Multiple interactions: a theory perspective

M. Diehl

Deutsches Elektronen-Synchroton DESY

LHC Physics discussion
Hamburg, 21 February 2011

Multiple interactions

- phenomenology based on simple, physically intuitive formula

$$\text{cross section} = \text{multiparton distributions}$$
$$\times \text{ hard-scattering cross sections}$$

Paver, Treleani 1982, 1984; Mekhfi 1985

  also underlies implementation in event generators

- questions:
  - to which extent can this formula be derived in QCD?
  - where and how does it need to be modified?
  - can factorization theorems for multiparton interactions be formulated and proven?

- no definitive answers to all points, but some results and identified problems

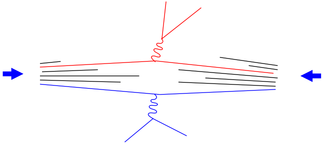
  MD, Schäfer arXiv:1102:3081

  MD, Ostermeier, Schäfer, in preparation

- ultimate goal: improved theory as a guide for phenomenology

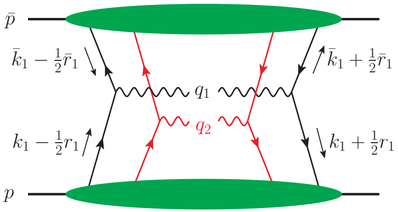
Theoretical framework

- require all interactions to have hard scale
  $\rightsquigarrow$ predictive power from factorization and pert. theory
- consider gauge boson pair production (pairs of γ^*, W, Z)



  - jet production $\gg$ complicated, gluon exchange between spectator partons and produced jets — Mulders, Rogers 2010
- keep transverse momentum of bosons differential
  - are interested in final-state distributions
  - allows discussion of Sudakov logarithms
    hope that will eventually be useful for event generators
  - need k_T dependent parton distributions
    Collins, Soper 1982; Ji, Ma, Yuan 2004; Collins, Rogers, Stasto 2007

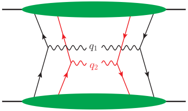
Basic results: space-time structure



- longitudinal parton momenta $x_i p$, $\bar{x}_i \bar{p}$ fixed by final state
  exactly as for single hard scattering
- transverse parton momenta not the same
  in amplitude and conjugate amplitude
- Fourier transform to impact parameter: $\boldsymbol{r}_1 \to \boldsymbol{y}$ and $\bar{\boldsymbol{r}}_1 \to \bar{\boldsymbol{y}}$
  $\boldsymbol{r}_1 + \bar{\boldsymbol{r}}_1 = \boldsymbol{0}$ (from momentum conservation) implies $\boldsymbol{y} = \bar{\boldsymbol{y}}$
- interpretation: $\boldsymbol{y}$ = transv. dist. between two scattering partons
  equal in both colliding protons

same procedure for partons with index 2

Basic structure: cross section



- get cross section formula

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \left[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i\bar{x}_i s\big)\right]$$

$$\times \left[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i)\right] \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

$\hat{\sigma}_i =$ parton-level cross section

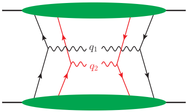
$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = k_T$ dependent two-parton distribution

- result follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2$ in cross sect. $\to$ collinear distributions

$$F(x_i, \boldsymbol{y}) = \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$$

recover usual cross section formula (slide 2)

Multiparton distributions



- naive interpretation of $F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$:
  have two partons with transv. momenta $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ at a transv. distance $\boldsymbol{y}$
- cannot be literally true because of uncertainty principle, but
  - in cross section variables enter as if naive interpretation were ok:

$$\frac{d\sigma}{\prod_{i=1}^{2} dx_i \, d\bar{x}_i \, d^2\boldsymbol{q}_i} = \Bigg[\prod_{i=1}^{2} \hat{\sigma}_i \int d^2\boldsymbol{k}_i \, d^2\bar{\boldsymbol{k}}_i \, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i) \Bigg] \int d^2\boldsymbol{y} \, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) \, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

  - formally: $F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$ is a Wigner distribution
- can define as operator matrix element

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2 \, q\big(\tfrac{1}{2} z_2\big) \, \bar{q}\big(y - \tfrac{1}{2} z_1\big) \Gamma_1 \, q\big(y + \tfrac{1}{2} z_1\big) | p \rangle$$

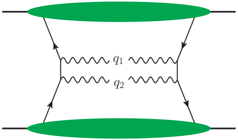
like for single-parton densities
$\rightsquigarrow$ rigorous basis to study scale evolution
possibility for lattice calculations

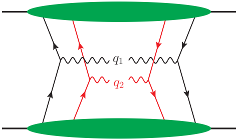
Power behavior: single versus double hard scattering

- from scattering formulae readily find

$$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\,d^2\boldsymbol{q}_1\;dx_2\,d\bar{x}_2\,d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2\Lambda^2}$$

$Q^2 \sim q_i^2,\ \Lambda^2 \sim \mathrm{GeV}^2$

for both 

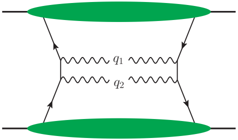
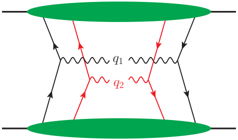
and 

$\Rightarrow$ double scattering not power suppressed

Power behavior: single versus double hard scattering

- from scattering formulae readily find

$$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\,d^2\boldsymbol{q}_1\;dx_2\,d\bar{x}_2\,d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2\Lambda^2} \qquad Q^2 \sim q_i^2,\ \Lambda^2 \sim \text{GeV}^2$$

for both  and 

$\Rightarrow$ double scattering not power suppressed

- but if integrate over $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ then

single: $$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\;dx_2\,d\bar{x}_2} \sim 1$$ since $\int d^2(\boldsymbol{q}_1 + \boldsymbol{q}_2) \sim \Lambda^2$ and $\int d^2(\boldsymbol{q}_1 - \boldsymbol{q}_2) \sim Q^2$

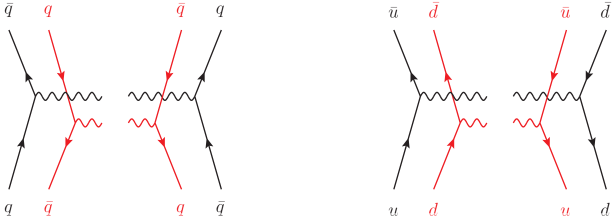
double: $$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\;dx_2\,d\bar{x}_2} \sim \frac{\Lambda^2}{Q^2}$$ since $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2 \sim \Lambda^4$

i.e. single hard scattering has larger phase space for transv. momenta previous formulae glossed over important details:

- quark flavors and quarks vs. antiquarks
- parton spin
- color
- scale dependence/evolution
- additional gluon exchange
  essential for factorization
  situation similar to single hard scattering, not discussed here

Interference effects

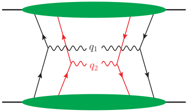
- so far: distributions with operators $\bar{q}_2\, q_2\; \bar{q}_1\, q_1 \;\rightsquigarrow\;$ double parton densities
  indices 1 and 2 refer to momentum fractions x_1, x_2
- but also have interference contributions (no probability interpretation)



operator: $\bar{q}_2\, \bar{q}_2\; q_1\, q_1$ operator: $\bar{u}_2\, d_2\; \bar{d}_1\, u_1$

- must be included in cross section formula
  but is discarded in existing estimates
- expect to decrease for small x_1, x_2, since does not mix with gluons

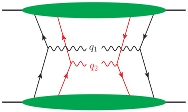
Spin structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}(-\tfrac{1}{2} z_2) \Gamma_2\, q(\tfrac{1}{2} z_2)\, \bar{q}(y - \tfrac{1}{2} z_1) \Gamma_1\, q(y + \tfrac{1}{2} z_1) | p \rangle$$

- at leading twist: $\Gamma_i = \frac{1}{2}\gamma^+,\ \frac{1}{2}\gamma^+\gamma_5,\ \frac{1}{2} i\sigma^{+\alpha}\gamma_5$
  $\Leftrightarrow$ unpolarized, long. polarized, transv. polarized quarks
  similar classification for gluons
- spin correlations even in unpolarized target, e.g.
  $$\Gamma_1 = \Gamma_2 = \tfrac{1}{2}\gamma^+\gamma_5 \quad \Leftrightarrow \quad q_1^{\uparrow} q_2^{\uparrow} + q_1^{\downarrow} q_2^{\downarrow} - q_1^{\uparrow} q_2^{\downarrow} - q_1^{\downarrow} q_2^{\uparrow}$$
  not suppressed by hard scattering in two-boson prod'n
- transverse spin correlations from $\Gamma_1 = \Gamma_2 = \frac{1}{2} i\sigma^{+\alpha}\gamma_5$
  $\rightsquigarrow$ $\cos 2\phi$ modulation between decay planes of the two bosons
  at low $\boldsymbol{q}_1, \boldsymbol{q}_2$ single hard scattering does not give $\cos 2\phi$ term
  in general: correlated decay planes

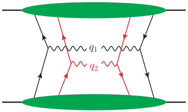
Spin structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}(-\tfrac{1}{2} z_2) \Gamma_2 \, q(\tfrac{1}{2} z_2) \, \bar{q}(y - \tfrac{1}{2} z_1) \Gamma_1 \, q(y + \tfrac{1}{2} z_1) | p \rangle$$

- at leading twist: $\Gamma_i = \frac{1}{2}\gamma^+, \; \frac{1}{2}\gamma^+\gamma_5, \; \frac{1}{2}i\sigma^{+\alpha}\gamma_5$
  $\Leftrightarrow$ unpolarized, long. polarized, transv. polarized quarks
  similar classification for gluons
- spin correlations even in unpolarized target, e.g.
  $$\Gamma_1 = \Gamma_2 = \tfrac{1}{2}\gamma^+\gamma_5 \quad \Leftrightarrow \quad q_1^\uparrow q_2^\uparrow + q_1^\downarrow q_2^\downarrow - q_1^\uparrow q_2^\downarrow - q_1^\downarrow q_2^\uparrow$$
  not suppressed by hard scattering in two-boson prod'n
- could naively expect spin effects to decrease for small x_1, x_2, but
  - will see counter-example on slide 16
  - for $x_1 \sim x_2 \ll 1$ spin correlations may be
    weak a parton and the proton (far away in rapidity)
    important between two partons (close in rapidity)

Color structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}(-\tfrac{1}{2} z_2) \Gamma_2\, q(\tfrac{1}{2} z_2)\; \bar{q}(y - \tfrac{1}{2} z_1) \Gamma_1\, q(y + \tfrac{1}{2} z_1) | p \rangle$$

- operators $\bar{q}_2\, q_2$ and $\bar{q}_1\, q_1$ can couple to color singlet or octet:

$$^1F \longrightarrow (\bar{q}_2 \mathbb{1} q_2)\, (\bar{q}_1 \mathbb{1} q_1)$$
$$^8F \longrightarrow (\bar{q}_2 t^a q_2)\, (\bar{q}_1 t^a q_1) = \tfrac{1}{2} (\bar{q}_2 \mathbb{1} q_1)\, (\bar{q}_1 \mathbb{1} q_2) - \tfrac{1}{2N_c}\, (\bar{q}_2 \mathbb{1} q_2)\, (\bar{q}_1 \mathbb{1} q_1)$$

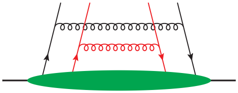
- in gauge boson pair production contrib's from $^1F\,^1F$ and $^8F\,^8F$
- color octet distributions essentially unknown
  (no probability interpretation as a guide)

Situation so far

- can extend basic cross section formula to include contributions from
  - spin correlations
    direct influence on rates and distributions
    may not all be small at low x_i
  - color correlations
  - interference distributions (quark-antiquark and flavor)
    expect to be small at low x_i
- many distributions needed to calculate multiparton scattering
  some of them qualitatively unknown

High q_T: more predictive power

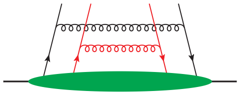
- consider region $\Lambda \ll q_T \ll Q$, with $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- k_T dependent distr'n $=$ hard scattering $\otimes$ collinear distr'n
  hard scattering closely related to DGLAP splitting functions
- ladder graphs: independent hard scatters for pair 1 and 2



- $|\boldsymbol{y}|$ of hadronic size
- color factors favor singlet distr's compared to octet ones

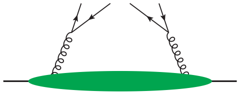
High q_T: more predictive power

- consider region $\Lambda \ll q_T \ll Q$, with $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- k_T dependent distr'n $=$ hard scattering $\otimes$ collinear distr'n
  hard scattering closely related to DGLAP splitting functions
- ladder graphs: independent hard scatters for pair 1 and 2



- $|\boldsymbol{y}|$ of hadronic size
- color factors favor singlet distr's compared to octet ones

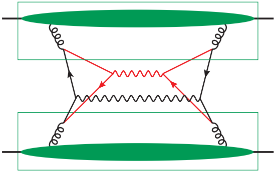
- splitting graphs



- perturbatively small $|\boldsymbol{y}|$
- longitudinal q and $\bar{q}$ spins fully correlated also large transv. spin correlation

- ladder graphs power suppressed by Λ^2/q_T^2 compared with splitting but have small-x enhancement

Problems with the splitting graphs



- contribution from splitting graphs in cross section gives divergent integrals $\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, \bar{F}(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$
- double counting problem between double scattering with splitting and single scattering at loop level
- possible solution:
  subtract splitting contribution from two-parton distr's when $\boldsymbol{y}$ is small
  will also modify their scale evolution remains to be worked out

Summary

- multiple hard interactions not power suppressed
  for cross section differential in $|\boldsymbol{q}_i| \ll Q$
- nontrivial spin and color structure
  interference in fermion number and quark flavor
  size of these effects presently unknown
- some simplification for transv. mom. $|\boldsymbol{q}_i| \gg \Lambda$
  - collinear distributions as input
  - enhancement of color singlet combinations

  further studies needed to make this quantitative
- need multi-parton distr's depending on transverse distance
  between partons
- should remove small $\boldsymbol{y}$ contribution in order to avoid
  divergences and double counting