Bubble wall velocities in local thermal equilibrium

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JCAP 03 (2022) 015 (WA, Björn Garbrecht, Carlos Tamarit)
arXiv:2303.10171 (WA, Benoit Laurent, Jorinde van de Vis)

17 May 2023, @ workshop “How fast does the bubble grow?”, DESY

Outline

- Total force on a bubble wall
- "Friction" in LTE and a new matching condition
- Bubble wall velocities in LTE
- Conclusions

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The coupled system

Coupled system: Scalar background + Plasma

[Moore & Prokopec '95]

EoM: $\Box\phi + \frac{\partial V(\phi)}{\partial\phi} + \sum_i \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3 2E_i} f_i(p,x) = 0$

$\frac{\mathrm{d}f_i}{\mathrm{d}t} = -C[f_i]$

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Difficult to solve! $\xrightarrow{\text{Two simple limits}}$

- $\gamma_w \to \infty$ [Bodeker & Moore '09, '17; Hoeche etl. '20; Azatove & Vanvlasselaer '20; Gouttenoire, Jinno & Sala 21']
- LTE [Mancha, Prokopec & Świeżewska '20; Balaji, Spannowsky & Tamarit '20; WA, Garbrecht & Tamarit '21; WA, Laurent & van de Vis '23]

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Taking $f_i(p,x) = f_i^{\mathrm{eq}}(p,x) + \delta f_i(p,x)$, we have

$$\Box\phi + \frac{\partial V_{\mathrm{eff}}(\phi,T)}{\partial\phi} + \sum_i \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi}\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(p,x) = 0$$

(equilibrium) (out-of-equilibrium)

dissipative friction

Conventional understanding: finite wall velocities require non-vanishing dissipative friction; not true!

Forces on the wall

Multiplying the EoM by $\mathrm{d}\phi/\mathrm{d}z$, and integrating over z, one has

$$\int \mathrm{d}z \frac{\mathrm{d}\phi}{\mathrm{d}z}\left(\Box\phi + \frac{\partial V_{\text{eff}}(\phi,T)}{\partial\phi} + \sum_i \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3 2E_i}\,\delta f_i(p,x)\right) = 0$$

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$$\frac{F_{\text{back}}}{A} = \int \mathrm{d}z \frac{\partial V_{\text{eff}}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z} - \sum_i \int \mathrm{d}z \frac{\mathrm{d}\phi}{\mathrm{d}z}\frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3 2E_i}\,\delta f_i(p,x)$$

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In thermal equilibrium: backreaction force does NOT necessarily vanish if the temperature is not constant across the bubble wall!

[Ignatius, Kajantie, Kurki-Suonio & Laine ’94; Espinosa, Konstandin, No & Servant ’10; Konstandin & No ’11]

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Two well-known matching conditions

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$$\omega_+ \gamma_+^2 v_+ = \omega_- \gamma_-^2 v_-$$

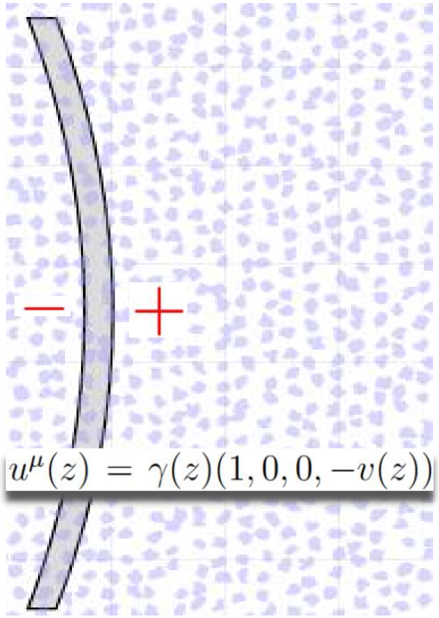
$$\omega_+ \gamma_+^2 v_+^2 + p_+ = \omega_- \gamma_-^2 v_-^2 + p_-$$

From the second condition, one can identify
(recall $\omega = Ts$, $\gamma^2 - 1 = \gamma^2 v^2$)

$$\frac{F_{\text{pressure}}}{A} \equiv -\Delta p\,, \qquad \boxed{\frac{F_{\text{back}}}{A} = \Delta\{(\gamma^2 - 1)Ts\}}$$

(thermal equilibrium)

[Balaji, Spannowsky & Tamarit '21]



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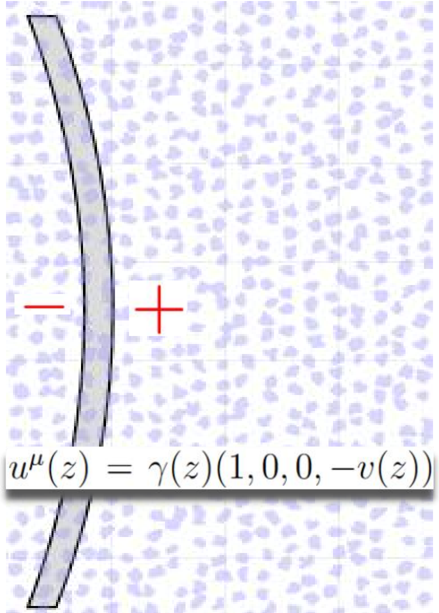

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[Balaji, Spannowsky & Tamarit ’21]

If one assumes constant temperature and fluid velocity, one would have

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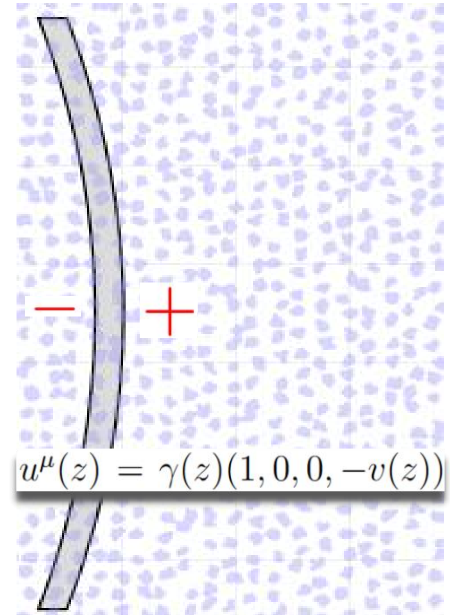

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However, as we showed, the temperature cannot be constant for nonvanishing backreaction force in LTE!

Argument from entropy conservation

In local thermal equilibrium, we have [Hindmarsh, Lüben, Lumma & Pauly '20]

$$\partial_\mu S^\mu \equiv \partial_\mu (s u^\mu) = 0 \quad \Rightarrow \quad s(z)\gamma(z)v(z) = \mathrm{const}$$

$$\gamma^2 - 1 = \gamma^2 v^2$$

$$\frac{F_{\mathrm{back}}}{A} = \Delta\{(\gamma^2 - 1)Ts\} \quad \Rightarrow \quad \frac{F_{\mathrm{back}}}{A} = \mathrm{const} \times \Delta\{\gamma v T\}$$

Further, dividing $\omega\gamma^2 v = \mathrm{const}$ by $s(z)\gamma(z)v(z) = \mathrm{const}$

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A bonus: a new matching condition in LTE!

$$\boxed{\gamma_+ T_+ = \gamma_- T_-}$$

[WA, Garbrecht & Tamarit '21]

We will use this new matching condition to determine the wall velocity in LTE

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General logic

Three matching conditions:

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Three hydrodynamic quantities:

$$(\cancel{T_+}, T_-, v_+, v_-)$$

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Most general equation of state:

$$\rho_s(T_+) = \frac{1}{3}a_+(T_+)\,T_+^4 + \epsilon_+(T_+)\,, \qquad p_s(T) = \frac{1}{3}a_+(T_+)\,T_+^4 - \epsilon_+(T_+)\,,$$
$$\rho_b(T) = \frac{1}{3}a_-(T_-)\,T_-^4 + \epsilon_-(T_-)\,, \qquad p_b(T) = \frac{1}{3}a_-(T_-)\,T_-^4 - \epsilon_-(T_-)\,.$$

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However, the temperature-dependence in the “bag parameters” makes it impossible to carry out (particle physics) model-independent analysis.

For example: $\dfrac{\Delta\epsilon}{a_+T_+^4}$ depends also on T_-

Simplifications: 1 bag model;
2 constant sound speeds (template model)

Bag model

The bag equation of state: assume no temperature dependence in all the "bag parameters"

First two matching conditions

$$v_+ v_- = \frac{1-(1-3\alpha_+)r}{3-3(1+\alpha_+)r}$$

$$\frac{v_+}{v_-} = \frac{3+(1-3\alpha_+)r}{1+3(1+\alpha_+)r}$$

no T_- dependence now

where $\alpha_+ \equiv \dfrac{\Delta\epsilon}{a_+ T_+^4}$, $\quad r \equiv \dfrac{a_+ T_+^4}{a_- T_-^4}$

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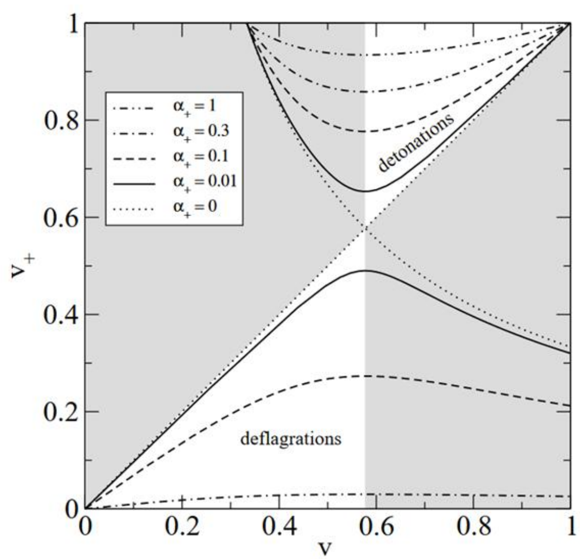
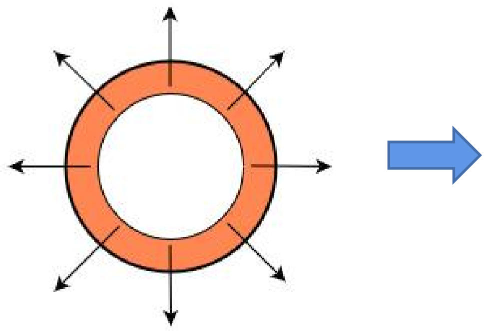
$$v_+ = \frac{1}{6(1+\alpha_+)v_-}\left[1+3v_-^2 \pm \sqrt{1+6\left(6\alpha_+^2+4\alpha_+-1\right)v_-^2+9v_-^4}\right]$$



[Espinosa, Konstandin, No & Servant '10]

Solutions in the bag model

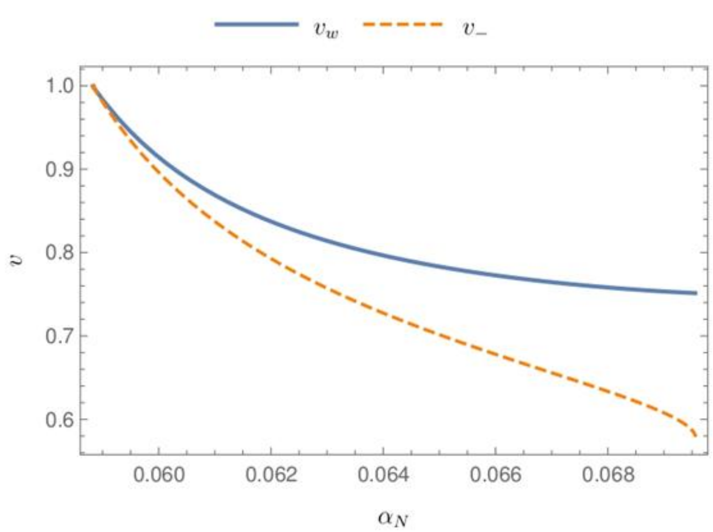
Detonations:



$$v_+ = v_w$$

$$T_+ = T_{\rm nuc}$$

$$\alpha_+ = \alpha_N \equiv \frac{\Delta\epsilon}{a_+ T_{\rm nuc}^4}$$



$$[\alpha_{\rm min} \approx 0.059, \alpha_{\rm max} \approx 0.07]$$

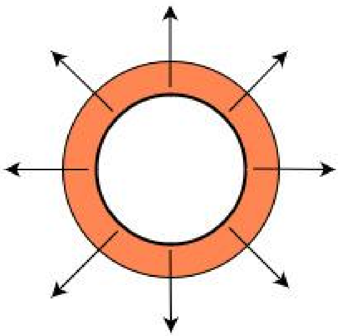
$$\boxed{b \equiv \frac{a_-}{a_+} = 0.85}$$

[Espinosa, Konstandin, No & Servant '10]

$$\Delta\mathcal{F} = (p_- - p_+)|_{T_{\rm nuc}} \geq 0 \Rightarrow \alpha_N \geq (1-b)/3 = 0.05$$

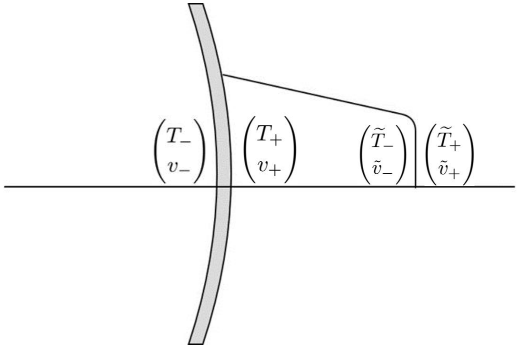
Solutions in the bag model

Deflagrations:



$$v_- = v_w$$

- Need to relate T_+, α_+ to $T_{\rm nuc}$ and α_N



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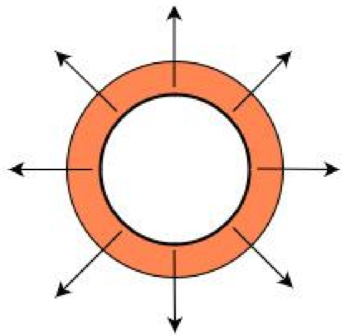

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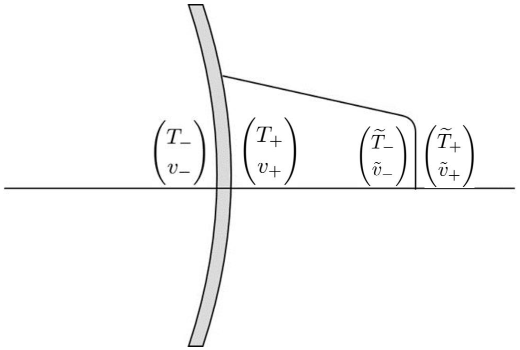

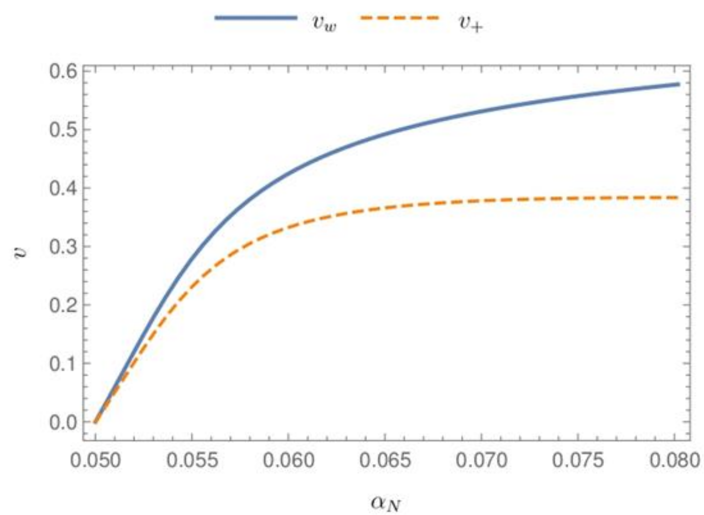

$[\alpha'_{\rm min} \approx 0.0500061, \alpha'_{\rm max} \approx 0.0789645]$

For $\alpha_N > \alpha'_{\rm max}$, no solutions exist for either deflagrations or detonations

Runaway for $\alpha_N > \alpha'_{\rm max}$ and in LTE,

Need to consider nonequilibrium effects

Constant sound speeds (the template model)

Assume constant sound speeds, one has [Leitao & Megevand ’14; Giese, Konstandin & van de Vis ’20]

$$e_s(T) = \frac{1}{3}a_+(\mu - 1)T^\mu + \epsilon\,, \qquad p_s(T) = \frac{1}{3}a_+T^\mu - \epsilon\,,$$
$$e_b(T) = \frac{1}{3}a_-(\nu - 1)T^\nu\,, \qquad p_b(T) = \frac{1}{3}a_-T^\nu\,.$$

where $\mu = 1 + \frac{1}{c_s^2}\,, \quad \nu = 1 + \frac{1}{c_b^2}$

➡ new hydrodynamic parameters: ($\mu\,,\nu\,,\Psi_n \equiv \frac{\omega_b(T_{\rm nuc})}{\omega_s(T_{\rm nuc})}$)

[WA, Laurent & van de Vis ’23]

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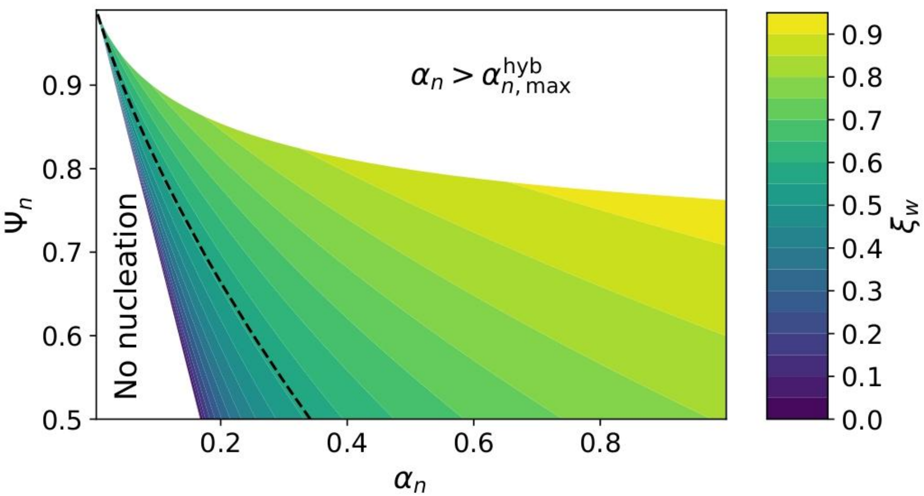
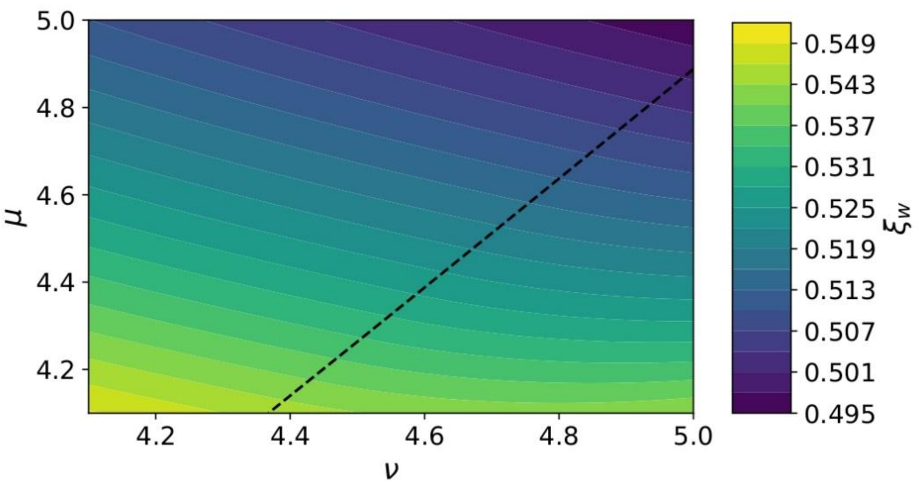


$(\mu = 4.1, \nu = 4.2)$



$(\alpha_n = 0.1, \Psi_n = 0.8)$

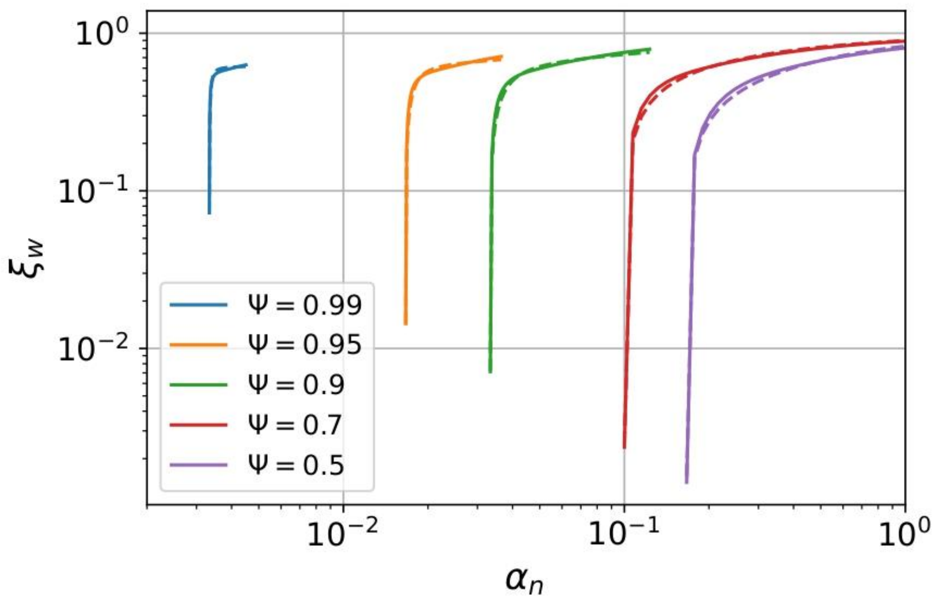
Fit for $\mu = 4, \nu = 4$

$$\xi_w^{\rm fit} = \left(|\xi_w^{\rm low}|^p + |\xi_w^{\rm high}|^p \right)^{1/p}$$

where

$$\xi_w^{\rm low} = \sqrt{\frac{3\alpha_n + \Psi_n - 1}{2(2 - 3\Psi_n + \Psi_n^3)}}, \qquad \xi_w^{\rm high} = v_J \left(1 - a \frac{(1-\Psi_n)^b}{\alpha_n} \right)$$

with $a = 0.2233, b = 1.704, p = -3.433$



Outline

- Total force on a bubble wall
- “Friction” in LTE and a new matching condition
- Bubble wall velocities in LTE
- Conclusions

Conclusions

- There can be an effective friction in LTE, but non-constant plasma temperature across the bubble wall is a necessary condition
- There is a new matching condition for the plasma hydrodynamic quantities in LTE $\gamma_+ T_+ = \gamma_- T_-$
- The new matching condition can be used to fully determine the wall velocity in LTE (only deflagration and hybrid solutions are "relevant")
- For $\mu = 4, \nu = 4$, we provided a fit formula for the wall velocity

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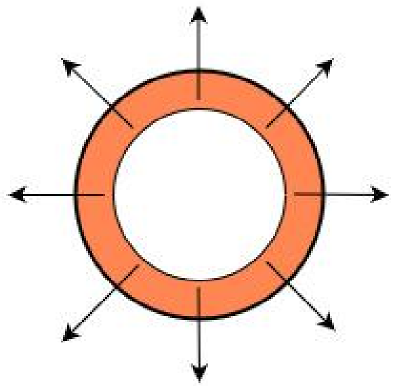
Thank you!

Backup slides

Detonations: equations



$v_w > c_s$

$$v_+ = v_w$$

$$T_+ = T_{\rm nuc}$$

$$\alpha_+ = \alpha_N \equiv \frac{\Delta\epsilon}{a_+ T_{\rm nuc}^4}$$

$$v_w v_- = \frac{1-(1-3\alpha_N)r}{3-3(1+\alpha_N)r}$$

$$\frac{v_w}{v_-} = \frac{3+(1-3\alpha_N)r}{1+3(1+\alpha_N)r}$$

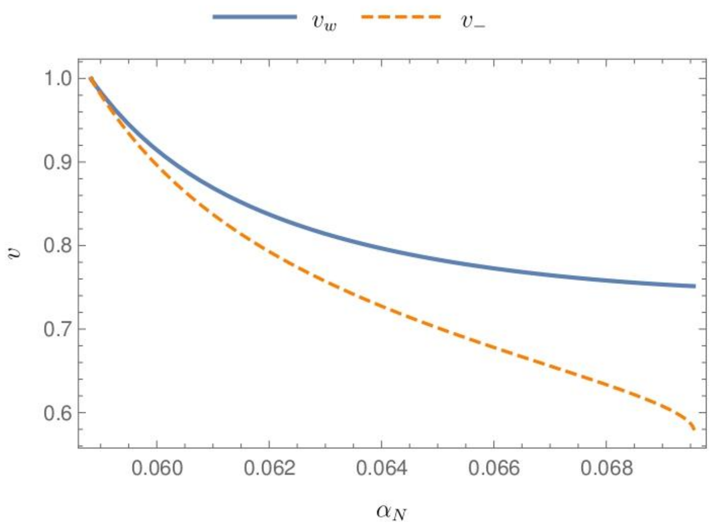
$$\gamma_w T_{\rm nuc} = \gamma_- T_-$$

$$r = \frac{a_+ T_{\rm nuc}^4}{a_- T_-^4}$$

$$v_- = \frac{1}{6v_w}\left[1-3\alpha_N+3(1+\alpha_N)v_w^2+\sqrt{(1-3\alpha_N+3(1+\alpha_N)v_w^2)^2-12v_w^2}\right]$$

$$v_- = v_w \frac{\left(\frac{\gamma_w}{\gamma_-}\right)^4 b + 3(1+\alpha_N)}{3\left(\frac{\gamma_w}{\gamma_-}\right)^4 b + (1-3\alpha_N)} \quad \text{where} \quad b = \frac{a_-}{a_+}$$

Detonations: numerical results

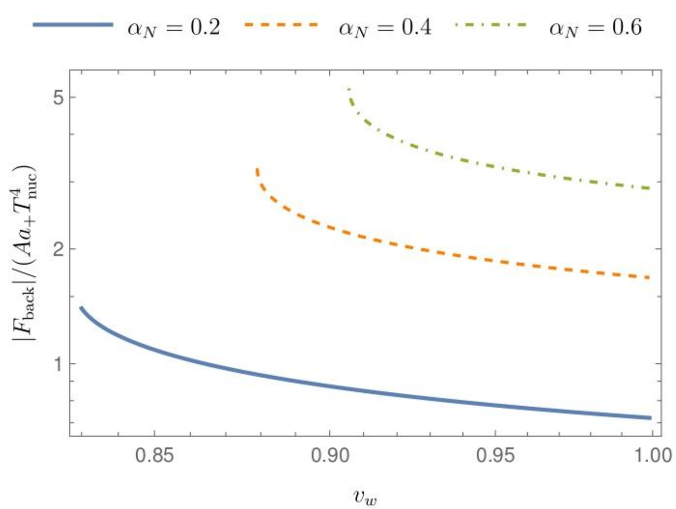


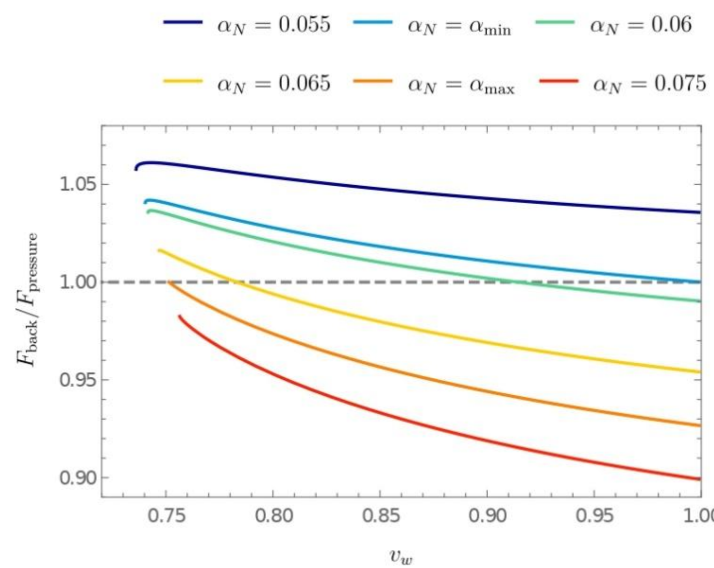
$$[\alpha_{\rm min} \approx 0.059, \alpha_{\rm max} \approx 0.07]$$

$$(b = 0.85\,)$$

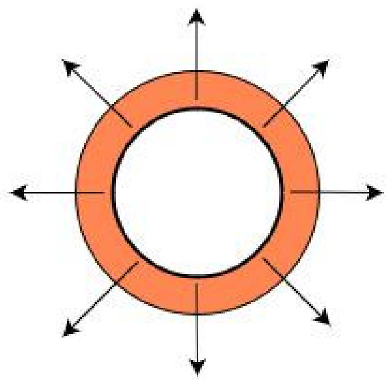
$$\Delta\mathcal{F} = (p_- - p_+)|_{T_{\rm nuc}} \geq 0 \Rightarrow \alpha_N \geq (1-b)/3 = 0.05$$

Balance of the forces:

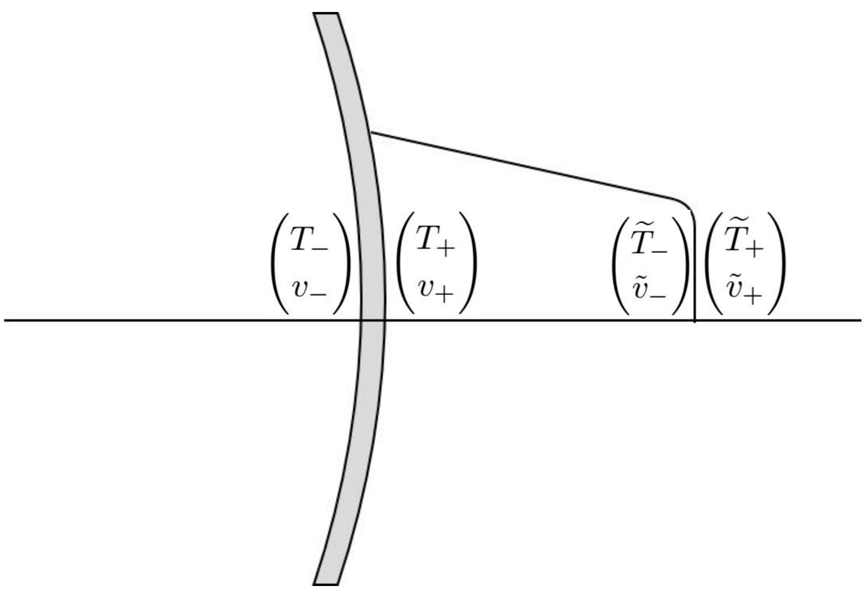




Deflagrations



$v_w < c_s$



$v_- = v_w \qquad \widetilde{T}_+ = T_{\rm nuc}$

$$v_- = v_w$$

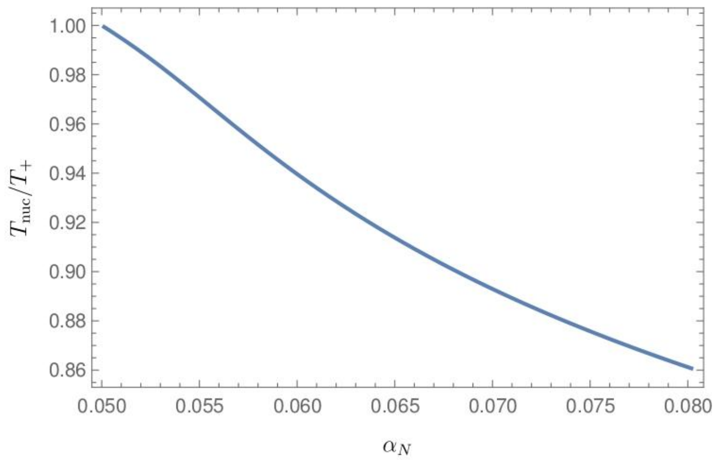
- Need to relate T_+, α_+ to $T_{\rm nuc}$ and α_N

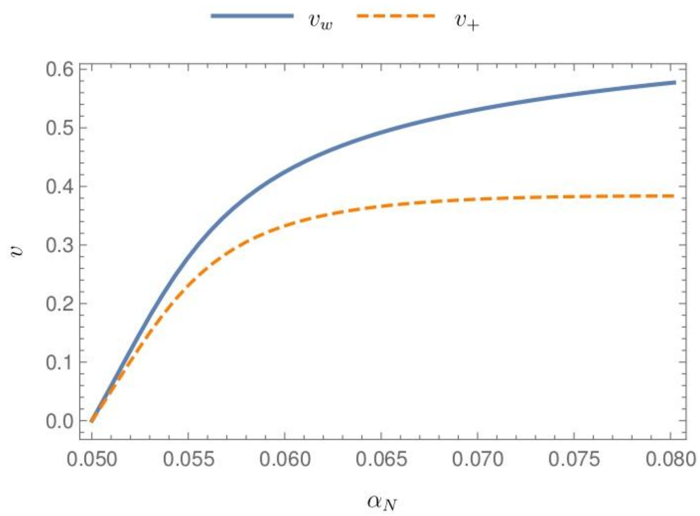
$$\alpha_+ = \frac{\Delta\epsilon}{a_+ T_+^4} = \frac{\Delta\epsilon}{a_{\rm nuc} T_{\rm nuc}^4} \frac{a_{\rm nuc} T_{\rm nuc}^4}{a_+ T_+^4} = \alpha_N \frac{q^4}{\tilde{b}}$$

$$\left(\tilde{b} = \frac{a_+}{a_{\rm nuc}}, \quad q = \frac{T_{\rm nuc}}{T_+} \right)$$

- Solve the fluid velocity and temperature profile away from the bubble wall
- Make use of the matching conditions for the shock-wave front

Deflagrations: numerical results





$[\alpha'_{\rm min} \approx 0.0500061, \alpha'_{\rm max} \approx 0.0789645]$

For $\alpha_N > \alpha'_{\rm max}$, no solutions exist for either deflagrations or detonations

Runaway for $\alpha_N > \alpha'_{\rm max}$ and in LTE