t)\gamma^\nu\} \\ &= \tfrac{1}{2}s^2 + st + (m_e^2 + m_t^2 - t)^2 \end{aligned}$$

Algebraic simplification

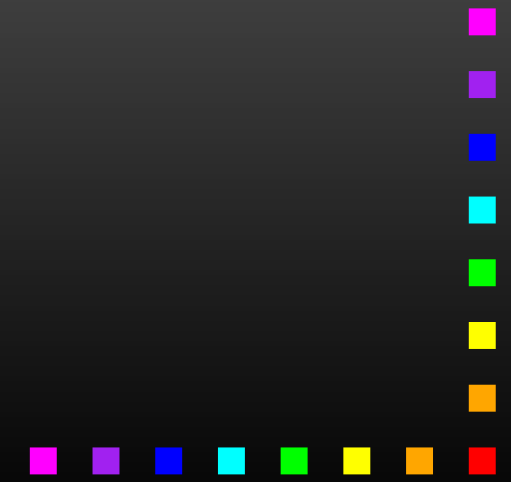
Feynman Diagram Cookbook

5. Write the results up as a ..... program
(put favourite language here)

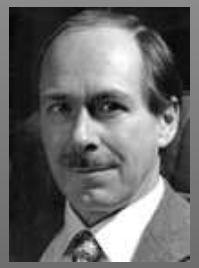
5a. Debug that program

6. Run it to produce numerical values

Programming



Recipe for Feynman Diagrams

Thanks to  and  (and others) we have a

Recipe for an ARBITRARY Feynman diagram up to one loop

①	Draw all possible types of diagrams	topological task
②	Figure out what particles can run on each type of diagram	combinatorical task
③	Translate the diagrams into formulas by applying the Feynman rules	database look-up
④	Contract the indices, take the traces, etc.	algebraic simplification
⑤	Write up the results as a computer program	programming
⑥	Run the program to get numerical results	waiting

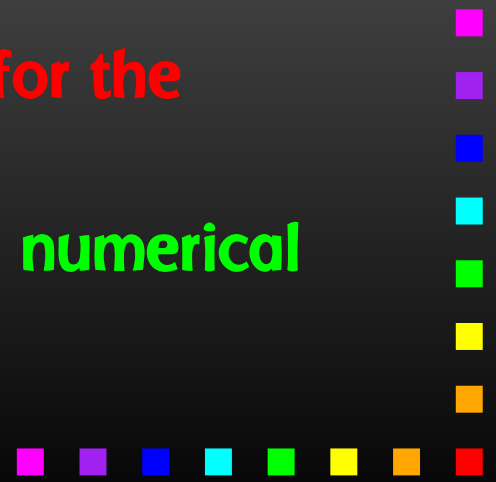
Programming Techniques

- Very different tasks at hand.
- Some objects must/should be handled symbolically, e.g. tensorial objects, Dirac traces, dimension (D vs. 4).
- Reliable results required even in the presence of large cancellations.
- Fast evaluation desirable (e.g. for Monte Carlos).

Hybrid Programming Techniques necessary

Symbolic manipulation (a.k.a. Computer Algebra) for the structural and algebraic operations.

Compiled high-level language (e.g. Fortran) for the numerical evaluation.



Automated Diagram Evaluation

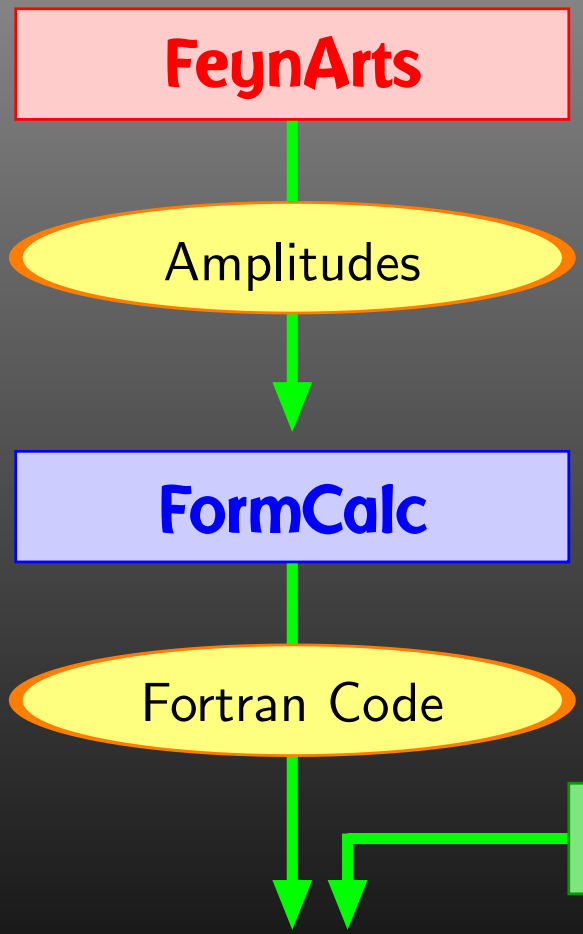


Diagram Generation:

- Create the topologies
- Insert fields
- Apply the Feynman rules
- Paint the diagrams

Algebraic Simplification:

- Contract indices
- Calculate traces
- Reduce tensor integrals
- Introduce abbreviations

Numerical Evaluation:

- Convert Mathematica output to Fortran code
- Supply a driver program
- Implementation of the integrals

Symbolic manipulation (Computer Algebra) for the structural and algebraic operations.

Compiled high-level language (Fortran) for the numerical evaluation.

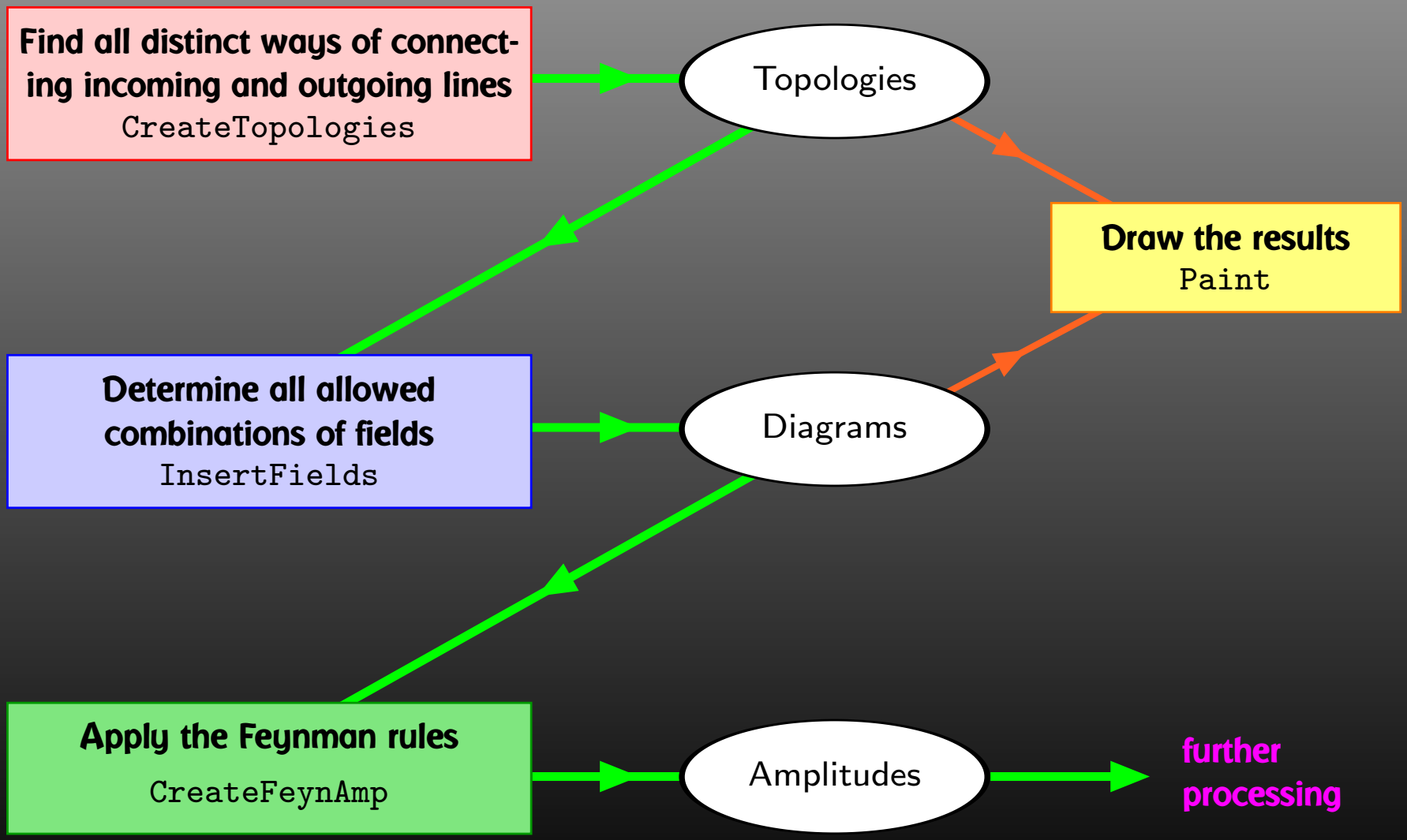
$|\mathcal{M}|^2 \longrightarrow$ Cross-sections, Decay rates, ...

Plan

Walk through the general setup of these programs and show some perhaps non-standard applications.

- **'Standard Candle'** – $e^+e^- \to t\bar{t}$,
- **Resumming a coupling** – Δ_b,
- **Example from flavour physics** – ΔM_s.

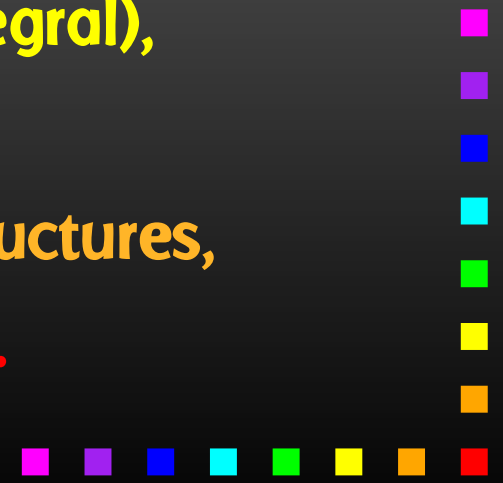
FeynArts


Find all distinct ways of connecting incoming and outgoing lines
CreateTopologies
Topologies
Draw the results
Paint
Determine all allowed combinations of fields
InsertFields
Diagrams
Apply the Feynman rules
CreateFeynAmp
Amplitudes
further processing

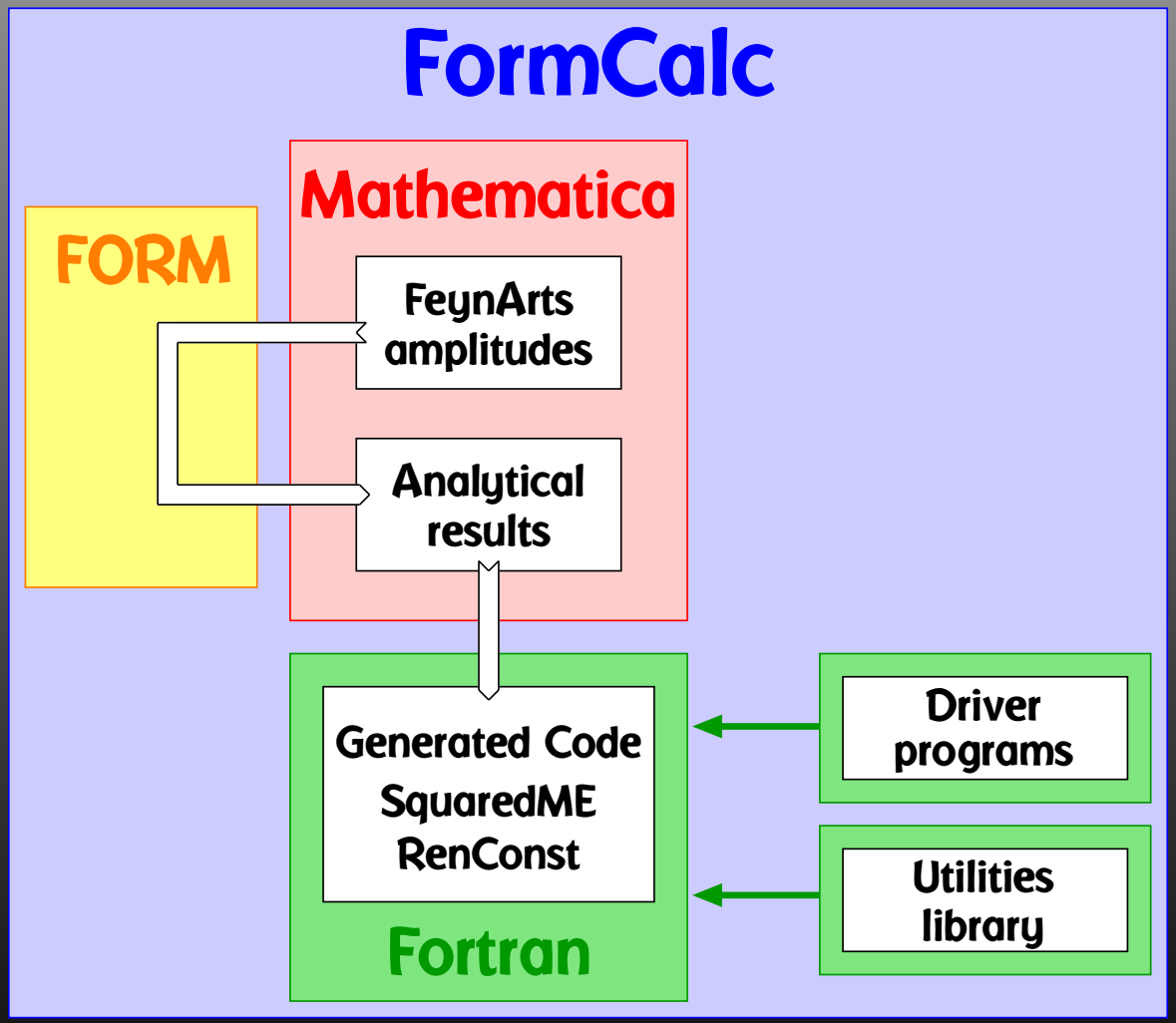
Algebraic Simplification

The amplitudes output by FeynArts so far are in no good shape for direct numerical evaluation. Some objects must/should be handled symbolically, e.g. tensorial objects, Dirac traces, dimension (D vs. 4).

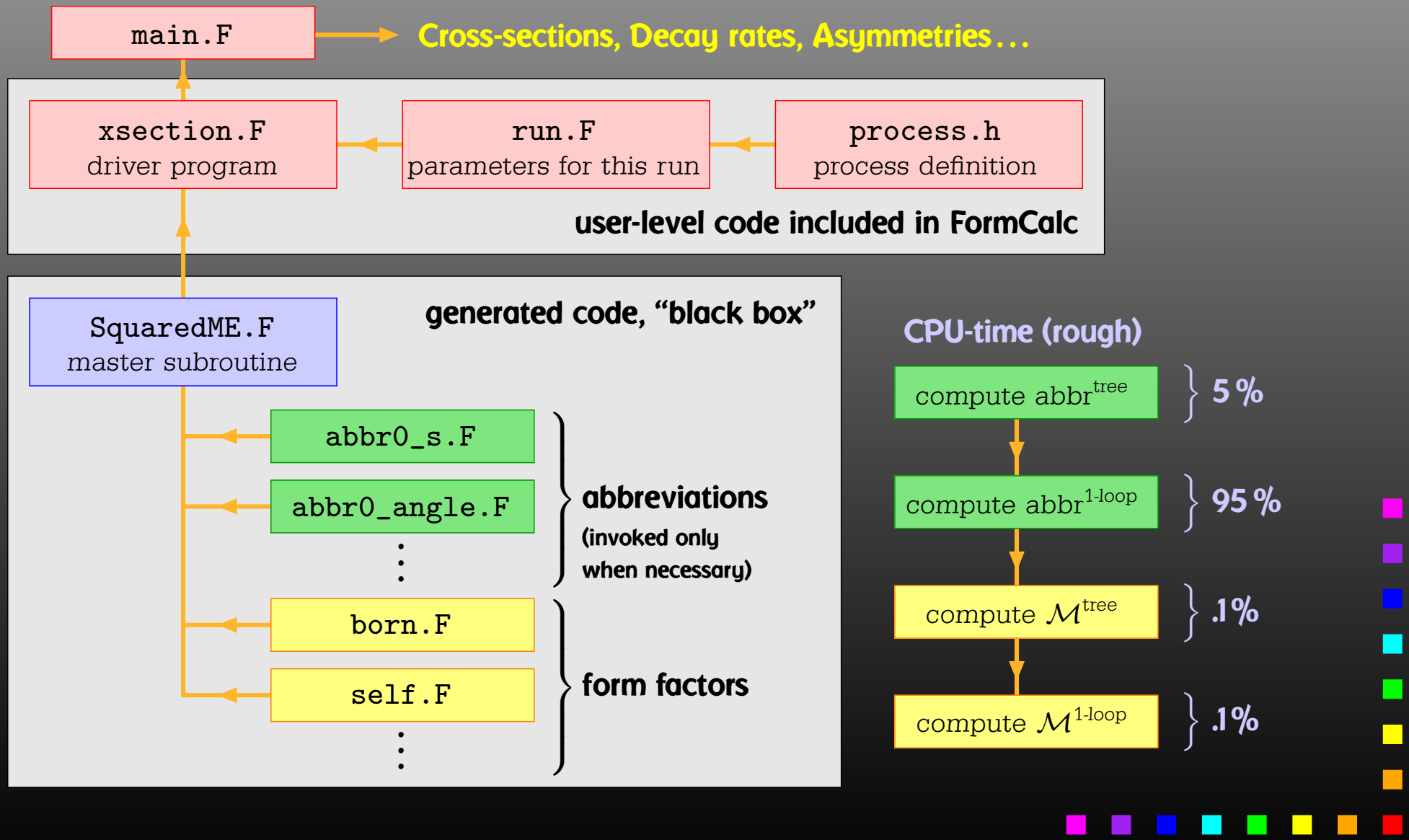
- contract indices as far as possible,
- evaluate fermion traces,
- perform the tensor reduction,
- add local terms arising from D·(divergent integral),
- simplify open fermion chains,
- simplify and compute the square of SU(N) structures,
- "compactify" the results as much as possible.



FormCalc Internals



Numerical Evaluation in Fortran 77



Three Levels of Fields

Generic level, e.g. `F, F, S`

$$C(F_1, F_2, S) = G_L \mathbb{P}_L + G_R \mathbb{P}_R \qquad \mathbb{P}_{R,L} = (\mathbb{1} \pm \gamma_5)/2$$

Kinematical structure completely fixed, most algebraic simplifications (e.g. tensor reduction) can be carried out.

Classes level, e.g. `-F[2], F[1], S[3]`

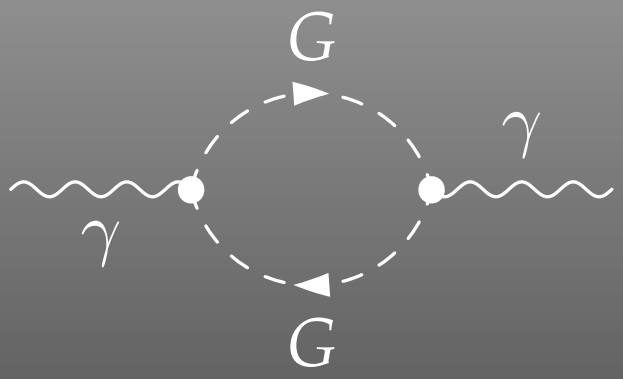
$$\bar{\ell}_i \nu_j G: \quad G_L = -\frac{\mathrm{i}\, e\, m_{\ell,i}}{\sqrt{2} \sin\theta_w M_W} \delta_{ij}\,, \quad G_R = 0$$

Coupling fixed except for i, j (can be summed in do-loop).

Particles level, e.g. `-F[2,{1}], F[1,{1}], S[3]`

insert fermion generation (1, 2, 3) for i and j

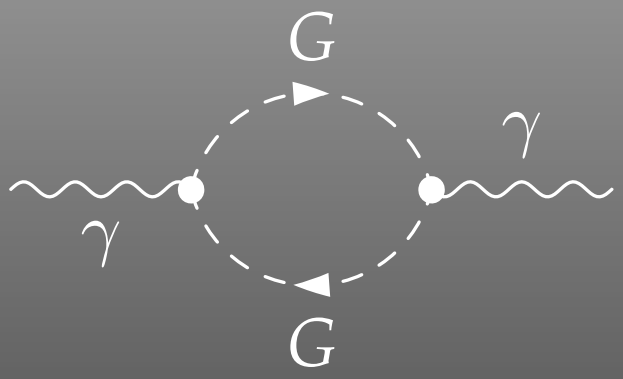
Sample CreateFeynAmp output



= FeynAmp[*identifier* ,
loop momenta ,
generic amplitude ,
insertions]

```
GraphID[Topology == 1, Generic == 1]
```

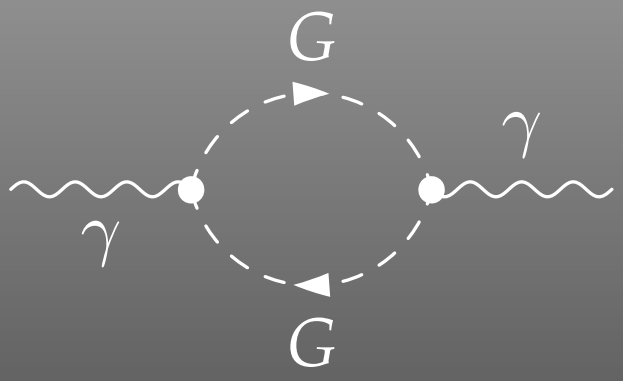
Sample CreateFeynAmp output



= FeynAmp[*identifier*,
loop momenta,
generic amplitude,
insertions]

```
Integral[q1]
```

Sample CreateFeynAmp output



= FeynAmp[*identifier*,
loop momenta,
generic amplitude,
insertions]

$\frac{\text{I}}{32\ \text{Pi}^4}$ RelativeCF ..... *prefactor*

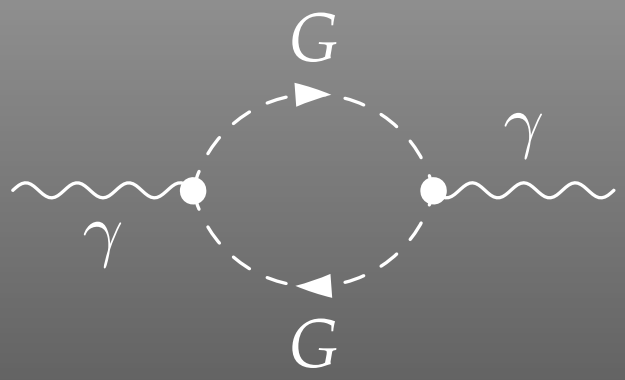
FeynAmpDenominator[$\frac{1}{\text{q1}^2 - \text{Mass[S[Gen3]]}^2}$, $\frac{1}{(-\text{p1} + \text{q1})^2 - \text{Mass[S[Gen4]]}^2}$] ..... *loop denominators*

(p1 - 2 q1)[Lor1] (-p1 + 2 q1)[Lor2] ..... *kin. coupling structure*

ep[V[1], p1, Lor1] ep*[V[1], k1, Lor2] ..... *polarization vectors*

$\text{G}^{(0)}_{\text{SSV}}$[(Mom[1] - Mom[2])[KI1[3]]]
$\text{G}^{(0)}_{\text{SSV}}$[(Mom[1] - Mom[2])[KI1[3]]], ... *coupling constants*

Sample CreateFeynAmp output



= FeynAmp[*identifier*,
loop momenta,
generic amplitude,
insertions]

```
{ Mass[S[Gen3]],
  Mass[S[Gen4]],
  G_SSV^(0)[(Mom[1] - Mom[2])[KI1[3]]],
  G_SSV^(0)[(Mom[1] - Mom[2])[KI1[3]]],
  RelativeCF } ->
Insertions[Classes][{MW, MW, I EL, -I EL, 2}]
```

Excursion: Programming Own Diagram Filters

Or, What if FeynArts' selection functions are not enough.

Observe the structure of inserted topologies:

TopologyList[__][t_1, t_2, ...]

t_i: Topology[_][__] -> Insertions[Generic][g_1, g_2, ...]

g_i: Graph[__][__] -> Insertion[Classes][c_1, c_2, ...]

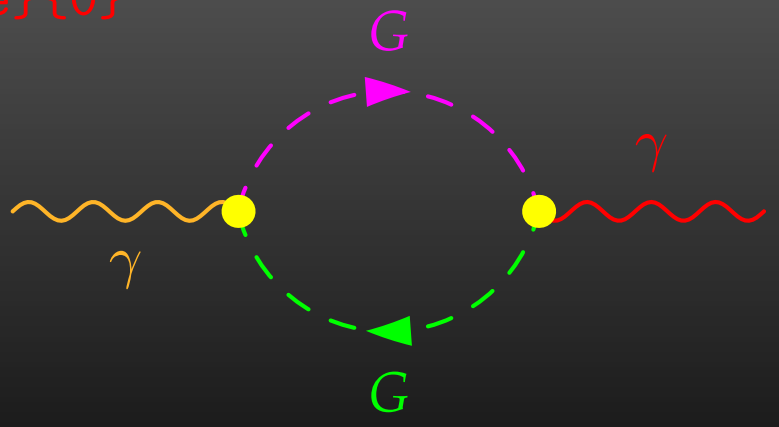
c_i: Graph[__][__] -> Insertion[Particles][p_1, p_2, ...]

Example: Select the diagrams with only fermion loops.

```
FermionLoop[t:TopologyList[___][__]] := FermionLoop/@ t

FermionLoop[(top:Topology[_][__]) -> ins:Insertions[Generic][__]] :=
  top -> TestLoops[top]/@ ins

TestLoops[top_][gi_ -> ci_] := (gi -> ci) /;
  MatchQ[Cases[top /. List@@ gi,
    Propagator[Loop[_]][v1_, v2_, field_] -> field], {F..}]

TestLoops[_][_] := Sequence[]
```

Sample Paint output

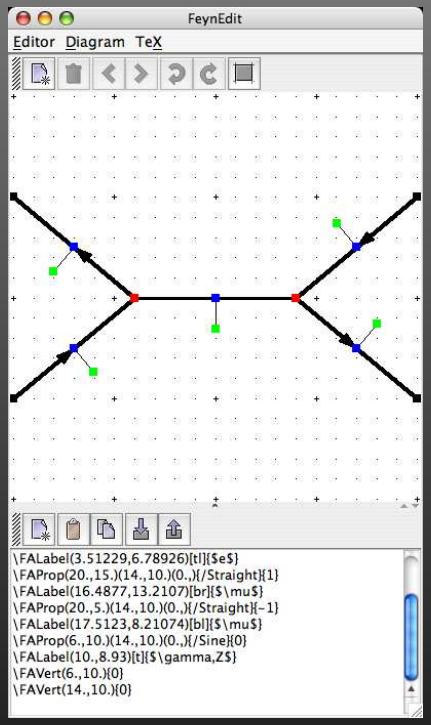
```
\begin{feynartspicture}(150,150)(1,1)
\FADiagram{}
\FAProp(6.,10.)(14.,10.)(0.8,){ScalarDash}{-1}
\FALabel(10.,5.73)[t]{$G$}
\FAProp(6.,10.)(14.,10.)(-0.8,){ScalarDash}{1}
\FALabel(10.,14.27)[b]{$G$}
\FAProp(0.,10.)(6.,10.)(0.,){Sine}{0}
\FALabel(3.,8.93)[t]{$\gamma$}
\FAProp(20.,10.)(14.,10.)(0.,){Sine}{0}
\FALabel(17.,11.07)[b]{$\gamma$}
\FAVert(6.,10.){0}
\FAVert(14.,10.){0}
\end{feynartspicture}
```



Technically: uses its own PostScript prologue.

Editing Feynman Diagrams

The elements of the diagram are easy to recognize and it is straightforward to make changes e.g. to the label text using any text editor.
It is less straightforward, however, to alter the geometry of the diagram, i.e. to move vertices and propagators.



The new tool FeynEdit lets the user:

- copy-and-paste the LaTeX code into the lower panel of the editor,
- visualize the diagram,
- modify it using the mouse, and finally
- copy-and-paste it back into the text.

FormCalc Output

A typical term in the output looks like

```
C0i[cc12, MW2, MW2, S, MW2, MZ2, MW2] *
  ( -4 Alfa2 MW2 CW2/SW2 S AbbSum16 +
    32 Alfa2 CW2/SW2 S^2 AbbSum28 +
    4 Alfa2 CW2/SW2 S^2 AbbSum30 -
    8 Alfa2 CW2/SW2 S^2 AbbSum7 +
    Alfa2 CW2/SW2 S(T-U) Abb1 +
    8 Alfa2 CW2/SW2 S(T-U) AbbSum29 )
```

= loop integral

= kinematical variables

= constants

= automatically introduced abbreviations

Abbreviations

Outright factorization is usually out of question.
Abbreviations are necessary to reduce size of expressions.

```
AbbSum29 = Abb2 + Abb22 + Abb23 + Abb3

Abb22 = Pair1 Pair3 Pair6

Pair3 = Pair[e[3], k[1]]
```

The full expression corresponding to `AbbSum29` is

```
Pair[e[1], e[2]] Pair[e[3], k[1]] Pair[e[4], k[1]] +
Pair[e[1], e[2]] Pair[e[3], k[2]] Pair[e[4], k[1]] +
Pair[e[1], e[2]] Pair[e[3], k[1]] Pair[e[4], k[2]] +
Pair[e[1], e[2]] Pair[e[3], k[2]] Pair[e[4], k[2]]
```

Excursion: Alternate FORM-Mathematica Link

FORM is able to handle very large expressions. To produce (pre-)simplified expressions, however, terms have to be wrapped in functions, to avoid immediate expansion:

```
a*(b + c)    ⟶  a*b + a*c
a*f(b + c)   ⟶  a*f(b + c)
```

The number of terms in a function is rather limited in FORM: on 32-bit systems to 32767.

Dilemma: FormCalc gets more sophisticated in pre-simplifying amplitudes while users want to compute larger amplitudes. Thus, recently many 'overflow' messages from FORM.

Solution: Send pre-simplified generic amplitude via external channel to Mathematica for introducing abbreviations. Significant reduction in size of intermediate expressions.

Tentukov, Vermaseren 2006

Effect on Intermediate Amplitudes

FORM $\longrightarrow$ Mathematica:

part of $uu \longrightarrow gg$ @ tree level

```
+Den[U,MU2]*(
  -8*SUNSum[Col5,3]*SUNT[Glu3,Col5,Col2]*SUNT[Glu4,Col1,Col5]*mul[Alfas*Pi]*
  abb[fme[WeylChain[DottedSpinor[k1,MU,-1],6,Spinor[k2,MU,1]]]*ec3.ec4
    -1/2*fme[WeylChain[DottedSpinor[k1,MU,-1],6,ec3,ec4,Spinor[k2,MU,1]]]
    +fme[WeylChain[DottedSpinor[k1,MU,-1],7,Spinor[k2,MU,1]]]*ec3.ec4
    -1/2*fme[WeylChain[DottedSpinor[k1,MU,-1],7,ec3,ec4,Spinor[k2,MU,1]]]]*MU
  -4*SUNSum[Col5,3]*SUNT[Glu3,Col5,Col2]*SUNT[Glu4,Col1,Col5]*mul[Alfas*Pi]*
  abb[fme[WeylChain[DottedSpinor[k1,MU,-1],6,ec3,ec4,k3,Spinor[k2,MU,1]]]
    -2*fme[WeylChain[DottedSpinor[k1,MU,-1],6,ec4,Spinor[k2,MU,1]]]*ec3.k2
    -2*fme[WeylChain[DottedSpinor[k1,MU,-1],6,k3,Spinor[k2,MU,1]]]*ec3.ec4
    +fme[WeylChain[DottedSpinor[k1,MU,-1],7,ec3,ec4,k3,Spinor[k2,MU,1]]]
    -2*fme[WeylChain[DottedSpinor[k1,MU,-1],7,ec4,Spinor[k2,MU,1]]]*ec3.k2
    -2*fme[WeylChain[DottedSpinor[k1,MU,-1],7,k3,Spinor[k2,MU,1]]]*ec3.ec4]
  +8*SUNSum[Col5,3]*SUNT[Glu3,Col5,Col2]*SUNT[Glu4,Col1,Col5]*mul[Alfas*MU*Pi]*
  abb[fme[WeylChain[DottedSpinor[k1,MU,-1],6,Spinor[k2,MU,1]]]*ec3.ec4
    -1/2*fme[WeylChain[DottedSpinor[k1,MU,-1],6,ec3,ec4,Spinor[k2,MU,1]]]
    +fme[WeylChain[DottedSpinor[k1,MU,-1],7,Spinor[k2,MU,1]]]*ec3.ec4
    -1/2*fme[WeylChain[DottedSpinor[k1,MU,-1],7,ec3,ec4,Spinor[k2,MU,1]]]] )
```

Mathematica $\longrightarrow$ FORM:

```
-4*Den(U,MU2)*SUNSum(Col5,3)*SUNT(Glu3,Col5,Col2)*SUNT(Glu4,Col1,Col5)*
  AbbSum5*Alfas*Pi
```

More Abbreviations

The `Abbreviate` Function allows to introduce abbreviations for arbitrary (sub-)expressions and extends the advantage of categorized evaluation.

The subexpressions are retrieved with `Subexpr[]`.

Abbreviations were so far restricted to one FormCalc session, e.g. one could not save intermediate results involving abbreviations and resume computation in a new session.

FormCalc 6 adds two functions to 'register' abbreviations and subexpressions from an earlier session:

```
RegisterAbbr[abbr]
RegisterSubexpr[subexpr]
```

Categories of Abbreviations

- Abbreviations are recursively defined in several levels.
- When generating Fortran code, FormCalc introduces another set of abbreviations for the loop integrals.

In general, the abbreviations are thus costly in CPU time.
It is key to a decent performance that the abbreviations are separated into different Categories:

- Abbreviations that depend on the helicities,
- Abbreviations that depend on angular variables,
- Abbreviations that depend only on $\sqrt{s}$.

Correct execution of the categories guarantees that almost no redundant evaluations are made and makes the generated code essentially as fast as hand-tuned code.

Choice of Language

Mentioning Fortran 77 as the programming language in many circles draws a "Weren't the dinosaurs extinct?" response.

But consider:

- Fortran was designed for 'number crunching,' i.e. efficient evaluation of large formulas.
- Good and free compilers are available.
- Fortran is still widely used in theoretical physics.
- The code is generated, so largely 'invisible' for the user.
- Linking Fortran 77 to C/C++ is pretty straightforward (particularly inside gcc), so is in some sense a 'smallest common denominator.'

Features of the Generated Code

- **Extensible:** default code serves (only) as an example. Other 'Frontends' can be supplied, e.g. HadCalc, sofox.
- **Modular:** largely autonomous pieces of code provide
  - kinematics,
  - model initialization,
  - convolution with PDFs.
- **Re-usable:** external program need only call `ProcessIni` (to set up the process) and `ParameterScan` (to set off the calculation).
- **Interactive:** Mathematica interface provides Mathematica function for cross-section/decay rate.
- **Parallel:** built-in distribution of parameter scans.

Parameter Scans

With the preprocessor definitions in `run.F` one can either

- assign a parameter a fixed value, as in

```
#define LOOP1 TB = 1.5D0
```

- declare a loop over a parameter, as in

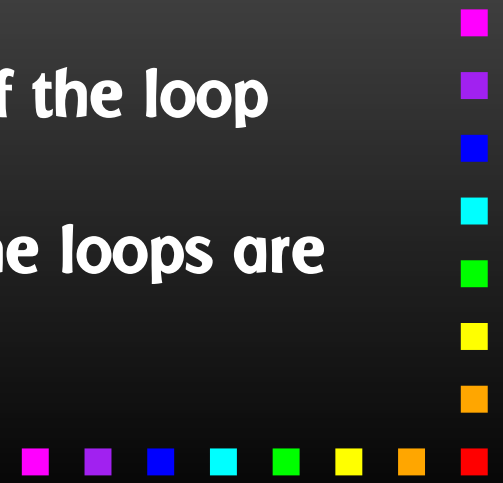
```
#define LOOP1 do 1 TB = 2,30,5
```

which computes the cross-section for `TB` values of 2 to 30 in steps of 5.

Main Program:

```
      LOOP1
      LOOP2
        ⋮
      (calculate
      cross-section)
1     continue
```

Scans are "embarrassingly parallel" - each pass of the loop can be calculated independently.
How to distribute the iterations automatically if the loops are a) user-defined b) usually nested?
Solution: Introduce a serial number



Unraveling Parameter Scans

```
      subroutine ParameterScan( range )
      integer serial
      serial = 0
      LOOP1
      LOOP2
        ⋮
      serial = serial + 1
      if( serial ∉ range ) goto 1
      (calculate cross-section)
1     continue
      end
```

Distribution on N machines is now simple:

- **Send serial numbers $1, N+1, 2N+1, \ldots$ on machine 1,**
- **Send serial numbers $2, N+2, 2N+2, \ldots$ on machine 2, etc.**

Shell-script Parallelization

Parameter scans can automatically be distributed on a cluster of computers:

- The machines are declared in a file `.submitrc`, e.g.

```
# Optional: Nice to start jobs with
nice 10
# Pentium 4 3000
pcl301
pcl301a
pcl305
# Dual Xeon 2660
pcl247b   2
pcl321    2
...
```

- The command line for distributing a job is *exactly the same* except that "`submit`" is prepended, e.g.

```
submit run uuuu 0,1000
```

External Fermion Lines

An amplitude containing external fermions has the form

$$\mathcal{M} = \sum_{i=1}^{n_F} c_i \, F_i \quad \text{where} \quad F_i = \text{(Product of)} \, \langle u | \, \Gamma_i \, | v \rangle \, .$$

$n_F =$ number of fermionic structures.

Textbook procedure: Trace Technique

$$|\mathcal{M}|^2 = \sum_{i,j=1}^{n_F} c_i^* \, c_j \, F_i^* F_j$$

where $F_i^* F_j = \langle v | \, \bar{\Gamma}_i \, | u \rangle \, \langle u | \, \Gamma_j \, | v \rangle = \mathrm{Tr}\big(\bar{\Gamma}_i \, | u \rangle \langle u | \, \Gamma_j \, | v \rangle \langle v | \big) \, .$

Problems with the Trace Technique

PRO: Trace technique is independent of any representation.

CON: For n_F F_i's there are n_F^2 $F_i^* F_j$'s.

Things get worse the more vectors are in the game: multi-particle final states, polarization effects . . .

Essentially $n_F \sim$ (# of vectors)! because all combinations of vectors can appear in the Γ_i.

Solution: Use Weyl–van der Waerden spinor formalism to compute the F_i's directly.

Sigma Chains

Define **Sigma matrices** and **2-dim. Spinors** as

$$\sigma_\mu = (\mathbb{1}, -\vec{\sigma})\,, \qquad \overline{\sigma}_\mu = (\mathbb{1}, +\vec{\sigma})\,,$$

$$\langle u|_{4\mathrm{d}} \equiv \left(\langle u_+|_{2\mathrm{d}}\,, \langle u_-|_{2\mathrm{d}}\right), \qquad |v\rangle_{4\mathrm{d}} \equiv \begin{pmatrix} |v_-\rangle_{2\mathrm{d}} \\ |v_+\rangle_{2\mathrm{d}} \end{pmatrix}.$$

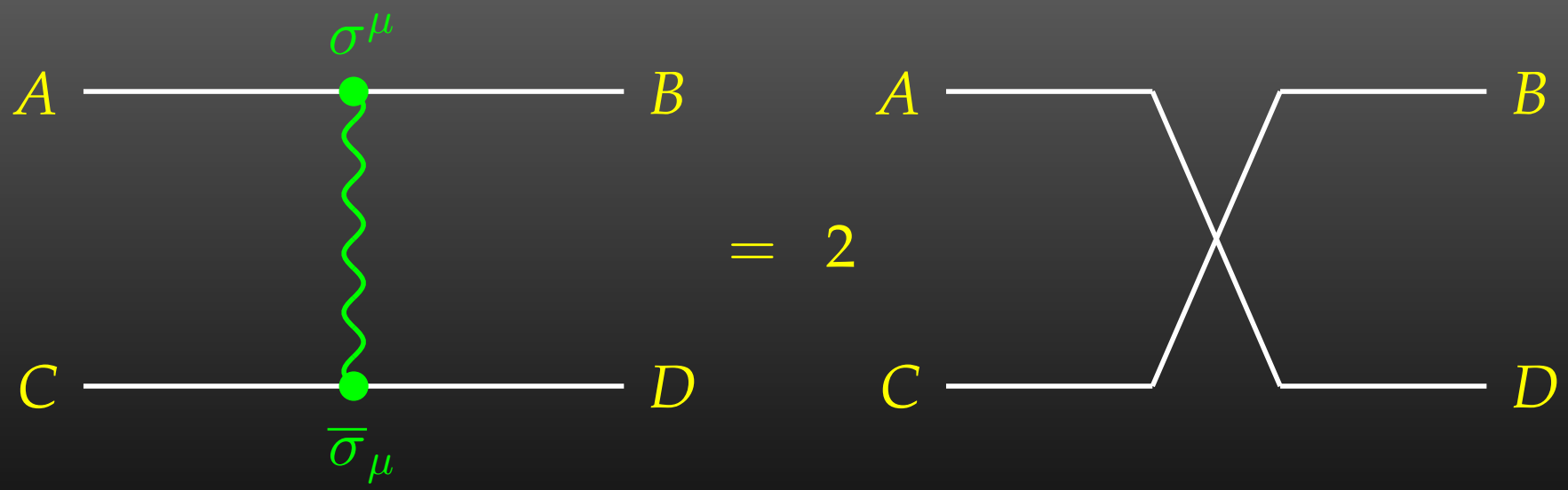
Using the chiral representation it is easy to show that every chiral 4-dim. Dirac chain can be converted to a *single* 2-dim. sigma chain:

$$\langle u|\,\omega_-\gamma_\mu\gamma_\nu\cdots|v\rangle = \langle u_-|\,\overline{\sigma}_\mu\sigma_\nu\cdots|v_\pm\rangle\,,$$
$$\langle u|\,\omega_+\gamma_\mu\gamma_\nu\cdots|v\rangle = \langle u_+|\,\sigma_\mu\overline{\sigma}_\nu\cdots|v_\mp\rangle\,.$$

Fierz Identities

With the Fierz identities for sigma matrices it is possible to remove all Lorentz contractions between sigma chains, e.g.

$$\langle A|\,\sigma_\mu\,|B\rangle\,\langle C|\,\overline{\sigma}^\mu\,|D\rangle \;=\; 2\,\langle A|D\rangle\,\langle C|B\rangle$$



Implementation

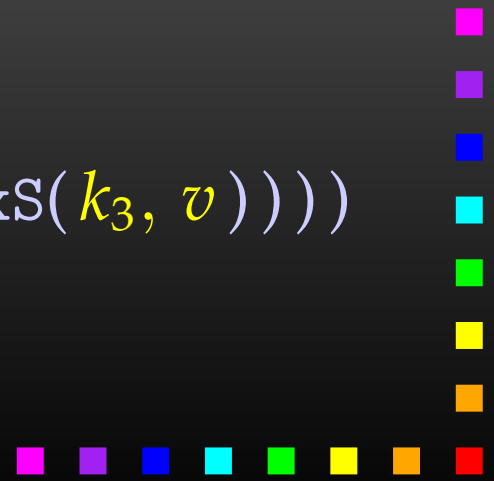
- **Objects (arrays):** $|u_\pm\rangle \sim \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad (\sigma \cdot k) \sim \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- **Operations (functions):**

$$\langle u|v\rangle \sim (u_1 \;\; u_2) \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \texttt{SxS}$$

$$(\overline{\sigma} \cdot k)\,|v\rangle \sim \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \texttt{VxS, BxS}$$

Sufficient to compute any sigma chain:

$$\langle u|\,\sigma_\mu \overline{\sigma}_\nu \sigma_\rho\,|v\rangle\, k_1^\mu k_2^\nu k_3^\rho = \texttt{SxS}(u, \texttt{VxS}(k_1, \texttt{BxS}(k_2, \texttt{VxS}(k_3, v))))$$



More Freebies

- Polarization does not 'cost' extra:
  = Get spin physics for free.
- Better numerical stability because components of k^μ are arranged as 'small' and 'large' matrix entries, viz.

$$\sigma_\mu k^\mu = \begin{pmatrix} k_0 + k_3 & k_1 - \mathrm{i}k_2 \\ k_1 + \mathrm{i}k_2 & k_0 - k_3 \end{pmatrix}$$

$$\downarrow$$

Large cancellations of the form $\sqrt{k^2 + m^2} - \sqrt{k^2}$ when $m \ll k$ are avoided: better precision for mass effects.

Mathematica Interface

The new **Mathematica Interface** turns the generated **stand-alone Fortran code** into a **Mathematica function for evaluating the cross-section or decay rate** as a function of user-selected model parameters.

The benefits of such a function are obvious, as the whole instrumentarium of Mathematica commands can be applied to them. Just think of

```
FindMinimum[sigma[TB, MA0], {{TB, 5}, {MA0, 250}}]
ContourPlot[sigma[TB, MA0], {TB, 5}, {MA0, 250}]
...
```

Mathematica Interface - Input

The changes to the code are minimal.

Example line in `run.F` for Stand-alone Fortran code:

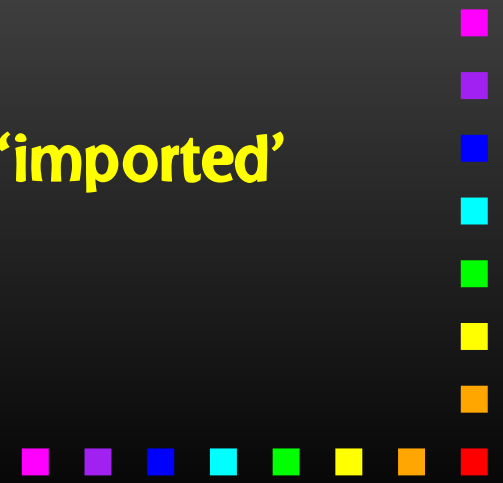
```
#define LOOP1 do 1 TB = 5, 50, 5
```

Change for the Mathematica Interface:

```
#define LOOP1 call MmaGetReal(TB)
```

The variable `TB` is 'imported' from Mathematica now, i.e. the cross-section function in Mathematica becomes a function of `TB` hereby.

The user has full control over which variables are 'imported' from Mathematica and which are set in Fortran.



Mathematica Interface - Output

Similar to the `MmaGetReal` invocations, the Fortran program can also **'export' variables to Mathematica.**

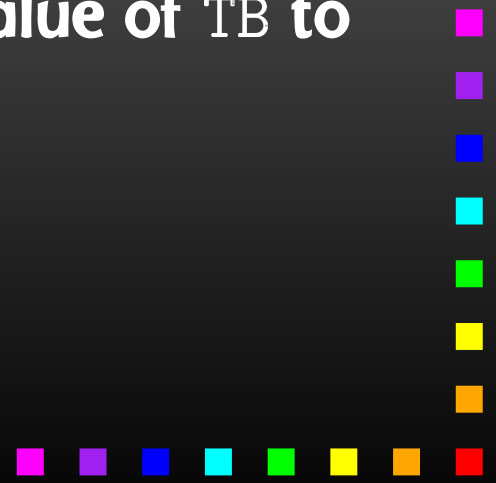
For example, the line that prints a parameter in the stand-alone code is

```
#define PRINT1 SHOW "TB", TB
```

becomes

```
#define PRINT1 call MmaPutReal("TB", TB)
```

for the Mathematica Interface and transmits the value of `TB` to Mathematica.



Mathematica Interface - Usage

Once the changes to `run.F` are made, the program `run` is compiled as usual:

```
./configure
make
```

It is then loaded in Mathematica with

```
Install["run"]
```

Now a Mathematica function of the same name, `run`, is available. There are two ways of invoking it:

Compute a differential cross-section at $\sqrt{s} =$ `sqrtS`:

```
run[sqrtS, arg1, arg2, ...]
```

Compute a total cross-section for `sqrtSfrom` $\leqslant \sqrt{s} \leqslant$ `sqrtSto`:

```
run[{sqrtSfrom, sqrtSto}, arg1, arg2, ...]
```

Mathematica Interface - Data Retrieval

The output of the function `run` is an integer which indicates how many records have been transferred. For example:

```
Para[1] = {TB -> 5., MA0 -> 250.}
Data[1] = {DataRow[{500.}, {0.0539684, 0.}, {2.30801 10^-21, 0.}],
           DataRow[{510.}, {0.0515943, 0.}, {4.50803 10^-22, 0.}]}
```

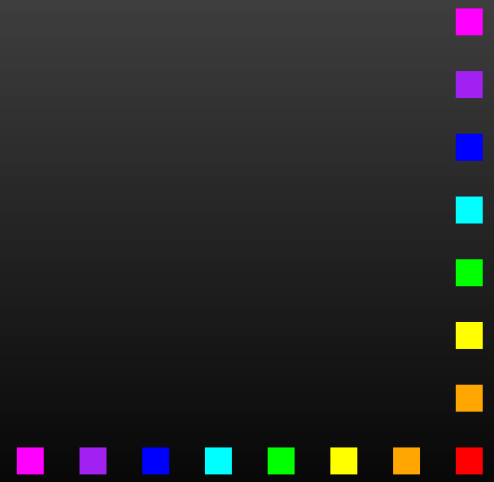
`Para` contains the parameters exported from the Fortran code.
`Data` contains:

- **the independent variables,**
  here e.g. `{500.}` $= \{\sqrt{s}\}$,
- **the cross-sections,**
  here e.g. `{0.0539684, 0.}` $= \{\sigma_{\text{tot}}^{\text{tree}}, \sigma_{\text{tot}}^{\text{1-loop}}\}$, and
- **the integration errors,**
  here e.g. `{2.30801 10^-21, 0.}` $= \{\Delta\sigma_{\text{tot}}^{\text{tree}}, \Delta\sigma_{\text{tot}}^{\text{1-loop}}\}$.

Code-generation Functions

FormCalc's code-generation functions are now public and disentangled from the rest of the code. They can be used to **write out an arbitrary Mathematica expression as optimized Fortran code:**

- *handle* = `OpenFortran["`*file.F*`"]`
  opens *file.F* as a Fortran file for writing,
- `WriteExpr[`*handle*`, {`*var* `->` *expr*`, ...}]`
  writes out Fortran code which calculates *expr* and stores the result in *var*,
- `Close[`*handle*`]`
  closes the file again.



Code generation

- **Expressions too large** for Fortran are split into parts, as in

```
var = part1
var = var + part2
...
```

- **High level of optimization,** e.g. common subexpressions are pulled out and computed in temporary variables.
- **Many ancillary functions,** e.g.

  `PrepareExpr`, `OnePassOrder`, `SplitSums`, `$Prefix`, `CommonDecl`, `SubroutineDecl`, **etc.**

  make code generation versatile and highly automatable, such that the resulting code needs few or no changes by hand.

The Model Files

One has to set up, once and for all, a

- **Generic Model File** (seldomly changed) containing the generic part of the couplings,

Example: the `FFS` coupling

$$C(F,F,S) = G_- \omega_- + G_+ \omega_+ = \vec{G} \cdot \begin{pmatrix} \omega_- \\ \omega_+ \end{pmatrix}$$

```
AnalyticalCoupling[s1 F[j1, p1], s2 F[j2, p2], s3 S[j3, p3]]
== G[1][s1 F[j1], s2 F[j2], s3 S[j3]] .
     { NonCommutative[ ChiralityProjector[-1] ],
       NonCommutative[ ChiralityProjector[+1] ] }
```

The Model Files

One has to set up, once and for all, a

- **Classes Model File** (for each model)
  declaring the particles and the allowed couplings

Example: the $\bar{\ell}_i \nu_j G$ coupling in the Standard Model

$$\vec{G}(\bar{\ell}_i, \nu_j, G) = \begin{pmatrix} G_- \\ G_+ \end{pmatrix} = \begin{pmatrix} -\frac{\mathrm{i}\, e\, m_{\ell,i}}{\sqrt{2} \sin\theta_w M_W} \delta_{ij} \\ 0 \end{pmatrix}$$

```
C[ -F[2,{i}], F[1,{j}], S[3] ]
== { {-I EL Mass[F[2,{i}]]/(Sqrt[2] SW MW) IndexDelta[i, j]},
     {0} }
```

Current Status of Model Files

Model Files presently available for FeynArts:

- SM [w/QCD], normal and background-field version.
  All one-loop counter terms included.
- MSSM [w/QCD].
  Counter terms by T. Fritzsche.
- Two-Higgs-Doublet Model.
  Counter terms not included yet.
- ModelMaker utility generates Model Files from the Lagrangian.
- "3rd-party packages" FeynRules and LanHEP generate Model Files for FeynArts and others.
- SARAH package derives SUSY Models.

Partial (Add-On) Model Files

FeynArts distinguishes

- **Basic Model Files** and
- **Partial (Add-On) Model Files.**

Basic Model Files, e.g. `SM.mod`, `MSSM.mod`, can be modified by Add-On Model Files. For example,

```
InsertFields[..., Model -> {"MSSMQCD", "FV"}]
```

This loads the Basic Model File `MSSMQCD.mod` and modifies it through the Add-On `FV.mod` (non-minimal flavour violation).

Model files can thus be built up from several parts.

Tweaking Model Files

Or, How to efficiently make changes in an existing model file.

Bad: Copy the model file, modify the copy. – Why?

- It is typically not very transparent what has changed.
- If the original model file changes (e.g. bug fixes), these do not automatically propagate into the derivative model file.

Better: Create a new model file which reads the old one and modifies the particles and coupling tables.

- `M$ClassesDescription` = list of particle definitions,
- `M$CouplingMatrices` = list of couplings.

Tweaking Model Files

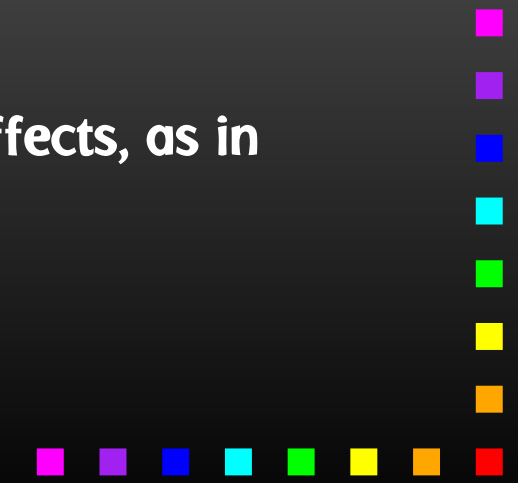
Example: Introduce enhancement factors for the b-$\bar{b}$-h_0 and b-$\bar{b}$-H_0 Yukawa couplings in the MSSM.

```
LoadModel["MSSM"]

EnhCoup[ (lhs:C[F[4,{g_,_}], -F[4,_], S[h:1|2]]) == rhs_ ] :=
  lhs == Hff[h,g] rhs

EnhCoup[other_] = other

M$CouplingMatrices = EnhCoup/@ M$CouplingMatrices
```

To see the effect, make a printout with the `WriteTeXFile` utility of FeynArts.

The `Hff[h,g]` can be defined to include e.g. resummation effects, as in

```
double precision Hff(2,3)
data Hff /6*1/
Hff(1,3) = 1 - CA/(SA*TB)*Delta_b
Hff(2,3) = 1 + SA/(CA*TB)*Delta_b
```



Linear Combinations of Fields

FeynArts can automatically linear-combine fields, i.e. one can specify the couplings in terms of gauge rather than mass eigenstates. For example:

```
M$ClassesDescription = { ...,
  F[11] = {...,
    Indices -> {Index[Neutralino]},
    Mixture -> ZNeu[Index[Neutralino],1] F[111] +
               ZNeu[Index[Neutralino],2] F[112] +
               ZNeu[Index[Neutralino],3] F[113] +
               ZNeu[Index[Neutralino],4] F[114]} }
```

Since `F[111]...F[114]` are not listed in `M$CouplingMatrices`, they drop out of the model completely.

Linear Combinations of Fields

Higher-order mixings can be added, too:

```
M$ClassesDescription = { ...,
  S[1] = {...},
  S[2] = {...},
  S[10] == {...,
    Indices -> {Index[Higgs]},
    Mixture -> UHiggs[Index[Higgs],1] S[1] +
               UHiggs[Index[Higgs],2] S[2],
    InsertOnly -> {External, Internal}} }
```

This time, `S[10]` ***and*** `S[1],S[2]` **appear in the coupling list (including all mixing couplings) because all three are listed in** `M$CouplingMatrices`**.**

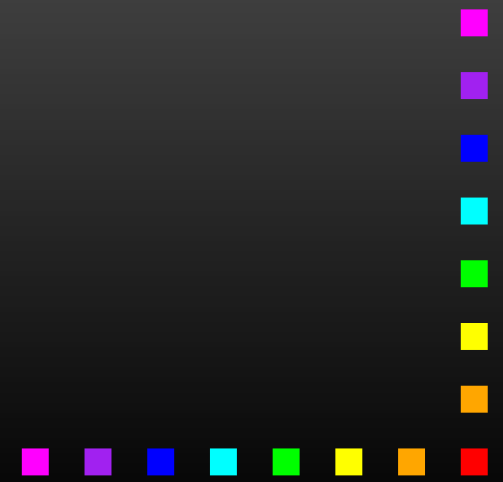
Due to the `InsertOnly`**,** `S[10]` **is inserted only on tree-level parts of the diagram, not in loops.**

Not the Cross-Section

Or, How to get things the Standard Setup won't give you.

Example: extract the Wilson coefficients for $b \to s\gamma$.

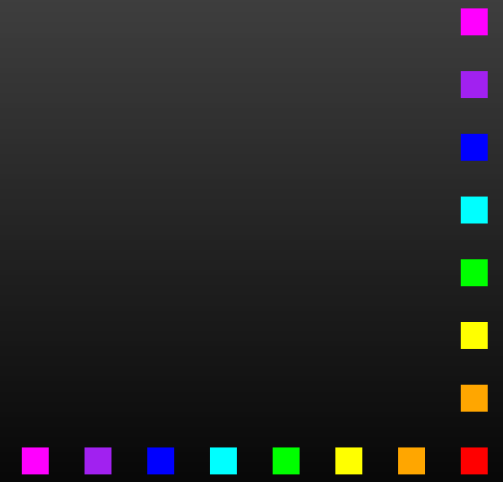
```
tops = CreateTopologies[1, 1 -> 2]
ins = InsertFields[tops, F[4,{3}] -> {F[4,{2}], V[1]}]
vert = CalcFeynAmp[CreateFeynAmp[ins], FermionChains -> Chiral]

mat[p_Plus] := mat/@ p

mat[r_. DiracChain[s2_Spinor, om_, mu_, s1:Spinor[p1_, m1_, _]]] :=
  I/(2 m1) mat[r DiracChain[sigmunu[om]]] +
  2/m1 r Pair[mu, p1] DiracChain[s2, om, s1]

mat[r_. DiracChain[sigmunu[om_]], SUNT[Col1, Col2]] :=
  r O7[om]/(EL MB/(16 Pi^2))

mat[r_. DiracChain[sigmunu[om_]], SUNT[Glu1, Col2, Col1]] :=
  r O8[om]/(GS MB/(16 Pi^2))

coeff = Plus@@ vert //. abbr /. Mat -> mat

c7 = Coefficient[coeff, O7[6]]
c8 = Coefficient[coeff, O8[6]]
```



Not the Cross-Section

Using FormCalc's output functions it is also pretty straightforward to generate your own Fortran code:

```
file = OpenFortran["bsgamma.F"]

WriteString[file,
  SubroutineDecl["bsgamma(C7,C8)"] <>
  "\tdouble complex C7, C8\n" <>
  "#include \"looptools.h\"\n"]

WriteExpr[file, {C7 -> c7, C8 -> c8}]

WriteString[file, "\tend\n"]

Close[file]
```



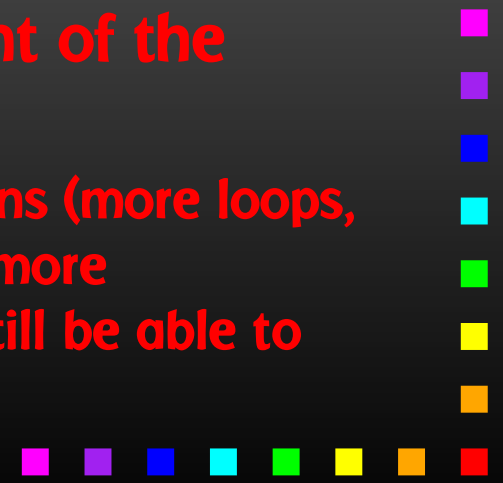
Dirac Chains in 4D

As numerical calculations are done mostly using Weyl-spinor chains, there has been a paradigm shift for Dirac chains to make them better suited for analytical purposes, e.g. the extraction of Wilson coefficients.

- Already in Version 5, Fierz methods have been implemented for Dirac chains, thus allowing the user to force the fermion chains into almost any desired order.
- Version 6 further adds the `Colour` method to the `FermionOrder` option of `CalcFeynAmp`, which brings the spinors into the same order as the external colour indices.
- Also new in Version 6: completely antisymmetrized Dirac chains, i.e. `DiracChain[-1, μ, ν]` $= \sigma_{\mu\nu}$.

Summary and Outlook

- Serious perturbative calculations these days can generally no longer be done by hand:
  - Required accuracy, Models with many particles, ...
- Hybrid programming techniques are necessary:
  - Computer algebra is an indispensable tool because many manipulations must be done symbolically.
  - Fast number crunching can only be achieved in a compiled language.
- Software engineering and further development of the existing packages is a must:
  - As we move on to ever more complex computations (more loops, more legs), the computer programs must become more "intelligent," i.e. must learn all possible tricks to still be able to handle the expressions.



Finally

Using FeynArts and FormCalc is a lot like driving a car:

- You have to decide where to go (this is often the hardest decision).
- You have to turn the ignition key, work gas and brakes, and steer.
- But you don't have to know, say, which valve has to open at which time to keep the motor running.
- On the other hand, you can only go where there are roads. You can't climb a mountain with your car.