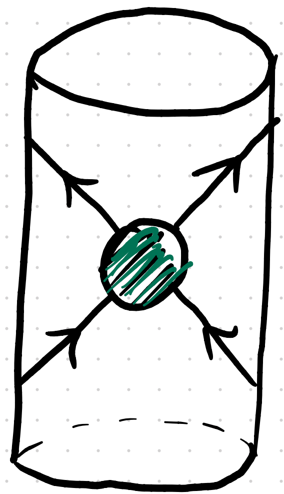
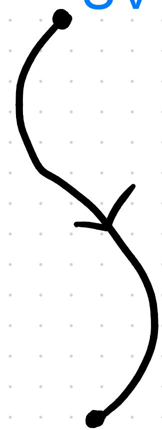
DESY

String Theory Seminar

November 21st, 2022

Bootstrapping RG Flows through AdS: sine-Gordon and friends





António Antunes, DESY

arXiv 2109.13261 + W.I.P.

w/ M.S.Costa, J.Penedones, A.Salgarkar, (B.C.van Rees / E. Lauria)

Main Idea

- We want to study QFTs undergoing RG Flow. Place it in a fixed AdS metric and use the radius as a scale.

- The Symmetries of AdS provide us with a one parameter family of boundary conformal correlators. Use the Bootstrap to obtain non-perturbative information.

Outline:

I) Motivation: QFT in AdS, Flat Space Limit, S-Matrix vs Conformal Bootstrap

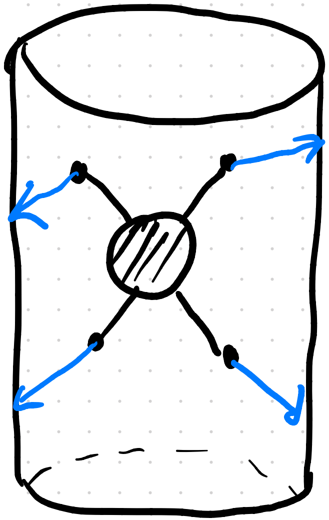
II) Bounds on sG Breathers (Z_2 Symmetry)

III) Bounds on sG Kinks (O(2) Symmetry)

IV) Staircase+Outlook

I) Massive QFT in AdS$_{d+1}$

$$ds^2 = \frac{L^2_{AdS}}{y^2}\left(dy^2 + dx^2\right)$$



$$\langle \phi(y_1, x_1) \ldots \phi(y_4, x_4) \rangle$$

Isometries SO(d+1,1)
same as Conformal(d)

$$\partial \text{AdS}_{d+1} = \text{CFT}_d$$

Bulk Fields ⟶ Boundary Operators

$$\phi_i(y,x) = \sum_k a_{ik}\, y^{\Delta_k} \mathcal{O}_k(x) + \ldots$$

$$m_i^2 L^2_{\text{AdS}} = \Delta_i(\Delta_i - d)$$

L^2_{AdS} IR reg
RG scale

maximally symmetric box

Asymptotic states

[PPTvRV] ‘16

Conformal correlators on the Boundary

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\mathcal{O}_1(x_3)\mathcal{O}_1(x_4)\rangle = \frac{\mathcal{A}(z,\bar{z})}{x_{12}^{2\Delta_1} x_{34}^{2\Delta_1}}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$$

Cross ratios

$$v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

Boundary OPE

$$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k c_{ijk}\, |x|^{\Delta_k-\Delta_i-\Delta_j}\, [\mathcal{O}_k(0) + desc$$

-We have all the bootstrap axioms:

Conformality, OPE, Unitarity ⟶ Conformal bootstrap:

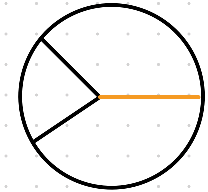
Non-perturbative (numerical) bounds on CFT data: $\Delta_i \quad c_{ijk}$

Non-local C(F)T

No Stress-Tensor <-> No graviton

Flat Space Limit

Flat space as $L_{\text{AdS}} \to \infty$



Massive particles: $\Delta_i \to \infty$

$$\frac{m_i}{m_1} = \lim_{\Delta_i \to \infty} \frac{\Delta_i}{\Delta_1}.$$

$$g_{112} = \# \lim_{\Delta_i \to \infty} c_{112} \Delta_1^{\frac{d-2}{4}} (\dots)^{\Delta_1}$$

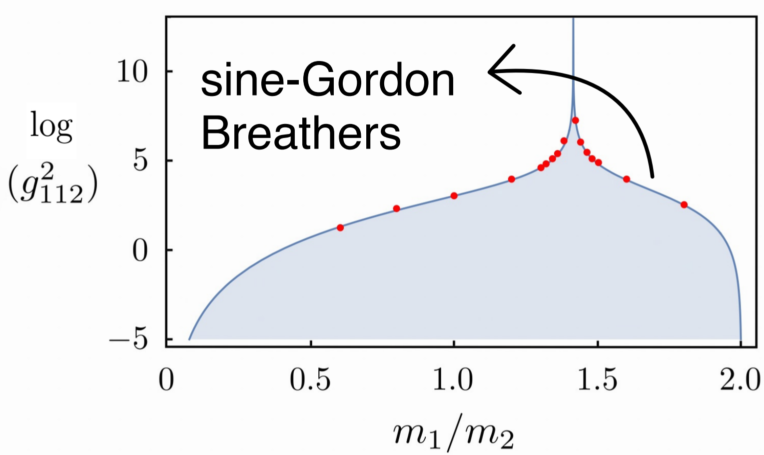
AdS_2 for simplicity. 1 2d Mandelstam s ; 1 1d cross-ratio z

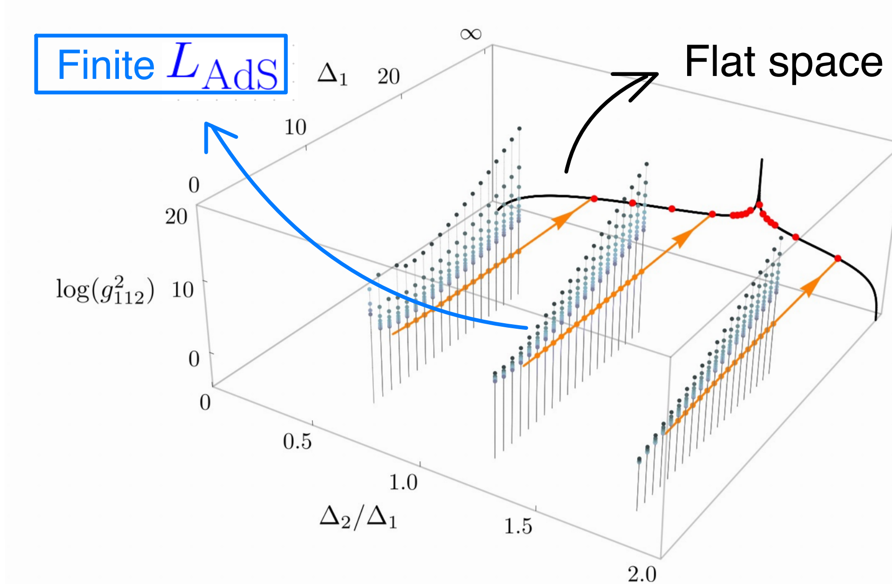
$$S(s) = \lim_{L_{\text{AdS}} \to \infty} \frac{\mathcal{A}(z)}{z^{2\Delta_1}}\Bigg|_{z=1-s/(4m^2)}$$

[KPvRZ] '20

2d S-Matrix Bootstrap	1d Conformal Bootstrap
Particle m_1 w) bound state m_2	Operator $\mathcal{O}_1$ w/ 'bound state' $\mathcal{O}_2$
What is the maximum coupling/residue g^2_{112} ?	$\mathcal{O}_1 \times \mathcal{O}_1 = 1 + c_{112}\mathcal{O}_2 + \ldots (\text{operators with } \Delta > 2\Delta_1) \ldots$
Is there a physical theory ?	Maximize $c_{112}(\Delta_1, \Delta_2)$ $\Delta_i \to \infty$

[PPTvRV]





General Logic

- Take the finite radius, non-perturbative bootstrap results seriously, not just as a data set to extrapolate upon.

- Attempt to see the flows as (approximately) saturating the bounds, even at finite radius.

II) Bounds on Breathers at finite L_{AdS}

$$S = \int_{AdS_2} d^2x\sqrt{g}\left[\frac{1}{2}(\partial\phi)^2 + \lambda\cos(\beta\phi)\right]$$

$$\phi \sim \phi + 2\pi/\beta$$

Relevant deformation if $\Delta_\beta = \beta^2/(4\pi) \leq 2$

UV

Free boson BCFT $\xleftarrow{0}$ $\lambda L_{\text{AdS}}^{2-\Delta_\beta}$ $\xrightarrow{\infty}$ Flat Space

IR

Dirichlet B.C.

$\mathcal{O}_1 = \partial_\perp\phi$ $\mathcal{O}_1 \times \mathcal{O}_1 = 1 + c_{112}\mathcal{O}_2$

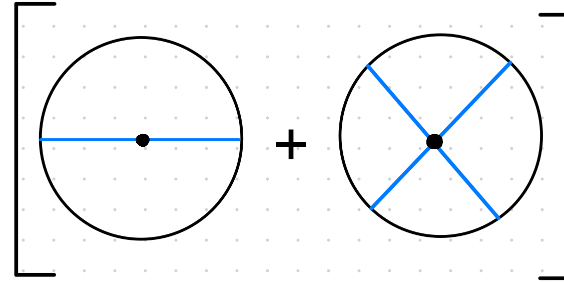
$\mathcal{O}_2 = (\partial_\perp\phi)^2$ Nearly bound state

$c_{112}^2 = 2$ $\Delta_1 = 1$ $\Delta_2 = 2$

Bosonic MFT maximizes c_{112}^2 UV ✓

Perturbative Corrections

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\mathcal{O}_1(x_3)\mathcal{O}_1(x_4)\rangle = \text{MFT} + \lambda$$



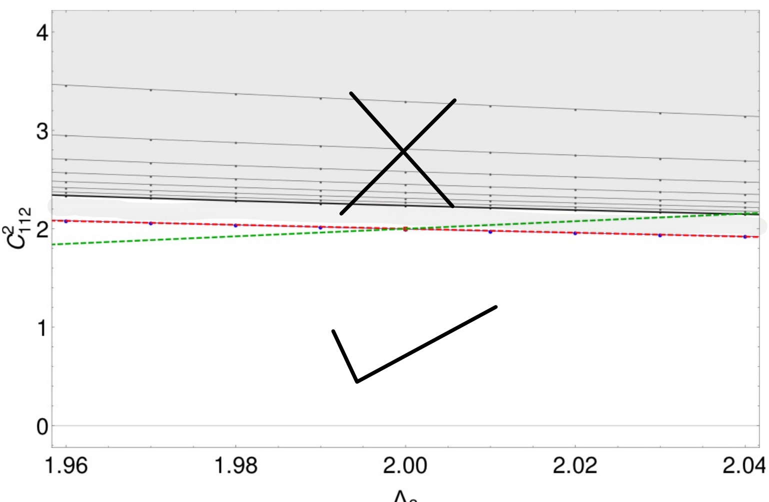
$$= \frac{1}{x_{12}^{2\Delta_1} x_{34}^{2\Delta_1}} \sum_{n=2,\ldots}^{\infty} c_{11n}^2 G_{\Delta_n}(z)$$

$$\Delta_1 = 1 + \gamma_1^{(1)}\lambda$$

$$\Delta_2 = 2 + \gamma_2^{(1)}\lambda$$

$$c_{112}^2 = 2 + c^{(1)}\lambda$$

$$\boxed{c_{112}^2 = 2 - 2\Delta_2 + 4\Delta_1}$$

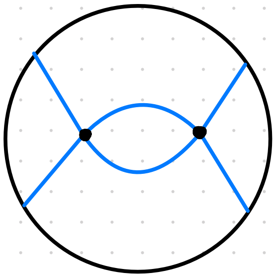


Bounds saturated!

Bonus: Bound on irrelevant coupling

$$\tilde{\lambda}(\partial\phi)^4 : \quad \tilde{\lambda} \geq 0$$

Second order: β expansion vs λ expansion

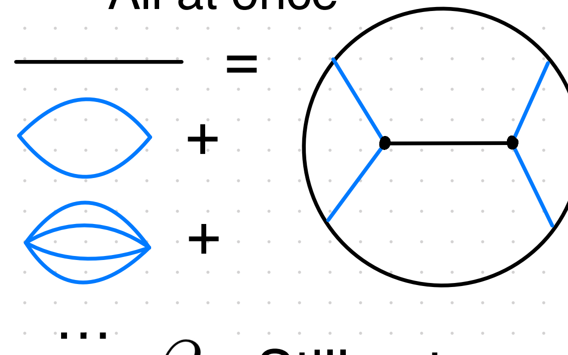


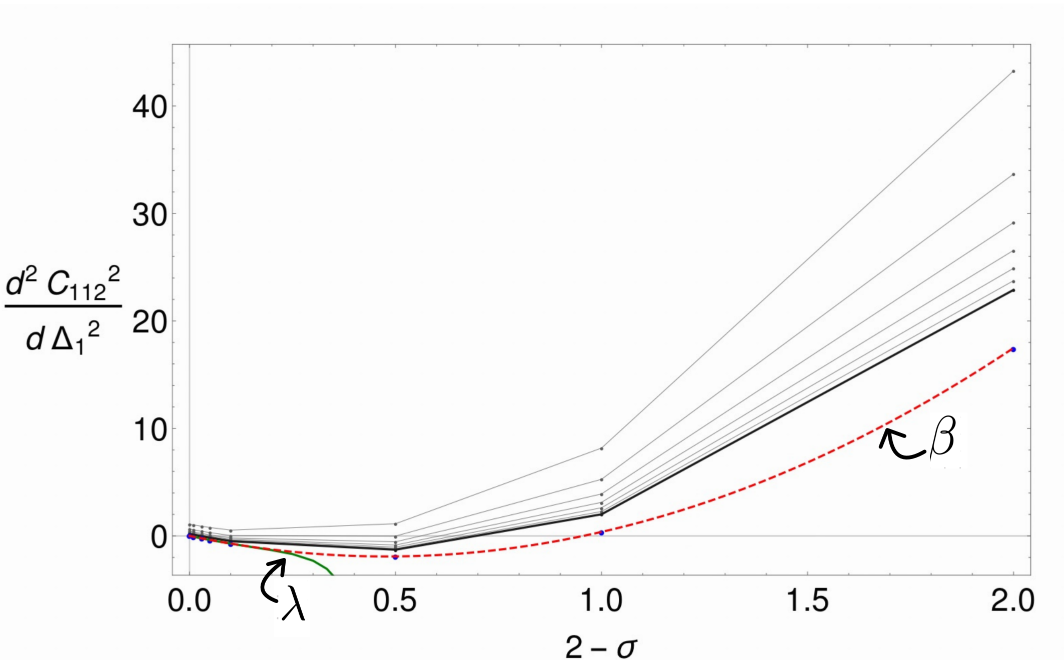
$$\beta^4\phi^4 + \#\beta^6\phi^6 + \dots$$

pure ϕ^4 / loop counting

$$\lambda(\phi^4 + \#\phi^6 + \dots)$$

All at once

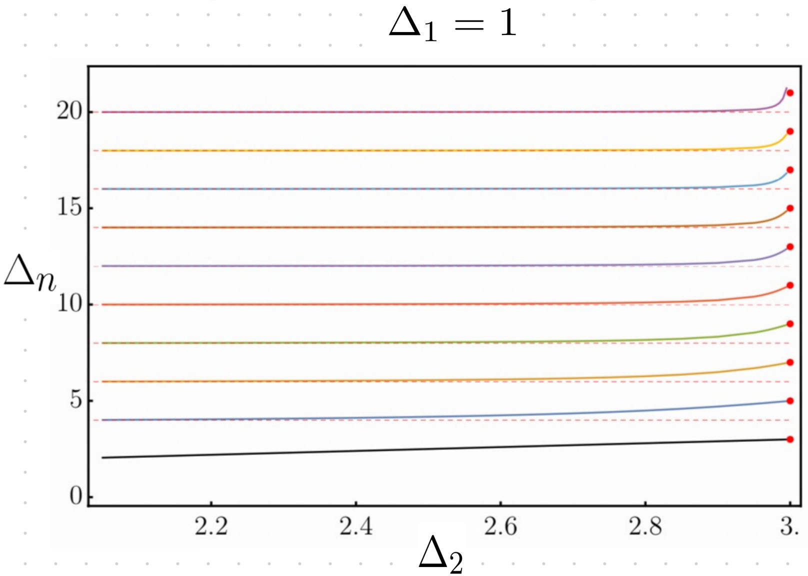


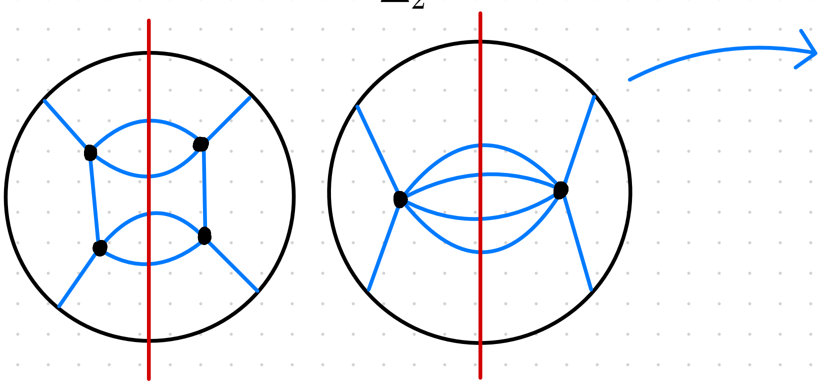


β Still extremal

λ Not anymore

Non-perturbative picture





We don't expect saturation!

Sparse extremal spectra from numerics.
'One operator per bin'/
Only 2-particle states.

[Paulos Zan] '19

In a real QFT in AdS:

$$\partial_\perp \phi \quad (\partial_\perp \phi)\Box(\partial_\perp \phi) \quad (\partial_\perp \phi)^4$$

Dense spectrum, multi-particle states.

Hidden until 3 loops

Higher Points?

Attempt to see more:
external $\mathcal{O}_2$

Multi-Correlator bounds

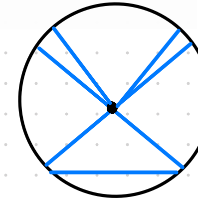
$\langle \mathcal{O}_1\mathcal{O}_1\mathcal{O}_1\mathcal{O}_1 \rangle$ $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle$

$\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_1\mathcal{O}_1 \rangle$

5d Space of Observables

$\{\Delta_1, \Delta_2, \Delta_{\text{gap}}, c_{112}, c_{222}\}$

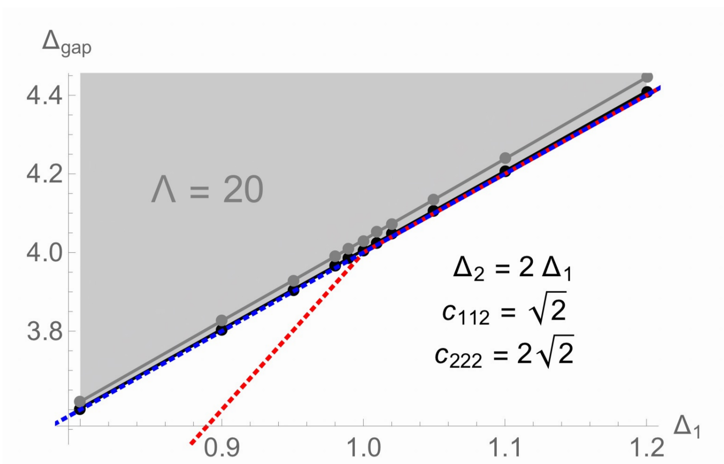
1st order:



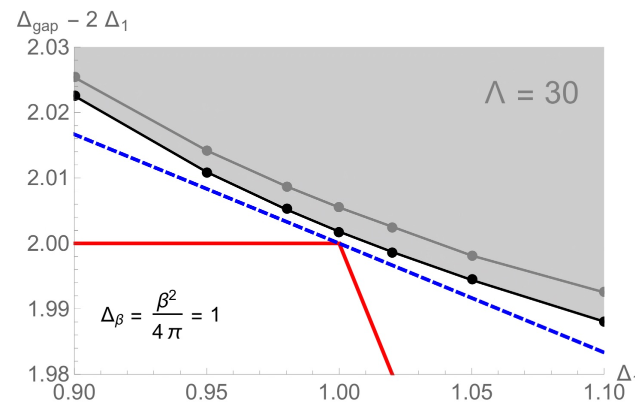
Unfortunately, bounds don't improve. Understood from perturbation theory + solutions without identity. Study gap.

$O_4 = (\partial_\perp \phi)\Box(\partial_\perp \phi)$ $\quad 2\Delta_1 + 2$

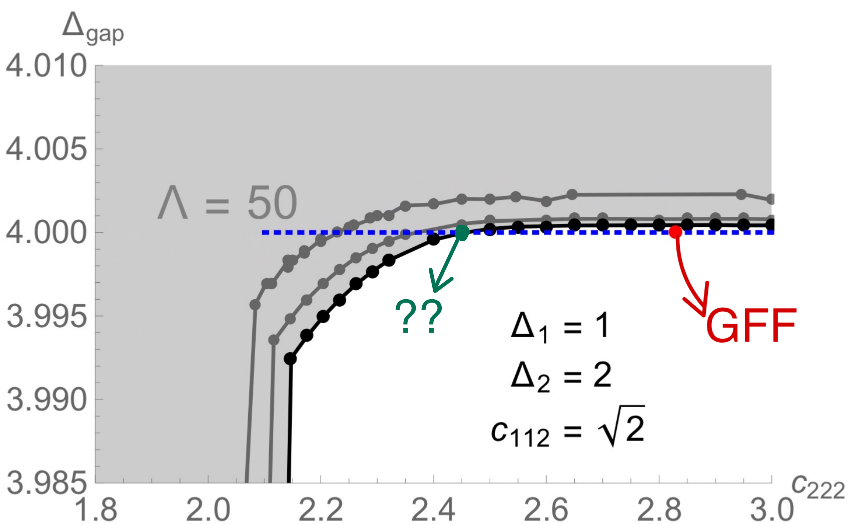
$\mathcal{O}_{4'} = (\partial_\perp \phi)^4$ $\quad 4\Delta_1$

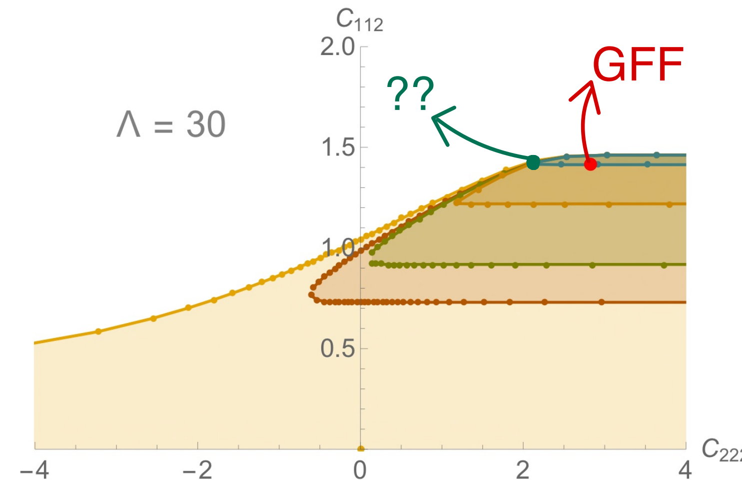


Resolving Mixing Explains sub-extr. ϕ^6



Bonus: ϕ^6, ϕ^8 only





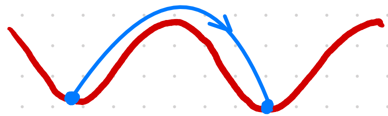
$\Delta_1 = 1$

$\Delta_2 = 2$

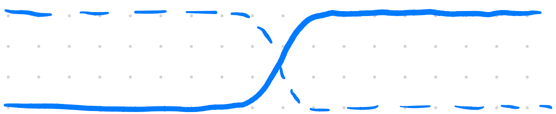
Vary Δ_{gap}

III) Bounds on Kinks

Breathers ϕ quanta

$\cos(\beta\phi) \longrightarrow$ 

Bound states



Kinks

$\overline{\text{Kinks}}$

Non-perturbative in ϕ

Breathers U(1)$_T$ neutral

$\pm$ charge under $Z_2 \subset O(2)$

$\begin{pmatrix} \text{K} \\ \overline{\text{K}} \end{pmatrix} \longrightarrow$ O(2)$_T$ doublet

Sensitive to β the inverse radius of ϕ

Excludes $\sum_n a_n \phi^{2n}$

What are the CFT kinks in the UV?

Compact boson has $O(2)_L \times O(2)_R$ sym.

$\cos(\beta\phi)$ explicitly breaks $\longrightarrow O(2)_T$

Charged bulk operators:

Momentum

Vertex Ops.

Winding

$$V_{n,m} = :e^{ip_L\phi_L + ip_R\phi_R}:$$

$$p_{L,R} = n\beta \pm \frac{2\pi}{\beta} m$$

$$\begin{pmatrix} K \\ \overline{K} \end{pmatrix} \rightarrow V_{0,\pm 1} = :e^{\pm\frac{2\pi i}{\beta}\tilde{\phi}}:$$

$$\tilde{\phi} = \phi_L - \phi_R$$

Bulk dimension π/β^2 (non-pert in small β)

Can tune to have no bound states

O(2) symmetric correlators

$i, j, k, l \in \{1, 2\}$

Irreps
S,T,A

$$x_{12}^{2\Delta_v} x_{34}^{2\Delta_v} \langle K_i(x_1) K_j(x_2) K_k(x_3) K_l(x_4) \rangle$$
$$= \delta_{ij}\delta_{kl}\, g_1(z) + \delta_{il}\delta_{jk}\, g_2(z) + \delta_{ik}\delta_{jl}\, g_3(z)$$

In the deep UV we can compute the boundary correlators:

$$g_{+-+-}(x_i) = \langle V_{0,+1}(x_1, y_1) V_{0,-1}(x_2, y_2) V_{0,+1}(x_3, y_3) V_{0,-1}(x_4, y_4) \rangle \Big|_{y_i \to 0}$$

$$g_{+-+-} = \frac{1}{(x_{12}x_{34})^{4\pi/\beta^2}} (1-z)^{-4\pi/\beta^2}$$

$$\Delta_v = 2\pi/\beta^2$$

$\longrightarrow$

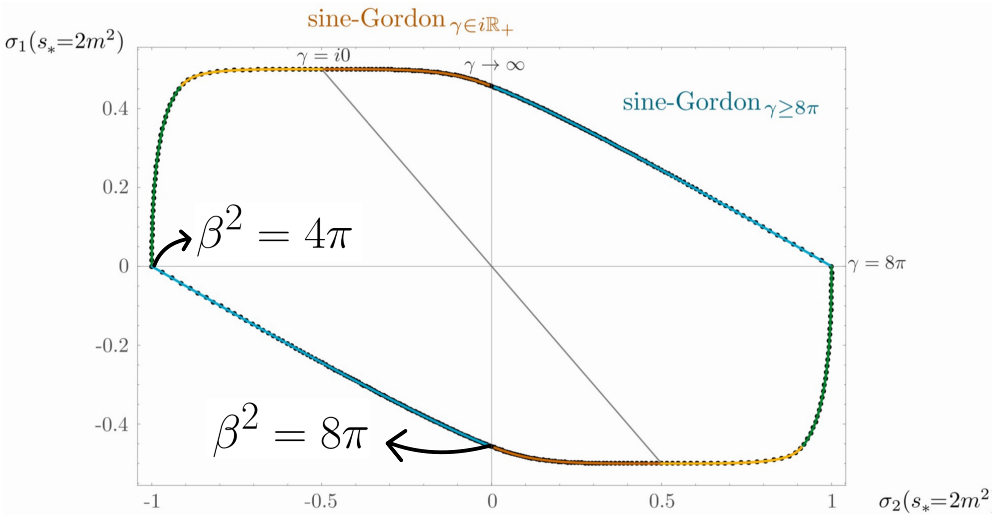
S $\rightarrow$ 0,2,4,… $(\partial_\perp \phi)^2$

A $\rightarrow$ 1,3,… $\partial_\perp \phi$

T $\rightarrow$ $4\Delta_v$ … $V_{0,\pm 2}$

What’s the bootstrap question?

Flat space kinks w/o bound states extremize $S_i(s^*)$



[CHKV] ‘19

Generalize to finite radius/ CFT?

Extremize $g_i(1/2) = g_i^*$

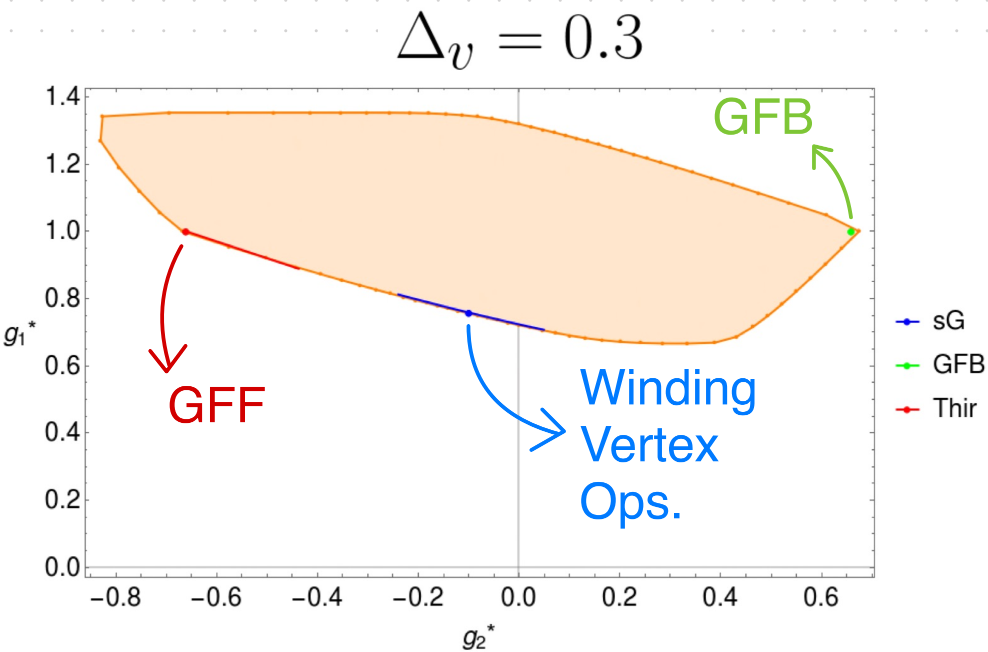
New conformal bootstrap problem.

Impose g_i^* as a constraint and bissect.

Assume no bound states ($\Delta \geq 2\Delta_v$)

Need to pick Δ_v for each radius. Start from no bound state range.

The O(2) CFT_1 slate and perturbations



Our free kinks saturate the bound, to first order in $\lambda \cos(\beta\phi)$ too.

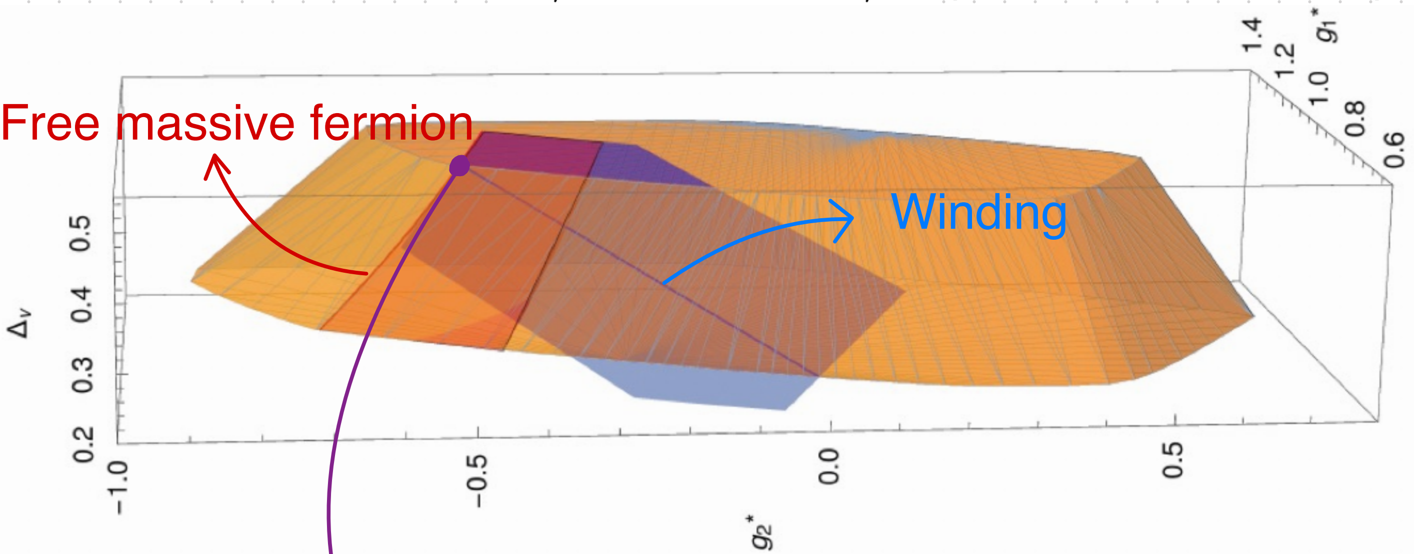
As do free fermions and their quartic Thirring perturbations.

$$\mathcal{L} = \lambda_f \left(\overline{\psi}\gamma^{\mu}_{flat}\psi\right)\left(\overline{\psi}\gamma_{\mu,flat}\psi\right)$$

+ free Dirac fermion

The O(2) menhir and Bosonization

Consider UV $1/4 \leq \Delta_v \leq 1/2$ (no bound states)



$\lambda \cos(\beta\phi)$

$\lambda_f j_f^\mu j_{\mu,f}$

Massless fermion +pert. = Winding w/ $\Delta_v = 1/2$ + pert.

(perturbative) boundary bosonization!

Thirring = sine-Gordon

Flat space limit

The menhir is actually an infinite tower as $\Delta_v\,,\ L_{\text{AdS}} \to \infty$

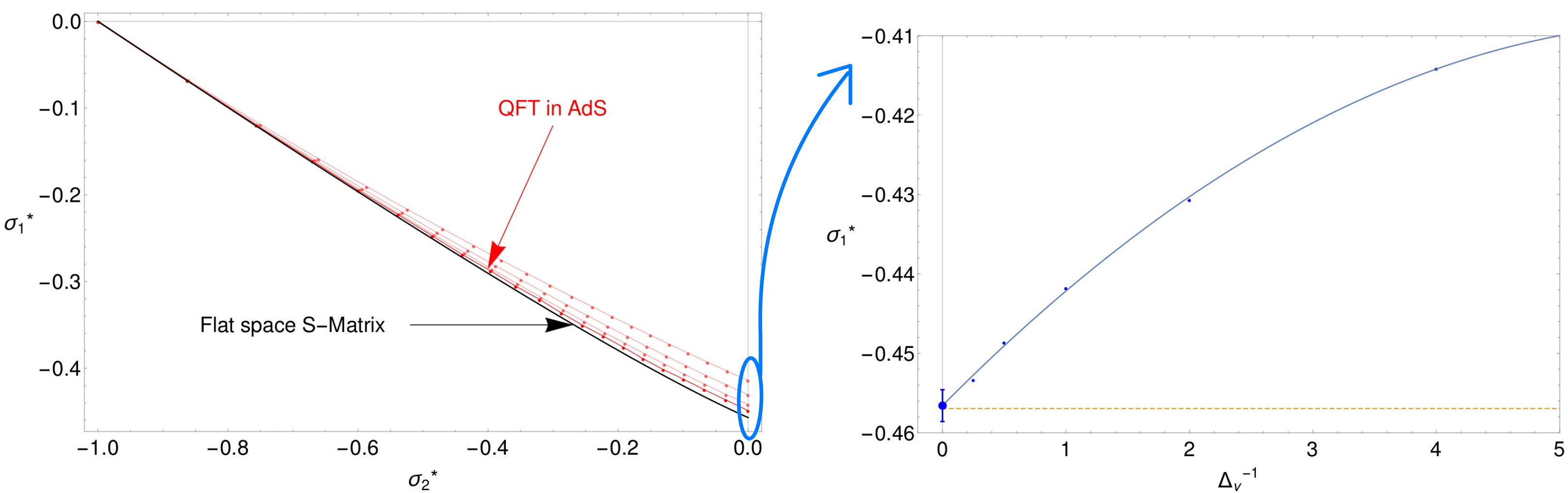
It is convenient to define

$$\sigma_1(s) = \left. z^{-2\Delta_v}\left(g_1(z)-1\right)\right|_{z=1-s/(4m^2)}$$

$$\sigma_2(s) = \left. z^{-2\Delta_v} g_2(z)\right|_{z=1-s/(4m^2)}$$

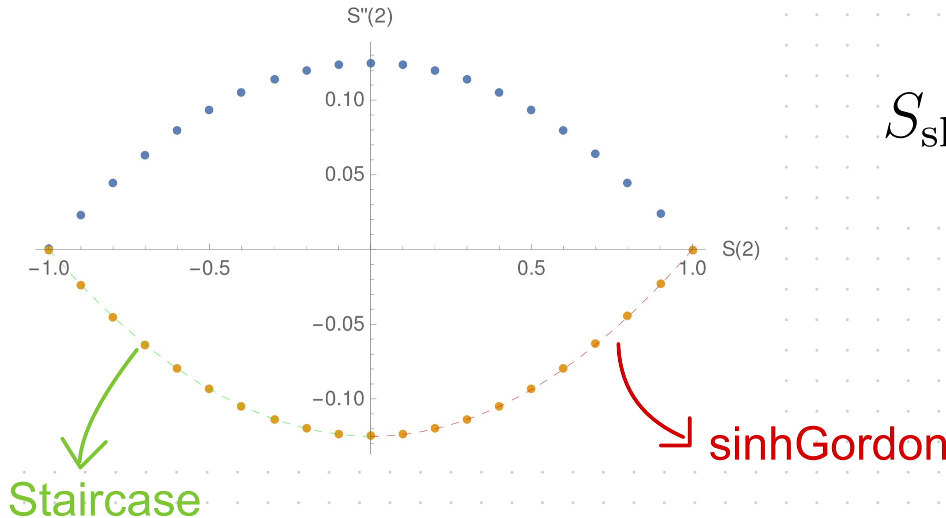
Approach S-Matrix as $L_{\text{AdS}} \to \infty$

Extrapolate to flat space by increasing ext. dimension



IV) Sinh-Gordon and Staircase model

Studying no bound states gave controllable setup. Back to breathers but without bound states. Sinh-Gordon and Staircase saturate flat space bounds.



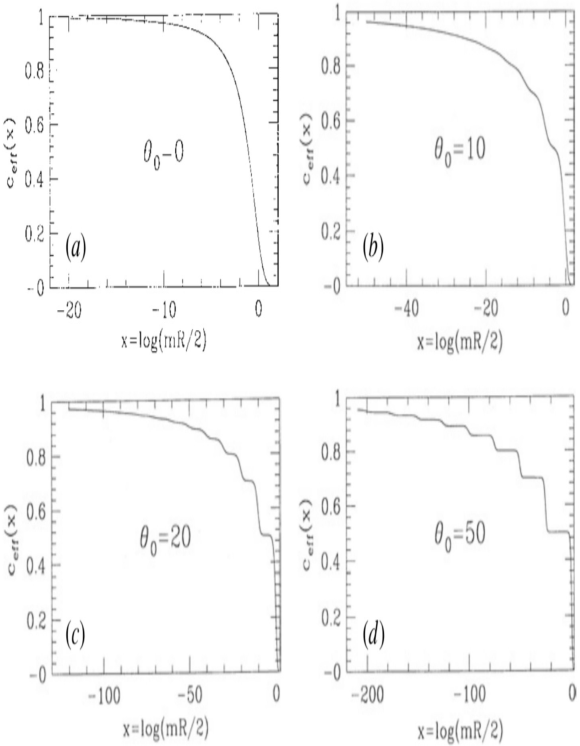
$$S_{\text{shG}}(\theta) = \frac{\sinh\theta - i\sin\gamma}{\sinh\theta + i\sin\gamma}$$

$$\gamma \to \frac{\pi}{2} + i\theta_0$$

$$g(1/2) \equiv g \text{ and } g''(1/2) \equiv g''$$

Staircase is interesting!
Goes through Minimal Model flows.
UV is mysterious.
Perfect for Bootstrap.

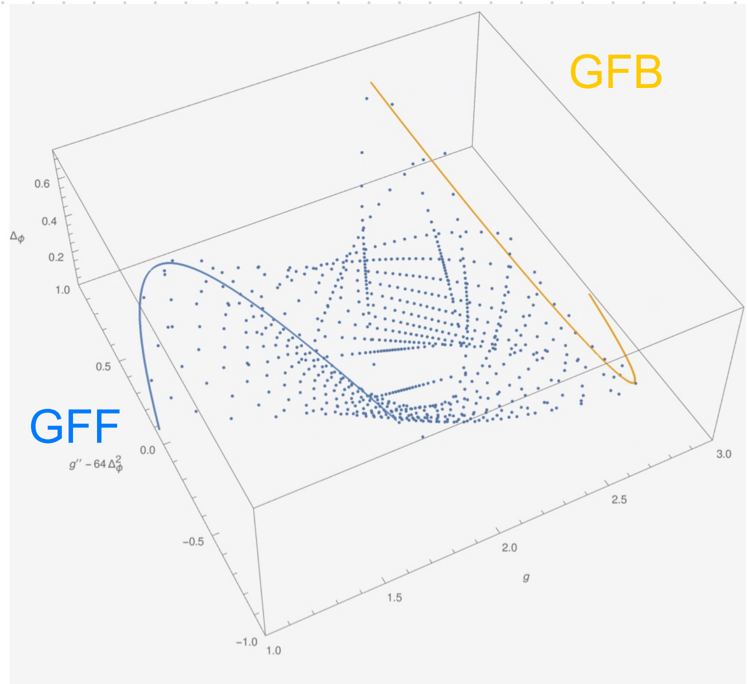
Climbing a conformal staircase



?

L_{AdS}

Want Dirichlet-like Z2 preserving (2,2) boundary conditions, containing lightest Op.



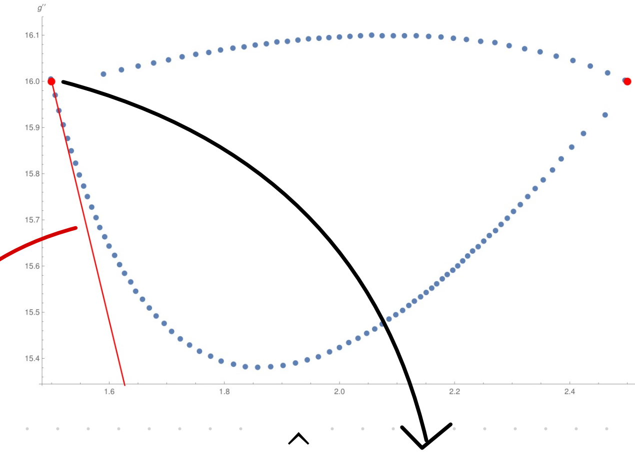
[Al. Zamolodchikov, unpublished]

Tricritical to Ising

‘Scattering’ Boundary Conditions for Minimal Models?

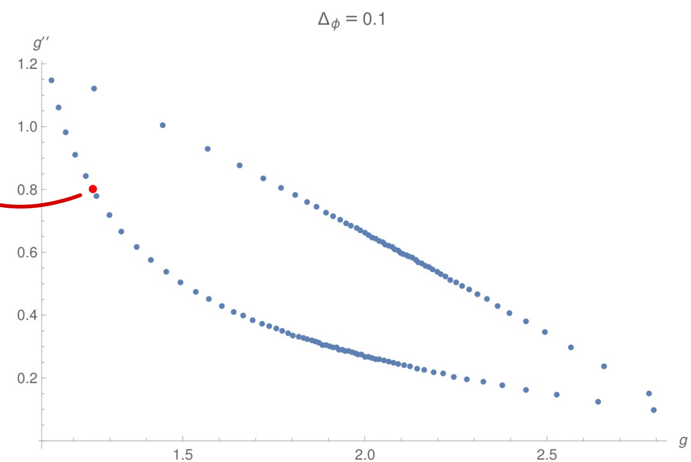
Fixed radius of AdS

$$\Delta_v = 1/2$$



TTb (Irrelevant flow to tricrit)

$\hat{\epsilon}$ in (2,2) Ising



$\hat{\epsilon}$ in (2,2) Tricrit Ising.

Does it persist all the way up to the UV? (large m)

Dimensions approaching zero?

Outlook

- More useful 2d bounds (O(N)), higher d bounds.
- Is there a systematic way to add more correlators and approach a realistic dense spectrum?
- Is there a sharp connection to integrability in AdS?

Thank you!

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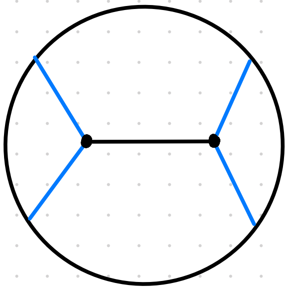
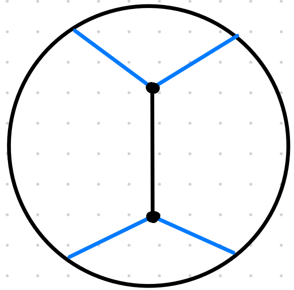
Extra Slides

Breather loops



Easy



Hard

 $= g_{\beta,\pm}(\zeta)$

$$g_{\beta,\pm}(X\cdot X') = \int_{-\infty}^{\infty} d\nu\, \tilde{g}_{\beta,\pm}(\nu)\Omega_{i\nu}(-\cosh(\rho))\,,$$

$$\int_{-\infty}^{\infty} d\nu\, \frac{\tilde{g}_{\beta,\pm}(\nu)}{(P_{12})^{\Delta}\,(P_{34})^{\Delta}}\, \frac{\Gamma^2_{\Delta-\frac{d}{4}-\frac{i\nu}{2}}\Gamma^2_{\Delta-\frac{d}{4}+\frac{i\nu}{2}}}{64\pi^{\frac{d}{2}+1}\Gamma^2_{\Delta}\Gamma^2_{1-\frac{d}{2}+\Delta}}\, \frac{\Gamma^4_{\frac{d}{4}+\frac{i\nu}{2}}\mathcal{G}_{\frac{d}{2}+i\nu}(z,\overline{z})}{\Gamma_{\frac{d}{2}+i\nu}\Gamma_{i\nu}}$$

Cross-channel infinite sums give trancendentality

$$\mathcal{G}_{\frac{d}{2}+i\nu}(z,\overline{z}) \;\sim\; z^{\Delta}\,{}_2F_1(\Delta,\Delta,2\Delta,z) \sim -\frac{\Gamma(2\Delta)\left(2\psi^{(0)}(\Delta)+\log(1-z)+2\gamma\right)}{\Gamma(\Delta)^2}$$

Unphysical solutions to crossing and multiple correlators

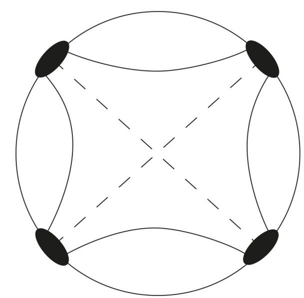
$\langle \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_1 \rangle$ $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_1 \mathcal{O}_1 \rangle$ $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$

c_{11n}^2 $c_{22n} c_{11n}$ c_{22n}^2

Mixes both

Decoupled

$$f_\chi(z) = \sum_{k,\, \Delta_k \geq 2\Delta_\phi} \mu_k^2\, g(\Delta_k, z)$$



Can add w/ arbitrary coefficient to c_{22n}^2 With tiny c_{11n}^2, cross term is arbitrary.

No better bound…

Kink correlators at first order

Winding

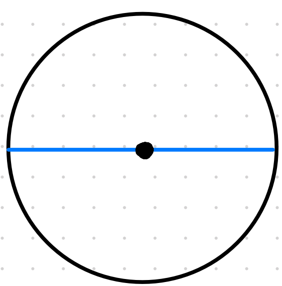
Momentum

$$\left\langle e^{i\alpha\phi(w_1)}e^{i\alpha\phi(w_1^*)}e^{-i\alpha\phi(w_2)}e^{-i\alpha\phi(w_2^*)}\left(e^{i\beta(\phi(w)-\phi(w^*))}+e^{-i\beta(\phi(w)-\phi(w^*))}-2\right)\right\rangle$$

$$\alpha\beta = 2\pi$$

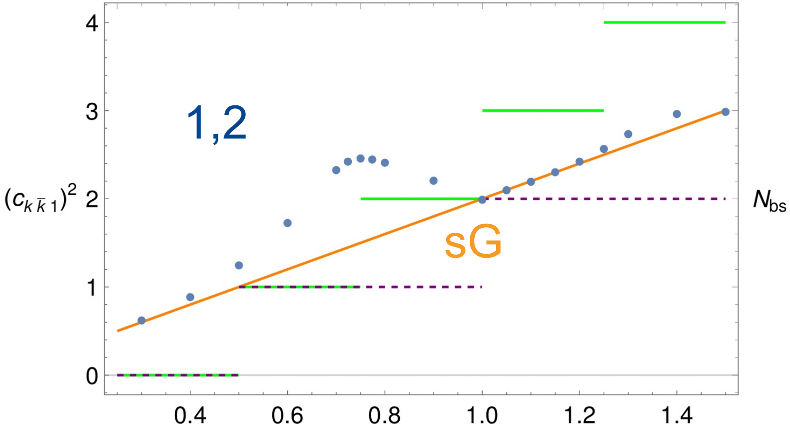
Rational integrand

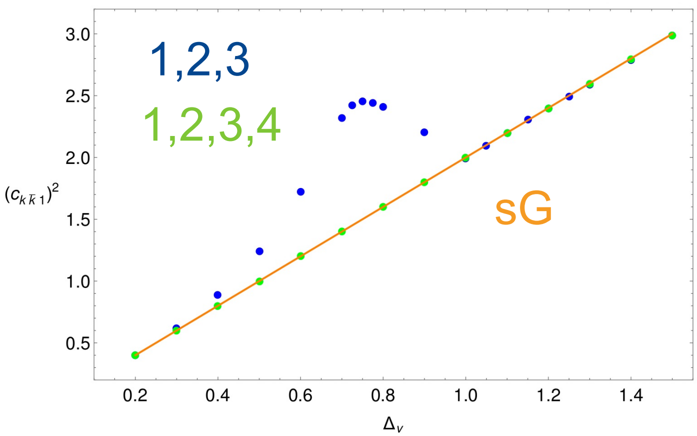
$$\int_{AdS_2}\frac{dxdy}{y^2}\frac{-2(x_{12})^2y^2}{(y^2+(x-x_1)^2)(y^2+(x-x_2)^2)}$$



$$\langle V_{\alpha_1}\dots V_{\alpha_n}\rangle_{\mathbb{R}^2}=\prod_{i<j}|z_i-z_j|^{\alpha_i\alpha_j/4\pi}$$

Kink anti-kink breather bounds





Hard, need to scan over bound state dimensions.

Bounds vary with number of bound states.