How fast does the bubble grow

Pressure on the bubble wall in the relativistic regime

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May 2023

VRIJE UNIVERSITEIT BRUSSEL

Tentative outline

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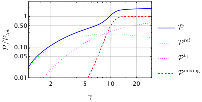

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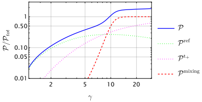
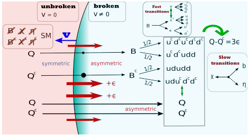


- EWBG from ultra-relativistic walls



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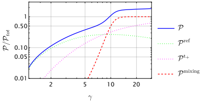

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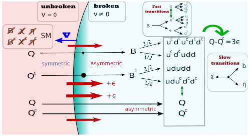

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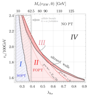

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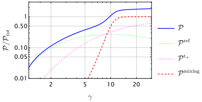

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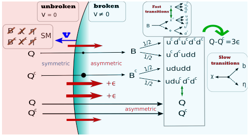

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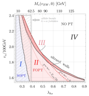

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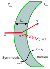

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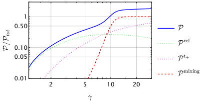

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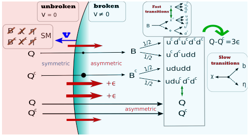

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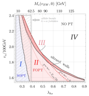

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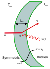

Phase transitions in the early universe

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FOPT and bubbles

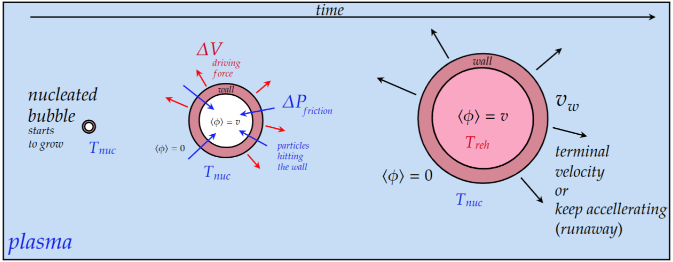

Figure: Credit: Giulio Barni, thanks to him

$$\Delta V = \underbrace{\Delta \mathcal{P}(\gamma = \gamma^{MAX})}_{??} \quad \text{(velocity)}$$

ultra-relativistic limit:

$$v_w \to c, \qquad \gamma_{wp} \equiv \frac{1}{\sqrt{1 - v_w^2}}$$

FOPT: Why do we even bother?

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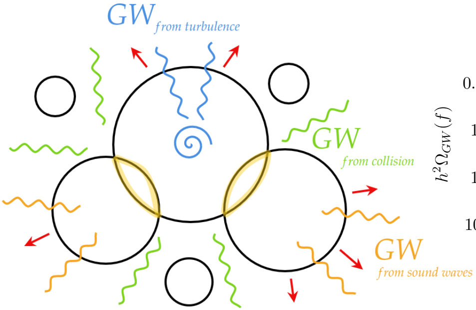
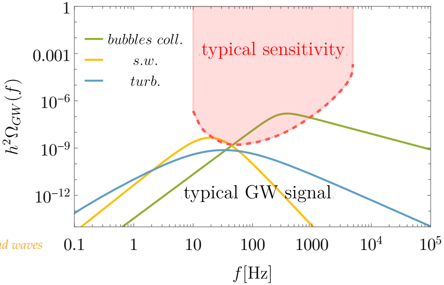
1. Bubbles can produce a stochastic GW background from
   - bubble collision
   - sound waves
   - turbulence





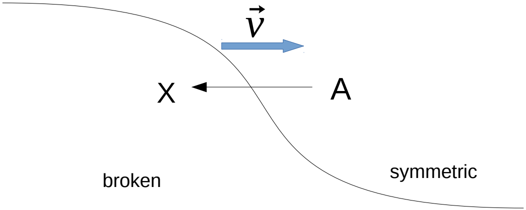
Primordial GWs could be observed soon (if they exist and/or if we will be able)!

Velocity

Final velocity $\gamma^{MAX} = \frac{1}{\sqrt{1-v_{MAX}^2}}$ of the wall set by

$$\Delta V = \Delta\mathcal{P}(\gamma^{MAX}) \quad \Rightarrow \quad \text{determination } \gamma^{MAX}$$

- ΔV independent of the velocity of the wall



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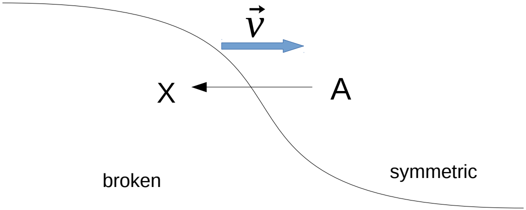

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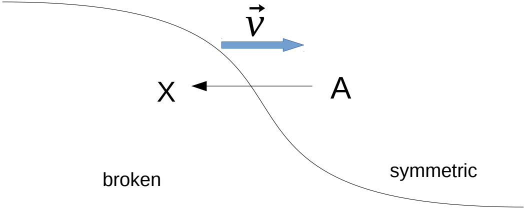
- Generic method: solve the full coupled system of Boltzmann equations

$$p^\mu \partial_\mu f_i + \frac{1}{2}\partial_z m_i[\phi]\partial_{p_z} f_i = \mathcal{C}[f_i, \phi]$$

$$\Box\phi + \frac{dV}{d\phi} + \sum_i \frac{dm_i^2[\phi]}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_i} f_i = 0$$



How to solve that ? Several simplification regime

- Expansion in perturbations (Original approach by Prokopec-Moore arXiv:hep-ph/9503296).

$$f_i = f_i^{\rm eq} + \delta f_i, \qquad f_i^{\rm eq} \gg \delta f_i$$

Solve order by order. Valid for *slow* walls!

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- Assume *Local thermal equilibrium* (Mancha-Prokopec-Swiezewska arXiv:2005.10875)

$$\text{LARGE } \mathcal{C}(g) \quad \Rightarrow \quad \Gamma_{scat} \gg \gamma v / L_w \quad \Rightarrow \quad \boxed{f_i \to f_i^{\text{eq}}}$$

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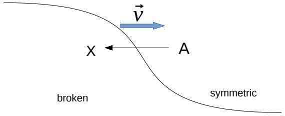
- Ballistic regime $\mathcal{C} \to 0$:



$$\text{SMALL } \mathcal{C} \quad \Rightarrow \quad \Gamma_{scat}(g) \ll \gamma v / L_w \quad \Rightarrow \quad f_i^{\rm eq} \ll \delta f_i$$

$$\boxed{\mathcal{P} = \int \frac{p_z d^3 p}{p_0 (2\pi)^3} f_A(p) \times \sum_X \int dP_{A\to X} (p_A^z - p_X^z)}$$

How monotonic is the pressure increase ?

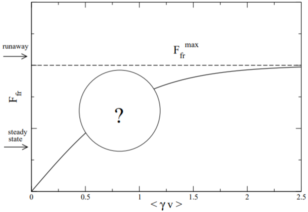


Figure: Espinosa et al arXiv:1004.4187

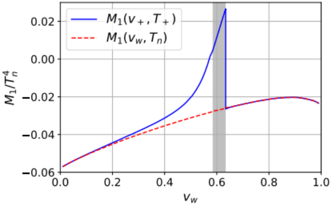
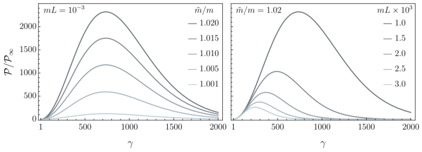


Figure: Cline et al. arXiv:2102.12490



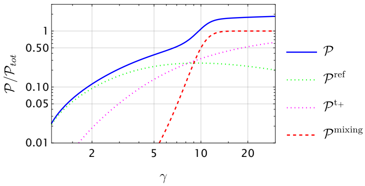
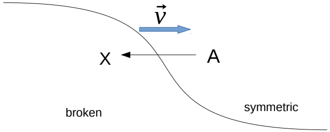
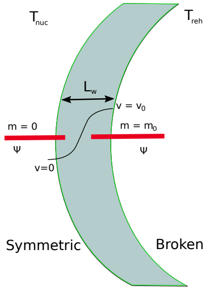


Figure: Garcia, Koszegi and Petrossian arXiv:2212.10572

Pressure from 1 to 1 Bodeker-Moore [0903.4099]

- $\mathcal{P} = \underbrace{\int \frac{p_z d^3p}{p_0 (2\pi)^3} f_A(p)}_{\propto \gamma_w T^3} \times \sum_X \int \underbrace{dP_{A\to X}}_{T\to 1} \underbrace{(p_A^z - p_X^z)}_{\propto v^2/T\gamma_w}$





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- $1 \to 1, A = X$ with $m_h > m_s$:

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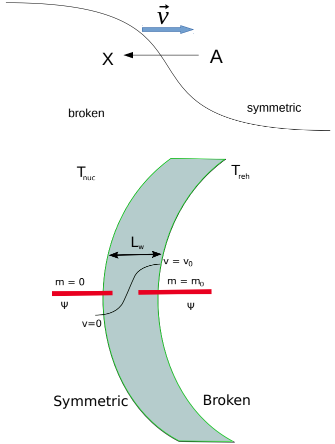

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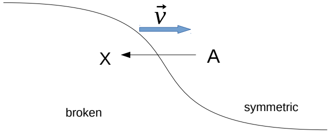
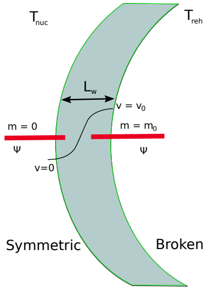
- LO relativistic pressure :

$$\int dP_{A\to A} \to 1, \qquad (p_h^z - p_s^z) \approx -\frac{\Delta m_i^2}{2E}$$

$$\Rightarrow \boxed{\mathcal{P}_{1\to 1} \to \sum_i \frac{\Delta m_i^2 T^2}{24}}, \qquad \Delta m_i^2 \equiv m_{h,i}^2 - m_{s,i}^2$$





Asymptotic regime of pressure Dine et al. Phys. Rev. D 46, 550

- Intermediary regime: reflected, transmitted and back-transmitted species:

$$\mathcal{P} = \mathcal{P}^{r} + \mathcal{P}^{t+} + \mathcal{P}^{t-}$$

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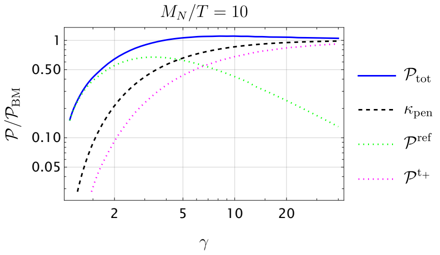

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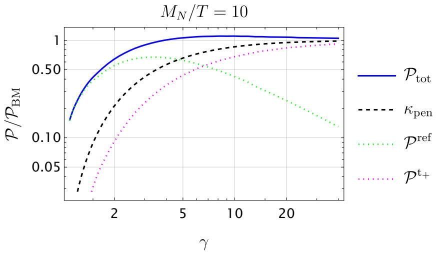

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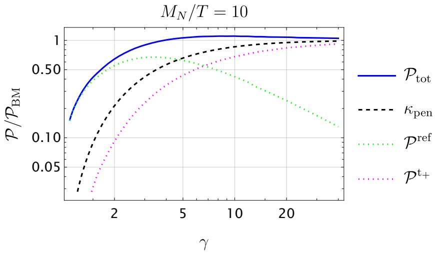

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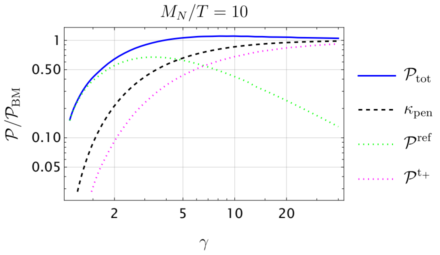

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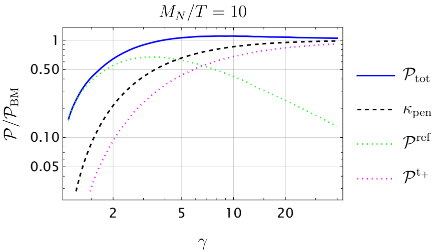
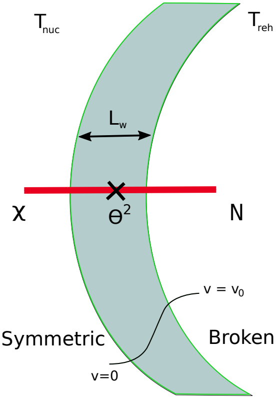
$$\gamma \propto \frac{R}{R_{\text{initial}}} \qquad \Rightarrow \qquad \gamma \to \frac{v}{H} \sim \frac{M_{pl}}{v} ??$$



Production of heavy states via mixing [2010.02590]:Idea

Scale of the transition and particles involved

Can heavy particles with $M \gg v$ be involved in the dynamics of the PT ?



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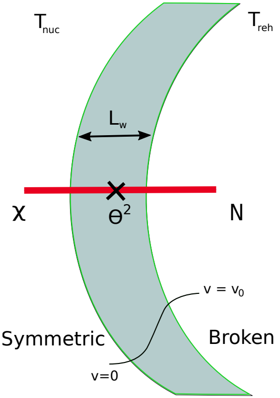

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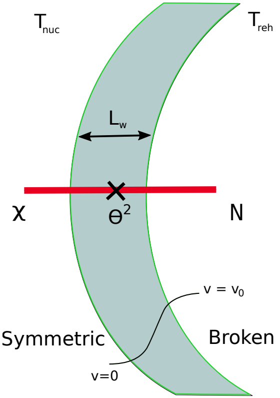

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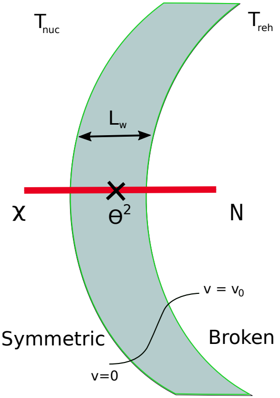

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- When no wall and $\langle\phi\rangle = v_\phi$: $\qquad \chi \to N$ forbidden

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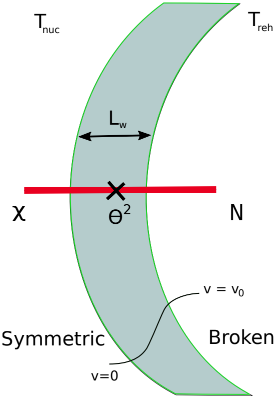

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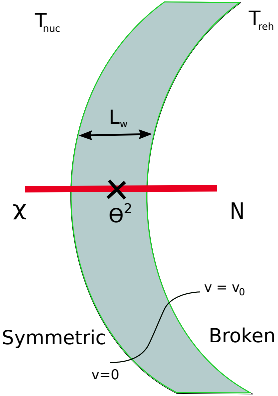
$$\text{No wall:} \int d^4x e^{ip\cdot x} \quad \propto \quad (2\pi)^4\delta^4(p), \quad p = p_N - p_\chi$$

- When no wall and $\langle\phi\rangle = v_\phi$: $\chi \to N$ forbidden
- With wall: $p^z = p^z_N - p^z_\chi$ not conserved: if $E > M$, $\chi \to N$ allowed

$$\int d^3x_\perp e^{ip_\perp\cdot x_\perp} \int \langle\phi\rangle(z)e^{izp_z}dz \propto (2\pi)^3\delta^3(p_\perp)\frac{\sin\Delta p_z L_w}{\Delta p_z L_w}$$

Production of heavy states via mixing [2010.02590]: computation



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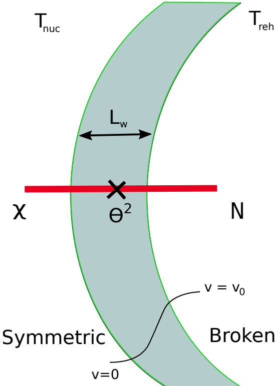

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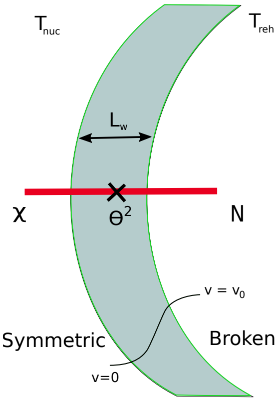

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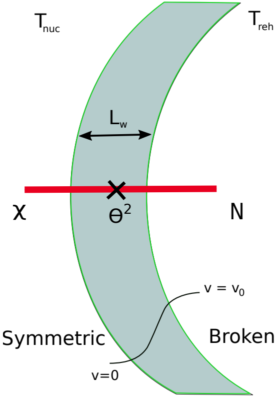

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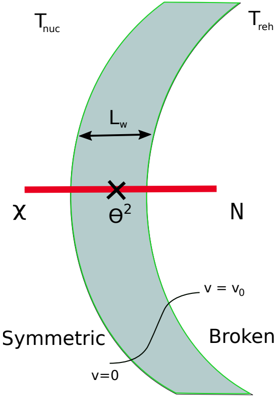

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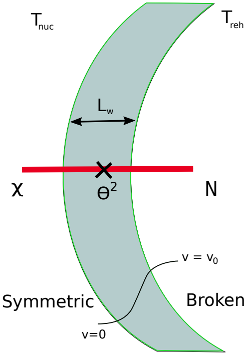
- Cosmological consequences: 1) Pressure on the bubble wall [2010.02590], 2) Non-thermal DM (1 to 2 splittings) [2101.05721] with Wen Yin, 3) Baryogenesis [arXiv:2106.14913] with Wen Yin ...

Pressure by mixing with heavy states Azatov, MV[2010.02590]

- Toy model; $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M_N\bar{N}N, \quad M_N \gg T_{\rm nuc}$



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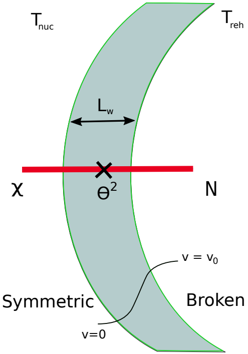

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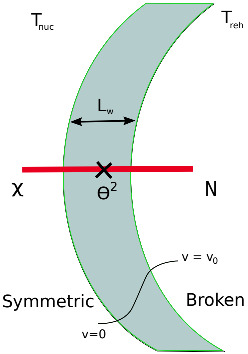

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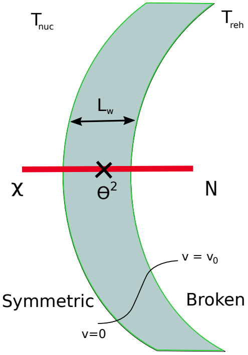

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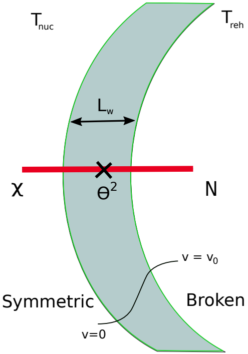

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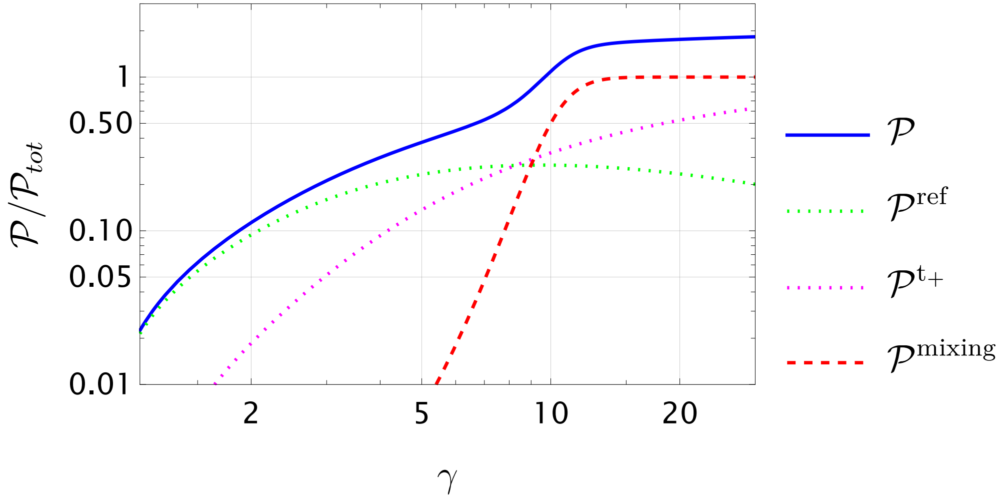
$$\Delta\mathcal{P}_{\text{mixing}} \approx \frac{Y^2T^2v^2}{48}\Theta(\gamma T - M^2 L_w)$$

Pressure depends on M only in the Θ-function

Mixing pressure



Pressure by splitting Bodeker-Moore [1703.08215]

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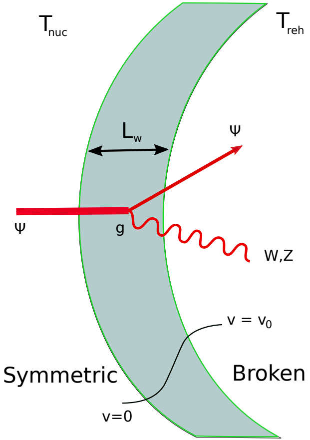

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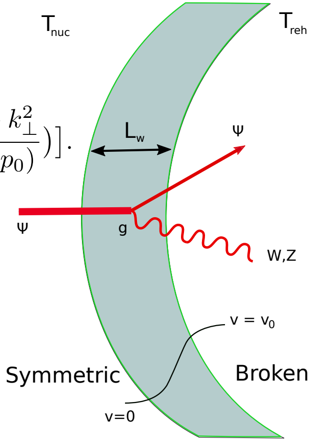
$$\chi_\psi(z)\chi_A^*(z)\chi_\psi^*(z) \sim \exp i\Big[\int_0^z \Big(\frac{m_\psi^2(z)}{2p_0} - \frac{m_V^2(z)+k_\perp^2}{2xp_0} - \frac{m_\psi^2(z)+k_\perp^2}{2(1-x)p_0}\Big)\Big].$$



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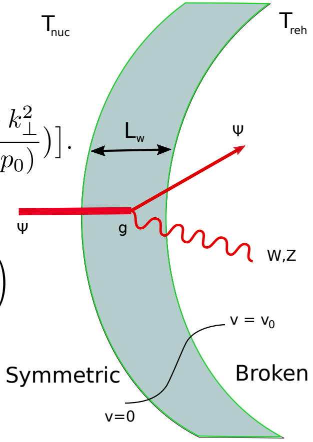

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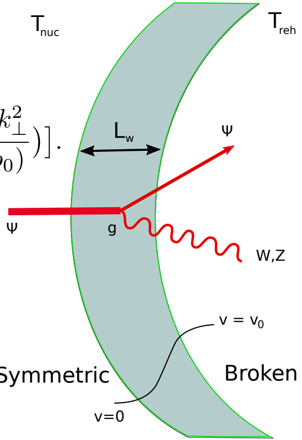

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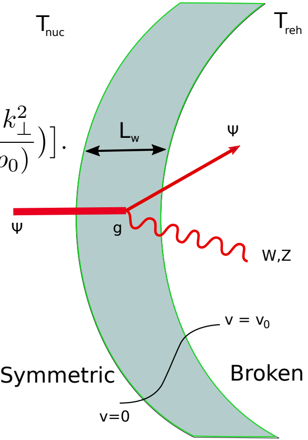

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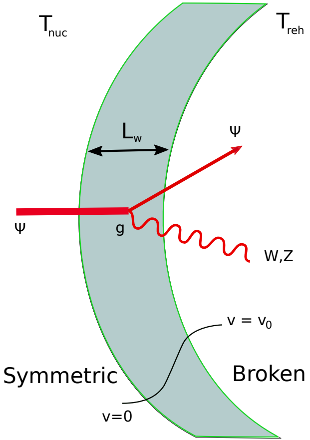

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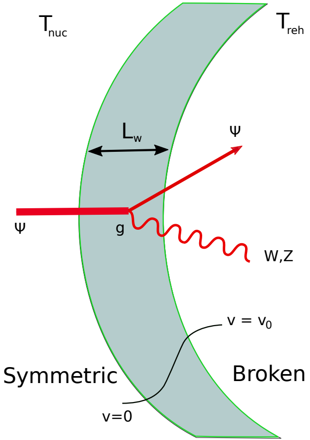

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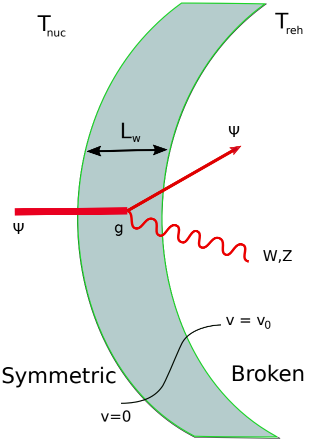

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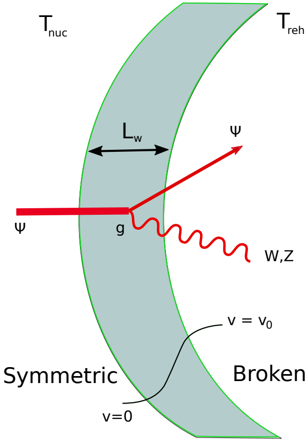

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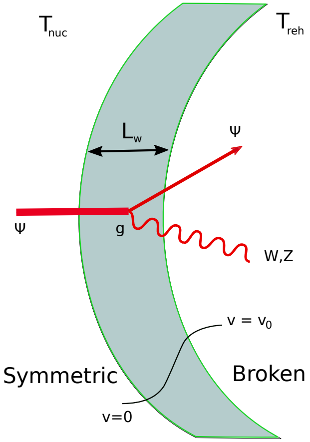

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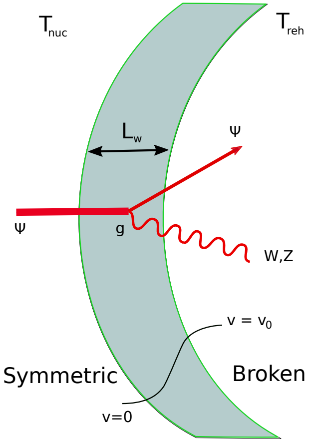

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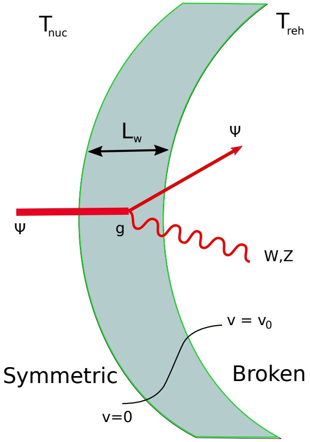

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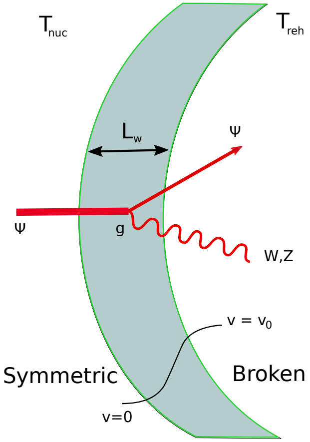

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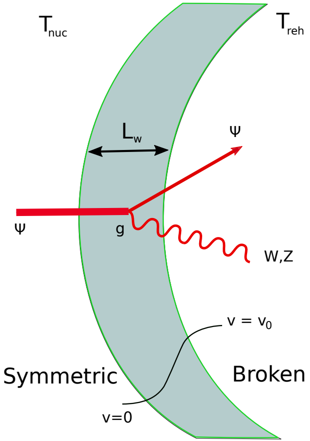
  $\mathcal{P}_{1\to N} \approx \mathcal{P}_{1\to 2} \times \log \frac{v}{T}$
  [arXiv:2007.10343]: incorrect application of Ward identities



Reflection of longitudinals: Garcia, Koszegi,Petrossian arXiv:2212.10572

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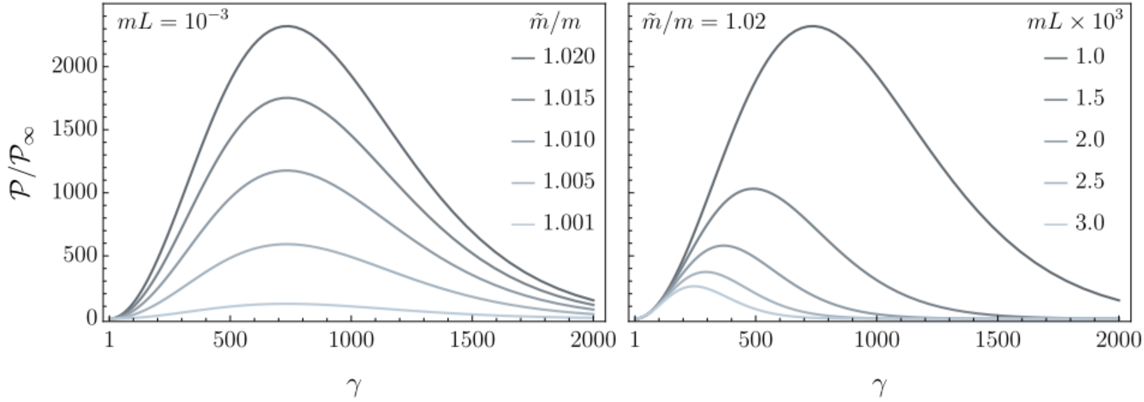
$$\boxed{\mathcal{P} \sim n_V \gamma^2 \frac{2}{3}\left(\frac{\Delta m_V^2}{2m_V^2}\right)^2 \quad 1 \ll \gamma \ll (m_V L)^{-1} \sim v/m_V}$$

Dynamic maximum pressure, Garcia, Koszegi and Petrossian arXiv:2212.10572



Summary on the velocity in the relativistic regime

- LO pressure by particle getting a mass

$$\Delta \mathcal{P}_{1\to 1} \to \sum_i \frac{\Delta m_i^2 T^2}{24}$$

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$$\mathcal{P} \sim n_V \gamma^2 \frac{2}{3}\left(\frac{\Delta m^2}{2m^2}\right)^2 \quad 1 \ll \gamma \ll \frac{v}{m_V}$$

EWBG from ultra-relativistic walls

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[arXiv:2106.14913] with *Aleksandr Azatov* and *Wen Yin*

Saving the soldier EWBG ?

Traditional EWBG

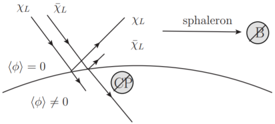


Figure: Credit:T.Konstandin [1302.6713]

- if *slow wall*:

$$Y_B \sim Y_t \times \underbrace{\Gamma_{\rm sph}/T}_{10^{-6}} \times \underbrace{\Delta\theta}_{\text{CP: EDM constraints}}$$

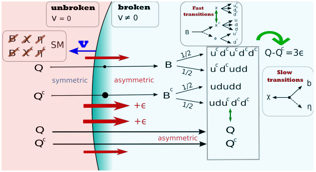
Ruled out ? White, Postma, Vd Vis: 2206.01120

- Hidding CP violation
- Breaking B explicitly

Challenges of B-breaking EWPT Baryogenesis

- Colored particles $M \gtrsim$ TeV: $e^{-(10-20)}$.
- Strongly coupled
- Unsuppressed wash-out

EWPT Baryogenesis with relativistic walls



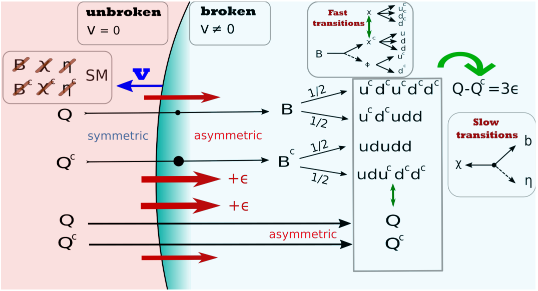
Ingredients:

- Breaks B by two units
- Works with relativistic bubble walls

Low energy baryogenesis with relativistic walls

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H)P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c d u}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2.$$

- χ_i Massive Majorana, η diquark, B_I heavy vectorlike b-like quarks. $B(\eta) = 2/3, B(\chi) = 1$.



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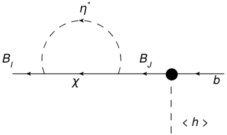

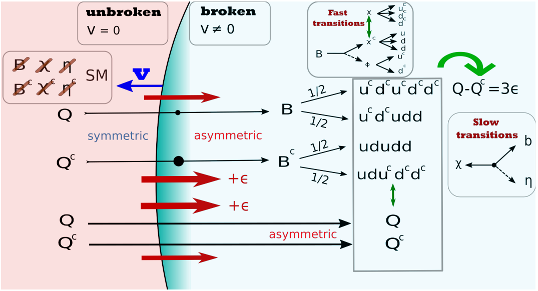

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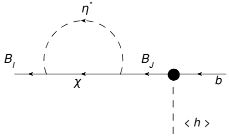
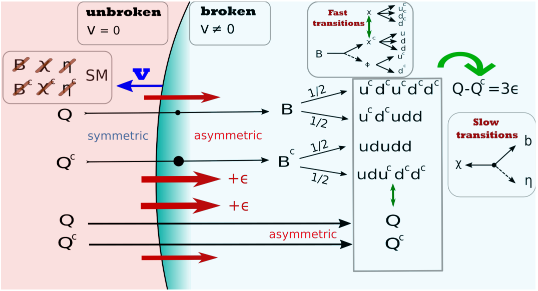


- Production: $\mathcal{P}(Q \to B_I) \neq \mathcal{P}(Q^c \to B_I^c)$

$$\Delta n_b = -\sum_I \Delta n_{B_I}$$



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- **Fast cascades**; 4 channels
  wash-out : $B \to (ddud^c u^c), \quad B^c \to (d^c d^c u^c du)$
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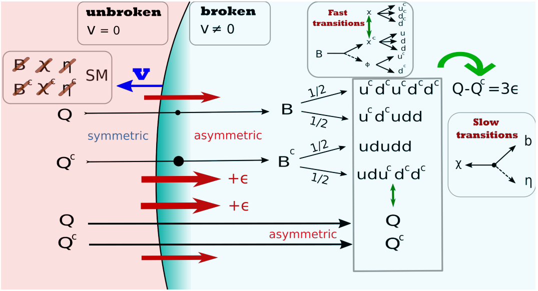

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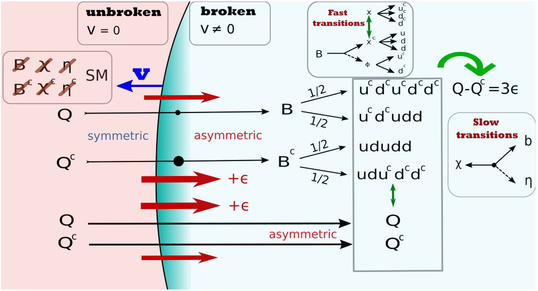
$$\boxed{3n_b^0 \sum_I \theta_I^2 \epsilon_I \times \frac{|y_I|^2}{|y_I|^2 + |Y_I|^2} \times \frac{2|\kappa|^2}{2|\kappa|^2 + |\sum y_I \theta_I|^2}}$$



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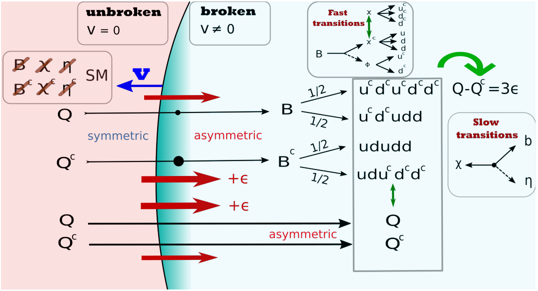
- Experimental signatures: $N \leftrightarrow \bar{N}$, Flavor, collider
  push: $\boxed{m_\chi \sim m_\eta \sim M_B \gtrsim 2 \text{ TeV}}$ and $d = b, u = t$



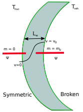
How tuned is EWPT with relativistic walls ?

How tuned is EWPT with relativistic walls ?

[2207.02230]: Azatov, Barni, Chackraborty, MV, Yin

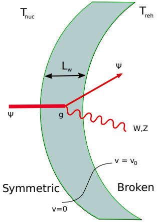
pressure during EWPT

Condition for relativistic wall $\boxed{\Delta V > 0.17 T_{\text{nuc}}^2 v_{EW}^2}$



NLO pressure from *TB*: [arXiv:2112.07686]: Gouttenoire, Jinno, Sala

$$\Delta \mathcal{P}_{\text{NLO}}^{SM} \approx \underbrace{\left[\sum_{abc} \nu_a g_a \beta_c C_{abc} \right]}_{\approx 150} \frac{\kappa \zeta(3)}{\pi^3} \times \alpha M_Z (v_{EW}) \gamma_{wp} T_{\text{nuc}}^3$$



Terminal velocity: $\gamma_w^{\text{terminal}} \approx 50 \times \left(\frac{40 \text{ GeV}}{T_{\text{nuc}}} \right)^3$

Maximal mass: $M^{MAX} \approx \sqrt{v_{EW} T_{\text{nuc}} \gamma_w} \approx 700 \text{ GeV} \times \left(\frac{40 \text{ GeV}}{T_{\text{nuc}}} \right)$

Intuition on supercooling and ultra-relativistic walls

- Only thing we need is long supercooling $T_{\rm nuc} \ll v_{EW}$: $\Delta V \propto \gamma^0, \quad \Delta \mathcal{P} \propto \gamma^{(0-1)}$

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$$V_{\rm tree} = -\frac{m_h^2}{2} h^2 + \frac{\lambda}{4} h^4, \qquad V_T(h) \propto \sum_i g_i^2 \frac{T^2 h^2}{24} - \frac{T h^3}{12\pi} \qquad \Rightarrow \boxed{T_{\rm min} \propto m_h} \quad \text{problem!}$$

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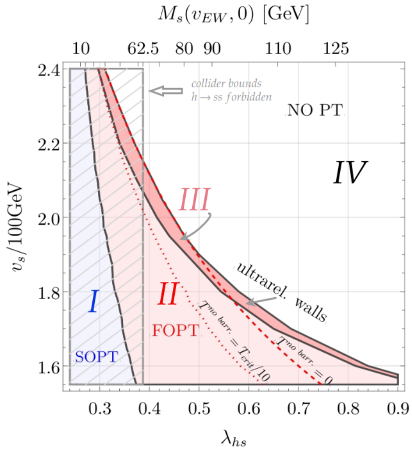
- **2-steps PT**: $\boxed{(0,0) \xrightarrow{SOPT} (0, v_s) \xrightarrow{FOPT} (v_{EW}, 0)}$
  In the second PT:

$$m_{\rm eff}^2(T) = -\frac{m_h^2}{2} + \frac{\lambda_{hs}}{2}v_s^2 + C \times T^2 \to 0$$

Relativistic EWPT: Parameter scan

[2207.02230]: Azatov, Barni, Chackraborty, MV, Yin



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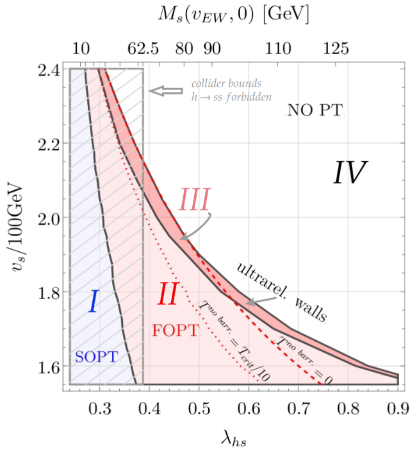
I. SOPT: there is **never a barrier** separating the two minima



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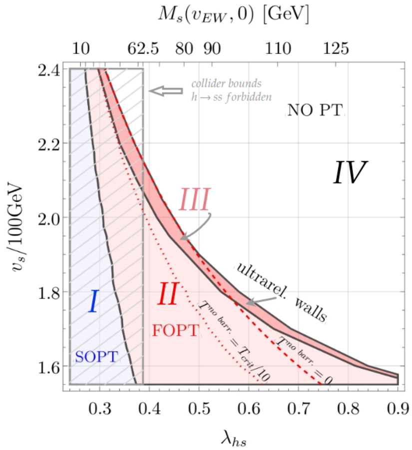

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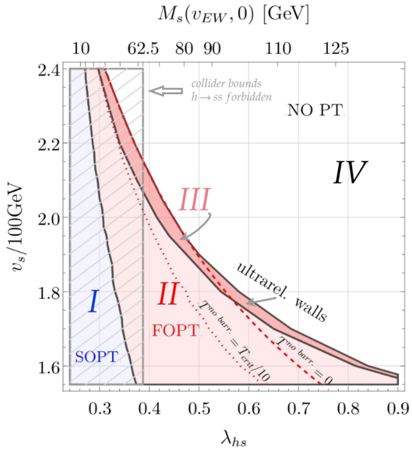
$\gamma_w \gg 1$ increasing λ_{hs} at fixed v_s.
[$\rightarrow$ barrier even at $T = 0$ above the red dashed line]



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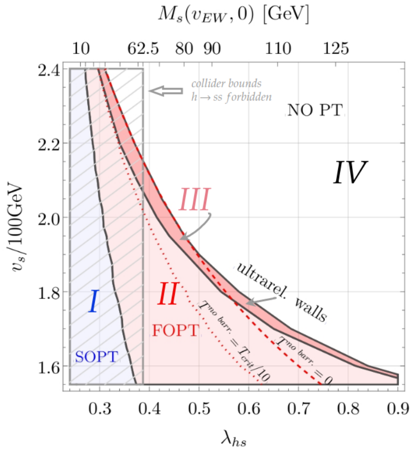
IV. No PT: the **system remains stuck** in the FV and never nucleates



How much tuning do we need ?

$$T_{\rm nuc} \approx T_{\rm instability} \equiv \frac{\sqrt{-\frac{m_h^2}{2} + \frac{\lambda_{hs}}{2} v_s^2}}{C}$$

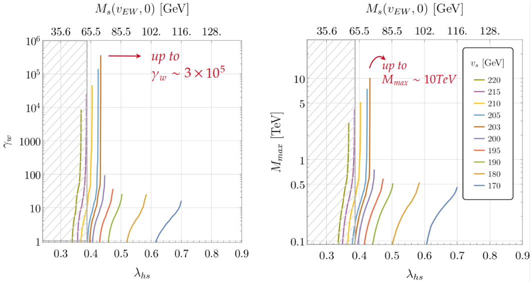
The tuning (*Giudice-Barbieri* definition):

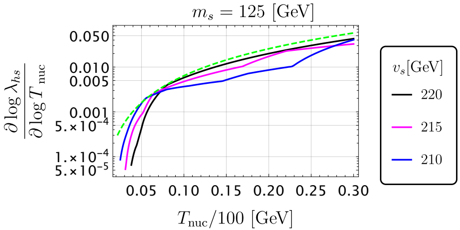
$$\text{tuning} \sim \frac{\partial \log \lambda_{hs}}{\partial \log T_{\rm nuc}/m_H} \propto \left(\frac{T_{\rm nuc}}{m_H}\right)^2$$

Relation between $T_{\rm nuc}$ and $M^{\rm MAX}$

$$M^{MAX} \approx 700 \text{ GeV} \times \left(\frac{40 \text{ GeV}}{T_{\rm nuc}}\right)\left(\frac{\Delta V}{v_{EW}^4}\right)^{1/8}$$

$$\Rightarrow \quad \text{tuning} \approx \left(\frac{250 \text{ GeV}}{M_{\rm heavy}}\right)^2$$





A closer look at NLO pressure

A closer look at NLO pressure

[arXiv:2305.xxxxx] with *Aleksandr Azatov*, *Giulio Barni* and *Rudin Petrossian*

The basis for the emission of GB: transverse modes

Equation of motion across the wall

$$\Box h = -V''(v)h$$

$$\Box \phi_2 = -2g\partial_\mu v A^\mu - \xi g^2 v^2 \phi_2 - V' \frac{\phi_2}{v}$$

$$\partial_\nu F^{\mu\nu} = \frac{1}{\xi}\partial^\mu(\partial_\nu A^\nu) + g^2 v^2 A^\mu - 2g\phi_2 \partial^\mu v$$

Impose unitary gauge $\xi \to \infty$

$$\Box h = -V''(h)h \qquad \partial_\nu F^{\mu\nu} = g^2 v^2 A^\mu \qquad \Rightarrow \partial_\mu \partial_\nu F^{\mu\nu} = \partial_\mu (v^2 A^\mu) = 0.$$

Vector field has three polarization degrees of freedom so that we can write down

$$A^\mu = \sum_{\lambda=1,2,3} \epsilon^\mu_\lambda a_\lambda(x). \qquad k_\mu = (k_0, k_\perp, 0, k_z)$$

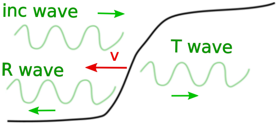
$$\mathsf{T} : \epsilon_1 = (0,0,1,0), \qquad \epsilon_2 = (k_\perp, k_0, 0, 0)/\sqrt{k_0^2 - k_\perp^2} \qquad \mathsf{L} : A_\mu^{(z-pol)} = \partial_n a + A_z, \quad n = 0,1,2$$

The basis of solutions for the emission of GB: transverse modes

Step wall framework for transverse modes:

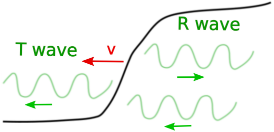
matching across the step wall: $a_T|_{<0} = a_T|_{>0}$ $\qquad \partial_z a_T|_{<0} = \partial_z a_T|_{>0}$

Right movers



$$A^z_{R,k} = e^{-ik^0 t}\chi_{R,k}(z) \equiv e^{-ik^0 t}\begin{cases} e^{ikz} + r_k e^{-ikz} & , z<0 \\ t_k\, e^{i\tilde{k}^z z} & , z>0 \end{cases}$$

Left movers



$$A^z_{L,k} = e^{-ik^0 t}\chi_{L,k}(z) \equiv e^{-ik^0 t}\sqrt{\frac{k}{\tilde{k}}}\begin{cases} t_k e^{-ik^z z} & , z<0 \\ e^{-i\tilde{k}z} - r_k e^{i\tilde{k}z} & , z>0 \end{cases}$$

$$r_{R,k} = \frac{k^z - \tilde{k}^z}{k^z + \tilde{k}^z}, \quad t_{R,k} = \frac{2k^z}{k^z + \tilde{k}^z} \quad r_{L,k} = \frac{\tilde{k}^z - k^z}{k^z + \tilde{k}^z}, \quad t_{L,k} = \frac{2\tilde{k}^z}{k^z + \tilde{k}^z}$$

Going to pressure and exchange of momentum

Building the pressure

$$\langle \Delta p_R \rangle = \int dP_{\phi\to\phi A_R} \Delta p^z_R = \frac{1}{2p^z}\int \frac{d^3k_R}{(2\pi)^3 2k^0}\int \frac{d^3q}{(2\pi)^3 2q^0}|\mathcal{M}_R|^2\,(2\pi)^3\delta^{\{0,\perp\}}(p-q-k_R)\,\underbrace{\Delta p^z_R}_{<1/L_w}$$

$$\langle \Delta p_L \rangle = \int dP_{\phi\to\phi A_L}\,\underbrace{\Delta p^z_L}_{<1/L_w}$$

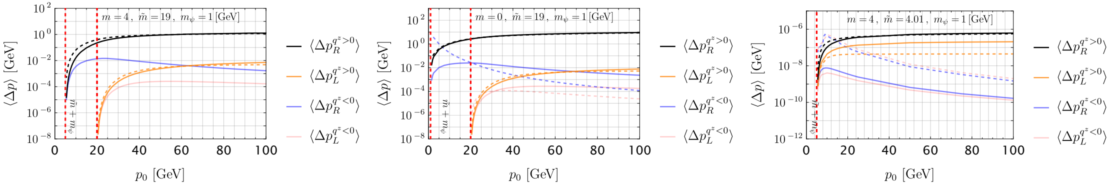
Pressure is (in the wall frame)

$$\mathcal{P} = \underbrace{\int \frac{d^3p}{(2\pi)^3} f_\phi(p)}_{\gamma_w T^3}(\langle \Delta p_R \rangle + \langle \Delta p_L \rangle)$$

$$\Delta p^z_R = |r_R|^2(p-q+k) + (1-|r_R|^2)(p-q-\tilde{k})\ ,$$
$$\Delta p^z_L = |r_L|^2(p-q-\tilde{k}) + (1-|r_L|^2)(p-q+k)\ ,$$

Transverse emission of GB

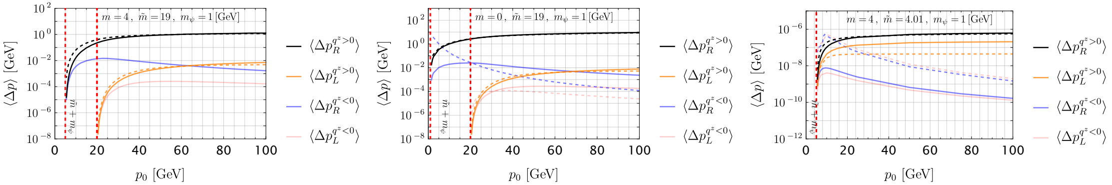
- $\langle \Delta p_R^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z>0}|^2 \cdot (p_z - q_z - \tilde{k}_z) \sim \frac{g^2(\tilde{m}^2 - m^2)^2}{m^3}$ Dominant



This recovers the usual result from previous computations!!

Transverse emission of GB

- $\langle \Delta p_R^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z>0}|^2 \cdot (p_z - q_z - \tilde{k}_z) \sim \frac{g^2(\tilde{m}^2 - m^2)^2}{m^3}$ Dominant
- $\langle \Delta p_L^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_L^{q_z>0}|^2 \cdot (p_z - q_z + k_z) \sim \frac{g^2(\tilde{m}^2 - m^2)^2}{\tilde{m}^3}$ Strong



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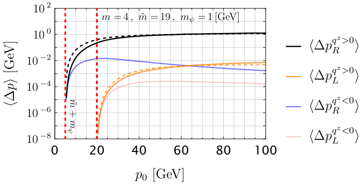
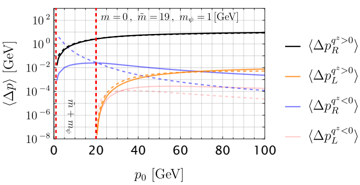
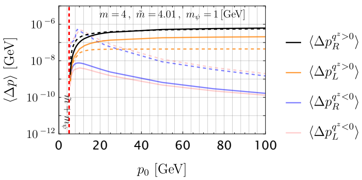
Transverse emission of GB

- $\langle \Delta p_R^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z>0}|^2 \cdot (p_z - q_z - \tilde{k}_z) \sim \frac{g^2(\tilde{m}^2-m^2)^2}{m^3}$ Dominant
- $\langle \Delta p_L^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_L^{q_z>0}|^2 \cdot (p_z - q_z + k_z) \sim \frac{g^2(\tilde{m}^2-m^2)^2}{\tilde{m}^3}$ Strong
- $\langle \Delta p_R^{q_z<0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z<0}|^2 \cdot (p_z + q_z - \tilde{k}_z) \sim \frac{g^2(\tilde{m}^2-m^2)^2}{p_0^3}$ Relevant



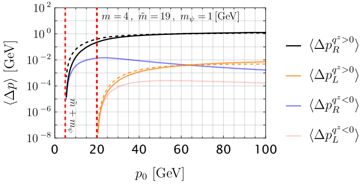


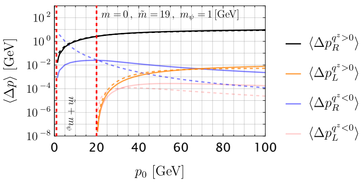


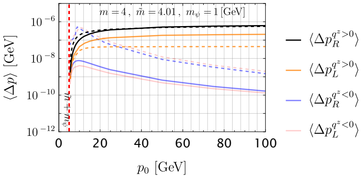
This recovers the usual result from previous computations!!

Transverse emission of GB

- $\langle \Delta p_R^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z>0}|^2 \cdot (p_z - q_z - \tilde{k}_z) \sim \frac{g^2(\tilde{m}^2 - m^2)^2}{m^3}$ Dominant
- $\langle \Delta p_L^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_L^{q_z>0}|^2 \cdot (p_z - q_z + k_z) \sim \frac{g^2(\tilde{m}^2 - m^2)^2}{\tilde{m}^3}$ Strong
- $\langle \Delta p_R^{q_z<0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z<0}|^2 \cdot (p_z + q_z - \tilde{k}_z) \sim \frac{g^2(\tilde{m}^2 - m^2)^2}{p_0^3}$ Relevant
- $\langle \Delta p_L^{q_z<0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_L^{q_z<0}|^2 \cdot (p_z + q_z + k_z) \sim \frac{g^2(\tilde{m}^2 - m^2)^2}{p_0^3}$ Negligible







This recovers the usual result from previous computations!!

What piece interact with the current j_μ??

Farrar-McIntosh [9412270]: $A_\mu^{(z-pol)} = \partial_n a + A_z, \quad n = 0,1,2 \quad$ (1 $\to$ 1 transitions)

$$-\partial_z^2 a + \partial_z A_z + g^2 v^2(z) a = 0 \qquad E^2 A_z - E^2 \partial_z a - g^2 v^2(z) A_z = 0$$

Eliminate A_z

$$A_z = \frac{E^2 \partial_z a}{E^2 - m^2} \qquad A_\mu^{z-pol} = \left(\partial_n a, \frac{E^2 \partial_z a}{E^2 - m^2}\right) = \partial_\mu a + \underbrace{\frac{m^2}{E^2}(0,0,0,A_z)}_{A^\mu_{\text{left}}}$$

Interacting piece

$$\Rightarrow \mathcal{M}_{\phi\to\phi A} \propto A_\mu j^\mu = \underbrace{j^\mu \partial_\mu a}_{\text{remove by hand}} + A^\mu_{\text{left}} j_\mu = \boxed{A^\mu_{\text{left}} j_\mu}$$

For v constant:

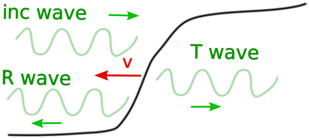
$$\epsilon_\mu^{z-pol} = \left(k_0, k_\perp, 0, \frac{E^2}{k_z}\right) \times \frac{k_z}{mE} = k_\mu \times \frac{k_z}{mE} + (0,0,0,\frac{m}{E})$$

$$\boxed{\epsilon^\nu_\lambda = \partial^\nu a + \frac{m}{E}(0,0,0,1)}$$

The basis for the emission of GB: longitudinal modes

matching across the step wall: $\partial_z A^z =$ continuous at $z = 0$ $\qquad m^2(z) A^z =$ continuous at $z = 0$.

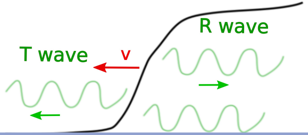
Right movers



$$A^z_{R,k} = e^{-ik^0 t}\chi_{R,k}(z) \equiv e^{-ik^0 t}\frac{E}{m}\begin{cases} e^{ikz} + r_k e^{-ikz} & , z < 0 \\ t_k \; e^{i\tilde{k}^z z} & , z > 0 \end{cases}$$

$$\mathrm{r}_k = \frac{\tilde{m}^2 k - m^2 \tilde{k}}{\tilde{m}^2 k + m^2 \tilde{k}} \, , \qquad t_k = \frac{2km^2}{\tilde{m}^2 k + m^2 \tilde{k}}$$

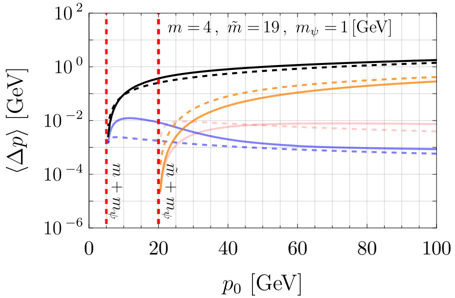
Left movers



$$A^z_{L,k} = e^{-ik^0 t}\chi_{L,k}(z) \equiv e^{-ik^0 t}\frac{E}{\tilde{m}}\sqrt{\frac{k}{\tilde{k}}}\begin{cases} \frac{\tilde{k}\tilde{m}^2}{km^2} t_k e^{-ik^z z} & , z < 0 \\ e^{-i\tilde{k}z} - r_k e^{i\tilde{k}z} & , z > 0 \end{cases}$$

Longitudinal emission of GB

- $\langle \Delta p_R^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z>0}|^2 \cdot (p_z - q_z + \tilde{k}_z) \sim \frac{g^2 p_0 m^2}{m_\psi^2} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2} \right)^2$ Dom



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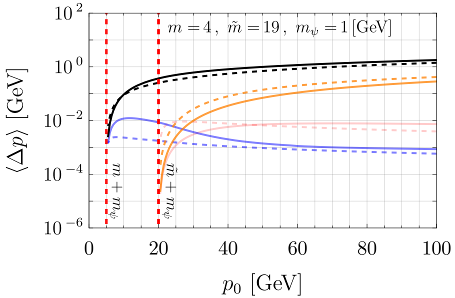
- $\langle \Delta p_L^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_L^{q_z>0}|^2 \cdot (p_z - q_z + k_z) \cdot \frac{4m^2}{\tilde{m}^2} \sim \frac{g^2 p_0 m^4}{m_\psi^2 \tilde{m}^2} \left(\frac{\Delta m^2}{m^2+\tilde{m}^2} \right)^2$ Dom



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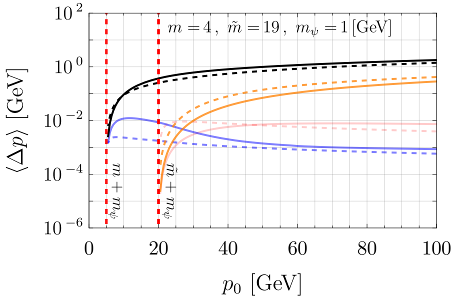
- $\langle \Delta p_R^{q_z<0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z<0}|^2 \cdot (p_z + q_z + k_z) \sim \frac{g^2 m^2}{p_0} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2} \right)^2$ Neg



Longitudinal emission of GB

- $\langle \Delta p_R^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z>0}|^2 \cdot (p_z - q_z + \tilde{k}_z) \sim \frac{g^2 p_0 m^2}{m_\psi^2} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2} \right)^2$ Dom
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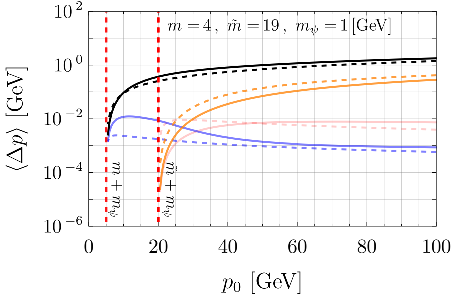
- $\langle \Delta p_R^{q_z<0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z<0}|^2 \cdot (p_z + q_z + k_z) \sim \frac{g^2 m^2}{p_0} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2} \right)^2$ Neg
- $\langle \Delta p_L^{q_z<0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_L^{q_z<0}|^2 \cdot (p_z + q_z - \tilde{k}_z) \sim \frac{g^2 m^2 m_\psi}{p_0^2} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2} \right)^2$ Neg



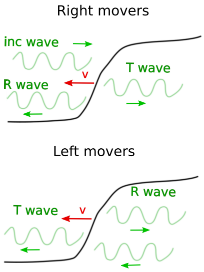
Longitudinal emission of GB: two limits

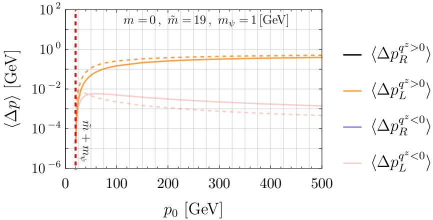
Take $\boxed{m \to 0}$, symmetry restored outside the bubble:

$$r_k \to 1, \qquad t_k \to 0, \qquad \text{Right-movers disappear}$$

$$\Delta p_L^{q_z>0} \to (p_z - q_z - \tilde{k}_z) \sim \frac{k_\perp^2 + (1-x)\tilde{m}^2 + x^2 m_\psi^2}{2x(1-x)p_0}$$

$$\langle \Delta p_L^{q_z>0} \rangle \sim \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_L^{q_z>0}|^2 \cdot (p_z - q_z - \tilde{k}_z) \sim g^2 \tilde{m}$$





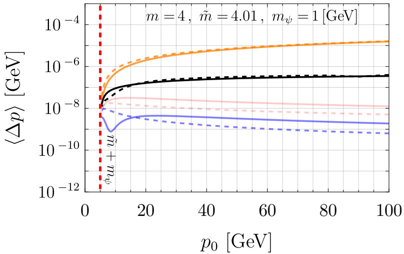
Longitudinal emission of GB: two limits

Take $\boxed{\tilde{m} \to m}$, small mass difference:$\Rightarrow |r_R|^2 = |r_L|^2 \sim \left|\frac{\tilde{m}^2 - m^2}{\tilde{m}^2 + m^2}\right|^2 \ll 1$

$$\Delta p^z_R = |r_R|^2 (p - q + k) + (1 - |r_R|^2)(p - q - \tilde{k}) \approx (p - q - \tilde{k})$$
$$\Delta p^z_L = |r_L|^2 (p - q - \tilde{k}) + (1 - |r_L|^2)(p - q + k) \approx (p - q + k)$$

$$\langle \Delta p_R^{q_z > 0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z>0}|^2 \cdot (p_z - q_z - \tilde{k}_z) \sim \frac{g^2 p_0 m^2}{m_\psi^2} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2}\right)^2$$

$$\langle \Delta p_L^{q_z > 0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_L^{q_z>0}|^2 \cdot (p_z - q_z + k_z) \sim \frac{4g^2 p_0 m^2}{m_\psi^2} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2}\right)^2 \qquad \text{Dominant}$$



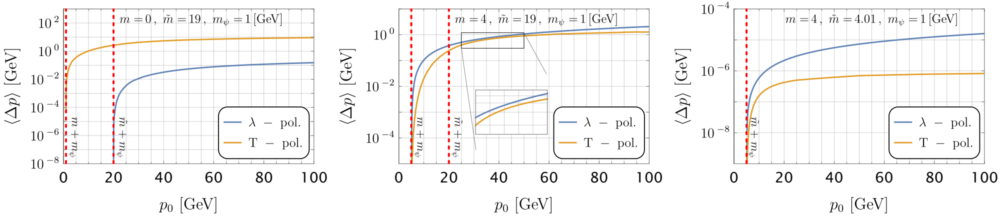
Comparison emission of GB: scaling

$$\mathcal{P}_L \approx \gamma_w^2 T^4 \frac{g^2 m^2}{m_\psi^2}\left(\frac{\Delta m^2}{m^2+\tilde{m}^2}\right)^2$$

What happens ???

$$\langle \Delta p_R^{q_z>0} \rangle = \int dk_\perp^2 \int dx \frac{p_0}{(2p_z)(2q_z)(2k_z)} \cdot |\mathcal{M}_R^{q_z>0}|^2 \cdot (p_z - q_z + \tilde{k}_z) \sim \frac{g^2 p_0 m^2}{m_\psi^2}\left(\frac{\Delta m^2}{m^2+\tilde{m}^2}\right)^2$$

$$M_R^\lambda \propto p_0 \Delta m^2 m \qquad (p_z - q_z + \tilde{k}_z) \propto x p_0 < 1/L_w$$



Take-home message

- **Validity of step wall**: $\Delta p_z \sim \gamma T < 1/L_w \sim m_H$

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- **Ordinary limit: transverse emission dominates Longitudinals with** $m \to 0$:

$$\mathcal{P} \sim \overbrace{(\underbrace{4}_{\text{ref}} + \underbrace{1}_{\text{trans}})\gamma_w T^3 g^3 \Delta m \log \Delta m / T_{\text{nuc}}}^{\text{Right-movers Transverse GB}} + \overbrace{(\underbrace{4}_{\text{ref}} + \underbrace{1}_{\text{trans}})\gamma_w T^3 g^3 \Delta m}^{\text{Right-movers longitudinal GB}}$$

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- **Longitudinals when** $m \neq 0$

$$\mathcal{P} \approx \gamma_w^2 T^4 \frac{g^2 m^2}{m_\psi^2} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2} \right)^2$$

until $\gamma \approx m_H / T$.

Take-home message

- **Validity of step wall**: $\Delta p_z \sim \gamma T < 1/L_w \sim m_H$
- **Ordinary limit: transverse emission dominates Longitudinals with** $m \to 0$:

$$\mathcal{P} \sim \overbrace{(\underbrace{4}_{\text{ref}} + \underbrace{1}_{\text{trans}})\gamma_w T^3 g^3 \Delta m \log \Delta m/T_{\text{nuc}}}^{\text{Right-movers Transverse GB}} + \overbrace{(\underbrace{4}_{\text{ref}} + \underbrace{1}_{\text{trans}})\gamma_w T^3 g^3 \Delta m}^{\text{Right-movers longitudinal GB}}$$

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$$\mathcal{P} \approx \gamma_w^2 T^4 \frac{g^2 m^2}{m_\psi^2} \left(\frac{\Delta m^2}{m^2 + \tilde{m}^2} \right)^2$$

  until $\gamma \approx m_H/T$.
- **Questions**: How to apply the same formalism to thick physical walls?

Back-up

Back-up

Baryogenesis

- **B-number violation**; B-violating interactions, sphalerons
- **CP-violation**; *physical* phase into the yukawa matrix
- **Out-of-equilibrium situation**; expansion of the universe, first-order phase transition

Electroweak baryogenesis

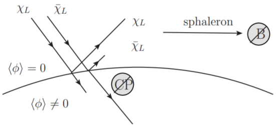


Figure: Credit:T.Konstandin [1302.6713]

Scattering of quarks with CP-violating yukawas off the *slow* bubble wall. B-violation via *sphalerons*

VS

Leptogenesis

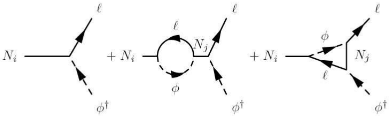


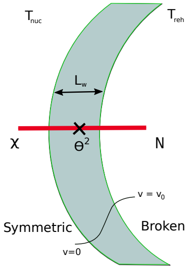
Figure: Credit:T.Konstandin [1302.6713]

Out-of-equilibrium decay of heavy L-violating RH neutrinos. B-violation via sphalerons. CP violation via *loops*

CP violation inside the bubble wall

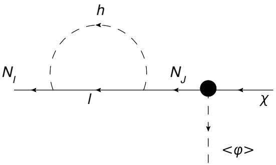
Ingredients: Higgs field H, φ scalar, 2 heavy N_I, SM $SU(2)_L$-fermions L_α, and χ_i light fermions

$$\mathcal{L} = i\bar{\chi}_i P_R \partial\!\!\!/ \chi_i + i\bar{N}_I \partial\!\!\!/ N_I - M_I \bar{N}_I N_I - Y_{iI}\varphi \bar{N}_I P_R \chi_i - y_{I\alpha}(H\bar{L}_\alpha)P_R N_I + h.c.$$



$$A(\chi_i \to N_I)_{\text{tree}} \propto Y_{iI}$$

$+$



$$A(\chi_i \to N_I)_{1-\text{loop}} \propto$$

$$+\sum_{\alpha,J} Y_{iJ} y^*_{\alpha J} y_{\alpha I} \times f^{(hl)}_{IJ}$$

$\Rightarrow$

$$\frac{\Gamma(\chi \to N_I) - \Gamma(\bar{\chi} \to \bar{N}_I)}{\Gamma(\chi \to N_I) + \Gamma(\bar{\chi} \to \bar{N}_I)} =$$

$$\frac{2\sum_{\alpha,J,i} \text{Im}(Y_{iI} Y^*_{iJ} y_{\alpha J} y^*_{\alpha I}) \text{Im} f^{(hl)}_{IJ}}{\sum_i |Y_{iI}|^2}.$$

and

$$\text{Im}[f^{(hl)}_{IJ}(x)] = \frac{1}{16\pi}\frac{\sqrt{x}}{1-x}, x = \frac{M_J^2}{M_I^2}$$

Conservation of current and longitudinal modes

$$J^\mu \partial_\mu \theta \qquad J^\mu = g(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger) \qquad \partial_\mu J^\mu = 0$$

- **Transverse**

$$\mathcal{M} = \frac{(p+q)_\mu k^\mu}{\Delta p_{inc}} + r_k \frac{(p+q)_\mu k_r^\mu}{\Delta p_r} - t_k \frac{(p+q)_\mu k_t^\mu}{\Delta p_t}$$

Conservation of current: $(m_\psi = \tilde{m}_\psi, m \neq \tilde{m}) \qquad \Rightarrow (p+q)_\mu k^\mu = (p+q)_z \Delta p_{inc}$

$$\mathcal{M} = (p+q)_z (1 + r_k - t_k) = 0!$$

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- **Longitudinals**

$$a|_{z<0} = \frac{k_z}{mE}(e^{ikz} - r_k e^{ik_r z}) \qquad a|_{z>0} = t_k \times \frac{\tilde{k}_z}{E\tilde{m}} e^{ik_t z}$$

$$\mathcal{M} = \frac{k_z}{Em}\frac{(p+q)_\mu k^\mu}{\Delta p_{inc}} - \frac{k_z}{Em} r_k \frac{(p+q)_\mu k_r^\mu}{\Delta p_r} - \frac{\tilde{k}_z}{E\tilde{m}} t_k \frac{(p+q)_\mu k_t^\mu}{\Delta p_t} = \frac{(p+q)_z}{E} \underbrace{\left(\frac{k_z}{m} - \frac{k_z}{m} r_k - \frac{\tilde{k}_z}{\tilde{m}} t_k\right)}_{=0}$$

Can γ_{wp} be large enough to produce ϕ of M_ϕ?

Transition strong enough : $\boxed{\Delta V > \Delta\mathcal{P}_{LO}}$

Transition sector *without* Gauge Bosons

$$\Delta\mathcal{P} = \Delta\mathcal{P}_{LO}$$

$$\Downarrow$$

Runaway regime: acceleration until collision

$$\Downarrow$$

$$\gamma_{w,\text{MAX}} \approx \frac{M_{\text{pl}} T_{\text{nuc}}}{v^2}$$

$$\Rightarrow \quad \boxed{M_\phi^{\text{MAX}} \sim T_{\text{nuc}} \left(\frac{M_{\text{pl}}}{v}\right)^{1/2}}$$

Transition sector *with* Gauge Bosons

$$\Delta\mathcal{P} = \Delta\mathcal{P}_{LO} + \Delta\mathcal{P}_{NLO}$$

$$\Downarrow$$

$$\gamma_{w,\text{MAX}} \approx \text{Min}\left[\frac{M_{\text{pl}} T_{\text{nuc}}}{v^2}, \frac{16\pi^2}{g_i g_{\text{gauge}}^3}\left(\frac{v}{T_{\text{nuc}}}\right)^3\right]$$

$$\Rightarrow \boxed{M_\phi^{\text{MAX}} \sim \text{Min}\left[T_{\text{nuc}}\left(\frac{M_{\text{pl}}}{v}\right)^{1/2}, 4\pi v\left(\frac{v}{T_{\text{nuc}}}\right)\right]}$$

Falkowski and No bubble wall production

Production of heavy states during the collision of bubbles. arXiv:1211.5615

- Can be non thermal DM: arXiv:1211.5615
- Or make a barygenesis mechanism: arXiv 1608.00583

Necessary ingredients

- Portal coupling similar to ours.
- Runaway bubble (otherwise, energy dissipated in the plasma): not operative in EWPT.
- Elastic collision (restoration of the false vacuum in between the bubble)

Constraints and experimental signatures on the EWBG proposed

1. **Neutron-anti-neutron oscillations**: baryon number violation by 2 units

$$\frac{1}{\Lambda_{n\bar{n}}^5}\overline{u^c d^c d^c} udd \equiv \frac{(\sum \kappa\theta_I y_I)^2}{M_\eta^4 m_\chi}\overline{u^c d^c d^c} udd \qquad \Rightarrow \qquad \delta m_{\bar{n}-n} \sim \frac{\Lambda_{QCD}^6}{M_\eta^4 m_\chi}(\sum \kappa\theta_I y_I)^2$$

Current bounds on this mixing mass are of order $\delta m_{\bar{n}-n} \lesssim 10^{-33}$

$$\Lambda_{n\bar{n}} \gtrsim 10^6 \text{GeV} \quad (M_\eta, m_\chi) \gtrsim 10^5 \text{GeV}$$

2. **Flavor violation**: Need to couple strongly only to t_R, b_R
3. **Contribution to electron EDM**:

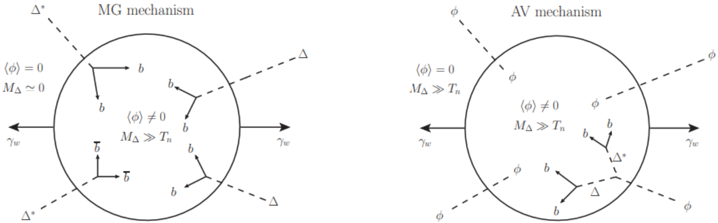
$$\frac{d_e}{e} \sim \frac{m_e (yYe)^2}{(4\pi)^6}\left(\frac{1}{\Lambda_{EDM}^2}\right) \sim 3\times 10^{-33} \times \left(\frac{10\text{TeV}}{\Lambda_{EDM}}\right)^2 \text{cm}$$

while experimental bound is $|d_e| < 1.1\times 10^{-29}\text{cm}\cdot e$

Comparison with proposal in arXiv:2106.15602

Baryogenesis with relativistic walls by Baldes et al. arXiv:2106.15602

- Relativistic walls $\gamma_{wp} \gg 1$
- scalar model $\Delta\mathcal{L} = -\frac{\lambda}{2}\phi^2 h^2 + \frac{M_\phi^2}{2}\phi^2$ with production of heavy scalar ϕ
- ϕ in $(3, 1, 2/3)$ of the SM and $\Delta\mathcal{L} = y_{di}\phi_i \bar{d}_R d'_R + y_{ui}\phi_i \bar{N}_R u^c_R$ with physics phase in y'
- CP and B violation in decay $\phi \to bb$



Full expression

PT leptogenesis: CP violation in production+decay

$$\frac{n_B - n_{\bar{B}}}{s} \simeq -\frac{28}{79} \times \frac{135\zeta(3) g_\chi}{8\pi^4 g_*} \times \sum_I \theta_I^2 \sum_{\alpha,J} \mathrm{Im}(Y_I Y_J^* y_{\alpha J} y_{\alpha I}^*) \mathrm{Im} f_{IJ}^{(hl)}$$

$$\times \left(\frac{2}{|Y_I|^2} - \frac{1}{\sum_\alpha |y_{\alpha I}|^2} \right) \left(\frac{T_{nuc}}{T_{reh}} \right)^3 \frac{\sum_\alpha |y_{\alpha I}|^2}{\sum_\alpha |y_{\alpha I}|^2 + |Y_I|^2}$$

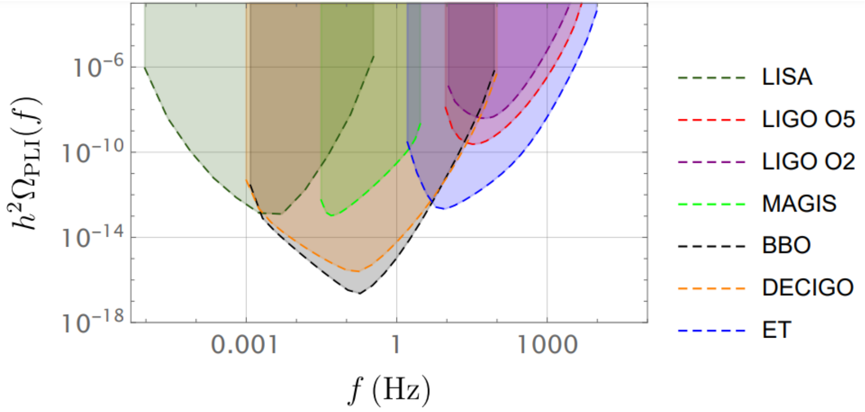
EWPT baryogenesis: CP violation in production+decay

$$\frac{\Delta n_{Baryon}}{s} \approx \frac{135\zeta(3)}{8\pi^4} \sum_{I,J} \theta_I^2 \frac{|y_I|^2}{|y_I|^2 + |Y_I|^2} \times \frac{g_b}{g_\star} \left(\frac{T_{\rm nuc}}{T_{reh}} \right)^3$$

$$\times \mathrm{Im}(Y_I Y_J^* y_I^* y_J) \left(-\frac{2\mathrm{Im}[f_B^{IJ}]}{|Y_I|^2} + \frac{4\mathrm{Im}[f_B^{IJ}]|_{m_{\chi,\eta}\to 0}}{|y_I|^2} \right).$$

Why do we even bother ?? Observation prospects of GW

$$\left(T_{\rm reh}, \quad v_w, \quad R_\star H, \quad \alpha = \frac{\Delta V}{\rho_{rad}}\right) \quad \Rightarrow \quad (\Omega^{GW}_{\rm peak}, \quad f^{GW}_{\rm peak})$$

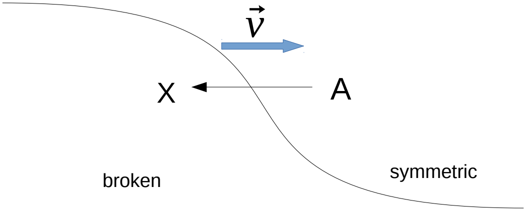


Velocity

Final velocity $\gamma^{MAX} = \frac{1}{\sqrt{1-v_{MAX}^2}}$ of the wall set by

$$\Delta V = \Delta \mathcal{P}(\gamma^{MAX}) \qquad \Rightarrow \qquad \text{determination } \gamma^{MAX}$$

- ΔV independent of the velocity of the wall



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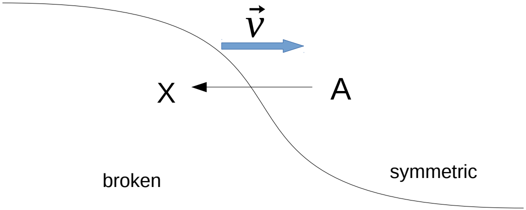

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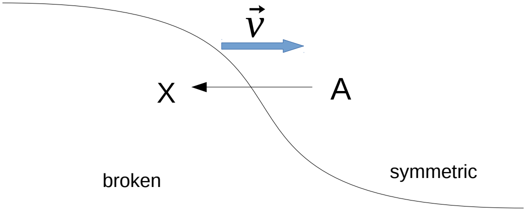
- $\Delta\mathcal{P}(\gamma^{MAX})$ very difficult to compute in general and depends on the velocity
- Generic method: solve the full coupled system of Boltzmann equations

$$p^\mu \partial_\mu f_i + \frac{1}{2}\partial_z m_i[\phi]\partial_{p_z} f_i = \mathcal{C}[f_i, \phi]$$

$$\Box\phi + \frac{dV}{d\phi} + \sum_i \frac{dm_i^2[\phi]}{d\phi}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_i} f_i = 0$$

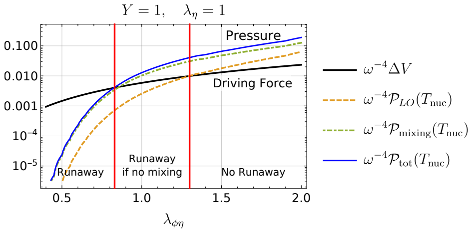


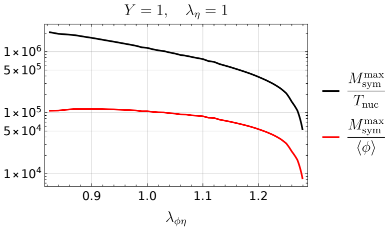
Toy model with active mixing pressure

Toy model with two scalars ϕ (PT field), η (spectator catalizes), heavy N and light χ fermions

$$\boxed{(\phi, \eta, \chi, N)}$$

$$\mathcal{L}_{UV} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - \frac{\tilde{m}_\eta^2 \eta^2}{2} - \frac{\tilde{\lambda}_\phi}{4}\phi^4 - \frac{\tilde{\lambda}_\eta}{4}\eta^4 - \frac{\tilde{\lambda}_{\phi\eta}}{2}\phi^2\eta^2$$
$$+ i\bar{\chi}\partial\!\!\!/\chi + i\bar{N}\partial\!\!\!/ N - M\bar{N}N - Y_{\text{mixing}}\bar{\chi}\phi N + h.c.$$





Summary on the velocity in the relativistic regime

- LO pressure by particle getting a mass

$$\Delta\mathcal{P}_{1\to 1} \to \sum_i \frac{\Delta m_i^2 T^2}{24}$$

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- $$\Delta\mathcal{P}_{tot}(\gamma^{\mathrm{MAX}}) = \Delta V \qquad \text{VS} \qquad \Delta\mathcal{P}_{tot}(\gamma \to \infty) < \Delta V \quad \text{Runaway}$$