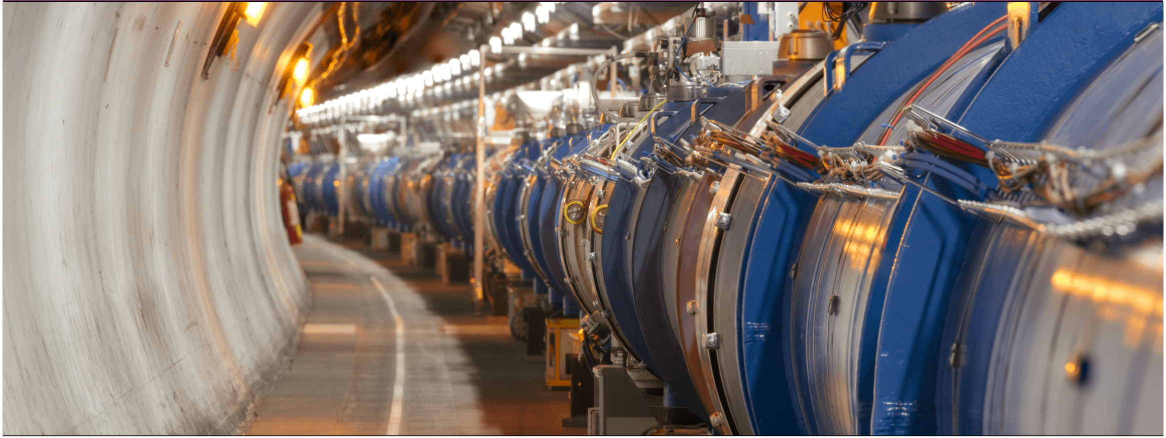
Quasi-3D Magneto-Thermal Quench Simulation of Superconducting Magnet Coils

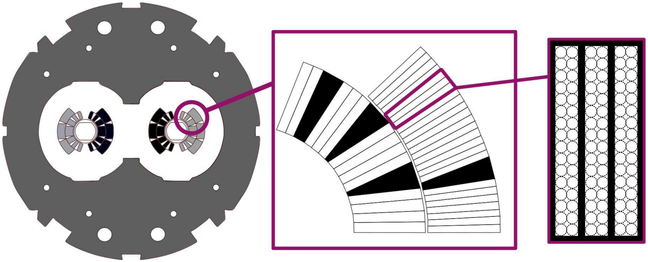
TECHNISCHE UNIVERSITÄT DARMSTADT

Laura D’Angelo, Yvonne Späck-Leigsnering and Herbert De Gersem



Superconducting Accelerator Magnets = Multi-Scale Problems

TECHNISCHE UNIVERSITÄT DARMSTADT



Challenges:

- Many geometrical details in cross-section while being very long: 1:20 ratio!
- Quench phenomenon occurs in mm-range along magnet

Goals of my work:

- Improvement of 3D quench simulations for superconducting accelerator magnets
- Improvement of spatial resolution while attaining reliable accuracy

Numerical Idea

TECHNISCHE UNIVERSITÄT DARMSTADT

Finite Element Method (FEM):

+ can resolve every geometry
- requires fine mesh for high accuracy

Spectral Element Method (SEM):

+ few elements for high accuracy
- simple domains

Numerical Idea

Finite Element Method (FEM):

+ can resolve every geometry
- requires fine mesh for high accuracy

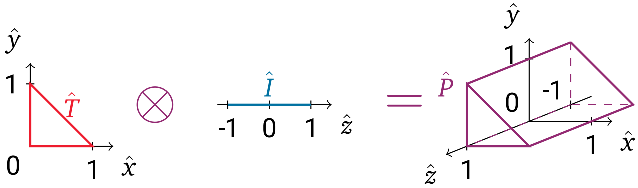
Spectral Element Method (SEM):

+ few elements for high accuracy
- simple domains

$\hookrightarrow$ Take the best of both worlds $\hookleftarrow$

$\downarrow$

Quasi-3D method with hybrid FE-SE basis functions:

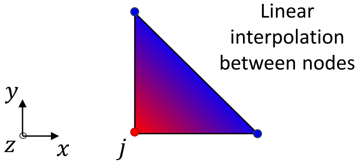


Agenda

- Motivation
- Numerical Modeling
- Application to Quench Simulation
- Conclusion

First Ingredient: 2D FE Nodal and Edge Functions

Nodal functions



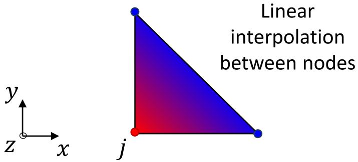
$$N_j(x,y) = \begin{cases} 1 & \text{on node } i = j \\ 0 & \text{on node } i \neq j \end{cases}$$

- Continuity in every direction
- Appropriate for **temperature**

$$T(x,y) \approx \sum_j u_j N_j(x,y)$$

First Ingredient: 2D FE Nodal and Edge Functions

Nodal functions

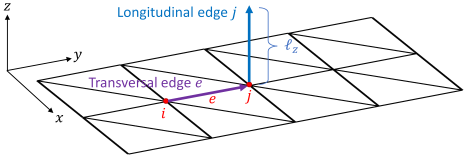


$$N_j(x,y) = \begin{cases} 1 & \text{on node } i = j \\ 0 & \text{on node } i \neq j \end{cases}$$

- Continuity in every direction
- Appropriate for **temperature**

$$T(x,y) \approx \sum_j u_j N_j(x,y)$$

Edge functions



$$\vec{w}_e^t(x,y) = N_i \nabla N_j - N_j \nabla N_i$$

$$\vec{w}_j^\ell(x,y) = {}^{N_j}/_{\ell_z}\, \vec{e}_z$$

- Only tangential continuity
- Appropriate for **magnetic vector potential**

$$\vec{A}(x,y) \approx \sum_e \widehat{a}_e^t\, \vec{w}_e^t(x,y) + \sum_j \widehat{a}_j^\ell\, \vec{w}_j^\ell(x,y)$$

Second Ingredient:?

Wishlist					
Sparse matrices					
Closed form					
C^0-continuity					
Easy BC incorporation					
$w(z) \equiv 1$					

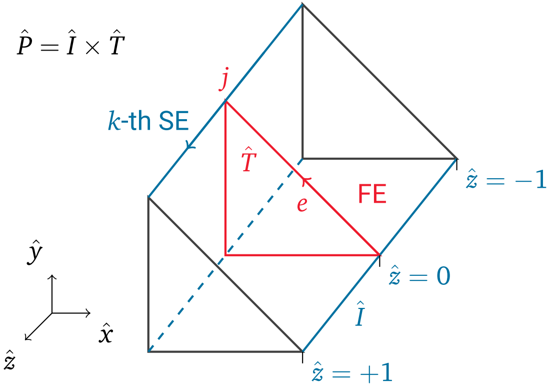
Second Ingredient: Modified Lobatto Polynomials!

Wishlist	Lagrange	Legendre	Chebyshev	Lobatto	Modified Lobatto
Sparse matrices	✗	✓	✓	✓	✓
Closed form	✓	✓	✓	✓	✓
C^0-continuity	✓	✗	✗	✗	✓
Easy BC incorporation	✓	✗	✗	✗	✓
$w(z) \equiv 1$	✓	✓	✗	✗	✓

Modified Lobatto: $$\phi_q(z) = \begin{cases} \frac{1-z}{2}, & q = 1 & \leftarrow \text{Boundary mode} \\ \frac{1-z^2}{4}\mathrm{LO}_{q-2}(z), & q = 2, \dots, N & \leftarrow \text{Interior modes} \\ \frac{1+z}{2}, & q = N+1 & \leftarrow \text{Boundary mode} \end{cases}$$

Quasi-3D Method: Ansatz

Discretization with prism elements:



Temperature:

$$T(x,y,z) \approx \sum_{jq} \tilde{u}_{jq}^{k} N_j(x,y)\phi_q^k(z) \quad \forall k$$

Magnetic vector potential:

$$\vec{A}(x,y,z) \approx \sum_{ejqw} \begin{bmatrix} \tilde{a}_{eq}^{t,k} \, \vec{w}_e^{t}(x,y)\phi_q^k(z) \\ \tilde{a}_{jw}^{\ell,k} \, \vec{w}_j^{\ell}(x,y)\phi_w^k(z) \end{bmatrix} \quad \forall k$$

Quasi-3D Method: Systems of Equations

Thermal system: $$\mathbf{K}_{\lambda}^{\text{Q3D}}\,\widetilde{\mathbf{u}} + \mathbf{M}_{C_\text{V}}^{\text{Q3D}}\,\partial_t\widetilde{\mathbf{u}} = \mathbf{q}_{\text{losses}}^{\text{Q3D}}$$

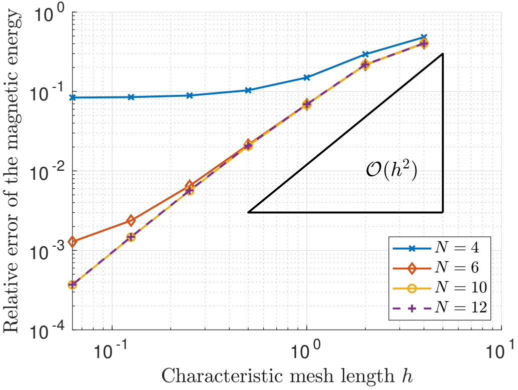
Magnetic system: $$\mathbf{K}_{\nu}^{\text{Q3D}}\,\widetilde{\mathbf{a}} + \mathbf{K}_{\nu\tau}^{\text{Q3D}}\,\partial_t\widetilde{\mathbf{a}} + \mathbf{M}_{\sigma}^{\text{Q3D}}\partial_t\widetilde{\mathbf{a}} = \widetilde{\mathbf{j}}^{\text{Q3D}}$$

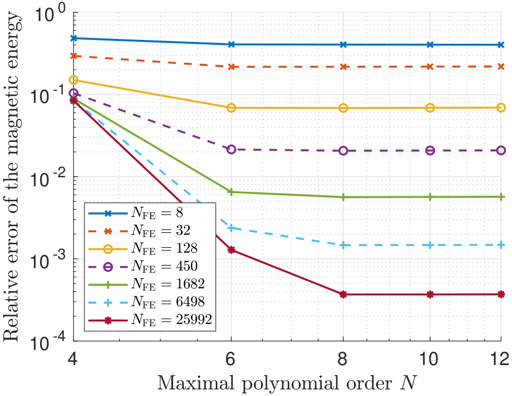
$$\widetilde{\mathbf{a}} = \begin{bmatrix} \widetilde{\mathbf{a}}^{t} \\ \widetilde{\mathbf{a}}^{\ell} \end{bmatrix}, \qquad \mathbf{K}_{\nu}^{\text{Q3D}} = \begin{bmatrix} \mathbf{K}^{\text{SE}} \otimes \mathbf{M}_{\nu}^{\text{FE},t} + \mathbf{M}^{\text{SE}} \otimes \mathbf{K}_{\nu}^{\text{FE},t} & -\mathbf{D}^{\text{SE}} \otimes \mathbf{C}_{\nu}^{\text{FE},t\ell} \\ \left(-\mathbf{D}^{\text{SE}} \otimes \mathbf{C}_{\nu}^{\text{FE},t\ell}\right)^{\top} & \mathbf{M}^{\text{SE}} \otimes \mathbf{K}_{\nu}^{\text{FE},\ell} \end{bmatrix}.$$

Kronecker tensor product: $$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}, \qquad \mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & a_{13}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & a_{23}\mathbf{B} \end{bmatrix}$$

Convergence Study (h- and p-refinement)

TECHNISCHE UNIVERSITÄT DARMSTADT



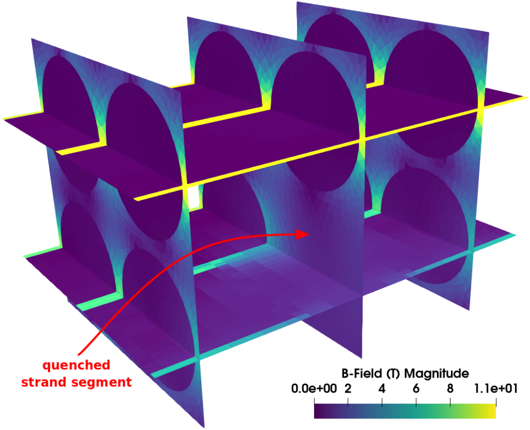


Application to Quench Simulation

TECHNISCHE UNIVERSITÄT DARMSTADT

	3D GetDP	Q3D solver
#FE	1,863,431	330
#SE	–	5
Polynomial order	–	6
Magnetic #DoF	1,741,669	21,979
Thermal #DoF	80,735	806
Computation time	1 h	5 min

↪ For a transient magneto-thermal nonlinear simulation of a superconducting wire



Conclusion

- Superconducting accelerator magnets = multi-scale problems
- Quasi-3D method: hybrid FE-SE ansatz functions
- Modified Lobatto polynomials = ideal for quasi-3D method
- Quasi-3D method implemented, verified and applied to quench simulation

What I did not talk about:

- Modeling quench phenomena $\rightarrow$ quench state model, coupling currents, ...
- External circuits $\rightarrow$ quasi-3D field-circuit coupling
- End winding models $\rightarrow$ boundary conditions & field-circuit coupling of end windings
- Nonlinear materials $\rightarrow$ efficient nonlinear procedure using tensor contraction

Reference

TECHNISCHE UNIVERSITÄT DARMSTADT

Publications regarding this work:

- L. A. M. D'Angelo, Y. Späck-Leigsnering and H. De Gersem: "Quasi-3D Magneto-Thermal Quench Simulation Scheme for Superconducting Accelerator Magnets". In: *IEEE Transactions on Applied Superconductivity*, vol. 32, no. 6, 2022.
- L. A. M. D'Angelo and H. De Gersem: "Quasi-3D Magnetic Field Simulation of Superconducting Devices with Translational Symmetry". In: *IET Science, Measurement & Technology*, vol. 15, pp. 319-327, 2021.
- J. Bundschuh, L. A. M. D'Angelo and H. De Gersem: "Quasi 3-D Spectral Wavelet Method for a Thermal Quench Simulation". In: *Journal of Mathematics in Industry*, vol. 11, no. 17, 2021.
- L. A. M. D'Angelo, J. Christ and H. De Gersem: "Quasi-3D Discretization of Thermal Hot-Spot Propagation in Superconducting Models". In: *IEEE Transactions on Applied Superconductivity*, vol. 30, no. 4, 2020.