Quantum Spectral Curve and string theory on $AdS_3 \times S^3 \times T^4$

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Overview

Quantum Spectral Curve

- Integrability-based framework for computing exact spectrum of single-trace local operators in planar N=4 SYM (spectrum of free closed strings on $AdS_5 \times S^5$)

- Also for ABJM ($AdS_4 \times CP^3$)

- Takes the form of a concise set of Riemann-Hilbert problems for Q-functions

String theory on $AdS_3 \times S^3 \times T^4$ with pure RR flux

- Integrable, but qualitatively different from the well-studied AdS_5 and AdS_4 models

- Unknown CFT_2 dual

- Worldsheet theory contains massless modes

Quantum Spectral Curve recently conjectured

[Ekhammar, Volin]

[Cavaglia, Gromov, Stefanski, Torreilli]

Punchline

- Solved QSC for string theory on AdS_3 x S^3 x T^4 with pure RR flux

- First ever predictions for generic unprotected string excitations

- Develop new tools to deal with massless modes in QSC

- Solve QSC numerically at finite coupling

- Solve QSC perturbatively at weak coupling

- Shed light on mysterious CFT_2 dual?

Outline

Review of string theory on $AdS_3 \times S^3 \times T^4$

Quantum Spectral Curve: From AdS_5 / CFT_4 to AdS_3 / CFT_2

Solving the AdS_3 / CFT_2 QSC

Summary + Outlook

Review of string theory on $AdS_3 \times S^3 \times T^4$

String theory on $AdS_3 \times S^3 \times T^4$ and holography

Maximally supersymmetric background: 16 supercharges

Background supports RR and NSNS flux

In contrast to $AdS_5 \times S^5$ and $AdS_4 \times CP^3$, dual CFT_2 is largely unknown

Pure NSNS flux – significant recent progress, can be studied using conventional worldsheet CFT methods

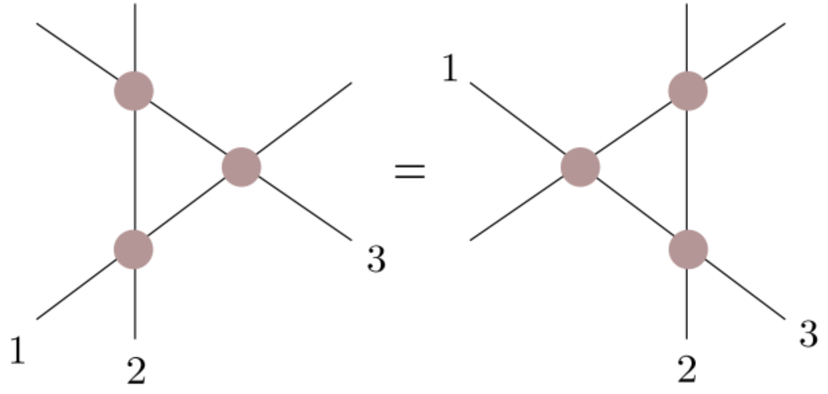
``k=1" unit of NSNS flux CFT dual known [Eberhardt, Gaberdiel, Gopakumar]

We focus on pure RR flux, closest analogue to $AdS_5 \times S^5$

String theory on $AdS_3 \times S^3 \times T^4$ and integrability

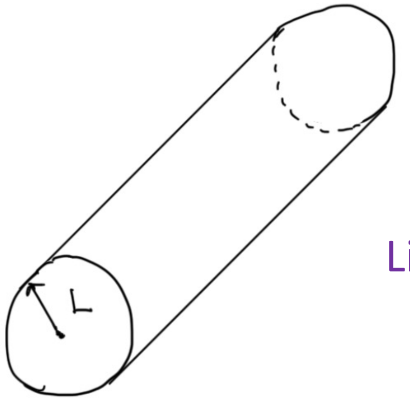
Isometries $AdS_3 \times S^3 \times T^4$

$$\mathfrak{so}(2,2) \oplus \mathfrak{so}(4) \simeq \mathfrak{su}(1,1)^2 \oplus \mathfrak{su}(2)^2 \subset \mathfrak{psu}(1,1|2)^2$$



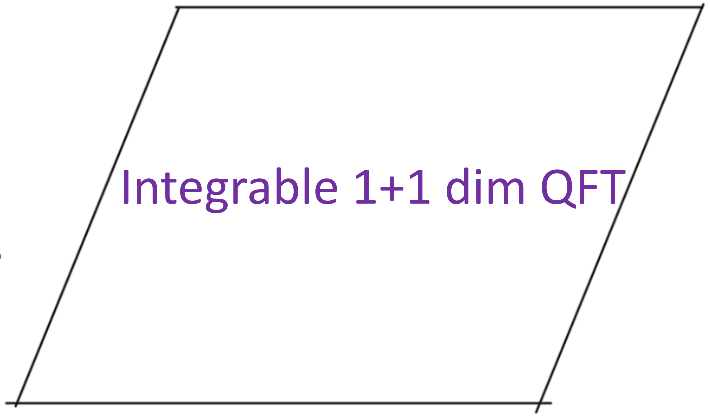
Classically integrable – infinitely many commuting conserved quantities

Expected to be quantum integrable

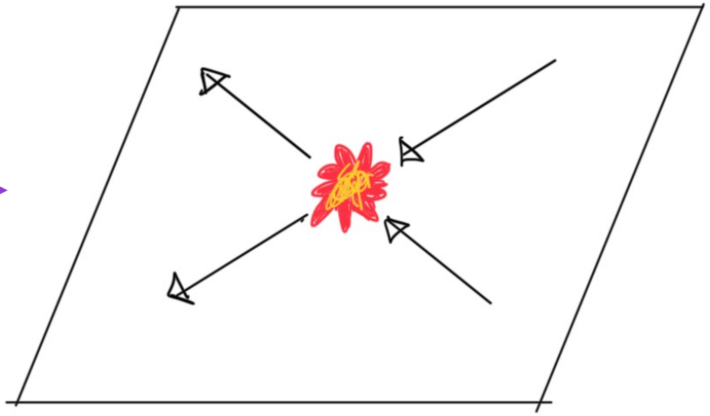


$L \to \infty$

Lightcone gauge



Integrable 1+1 dim QFT



Exact S-matrices known [Borsato, Ohlsson Sax, Sfondrini, Stefanski]

AdS_5: massive

AdS_3: also massless modes

Spectrum for large strings described by Asymptotic Bethe Ansatz (ABA) equations

ABA only valid for large operators

$$L \to \infty, \quad \Delta \sim L$$

in a tiny region around this limit

After that: ``wrapping effects”

$\mathcal{O}(e^{-L})$ for massive

$\mathcal{O}(1/L)$ for massless [Abbott, Aniceto]

Problem even in the massive sector – virtual massless particles

[Abbott, Aniceto]

Massive sector

psu(1,1|2)

$$1 = \prod_{j=1}^{K_2} \frac{y_{1,k} - x_j^+}{y_{1,k} - x_j^-} \prod_{j=1}^{K_{\bar{2}}} \frac{1 - \frac{1}{y_{1,k}\bar{x}_j^-}}{1 - \frac{1}{y_{1,k}\bar{x}_j^+}},$$

$$\left(\frac{x_k^+}{x_k^-}\right)^L = \prod_{j\neq k}^{K_2} \frac{x_k^+ - x_j^-}{x_k^- - x_j^+} \frac{1 - \frac{1}{x_k^+ x_j^-}}{1 - \frac{1}{x_k^- x_j^+}} \sigma^2(x_k, x_j) \prod_{j=1}^{K_1} \frac{x_k^- - y_{1,j}}{x_k^+ - y_{1,j}} \prod_{j=1}^{K_3} \frac{x_k^- - y_{3,j}}{x_k^+ - y_{3,j}}$$

$$\times \prod_{j=1}^{K_{\bar{2}}} \frac{1 - \frac{1}{x_k^+ \bar{x}_j^+}}{1 - \frac{1}{x_k^- \bar{x}_j^-}} \frac{1 - \frac{1}{x_k^+ \bar{x}_j^-}}{1 - \frac{1}{x_k^- \bar{x}_j^+}} \tilde{\sigma}^2(x_k, \bar{x}_j) \prod_{j=1}^{K_{\bar{1}}} \frac{1 - \frac{1}{x_k^- y_{\bar{1},j}}}{1 - \frac{1}{x_k^+ y_{\bar{1},j}}} \prod_{j=1}^{K_{\bar{3}}} \frac{1 - \frac{1}{x_k^- y_{\bar{3},j}}}{1 - \frac{1}{x_k^+ y_{\bar{3},j}}},$$

$$1 = \prod_{j=1}^{K_2} \frac{y_{3,k} - x_j^+}{y_{3,k} - x_j^-} \prod_{j=1}^{K_{\bar{2}}} \frac{1 - \frac{1}{y_{3,k}\bar{x}_j^-}}{1 - \frac{1}{y_{3,k}\bar{x}_j^+}},$$

psu(1,1|2)

$$1 = \prod_{j=1}^{K_{\bar{2}}} \frac{y_{\bar{1},k} - \bar{x}_j^-}{y_{\bar{1},k} - \bar{x}_j^+} \prod_{j=1}^{K_2} \frac{1 - \frac{1}{y_{\bar{1},k} x_j^+}}{1 - \frac{1}{y_{\bar{1},k} x_j^-}},$$

$$\left(\frac{\bar{x}_k^+}{\bar{x}_k^-}\right)^L = \prod_{j\neq k}^{K_{\bar{2}}} \frac{\bar{x}_k^- - \bar{x}_j^+}{\bar{x}_k^+ - \bar{x}_j^-} \frac{1 - \frac{1}{\bar{x}_k^+ \bar{x}_j^-}}{1 - \frac{1}{\bar{x}_k^- \bar{x}_j^+}} \sigma^2(\bar{x}_k, \bar{x}_j) \prod_{j=1}^{K_{\bar{1}}} \frac{\bar{x}_k^+ - y_{\bar{1},j}}{\bar{x}_k^- - y_{\bar{1},j}} \prod_{j=1}^{K_{\bar{3}}} \frac{\bar{x}_k^+ - y_{\bar{3},j}}{\bar{x}_k^- - y_{\bar{3},j}}$$

$$\times \prod_{j=1}^{K_2} \frac{1 - \frac{1}{\bar{x}_k^- x_j^-}}{1 - \frac{1}{\bar{x}_k^+ x_j^+}} \frac{1 - \frac{1}{\bar{x}_k^+ x_j^-}}{1 - \frac{1}{\bar{x}_k^- x_j^+}} \tilde{\sigma}^2(\bar{x}_k, x_j) \prod_{j=1}^{K_1} \frac{1 - \frac{1}{\bar{x}_k^+ y_{1,j}}}{1 - \frac{1}{\bar{x}_k^- y_{1,j}}} \prod_{j=1}^{K_3} \frac{1 - \frac{1}{\bar{x}_k^+ y_{3,j}}}{1 - \frac{1}{\bar{x}_k^- y_{3,j}}},$$

$$1 = \prod_{j=1}^{K_{\bar{2}}} \frac{y_{\bar{3},k} - \bar{x}_j^-}{y_{\bar{3},k} - \bar{x}_j^+} \prod_{j=1}^{K_2} \frac{1 - \frac{1}{y_{\bar{3},k} x_j^+}}{1 - \frac{1}{y_{\bar{3},k} x_j^-}}.$$

Wrapping effects captured by TBA

Wrapping effects can be taken into account via Thermodynamic Bethe Ansatz (TBA)

Successfully done for AdS_5 and AdS_4 and recast as Quantum Spectral Curve

TBA → Y-system → T-system → Quantum Spectral Curve

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TBA → Y-system → T-system → Quantum Spectral Curve

AdS_3: significant technical difficulties coming from massless modes, only recently resolved

[Frolov, Sfondrini]

Quantum Spectral Curve proposed based on symmetry + analogy with previous known cases

Opened the way to precision spectroscopy

[Ekhammar, Volin]

[Cavaglia, Gromov, Stefanski, Torreilli]

Review of Quantum Spectral Curve

Quantum Spectral Curve

A complex analysis problem for functions $Q_i(u)$

single complex variable

Modern formulation of spectral problem of integrable systems

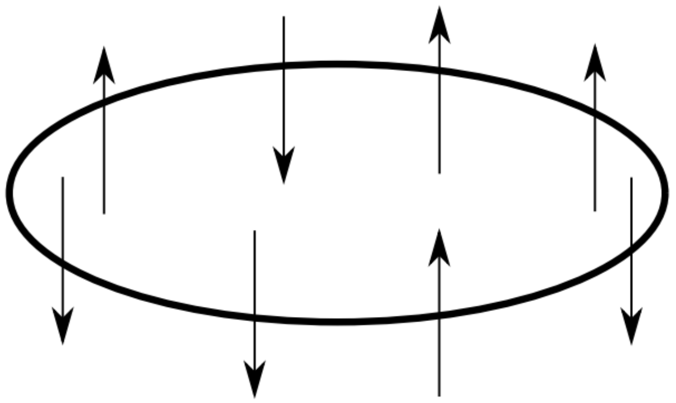
- su(2) Heisenberg spin chain (discrete lattice models)
- KdV hierarchy (1+1 dim QFT)
- N=4 SYM (4d gauge theory)

su(2) invariant Heisenberg XXX spin chain

$$H = \frac{1}{4}\sum_{\alpha,n} \sigma_n^\alpha \otimes \sigma_{n+1}^\alpha - 1$$

$$\alpha = x, y, z$$

Periodic boundary conditions



QQ relations

$$Q_{12}Q_{\varnothing} = Q_1^+ Q_2^- - Q_1^- Q_2^+$$

$$E = \lim_{u\to 0} \partial_u \log \frac{Q_1^+}{Q_1^-}$$

Analytic properties

$$Q_{12} = u^L$$

$$Q_{\varnothing} = 1$$

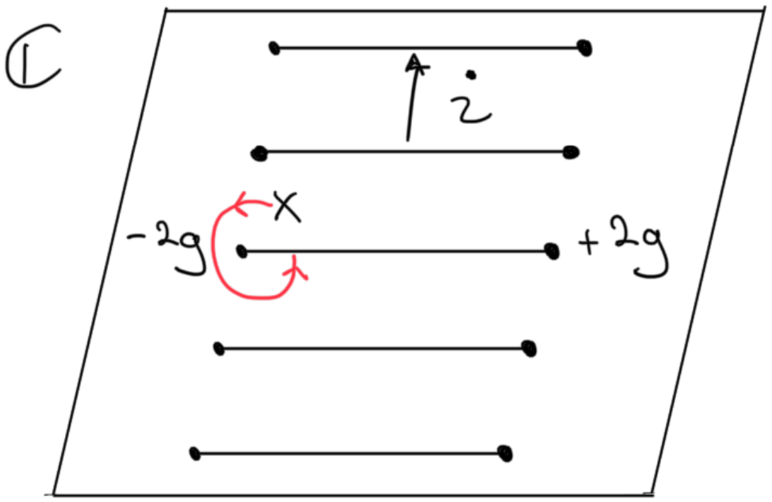
Q_1, Q_2 } polynomial

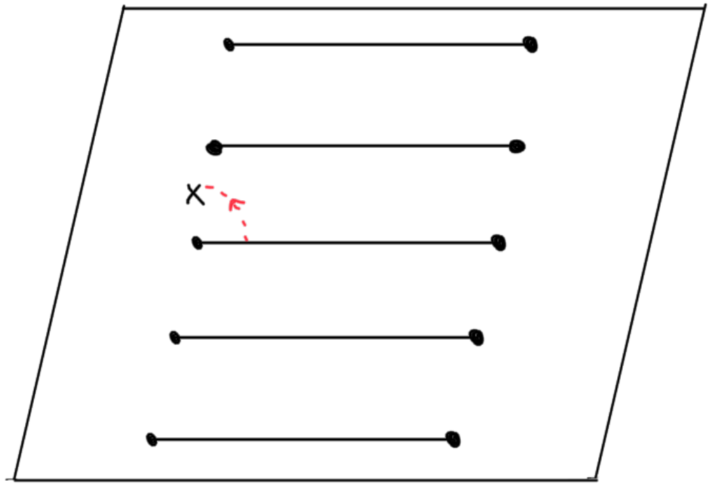
$$f^{\pm} := f\left(u \pm \frac{i}{2}\right)$$

``Completeness”
Bijection between highest-weight eigenvectors and solutions of Q-system

[Mukhin, Tarasov, Varchenko]
[Chernyak, Leurent, Volin]

Planar N=4 Supersymmetric Yang-Mills





[Gromov, Kazakov, Leurent, Volin]

QQ relations fixed by symmetry

$$Q_{A|I}Q_{Aab|I} = Q^+_{Aa|I}Q^-_{Ab|I} - Q^-_{Aa|I}Q^+_{Ab|I}$$

$$f^{\pm} := f\left(u \pm \frac{i}{2}\right)$$

't Hooft coupling

$$g = \frac{\sqrt{\lambda}}{4\pi}$$

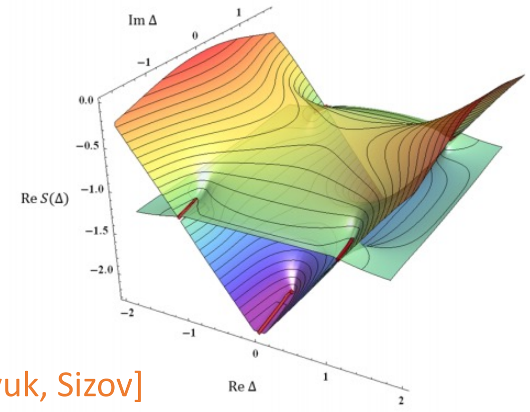
Analytic continuation a symmetry of QQ relations

Global charges of state in asymptotics

$$Q(u) \sim u^{\Delta}$$

QSC: The triumph of integrability

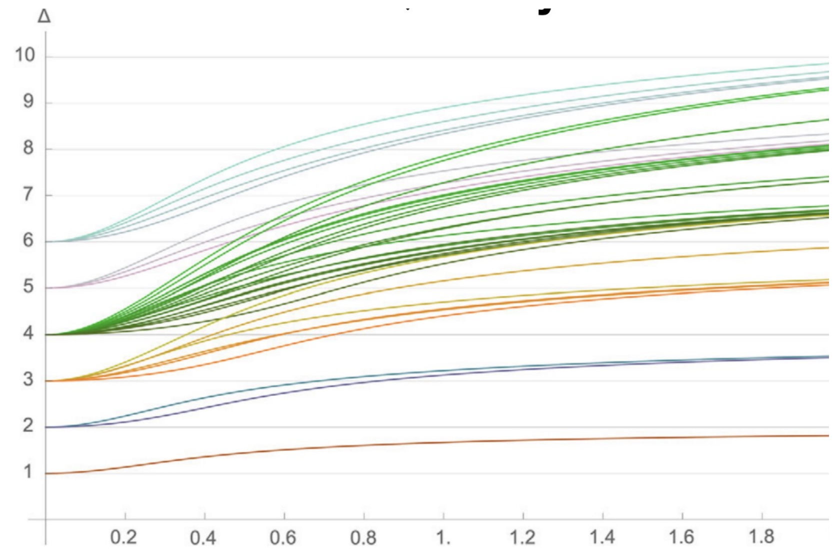
Numerics



[Gromov, Levkovich-Maslyuk, Sizov]

[Grabner, Gromov, Julius] [Cavaglia, Gromov, Julius, Preti]

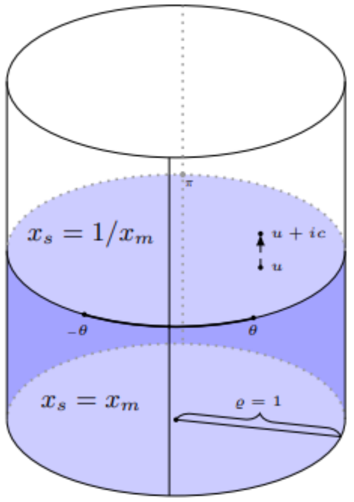
Defect CFT on Wilson line



Perturbative tool

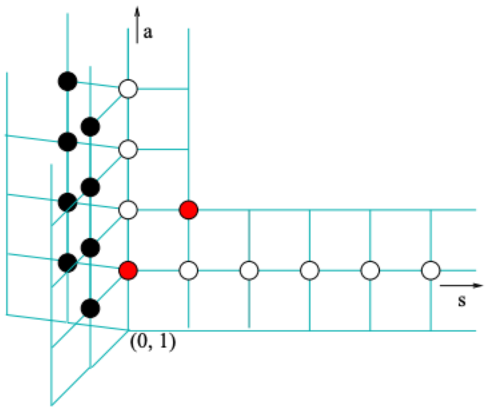
$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \cdots + \#g^{20} + \mathcal{O}(g^{22})$$

[Marboe, Volin]



Quantum deformation

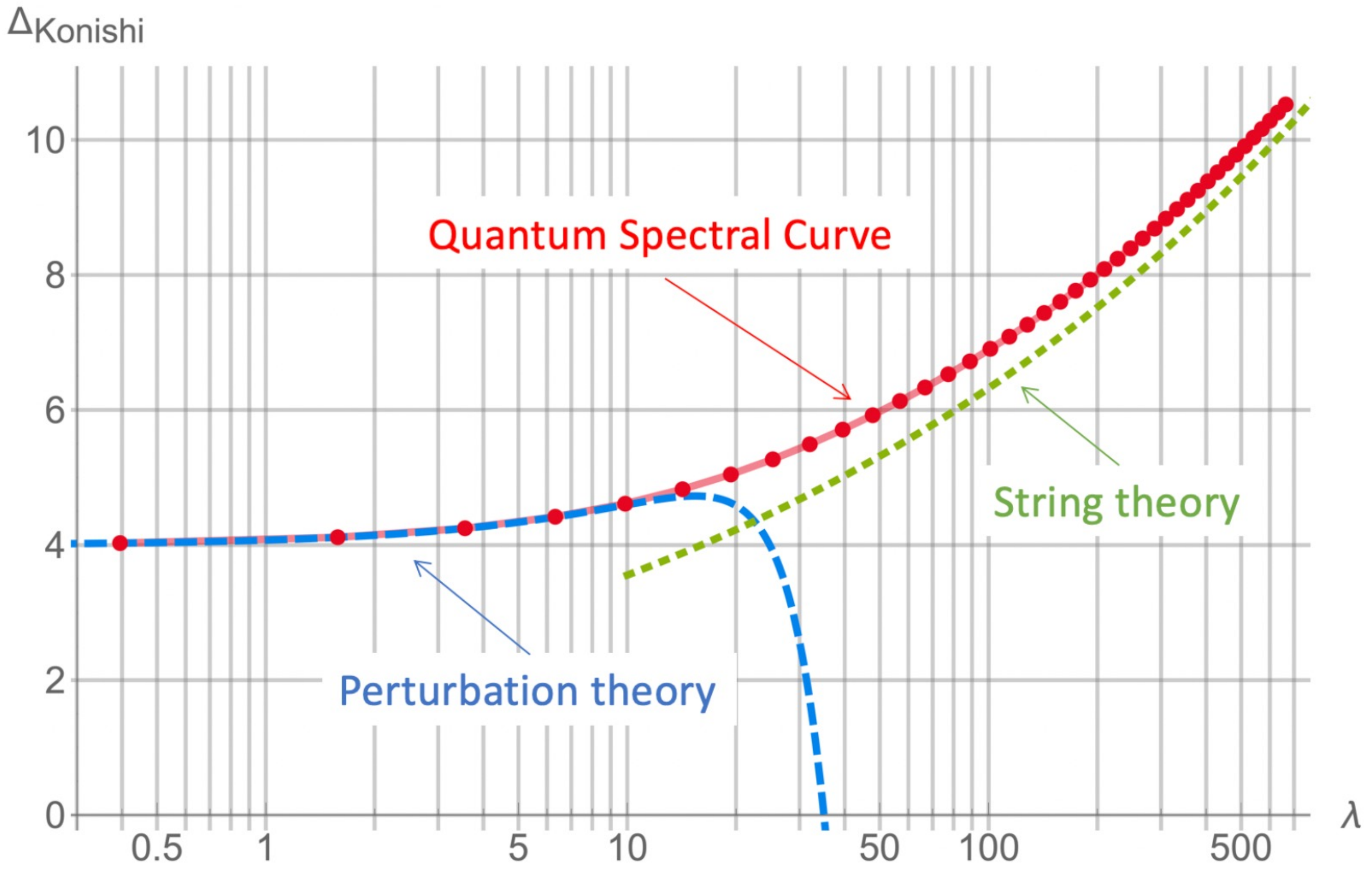
[Klabbers, Van Tongeren]



ABJM

[Bombardelli, Cavaglia, Fioravanti, Gromov, Tateo]

Any question about the planar spectrum can be answered!



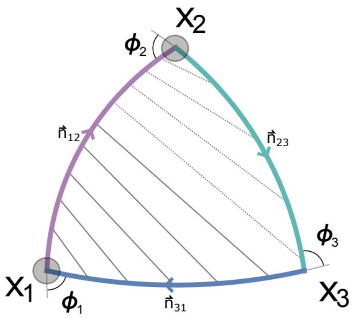
Δ_{Konishi}
10
8
6
4
2
0
0.5
1
5
10
50
100
500
λ
Quantum Spectral Curve
String theory
Perturbation theory

QSC even gives correlation functions!

Separation of Variables

$$\frac{\langle\Psi|\partial_\phi\left(2\sin\phi\,\hat{D}\right)|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \pm\frac{\left(\!\left(u\,Q^2 \right)\!\right)}{\left(\!\left(Q^2 \right)\!\right)}$$

$$\left(\!\left(f \right)\!\right) = \int_\gamma \mathrm{d}u\,\mu(u)\,f(u)$$



[Cavaglia, Gromov, Levkovich-Maslyuk]

[Giombi, Komatsu]

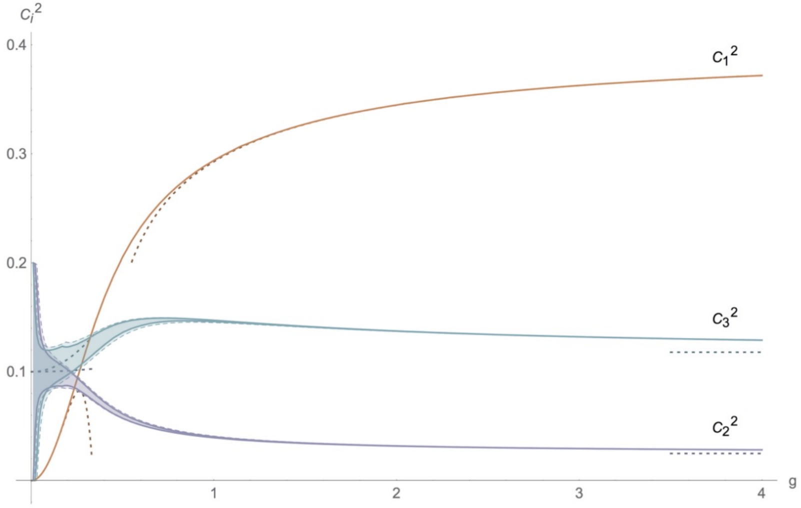
[Bercini, Homrich, Vieira]

[Cavaglia, Gromov, Julius, Preti]

[Caron-Huot, Coronado, Trinh, Zahraee]

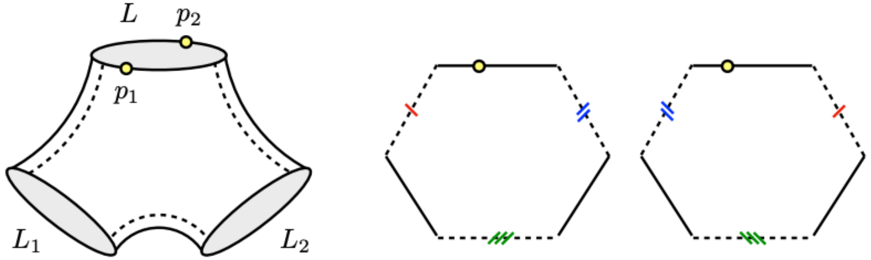
``Bootstrability"

QSC + numerical bootstrap



QSC + Hexagons

[Basso, Georgoudis, Sueiro]



QSC for AdS_3 conjectured based on symmetries + analogy with AdS_5 x S^5

[Gromov, Kazakov, Leurent, Volin]

AdS_5: QSC takes the form of PSU(2,2|4) ~ SL(4|4) Q-system

$$Q_{A|I} \qquad A \subset \{1,2,3,4\}$$
$$I \subset \{1,2,3,4\}$$

QQ relations

$$Q_{A|I}Q_{Aab|I} = Q^+_{Aa|I}Q^-_{Ab|I} - Q^-_{Aa|I}Q^+_{Ab|I}$$ + others

$$f^\pm := f\left(u \pm \frac{i}{2}\right)$$

Distinguished Q-functions

$$\mathbf{P}_a := Q_{a|\varnothing} \qquad S^5$$

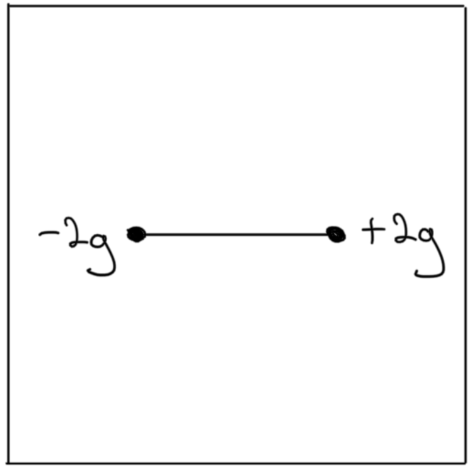
$$\mathbf{Q}_i := Q_{\varnothing|i} \qquad AdS_5$$

Hodge duals $\mathbf{P}^a$

Analytic continuation

$$\mathbf{P}^\gamma_a = \mu_{ab}\mathbf{P}^b$$

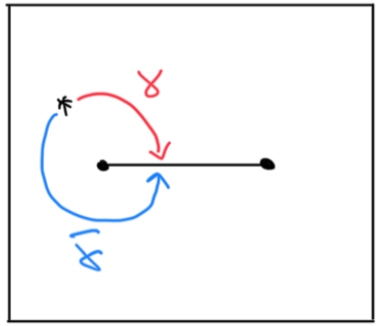
Defining sheet of P



Power-like asymptotics

$$\mathbf{P}_a(u) \sim A_a u^{M_a} \qquad \mathbf{Q}_i(u) \sim B_i u^{\hat{M}_i}$$

Determined by global charges on AdS_5 x S^5



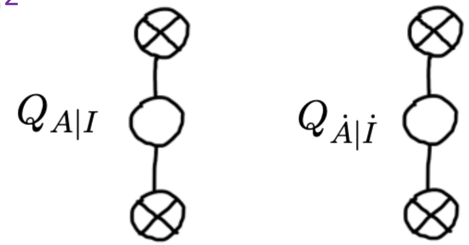
$$\bar{\gamma} = \gamma$$

All branch points quadratic

QSC for AdS_3 conjectured based on symmetries + analogy with $AdS_5 \times S^5$

Global symmetry $psu(1,1|2)^2$

Introduce two Q-systems



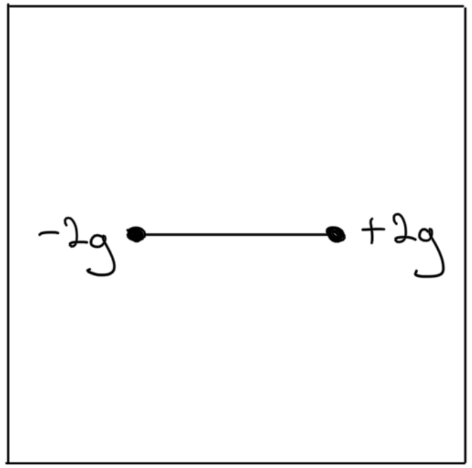
[Ekhammar, Volin]

[Cavaglia, Gromov, Stefanski, Torreilli]

Distinguished Q-functions $\mathbf{P}_a$ $\mathbf{P}_{\dot{a}}$ $\mathbf{Q}_k$ $\mathbf{Q}_{\dot{k}}$ + Hodge duals

$$a, \dot{a} \in \{1, 2\}$$

Defining sheet of P



Power-like asymptotics

$$\mathbf{P}_a(u) \sim A_a u^{M_a}$$

$$\mathbf{Q}_i(u) \sim B_i u^{\hat{M}_i}$$

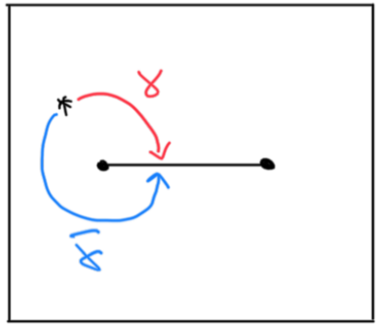
+ dotted

Analytic continuation relates the two Q-systems

$$\mathbf{P}_a^{\gamma} = \mu_a{}^{\dot{c}} \mathbf{P}_{\dot{c}}$$

Cuts cannot be quadratic!

$$\bar{\gamma} \neq \gamma$$

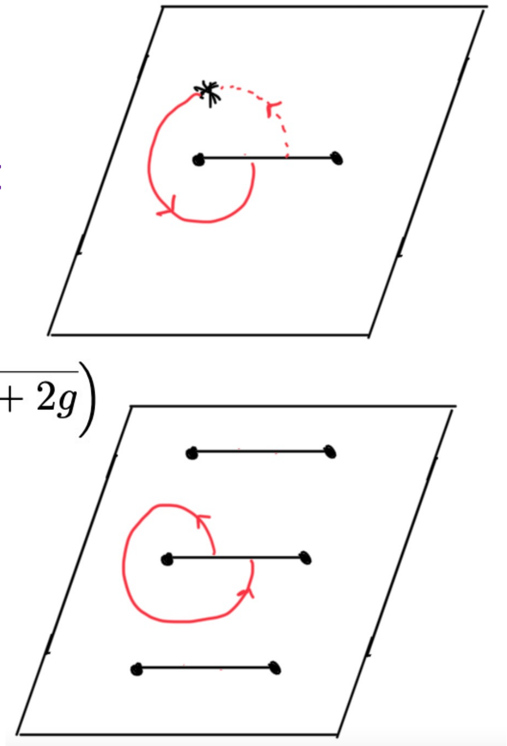


Riemann Surface of P functions

In AdS_5 all branch cuts are quadratic, i.e. square root type

AdS_5

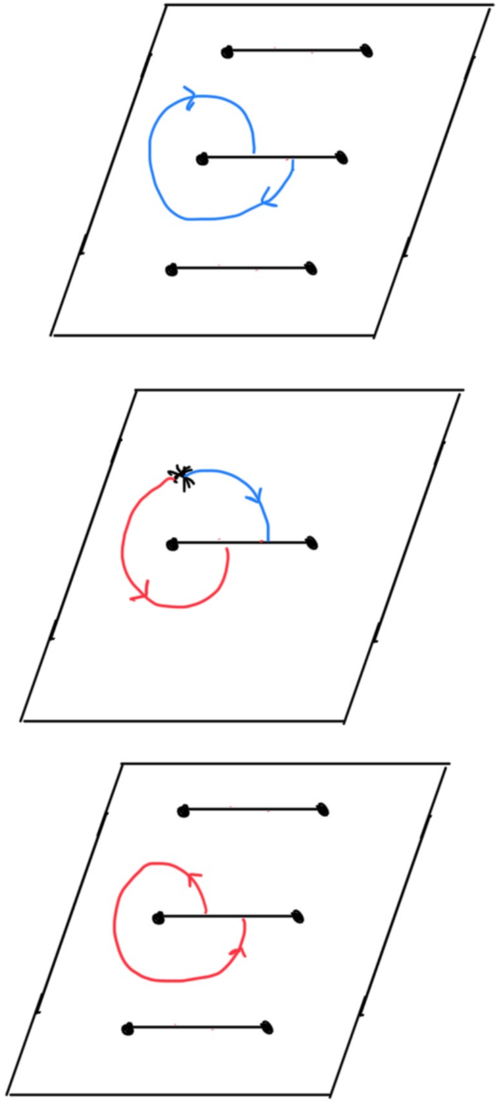
Defining sheet



$$x(u) = \frac{1}{2g}\left(u + \sqrt{u-2g}\sqrt{u+2g}\right)$$

AdS_3

Defining sheet



$$\log\left(\frac{u-2g}{u+2g}\right)$$

In AdS_3 we are forced to deal with logarithmic branch cuts

Solving the AdS_3 Quantum Spectral Curve

Set-up

We consider an analogue of the SL(2) sector of N=4 SYM

excitations

length

$$\mathcal{O} = \mathrm{Tr}\left(D^S Z^L\right) + \dots$$

permutations

+ parity symmetric

Konishi in N=4, L=2, S=2

covariant derivative

scalar field

Weak coupling – SL(2) spin chain, length L, S magnons

QSC drastically simplifies in this sector

Set-up

$$\chi^{ab} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix};$$

AdS_5 x S^5 QSC in SL(2) sector + parity invariance

Left / Right symmetry

$$\mathbf{P}^a(u) = \chi^{ac}\mathbf{P}_c(u)$$

Parity

$$\mathbf{P}_a(-u) = \pm\mathbf{P}_a(u)$$

AdS_3: don't know the CFT_2 dual, so no way to construct a parity-symmetric SL(2) sector

But we can define it at the level of QSC

Left / Right symmetry

$$\mathbf{P}^a = -\varepsilon^{ab}\mathbf{P}_b$$

Parity

$$\mathbf{P}_a(-u) = \pm\mathbf{P}_a(u)$$

Review of AdS_5 numerical algorithm

Cutoff ~ 5

Initial real parameters $c_{a,n}$ and Δ

Input $$\mathbf{P}_a = A_a (gx)^{M_a} \left(1 + \sum_{n=1}^{\infty} \frac{c_{a,n}}{x^n}\right)$$

Known function of Δ

Known function of Δ

Implement gluing $\tilde{\mathbf{Q}}_i(u) = \alpha_i^j \mathbf{Q}_j(-u)$ → $\tilde{\mathbf{Q}}_1(u) = \alpha_{13}\mathbf{Q}_3(-u), \quad \tilde{\mathbf{Q}}_3(u) = \alpha_{31}\mathbf{Q}_1(-u)$

Recast as optimisation problem:

$\tilde{\mathbf{Q}}_1(u)/\mathbf{Q}_3(-u)$ must be constant on cut. →

Compute at sampling points on cut
Variance from mean value must be 0
Hence minimize variance by updating
$c_{a,n}$ and Δ to their true values

Non-quadratic cuts present a significant challenge

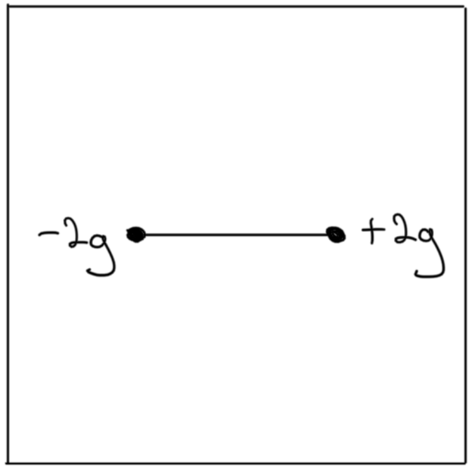
P-functions on the defining sheet in AdS_5

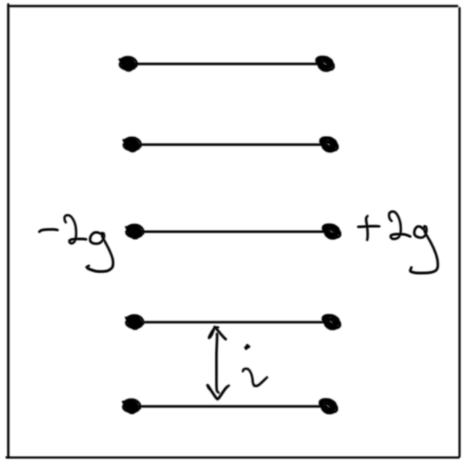
- Power-like asymptotics $\mathbf{P}_a(u) \sim A_a u^{M_a}$
- Square root branch cut [-2g,2g]
- No other singularities

Such functions admit a convergent expansion

$$\mathbf{P}_a = A_a (gx)^{M_a} \left(1 + \sum_{n=1}^{\infty} \frac{c_{a,n}}{x^n}\right)$$

Converges everywhere on defining sheet and a finite region of next sheet





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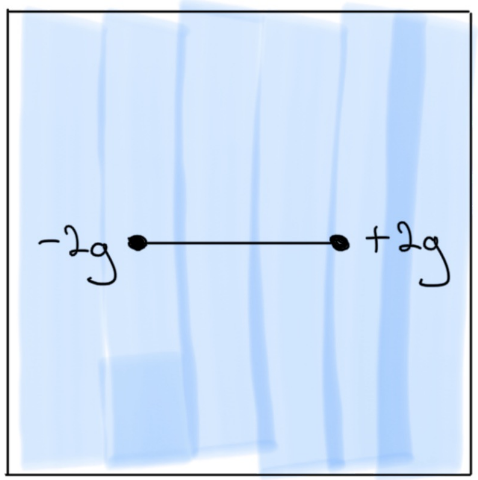
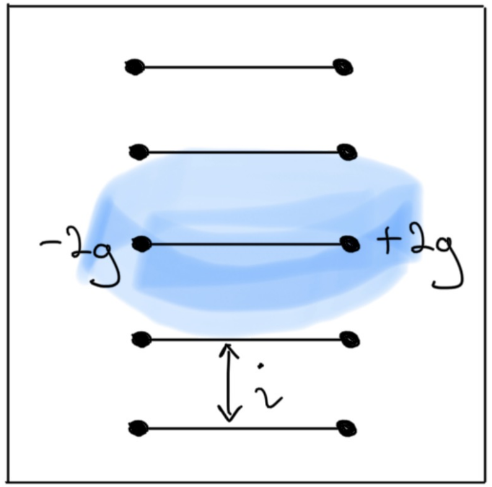




Non-quadratic cuts present a significant challenge

P-functions on the defining sheet in AdS_3

- Power-like asymptotics $\mathbf{P}_a(u) \sim A_a u^{M_a}$

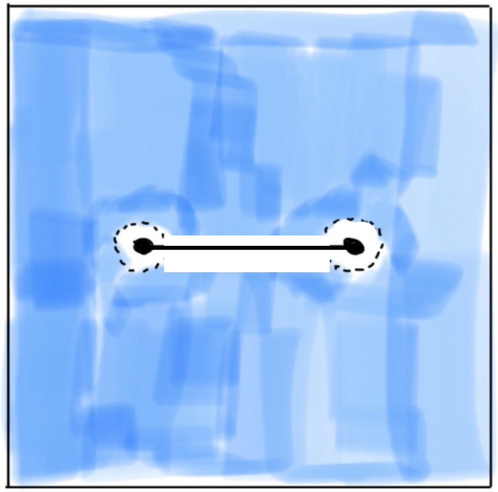
- Branch cut [-2g,2g] no longer square root

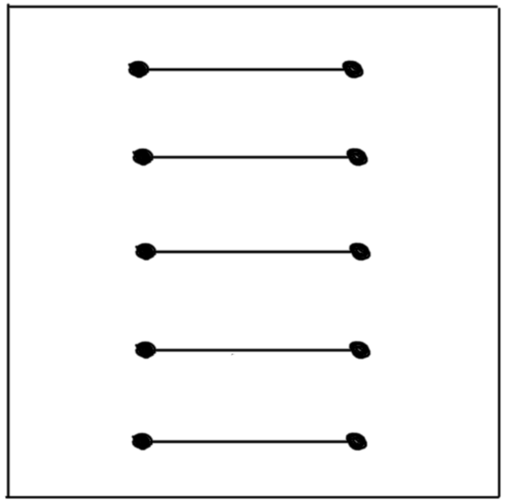
- No other singularities

Can still use

$$\mathbf{P}_a = A_a (gx)^{M_a} \left(1 + \sum_{n=1}^{\infty} \frac{c_{a,n}}{x^n}\right)$$

But very poor convergence near branch cut and no convergence on other sheets





Significant problem for closing QSC equations via gluing!

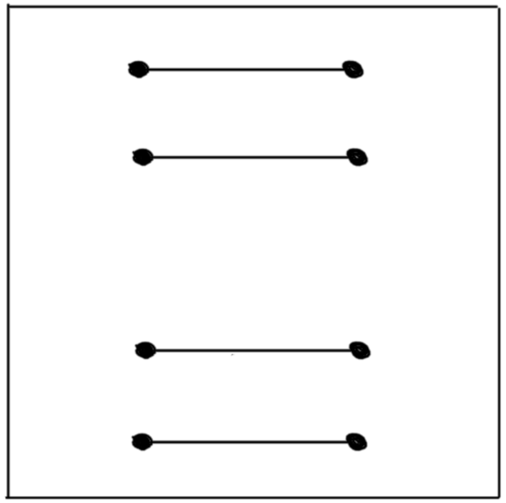
Solution: Construct a new object with square root branch points
Solve QSC with this object

Analytic continuation takes P-functions to their dotted cousins $\mathbf{P}_a^{\gamma} = \mu_a{}^{\dot{c}} \mathbf{P}_{\dot{c}}$

Going around the cut twice brings us back to the original P-functions $\mathbf{P}_a^{2\gamma} = W_a{}^b \mathbf{P}_b$

Completely fixed in terms of P!

W has no cut on the real axis!



We introduce

$$\mathbb{P}_a \equiv \left(W^l\right)_a{}^b \mathbf{P}_b$$

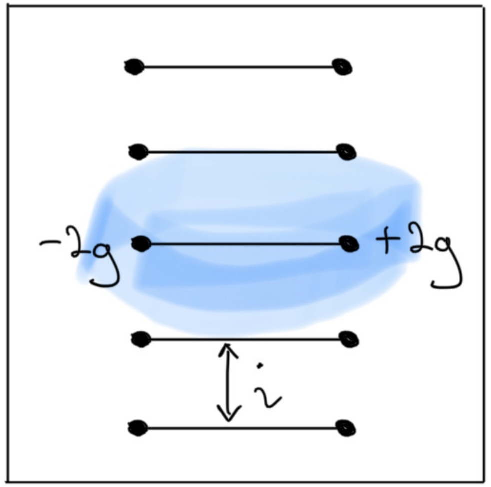
$$l = \frac{i}{2\pi} \log\left(\frac{x-1}{x+1}\right)$$

$$\mathbb{P}^{2\gamma} = W^{l-1}\mathbf{P}^{2\gamma} = W^{l-1} W \mathbf{P} = \mathbb{P}$$

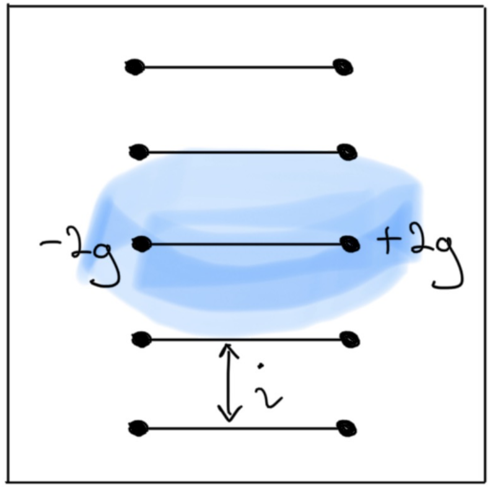
Cut on the real axis is quadratic!

In the vicinity of the branch cut on the real axis

$$\mathbb{P}_a = \sum_{n=-\infty}^{\infty} d_{a,n} x^n$$



$$\mathbb{P}_a^{\gamma} = \sum_{n=-\infty}^{\infty} \frac{d_{a,n}}{x^n}$$



We can now compare!

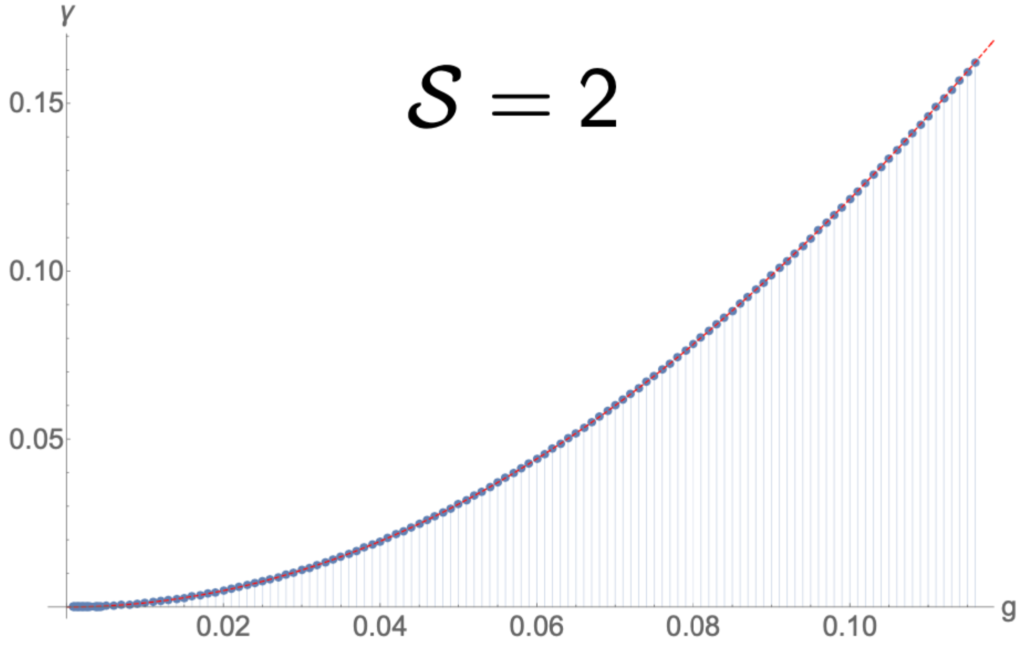
$$\mathbf{P}_a = A_a (gx)^{M_a} \left(1 + \sum_{n=1}^{\infty} \frac{c_{a,n}}{x^n}\right) \longrightarrow W \longrightarrow \mathbb{P} \longrightarrow d_{a,n}$$

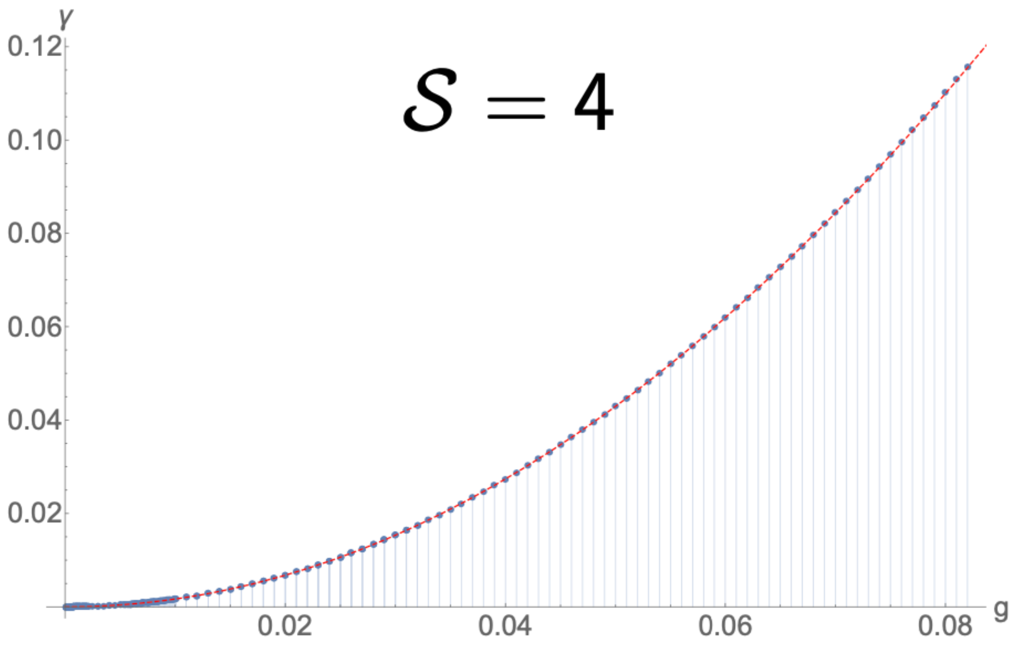
$$\mathbf{P}_a^{\gamma} = \mu_a{}^{c} \mathbf{P}_{\dot{c}} \longrightarrow \mathbb{P}^{\gamma} = W \mathbf{P}^{\gamma} \longrightarrow d_{a,n}$$

Computed in two different ways, must be equal on true solution of QSC!

Leads to optimisation problem for $c_{a,n}$ and Δ

Numerical algorithm converges!





Dimension seen to be a continuous function of g.
Strongly supports the QSC proposal – not an empty mathematical object!

We can now compare numerics with perturbation theory (small g expansion)

$$l = \frac{i}{2\pi} \log\left(\frac{x-1}{x+1}\right)$$

$$\mathbb{P}_a \equiv \left(W^l\right)_a^{\ b} \mathbf{P}_b \longrightarrow \mathbf{P} = W^{-l}\mathbb{P} = \sum_{n=0}^{\infty} l^n \mathbf{P}^{(n)}$$

$$\mathbf{P}^{(0)} = \mathbb{P}$$

Under excellent control from numerics!

$$W = 1 + \mathcal{O}(g)$$

$$\mathbf{P}^{(n)}/\mathbf{P}^{(n-1)} = \mathcal{O}(g)$$

$$l = \mathcal{O}(g)$$

Can consistently expand in g and determine dimension using methods from AdS_5 [Marboe, Volin]

Unlike AdS_5 we have no field theory to compare with

Results

$$\gamma_{S=2} = 12\,g^2 + \frac{864}{35\pi}g^3 + (-48 - \frac{576}{7\pi^2})g^4 + \left(-\frac{405504}{875\pi^3} - \frac{51552}{143\pi}\right)g^5$$
$$+ (444 - \frac{70665216}{4375\pi^4} + \frac{230121984}{175175\pi^2})g^6$$
$$+ (-\frac{16896}{35\pi}\zeta_3 - \frac{4965482496}{21875\pi^5} + \frac{6791453184}{875875\pi^3} + \frac{1102677696}{146965\pi})g^7$$
$$+ (-288\zeta_3 + \frac{1898496}{1225\pi^2}\zeta_3 - 576\zeta_5 - 5844$$
$$- \frac{302725824512}{109375\pi^6} - \frac{9030729728}{25025\pi^4} + \frac{25695082110528}{282907625\pi^2})g^8$$

Perfect agreement with numerics!

Comparing with asymptotic Bethe ansatz:

$$\gamma^{ABA}_{S=2} = 12g^2 + \mathcal{O}(g)^4 \implies \frac{864}{35\pi}g^3$$ non-trivial correction!

Comments on the results

$$\gamma_{S=2} = 12\,g^2 + \frac{864}{35\pi}g^3 + (-48 - \frac{576}{7\pi^2})g^4 + \left(-\frac{405504}{875\pi^3} - \frac{51552}{143\pi}\right)g^5$$
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Even and odd powers of g – in AdS_5 only even powers appear

Powers of $1/\pi$ related to log-type monodromies (massless modes)

Curious feature: $1/\pi$ a coupling constant for massless modes?

$\pi \to \infty$ reproduce N=4 SYM at g^2 and g^4 !

Summary & Outlook

Summary

- Solved conjectured QSC for string theory on $AdS_3 \times S^3 \times T^4$ with pure RR flux

- Solutions exist! Solutions isolated (no continuous parameters for fixed coupling)

- Provides a highly non-trivial test for the validity of the proposed QSC

- First ever predictions for generic unprotected string excitations

  - New tools necessary to deal with massless modes – QSC harder, but still solvable

- New features in perturbative expansion vs AdS_5: inverse powers of π

Outlook

- We consider pure RR flux. Can we extend to mixed flux? Pure NSNS?

- Comparisons with TBA of Frolov + Sfondrini

- Smaller system - ideal playground for Separation of Variables approach to correlators

- 2d CFT – ideal playground for bootstrability approach to correlators

Thank you!