CRC-TR 211
Strong-interaction matter under extreme conditions

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Phase transition thermodynamic parameters at high precision[*]

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How fast does the bubble grow? – workshop
DESY, Hamburg, 05/2023

[*] P. Schicho, T. V. I. Tenkanen, and G. White, *Combining thermal resummation and gauge invariance for electroweak phase transition,* JHEP **11** (2022) 047 [2203.04284], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

- Baryogenesis — Baryon asymmetry of the universe
- Colliding bubbles — Gravitational wave (GW) production

In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

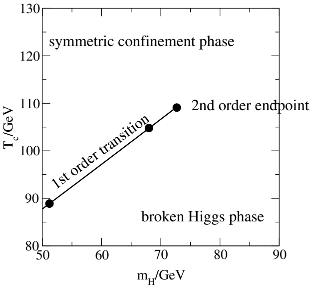

figure by M. Laine, *Electroweak phase transition beyond the standard model*, in *4th International Conference on Strong and Electroweak Matter*, pp. 58–69, 6, 2000 [hep-ph/0010275]

The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

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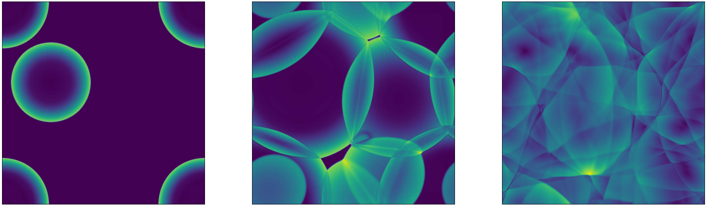
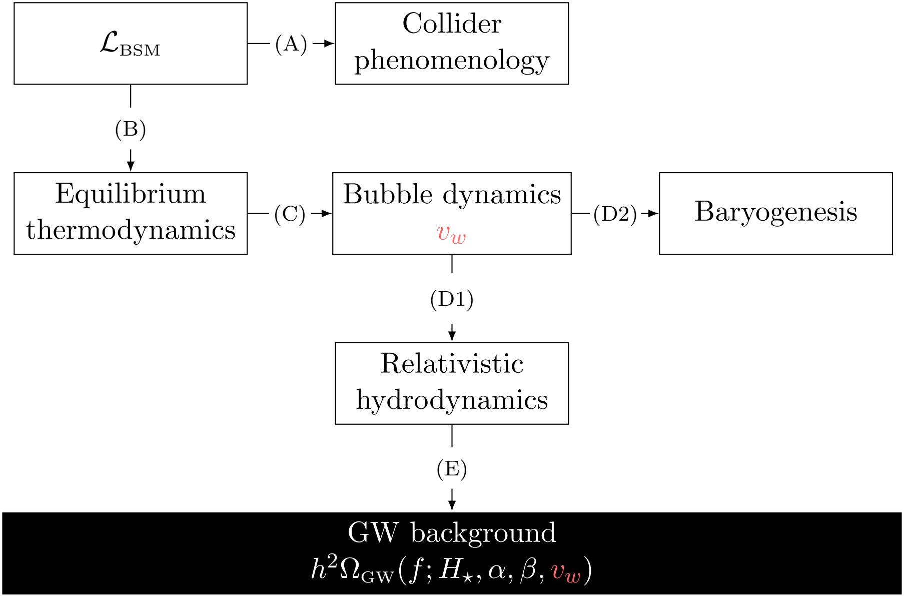
In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions. Study **BSM physics** near EW scale in context of phase transitions:

- Light fields strongly coupled to Higgs
- Collider targets. BSM testing pipeline: Collider phenomenology



figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

Uncertainties of the gravitational wave pipeline



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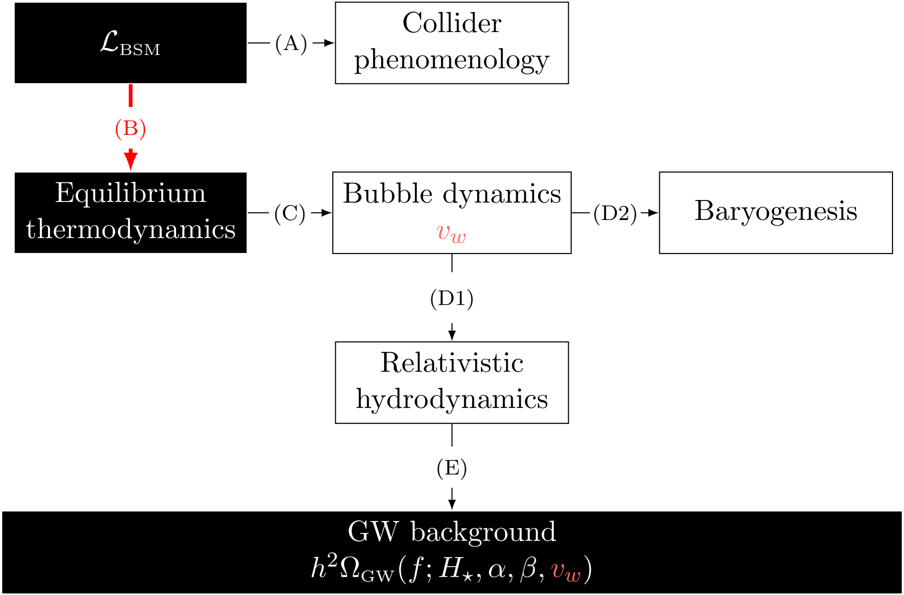
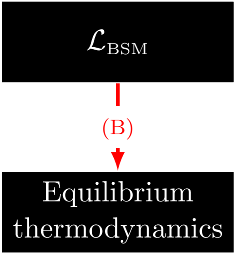
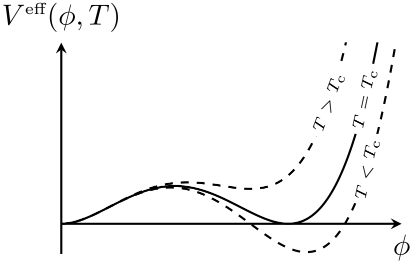


The effective potential in perturbation theory[1]





(B): Equilibrium thermodynamics as a function of BSM theory parameters. Encoded in **effective potential**, $V^{\rm eff}$. Origin of uncertainty.

Important for v_w in the context of local thermal equilibrium (LTE).

☆cf. **talks** by **B. Laurent Tue 10:45**, **O. Gould Wed 11:20**, and **J. Hirvonen Wed 14:40**

[1] R. Jackiw, *Functional evaluation of the effective potential*, Phys. Rev. D **9** (1974) 1686

Theoretical predictions are not robust

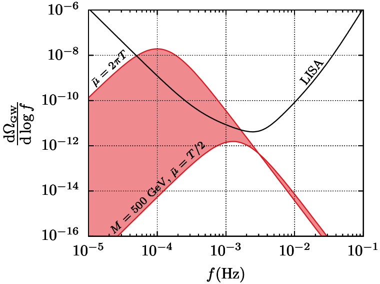
$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[2] as $\Omega_{\rm GW}$ depends strongly on the transition temperature, T_*, in simulation fits:

$$\Omega_{\rm GW} \propto \frac{(\Delta V_*)^2}{T_*^8}$$

- Ensure (improve) quantitative precision at finite T?

Minimal SM extensions e.g.:

- SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$



[2] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

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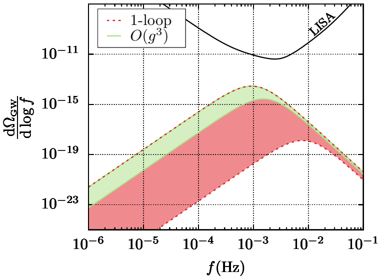
Minimal SM extensions e.g.:

- SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$
- xSM: SM + singlet



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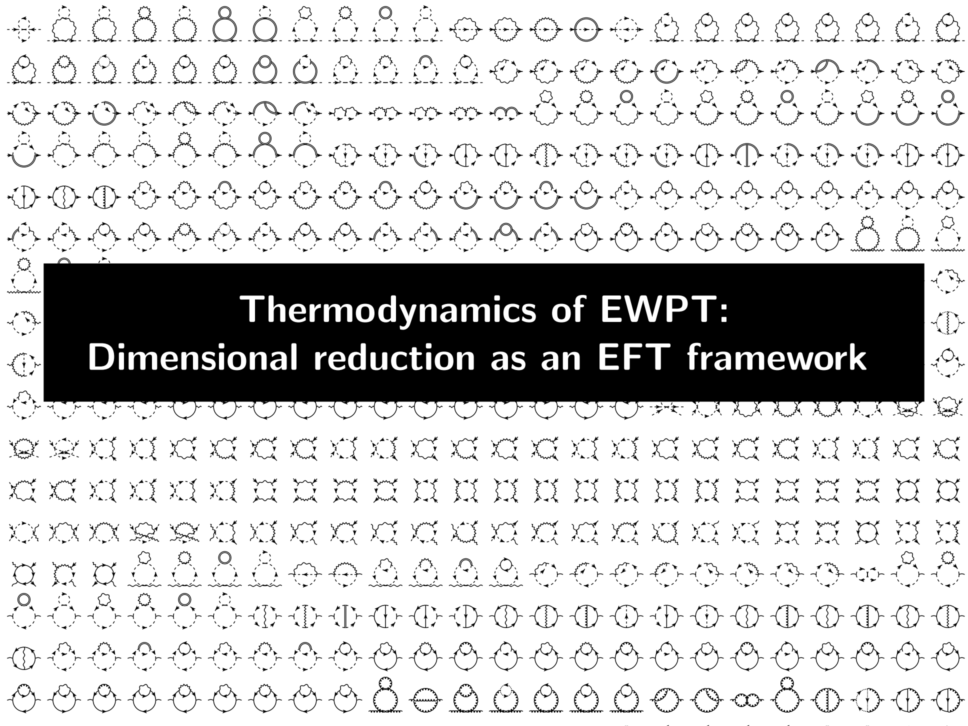
Thermodynamics of EWPT: Dimensional reduction as an EFT framework



Equilibrium Thermodynamics: Imaginary Time Formalism

$\rho(\beta) = e^{-\beta\mathcal{H}} \rightarrow \mathcal{U}(t) = e^{-i\mathcal{H}t}$. Relating density operator to time evolution corresponds to path integral over imaginary-time $t \rightarrow -i\tau$,

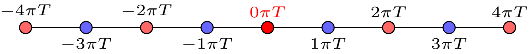
$$\mathcal{Z} = C \int_{\mathrm{b.c.}} \mathcal{D}\phi \, \exp\left[- \int_0^{\beta=1/T} \mathrm{d}\tau \int_{\mathbf{x}} \mathcal{L}_{\mathrm{E}} \right] , \quad \phi(0, \mathbf{x}) = \pm\phi(\beta, \mathbf{x}) \, .$$

(Anti-)periodic bosonic(fermionic) fields at boundaries $\rightarrow$ compact time direction: $\mathbb{R}^3 \times S^1_\beta$.

Finite-τ and (b.c.) induce a discrete Fourier sum for time component $P = (\omega_n, \mathbf{p})$ with **Matsubara frequencies**

$$\omega_n = \begin{cases} 2n\pi T & \text{bosonic} \\ (2n+1)\pi T & \text{fermionic} \end{cases}$$

Absent zero mode $\omega_{n=0}$ for fermions:



Multi-scale hierarchy in hot gauge theories

Evaluated **Matsubara sums** yield Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}}|p| \sim g^2/2$.

Theory separates scales rigorously:

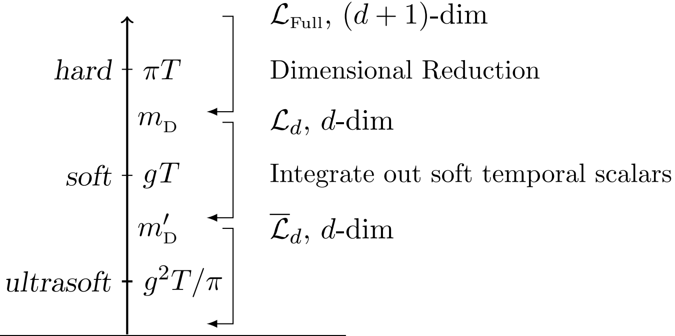
$$|p| \sim \begin{cases} \pi T & \textit{hard}\ \text{scale} \\ gT & \textit{soft}\ \text{scale} \\ g^{3/2}T & \boxed{\textit{supersoft}\ \text{scale} \quad \text{symmetry breaking}} \\ g^2 T/\pi & \textit{ultrasoft}\ \text{scale} \end{cases}$$

Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\mathrm{B}}(g^2 T) \sim \mathcal{O}(1)$. Ultrasoft bosons are non-perturbative at finite T: Linde IR problem.[3]

[3] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas,* Phys. Lett. B **96** (1980) 289

Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.
Incorporates an all order thermal resummation to by-pass IR problem.
Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[4] using e.g. `DRalgo`[5]

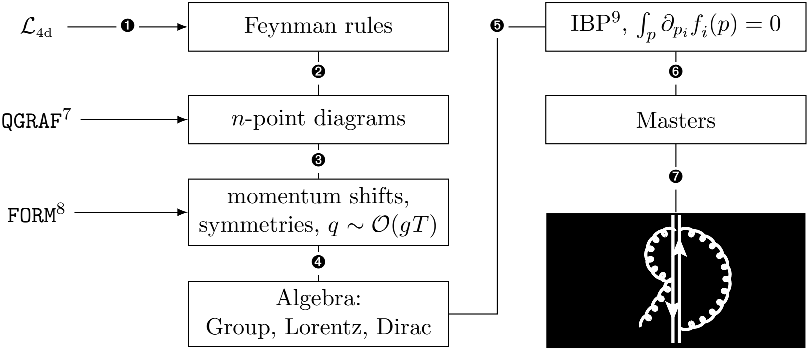


[4] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020]

[5] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

A Dimensional Reduction algorithm (DRalgo)

State-of-the-art Mathematica package DRalgo.[6]



[6] github.com/DR-algo/DRalgo, A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

[7] P. Nogueira, *Automatic Feynman Graph Generation,* J. Comput. Phys. **105** (1993) 279

[8] B. Ruijl, T. Ueda, and J. Vermaseren, *FORM version 4.2* arXiv (2017) [1707.06453]

[9] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations,* Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033]

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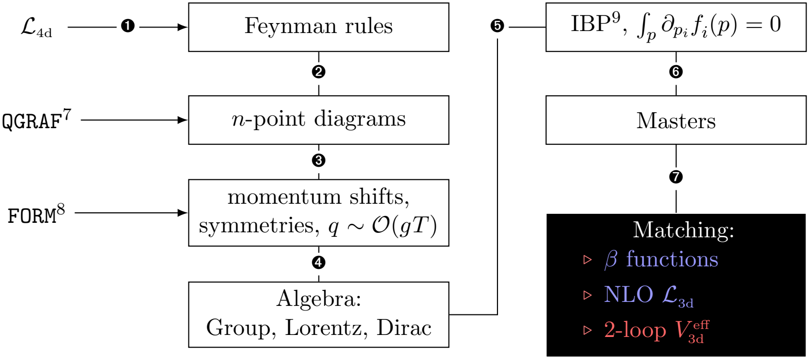

[6] `github.com/DR-algo/DRalgo`, A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

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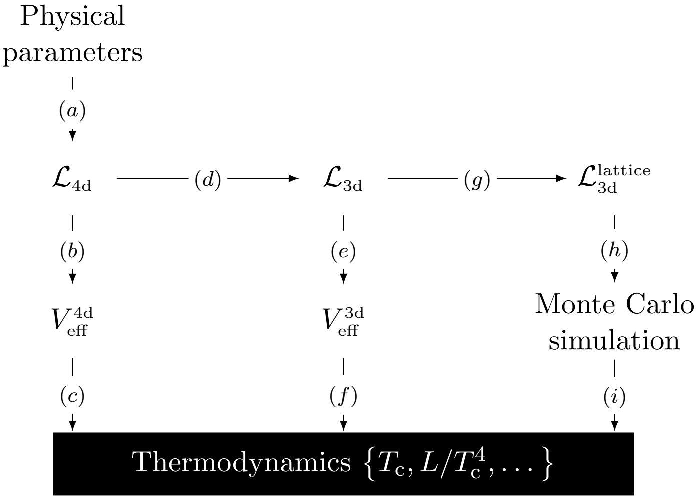
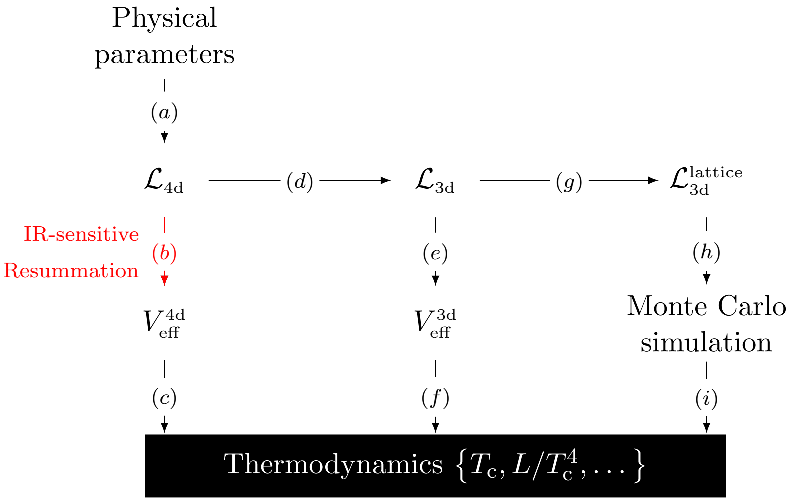
Thermodynamics of electroweak phase transition

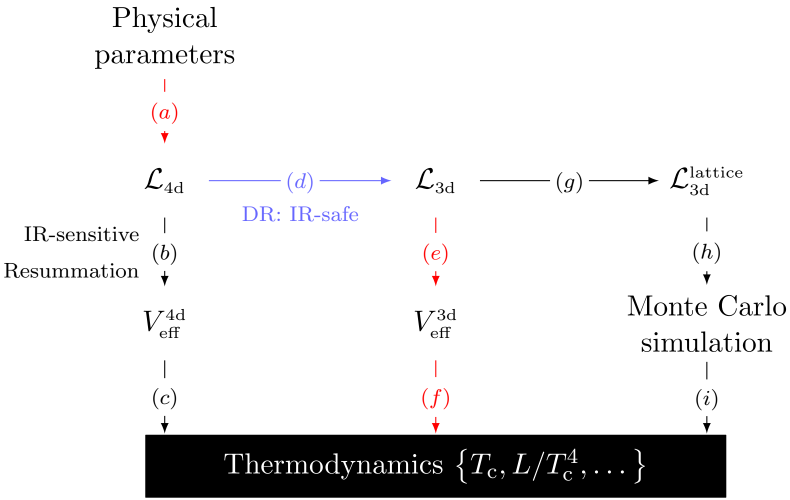


Thermodynamics of electroweak phase transition



▹ $4d$ approach: $(a) \to (b) \to (c)$

Thermodynamics of electroweak phase transition



- ▷ $4d$ approach: $(a) \to (b) \to (c)$
- ▷ Perturbative $3d$ approach: $(a) \to (d) \to (e) \to (f)$

Improving accuracy of EWPT: NLO dimensional reduction and beyond

Dimensionally reduced effective theory for hot QCD

QCD described by 3-dimensional super-renormalisable theory

$$S_{\rm EQCD} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}_{\rm EQCD} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\} .$$

"Electrostatic QCD" (EQCD) at high T with $D_i = \partial_i - i g_{\rm E} A_i$:

$$\mathcal{L}_{\rm EQCD} \equiv \frac{1}{2}{\rm Tr}\, F_{ij}F_{ij} + {\rm Tr}\,[D_i, A_0][D_i, A_0] + m_{\rm D}^2 {\rm Tr}\, A_0^2 + \lambda_{\rm E}({\rm Tr}\, A_0^2)^2 .$$

Developed to study

- high-T thermodynamics,[10]
- soft light-cone observables.[11]

[10] P. Ginsparg, *First and second order phase transitions in gauge theories at finite temperature,* Nucl. Phys. B **170** (1980) 388, T. Appelquist and R. D. Pisarski, *High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics,* Phys. Rev. D **23** (1981) 2305

[11] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603], J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]

EFT step 1: hot QCD $\to$ EQCD

DR step 1 fixes high-T EQCD. EFT for Electrostatic modes ($D_i = \partial_i - i g_{\rm E} A_i$), Describes hot QCD IR dynamics and contains UV in matching coefficients:[12]

$$g_{\rm E}^2 = \underbrace{\boxed{\text{tree-level}\; T^2 g^2}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\text{1-loop}\; \# g^4}}_{\mathcal{O}(g^4)} + \underbrace{\boxed{\text{2-loop}\; \# g^6}}_{\mathcal{O}(g^6)} + \underbrace{\boxed{\text{3-loop}\; \# g^8}}_{\mathcal{O}(g^8)} + \mathcal{O}(g^{10}) \,,$$

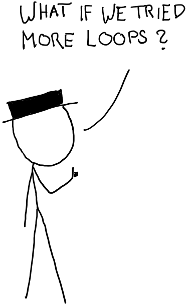
$$m_{\rm D}^2 = \underbrace{\boxed{\text{tree-level} + \text{1-loop}\; \# g^2 T^2}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\text{2-loop}\; \# g^4}}_{\mathcal{O}(g^4)} + \underbrace{\boxed{\text{3-loop}\; \# g^6}}_{\mathcal{O}(g^6)} + \mathcal{O}(g^8) \,,$$

$$\lambda_{\rm E} = \underbrace{\boxed{\text{tree-level}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\text{1-loop}\; \# g^4}}_{\mathcal{O}(g^4)} + \underbrace{\boxed{\text{2-loop}\; \# g^6}}_{\mathcal{O}(g^6)} + \mathcal{O}(g^8) \,.$$

[12] I. Ghişoiu, *Three-loop Debye mass and effective coupling in thermal QCD*, PhD thesis, Universität Bielefeld, Jan, 2013, I. Ghişoiu, J. Möller, and Y. Schröder, *Debye screening mass of hot Yang-Mills theory to three-loop order*, JHEP **2015** (2015) 121 [1509.08727], K. Kajantie, M. Laine, K. Rummukainen, and M. Shaposhnikov, *3d SU(N) + adjoint Higgs theory and finite temperature QCD*, Nucl. Phys. B **503** (1997) 357 [9704416]

Improving accuracy of dimensional reduction

- Increase precision of matching parameters: higher-loops

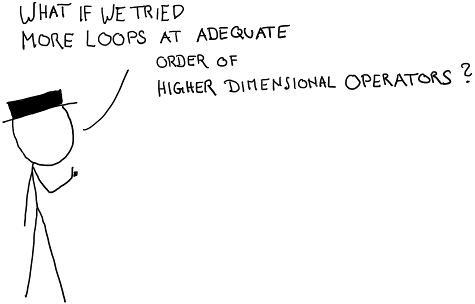


[13] T. Appelquist and J. Carazzone, *Infrared singularities and massive fields,* Phys. Rev. D **11** (1975) 2856, N. Landsman, *Limitations to dimensional reduction at high temperature,* Nucl. Phys. B **322** (1989) 498

Improving accuracy of dimensional reduction

- Increase precision of matching parameters: higher-loops
- Increase EFT validity: higher-dimensional operators.

These are related at finite temperature![13]



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Dimension-six operators in EQCD

1-loop sum-integral yields finite contributions

$$\sum\!\!\!\!\!\!\!\int_P^{\prime} \frac{1}{P^6} = \frac{\zeta_3}{128\pi^4 T^2}\left[1+\mathcal{O}(\epsilon)\right] .$$

Augment $\mathcal{L}_{\rm EQCD}$ by dim-6 operators[14] and colour trace in adjoint rep:[15]

$$\begin{aligned}\delta\mathcal{L}_{\rm EQCD}[A] = \left(\frac{2g_{\rm E}^2\zeta_3}{128\pi^4 T^2}\right) {\rm Tr}\Big\{ & c_1\,(D_\mu F_{\mu\nu})^2 + c_2\,(D_\mu F_{\mu 0})^2 \\ & + ig_{\rm E}\big[c_3\,F_{\mu\nu}F_{\nu\rho}F_{\rho\mu} + c_4\,F_{0\mu}F_{\mu\nu}F_{\nu 0} + c_5\,A_0(D_\mu F_{\mu\nu})F_{0\nu}\big] \\ & + g_{\rm E}^2\big[c_6\,A_0^2 F_{\mu\nu}^2 + c_7\,A_0 F_{\mu\nu}A_0 F_{\mu\nu} + c_8\,A_0^2 F_{0\mu}^2 + c_9\,A_0 F_{0\mu}A_0 F_{0\mu}\big] \\ & + g_{\rm E}^4\big[c_{10} A_0^6\big]\Big\} .\end{aligned}$$

Redundancies of coefficients leave physics invariant and are practical for cross-checks:

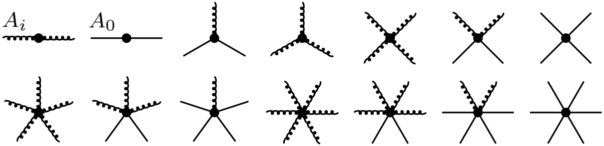
$$c_i^{\rm new} \equiv c_i + \delta c_i , \quad i = 4,\ldots,7 .$$

[14] S. Chapman, *New dimensionally reduced effective action for QCD at high temperature*, Phys. Rev. D **50** (1994) 5308 [9407313]

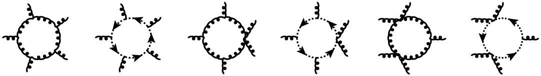
[15] ${\rm Tr}\,(AB) = A_{ab}B_{ba}$ with e.g. $(A_0)_{ab} = -if^{abc}A_0^c$ and $X^{abcd} = f^{m_4 a m_1}\cdots f^{m_3 d m_4}$ etc.

Vertex structures and matching

$\mathcal{L}_{\mathrm{EQCD}} + \delta\mathcal{L}_{\mathrm{EQCD}}$ is non-super-renormalisable.



Determine coefficients $c_i(d)$ in d-dimensions in background field gauge[16]. Evaluate (2–6)-point vertices at one-loop order in hot YM $\rightarrow$ uniqueness.



Done in d-dimensions for Yang-Mills.[17]
Todo: extend to general models.

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[16] L. Abbott, *The background field method beyond one loop,* Nucl. Phys. B **185** (1981) 189

[17] M. Laine, P. Schicho, and Y. Schröder, *Soft thermal contributions to 3-loop gauge coupling,* JHEP **2018** (2018) 37 [1803.08689]

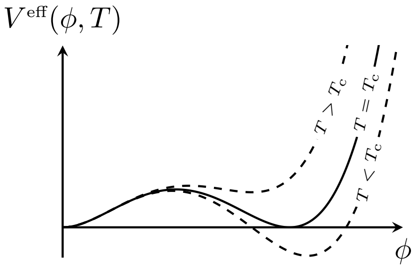
Improving accuracy of EWPT: Effective potential

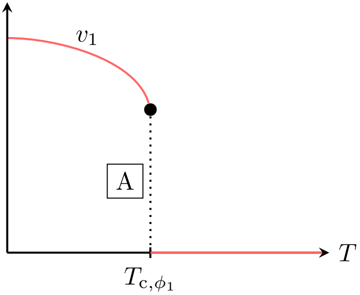
The effective potential in perturbation theory

receives thermal corrections $\Pi_T \sim \gamma T^2$ with $\gamma \sim g^n$. Close to critical temperature T_c:

$$V^{\text{eff}} \simeq \frac{1}{2}(-\mu^2 + \Pi_T)\phi^2 + \frac{1}{2}\lambda\phi^4 + \#\phi^3 + \dots$$

$$(-\mu^2 + g^n T^2) \sim \underbrace{0 \times (gT)^2}_{\text{soft}} + \underbrace{\#(g^2T)^2}_{\text{ultrasoft}} .$$





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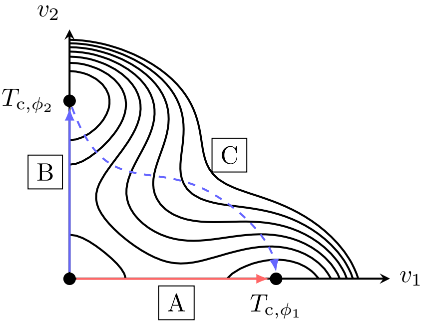
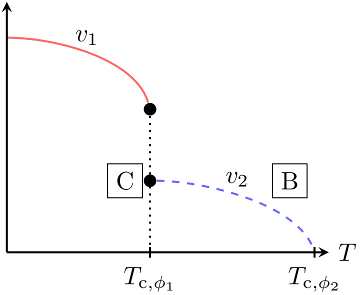
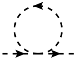
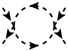
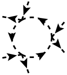




The thermal effective potential at LO

$$V^{\text{eff}} = V^{\text{eff}}_{\text{tree}} + V^{\text{eff}}_{1\ell}\,.$$

At 1-loop sum over n-point functions at $Q_i = 0$ external momenta

$V^{\text{eff}}_{1\ell} =$  $+ \frac{1}{2}$  $+ \frac{1}{3}$  $+ \ldots \Big|_{Q_i=0}$

$$= \frac{1}{2} \sum\!\!\!\!\!\!\!\int_P \ln\left(P^2 + m^2\right)$$

$$V^{\text{eff}}_{1\ell} = \underbrace{\frac{1}{2}\int_P \ln(P^2 + m^2)}_{\equiv V_{\text{CW}}(m)} \underbrace{-T\int_p \ln\Big(1 \mp n_{\text{B/F}}(E_p, T)\Big)}_{\equiv V_{T,b/f}\left(\frac{m^2}{T^2}\right)}$$

$$= \underbrace{\frac{T}{2}\int_p \ln(p^2 + m^2)}_{\equiv T V_{\text{soft}}(m)} + \underbrace{\frac{1}{2} \sum\!\!\!\!\!\!\!\int_{P/\{P\}}{}' \ln(P^2 + m^2)}_{\equiv V_{\text{hard}}(m)}\,.$$

Renormalization scale (in)dependence at finite T

Zero temperature

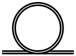
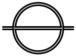
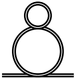
$$V^{\text{eff}}(\phi,\bar{\mu}) = \boxed{\underset{\mathcal{O}(g^2)}{V^{\text{eff}}_{\text{tree}}}} + \boxed{\underset{\mathcal{O}(g^4)}{V_{\text{CW},1\ell}}}\,, \qquad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\Big(V^{\text{eff}}_{\text{tree}} + V_{\text{CW},1\ell}\Big) = 0\,.$$

At finite temperature[18]

$$V^{\text{eff}}_{\text{res.}}(\phi,T,\bar{\mu}) = \boxed{\underset{\mathcal{O}(g^2)}{V^{\text{eff}}_{\text{tree}}}} + \boxed{\underset{\mathcal{O}(g^3)}{V_{\text{res.,soft}}}} + \boxed{\underset{\mathcal{O}(g^2T^2)+\mathcal{O}(g^4)}{V_{\text{hard}}}}\,,$$

running of 1-loop thermal masses is of the same order as 2-loop thermal-mass logarithms.

Automatically included in dimensionally reduced $3d$-approach:

$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}$  $\sim \mu\frac{\mathrm{d}}{\mathrm{d}\mu}$  $\sim$  $\sim$  $\sim \mathcal{O}(g^4T^2)$

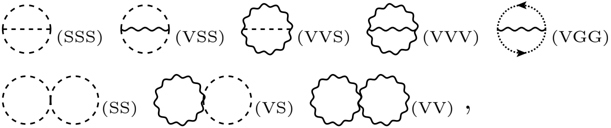
[18] O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

The effective potential at NLO and beyond

$$V^{\text{eff}} = V^{\text{eff}}_{\text{tree}} + V^{\text{eff}}_{1\ell} + V^{\text{eff}}_{2\ell} + V^{\text{eff}}_{3\ell} \ .$$

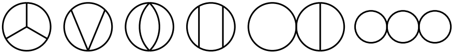
Computing 2-loop[19] and 3-loop[20] V^{eff} via vacuum integrals in $3d$ EFT:

$V^{\text{eff}}_{2\ell} \supset$



,

$V^{\text{eff}}_{3\ell} \supset$



.

Todo: extend 3-loop V^{eff} to general models.

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[19] K. Farakos, K. Kajantie, K. Rummukainen, and M. E. Shaposhnikov, *3-D physics and the electroweak phase transition: Perturbation theory,* Nucl. Phys. B **425** (1994) 67 [hep-ph/9404201], M. Laine, *The Two loop effective potential of the 3-d SU(2) Higgs model in a general covariant gauge,* Phys. Lett. B **335** (1994) 173 [hep-ph/9406268], L. Niemi, M. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Thermodynamics of a two-step electroweak phase transition,* [2005.11332], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops,* Phys. Rev. D **103** (2021) 115035 [2103.07467]

[20] A. K. Rajantie, *Feynman diagrams to three loops in three-dimensional field theory,* Nucl. Phys. B **480** (1996) 729 [hep-ph/9606216]

A minimal scheme for gauge invariance and resummation

1. Determine 3d EFT at NLO (gauge-invariant)
2. Compute V_{3d}^{eff} within 3d EFT at 1-loop level
3. Determine T_c, condensates e.g. $\langle\phi^\dagger\phi\rangle$, and latent heat

Minimum of V^{eff} is gauge parameter independent (Nielsen identities[21]); use $\hbar$-expansion. Improve previous schemes.[22]

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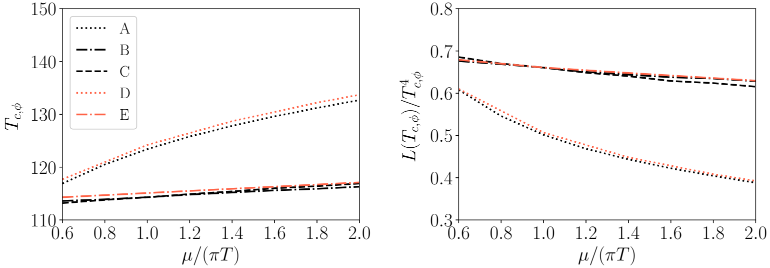
[21] N. Nielsen, *On the gauge dependence of spontaneous symmetry breaking in gauge theories*, Nucl. Phys. B **101** (1975) 173

[22] PRM scheme: H. H. Patel and M. J. Ramsey-Musolf, *Baryon Washout, Electroweak Phase Transition, and Perturbation Theory*, JHEP **2011** (2011) 29 [1101.4665]

Increasing accuracy to $\mathcal{O}(g^4)$: cxSM (complex singlet)

Augment SM with **complex singlet scalar**[23], $\mathbb{S} \to v_{\mathbb{S}} + \mathbb{S} + iA$ at

Benchmark	$M_{\mathbb{S}}$	M_A	λ_p	$\lambda_{\mathbb{S}}$
BM1	62.5 GeV	62.5 GeV	0.55	0.5



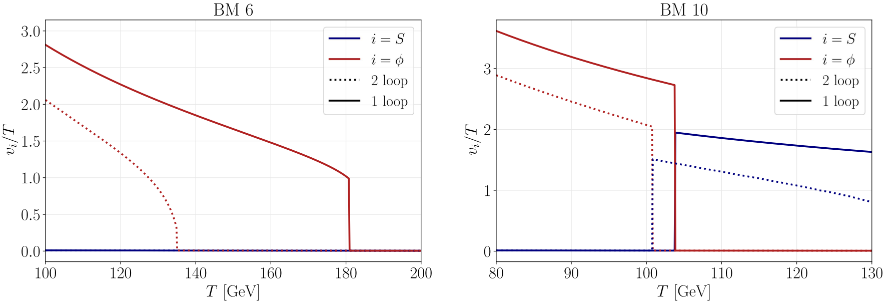
- A, D: 1-loop level dimensional reduction
- B, E: 2-loop level dimensional reduction
- C: as B, with varying $\mu_3 = \mu/(\pi T) g_3^2$

[23] P. Schicho, T. V. I. Tenkanen, and G. White, *Combining thermal resummation and gauge invariance for electroweak phase transition*, JHEP **11** (2022) 047 [2203.04284], C.-W. Chiang, M. J. Ramsey-Musolf, and E. Senaha, *Standard Model with a Complex Scalar Singlet: Cosmological Implications and Theoretical Considerations*, Phys. Rev. D **97** (2017) 1 [1707.09960]

Transitions in the xSM (real singlet)

Monitor Higgs (v) and **real singlet** (x) VEV after shift $\mathbb{S} \to x + \mathbb{S}$. 2-loop corrections are significant.[24]

Benchmark	$M_{\mathbb{S}}$	λ_p	$\lambda_{\mathbb{S}}$
BM6	350 GeV	3.5	0.3
BM10	325 GeV	3.5	0.3



[24] Plots courtesy of Daniel Schmitt as well as L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Conclusions

Precision thermodynamics of BSM theories:

- reliably describe cosmological FOPT and GW production,
- practical approach: **Effective Theories**.

Precision cosmology with dimensionally reduced 3d EFT:

- multi-loop sport – automatic all-order high-T resummation,
- analytic fermions, numerical on the lattice at $T_{\rm c} \sim 100$ GeV,
- systematic higher-loop/operator improvement,
- **universality**,
- apply to **supercooled phase transitions**(?),
- accurate description of the phase transition.[a]

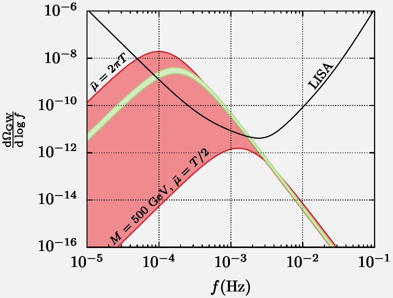
[a] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions,* JHEP **04** (2021) 055 [2009.10080]

Conclusions

Precision thermodynamics of BSM theories:

- reliably describe cosmological FOPT and GW production,
- practical approach: **Effective Theories**.

Precision cosmology with dimensionally reduced 3d EFT:

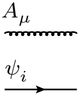


Overfull hbox (badness 10000)

Differences to zero temperature

$(d+1) \to d$-dimensional theory with infinite tower of massive modes.

Euclidean free-particle propagator for bosonic(fermionic) fields:



$$\left.\right\} \propto \frac{1}{P^2+m^2} = \frac{1}{\mathbf{p}^2+\omega_n^2+m^2}\,, \qquad P = (\omega_n, \mathbf{p})\,.$$

Same diagrams as in zero temperature QFT, go over to Euclidean space, and substitute Euclidean frequency integrals by sums

$$\int \frac{\mathrm{d}^{d+1}p}{(2\pi)^{d+1}} f(p) \to T\sum_n \int \frac{\mathrm{d}^d p}{(2\pi)^d} f(\omega_n, \mathbf{p}) = \sum\!\!\!\!\!\!\!\int_P f(\omega_n, \mathbf{p})\,.$$

- ▹ Ultraviolet (UV) contained at $T=0$
- ▹ Infrared (IR) sensitivity worsened $\to$ field in reduced spacetime dimension

Resummation

Dynamically generated masses through collective plasma effects

$$m_T = g^n T + m\ .$$

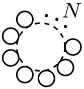
Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|} \geq \frac{g^2 T}{m}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}} |p| \sim g^2/2$.

Cure IR-sensitive contributions at $m_T \sim gT$ by thermal resummation:

$$V^{\mathrm{eff}} \supset$$



$$\propto \quad g^{2N} \left[{m_T}^{3-2N} T \right] \left[\frac{T^2}{12} \right]^N \propto m^3 T \left[\frac{gT}{m_T} \right]^{2N}$$

For $m_T \leq g^2 T$ weak expansion breaks down. At finite T, light bosons are non-perturbative.

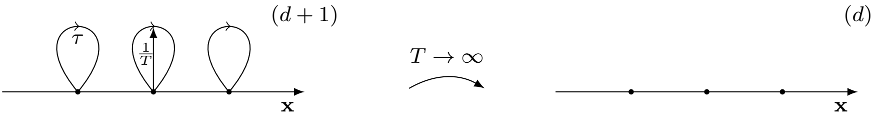
Effective Theory (EFT): Definition

Framework for theory with scale hierarchy: Effective Field Theory.

1. Identify soft degrees of freedom.
2. Construct most general low-energy Lagrangian.
3. *Match* Green's functions $\rightarrow$ determine EFT coupling constants.

Perturbative and IR safe: Matching in the IR (regime of mutual validity), UV incorporated in EFT coefficients.

Modes with wavelengths $|\mathbf{x}|, |x_0| \gg \beta$ or $\omega_n^2 + m^2 \ll T^2$ *effectively* live in 3-dimensions.



EFT step 2: EQCD → MQCD

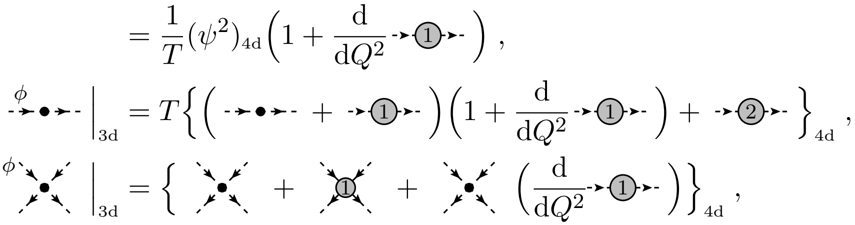
DR step 2 fixes high-T MQCD. EFT for Magnetostatic modes aka 3d pure Yang-Mills ($D_i = \partial_i - i g_{\rm M} A_i$). Describes EQCD IR dynamics and contains UV in matching coefficients:[25]

$$g_{\rm M}^2 = + \underbrace{\boxed{\overset{\text{1-loop}}{\# \frac{g_{\rm E}^4}{m_{\rm D}}}}}_{\mathcal{O}(g^3)} + \boxed{\overset{\text{2-loop}}{\# \frac{g_{\rm E}^6}{m_{\rm D}^2}}} + \boxed{\overset{\text{3-loop}}{\# \frac{g_{\rm E}^8}{m_{\rm D}^3}}} + \dots ,$$

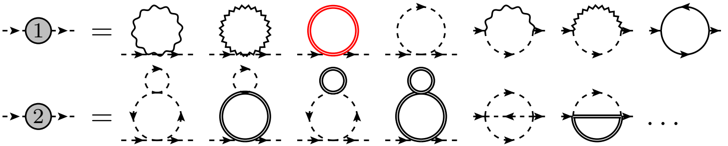
$$\mathcal{L}_{\rm MQCD} = \mathcal{L}_{\rm 3d\ Yang\text{-}Mills} \equiv \frac{1}{2} {\rm Tr}\, F_{ij} F_{ij} \ .$$

[25] M. Laine and Y. Schröder, *Two-loop QCD gauge coupling at high temperatures,* JHEP **03** (2005) 067 [hep-ph/0503061], M. Laine, P. Schicho, and Y. Schröder, *Soft thermal contributions to 3-loop gauge coupling,* JHEP **2018** (2018) 37 [1803.08689]

$$(\psi^2)_{3d} = \frac{1}{T}(\psi^2)_{4d} Z_\psi^{-1}$$



where



The Nielsen identities[26]

(A useful tool for showing gauge invariance.) Vary effective potential with gauge parameter

$$\xi \frac{\partial S^{\text{eff}}}{\partial \xi} = -\int_{\mathbf{x}} \frac{\delta S^{\text{eff}}}{\delta \phi(x)} \, \mathcal{C}(x) \, ,$$

use derivative expansion of functional

$$\mathcal{C}(x) = C(\phi) + D(\phi)(\partial_\mu \phi)^2 - \partial_\mu \big(\tilde{D}(\phi)\partial_\mu \phi\big) + \mathcal{O}(\partial^4)$$

results in Nielsen identities

$$\xi \frac{\partial}{\partial \xi} V^{\text{eff}} = -C \frac{\partial}{\partial \phi} V^{\text{eff}} \, , \qquad (1)$$

$$\xi \frac{\partial}{\partial \xi} Z = -C \frac{\partial}{\partial \phi} Z - 2Z \frac{\partial}{\partial \phi} C - D \frac{\partial}{\partial \phi} V^{\text{eff}} - \tilde{D} \frac{\partial^2}{\partial \phi^2} V^{\text{eff}} \, . \qquad (2)$$

Identity (1): EWSB is gauge invariant.

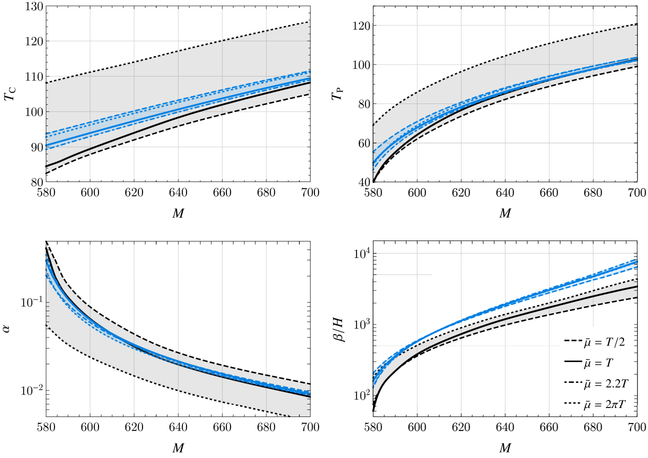
[26] N. Nielsen, *On the gauge dependence of spontaneous symmetry breaking in gauge theories*, Nucl. Phys. B **101** (1975) 173

Increasing accuracy to $\mathcal{O}(g^4)$: SMEFT

Also include dim-6 operator in full SM → "SMEFT"

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{M^2}(\phi^\dagger\phi)^3$$

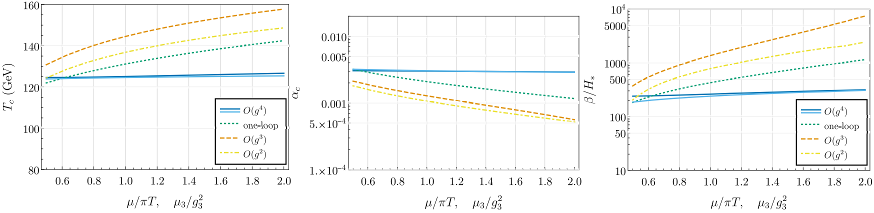
Dependence on $\bar{\mu}$ in the 3d approach and the 4d approach



Increasing accuracy to $\mathcal{O}(g^4)$: xSM

Augment SM with real singlet scalar[27] at

$$\text{BM1}: \quad \{M_\sigma, \lambda_m, \lambda_\sigma\} = \{160\ \text{GeV}, 1.1, 0.45\}$$



[27] O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions,* JHEP **06** (2021) 069 [2104.04399], P. M. Schicho, T. V. I. Tenkanen, and J. Österman, *Robust approach to thermal resummation: Standard Model meets a singlet,* JHEP **06** (2021) 130 [2102.11145], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops,* Phys. Rev. D **103** (2021) 115035 [2103.07467]