

Letter: non perturbative Sudakov

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PB evolution equation

- Evolution equation for parton density (not momentum weighted)

$$f_a(x, \mu^2) = \Delta_a^S(\mu^2) f_a(x, \mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a^S(\mu^2)}{\Delta_a^S(\mu'^2)} \int_x^{z_M} \frac{dz}{z} P_{ab}(\alpha_s(\mu'^2), z) f_b\left(\frac{x}{z}, \mu'^2\right)$$

- Sudakov form factor

$$\Delta_a^S(\mu^2, \mu_0^2) = \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz P_{ab}(\alpha_s(\mu'^2), z) \right)$$

- For transverse momentum dependent pdfs:

$$\mathcal{A}_a(x, \mathbf{k}, \mu^2) = \Delta_a^S(\mu^2) \mathcal{A}_a(x, \mathbf{k}, \mu_0^2) + \int \frac{d^2 \mathbf{q}'}{\pi \mathbf{q}'^2} \frac{\Delta_a^S(\mu^2)}{\Delta_a^S(\mathbf{q}'^2)} \Theta(\mu^2 - \mathbf{q}'^2) \Theta(\mathbf{q}'^2 - \mu_0^2) \times \left[\int_x^{z_M} \frac{dz}{z} P_{ab}(\alpha_s(\mathbf{q}'^2), z) \mathcal{A}_a\left(\frac{x}{z}, \mathbf{k} + (1-z)\mathbf{q}', \mathbf{q}'^2\right) \right]$$

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- Splitting function

$$P_{ab}(\alpha_s, z) = d_a(\alpha_s)\delta(1-z) + k_a(\alpha_s)\frac{1}{1-z} + R_{ab}(\alpha_s, z)$$

- Sudakov, with virtual term

$$\Delta_a^S(\mu^2, \mu_0^2) = \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[\int_0^{z_M} dz \frac{k_q}{1-z} - d_q \right] \right)$$

- split this into different pieces

$$\begin{aligned} \Delta_a^S(\mu^2, \mu_0^2) &= \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[\int_0^{z_{dyn}} dz \frac{k_q}{1-z} - d_q \right] \right) \\ &\times \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_{z_{dyn}}^{z_M} dz \frac{k_q}{1-z} \right) \end{aligned}$$

The $q_T \geq q_0$ region

- “perturbative region”

$$\Delta_a^S(\mu^2, \mu_0^2) = \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[\int_0^{z_{dyn}} dz \frac{k_q}{1-z} - d_q \right] \right).$$

- leads to CSS Sudakov structure

$$\begin{aligned} \log \Delta_a^S(\mu^2, \mu_0^2) &= - \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[\int_0^{z_{dyn}} dz \frac{k_a}{1-z} - d_a \right] \\ &= - \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[\frac{1}{2} k_a \log \left(\frac{\mu'^2}{q_0^2} \right) - d_a \right] \\ &= - \frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[\log \left(\frac{q_0^2}{\mu'^2} \right) A + B \right] \end{aligned}$$

The $q_T \leq q_0$ region

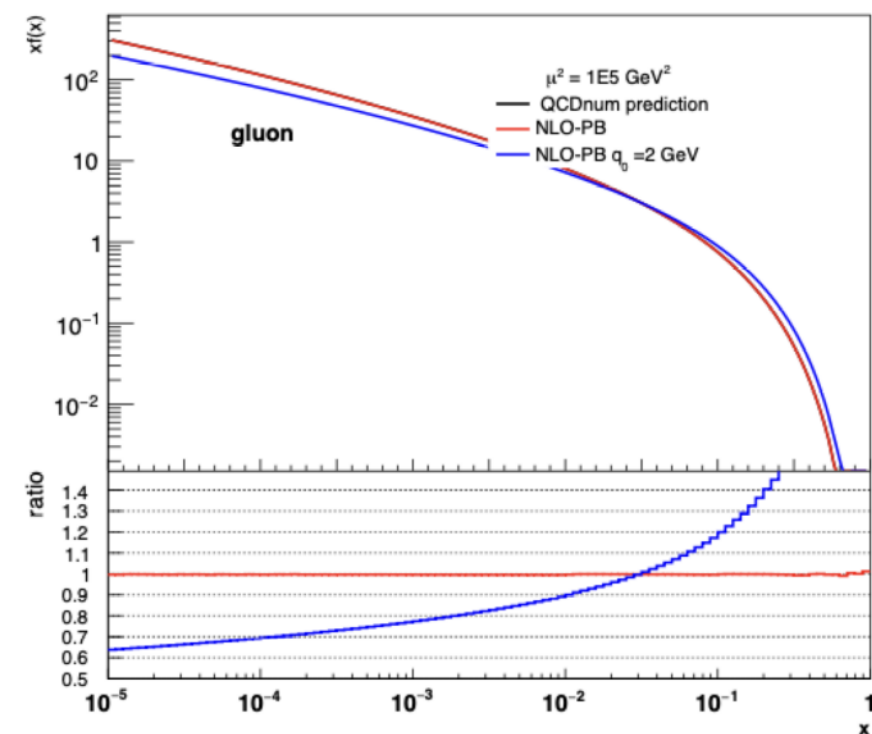
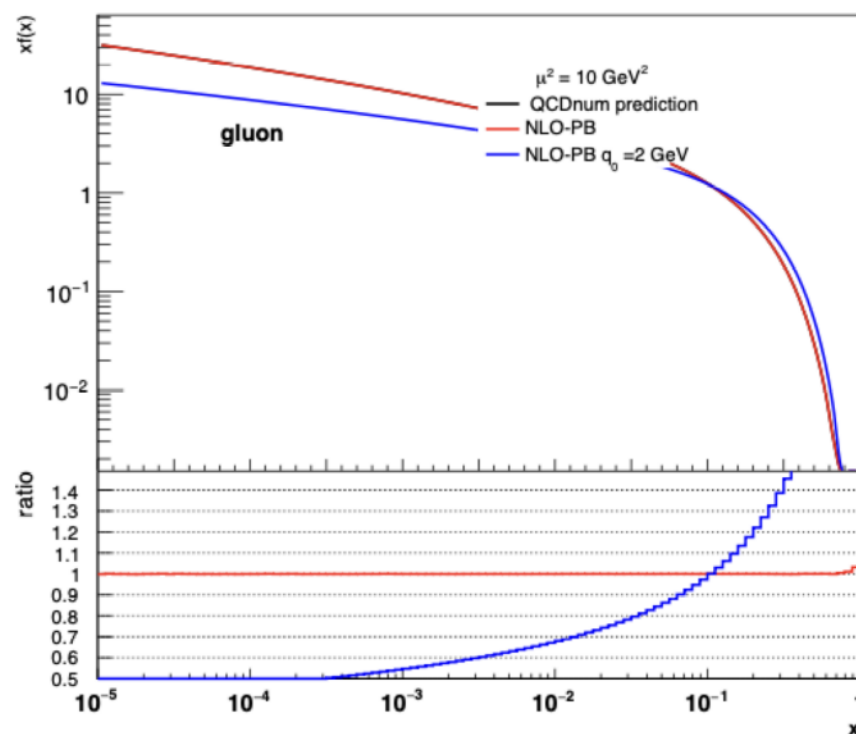
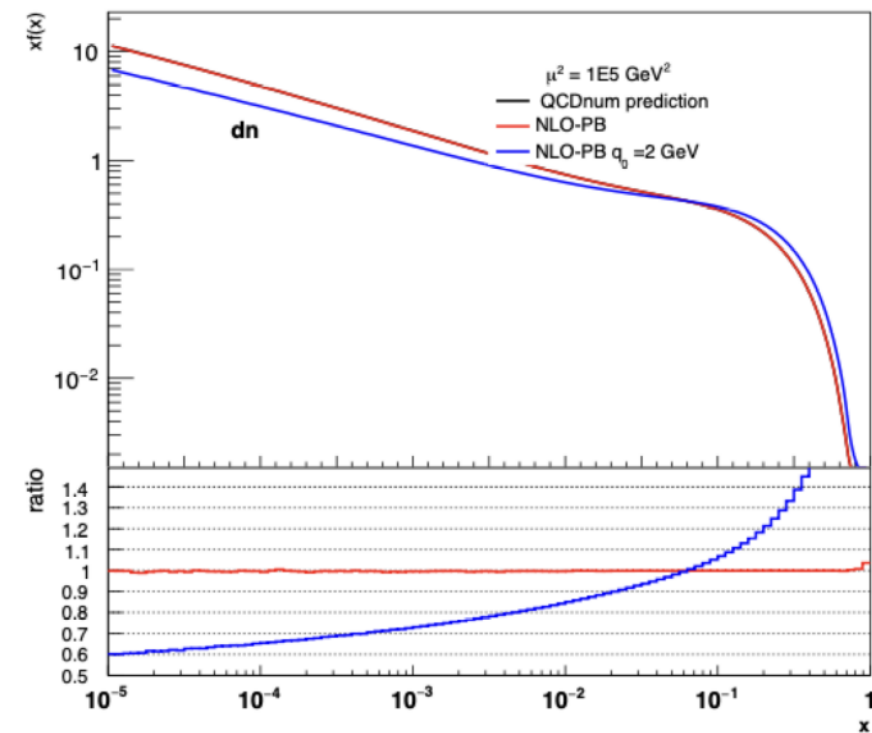
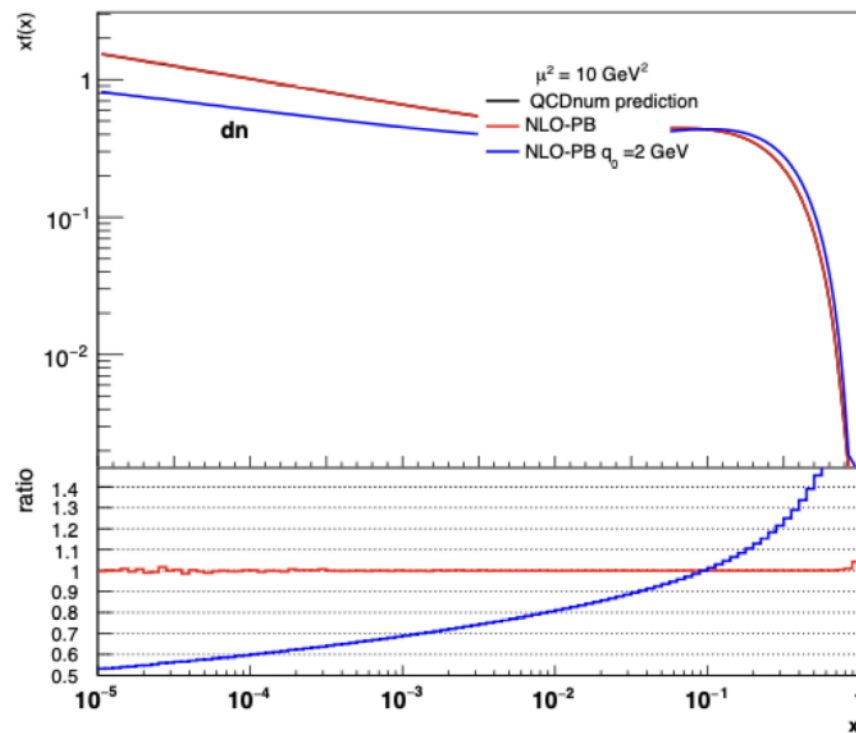
- “non-perturbative” region (very large z)

$$\Delta_a^{non-pert.}(\mu, \mu_0) = \exp \left(- \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_{z_{dyn}}^{z_M} dz \frac{k_q}{1-z} \right)$$

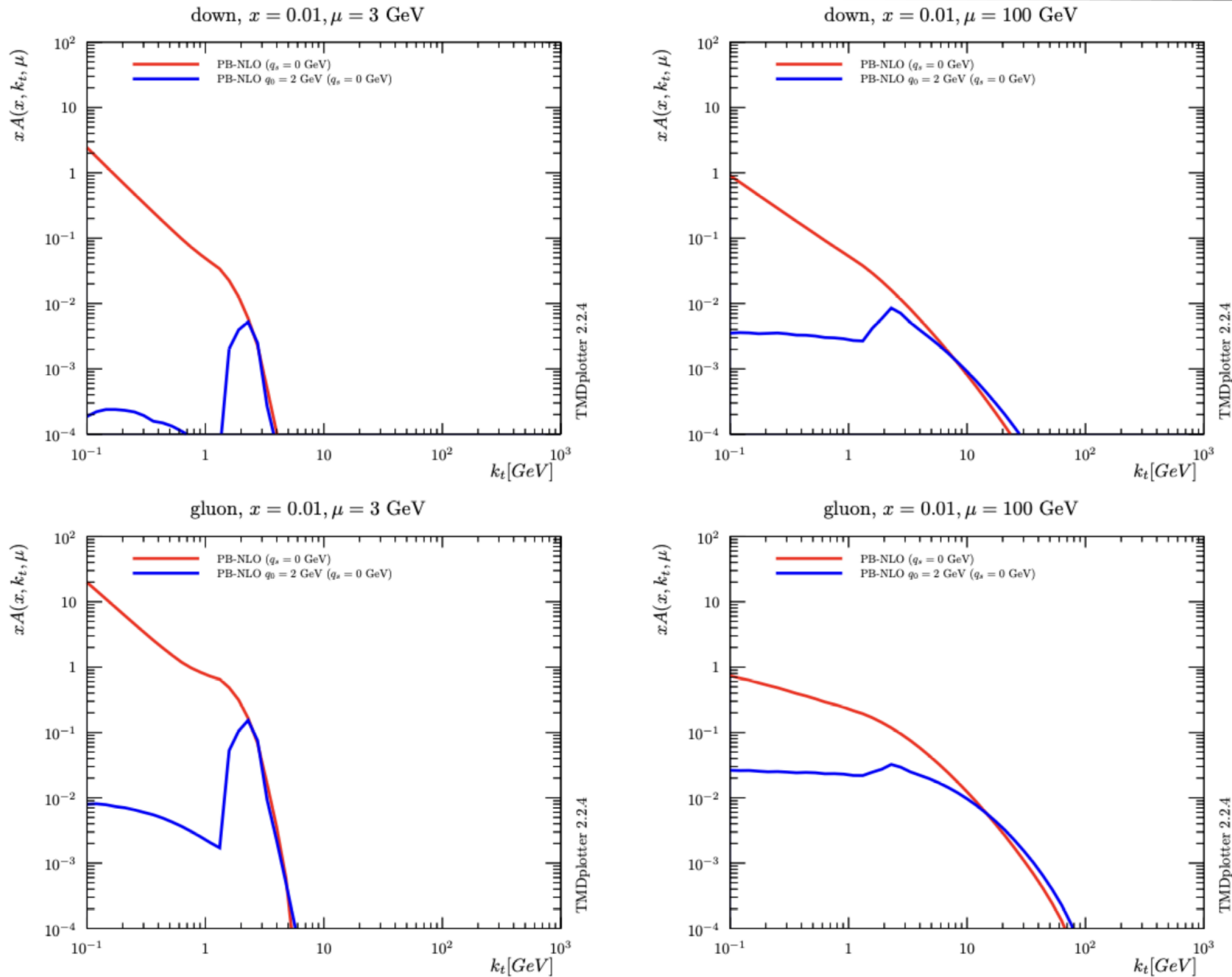
$$\begin{aligned} \log \Delta_a^{non-pert}(\mu^2, \mu_0^2) &= - \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{1}{2} k_q \log \left(\frac{q_0^2}{\mu'^2 \epsilon^2} \right) \\ &= - \log \left(\frac{\mu^2}{\mu_0^2} \right) g_1(\alpha_s, q_0) \end{aligned}$$

- gives a structure as CSS “non-pert” Sudakov

Numerical results: integrated distributions



Numerical results: TMD distributions



- Effect of z_{dyn} clearly visible

Conclusion

- Introduction of z_{dyn} allows to recover CSS structure in PB Sudakov Form Factor
- z_{dyn} makes an important effect on inclusive and TMD distributions
- Integral $z > z_{\text{dyn}}$ is calculable, although non-perturbative, gives the same structure as CSS NP factor
 - comes automatically from PB

AOB

- Further news ?