Hydrodynamic backreaction forces in expanding bubbles

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The aim:

Provide **new insights** and **explicit calculations** for hydrodynamic effects leading to **friction-like behaviour in local equilibrium**

The novelty:

We **relate previous results** in the literature and provide a new **understanding** in terms of **entropy conservation**

We confirm directly the friction effect by studying time-dependent solutions, and **relate local friction** to the **field-dependence of enthalpy**

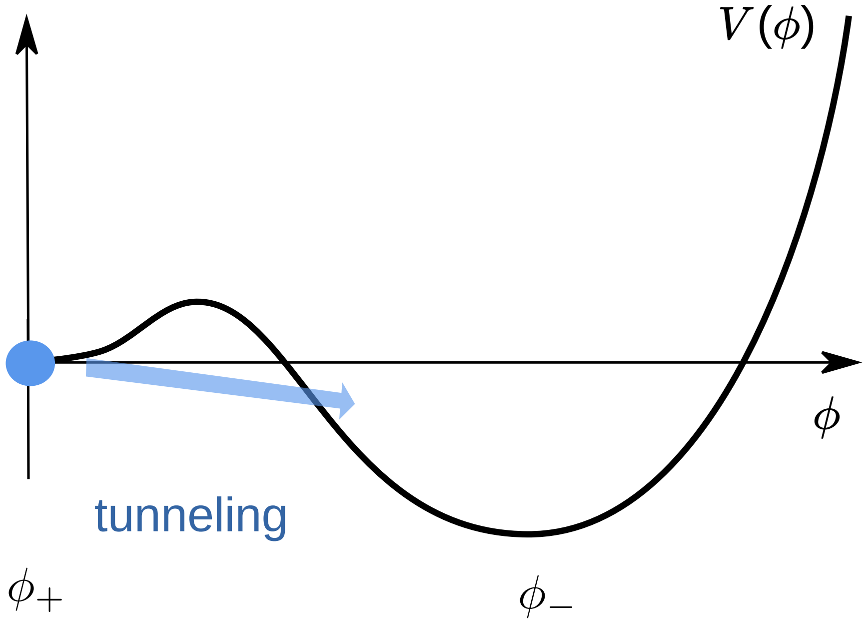
We also illustrate the effect for **detonations** in the wall frame, for which the **friction decreases with the wall velocity**

The plan:

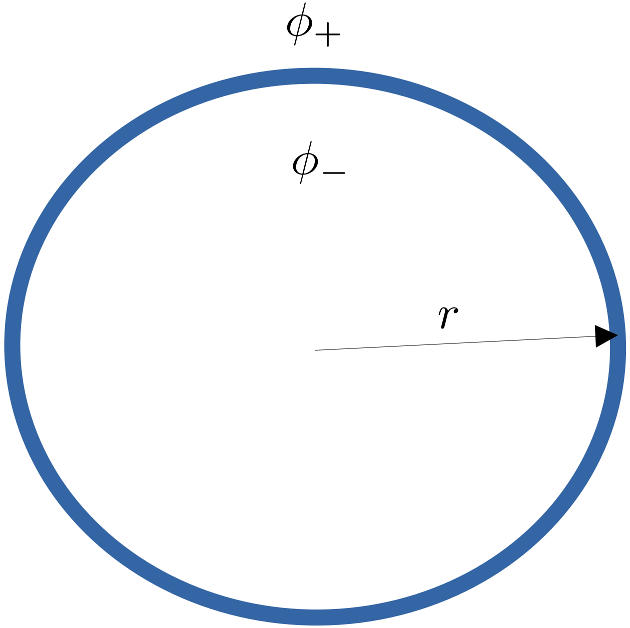
Usual understanding of bubble friction
Friction in local equilibrium: previous literature
Friction in local equilibrium from local stress-energy conservation

Usual understanding of bubble friction

Vacuum bubbles



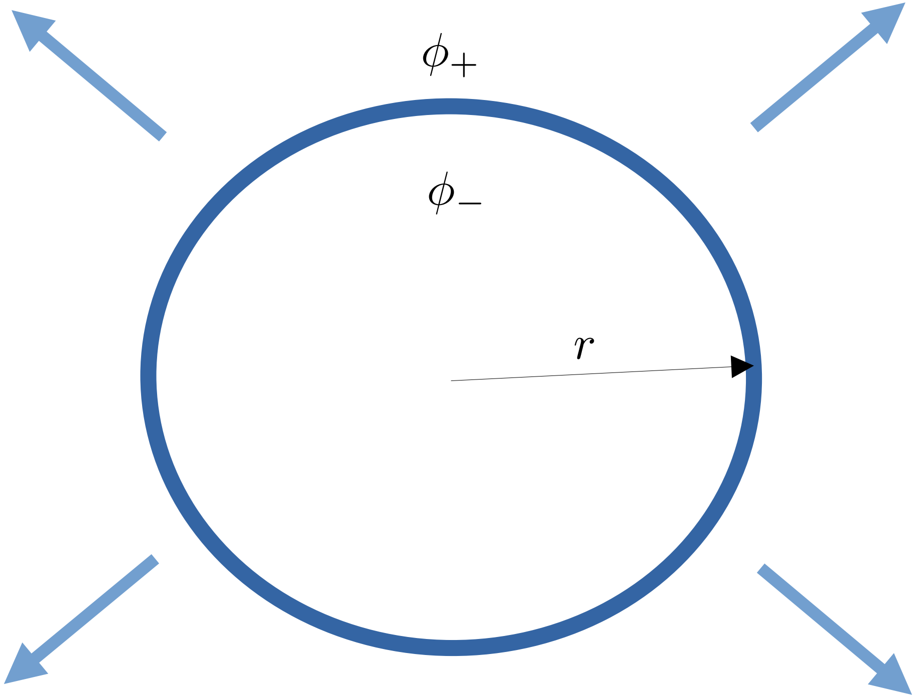
Vacuum bubbles



$$V(\phi_+) - V(\phi_-) \sim \frac{\Delta E}{\Delta V} \sim \frac{(\text{Force})(\Delta r)}{\Delta V} \sim -\Delta p > 0$$

→ **Accelerated expansion**

Vacuum bubbles



$$V(\phi_+) - V(\phi_-) \sim \frac{\Delta E}{\Delta V} \sim \frac{(\mathrm{Force})(\Delta r)}{\Delta V} \sim -\Delta p > 0$$

→ **Accelerated expansion**

Vacuum bubbles

- In the vacuum, the **scalar equation of motion** is:

$$\Box\phi + \frac{\partial V(\phi)}{\partial\phi} = 0$$

- **Lorentz invariance** of equation allows for **bubble solutions invariant under boosts**:

$$\phi(\vec{x}, t) = \phi(x^2) = \phi(t^2 - r^2)$$

- A bubble front with $\phi = const$ is localized at $t^2 - R^2_{\rm bubble} = const'$

$$R_{\rm bubble} = \sqrt{t^2 - {\rm const}'^2}$$

▶ **Uniformely accelerated motion**, approaching asymptotically the **speed of light**

Bubbles in a plasma

- Energy considerations have to be based on the **free-energy density**

$$Z[T \equiv 1/\beta] = \mathrm{Tr}(e^{-\beta H}) = e^{-\beta F} = e^{-\beta V f}$$

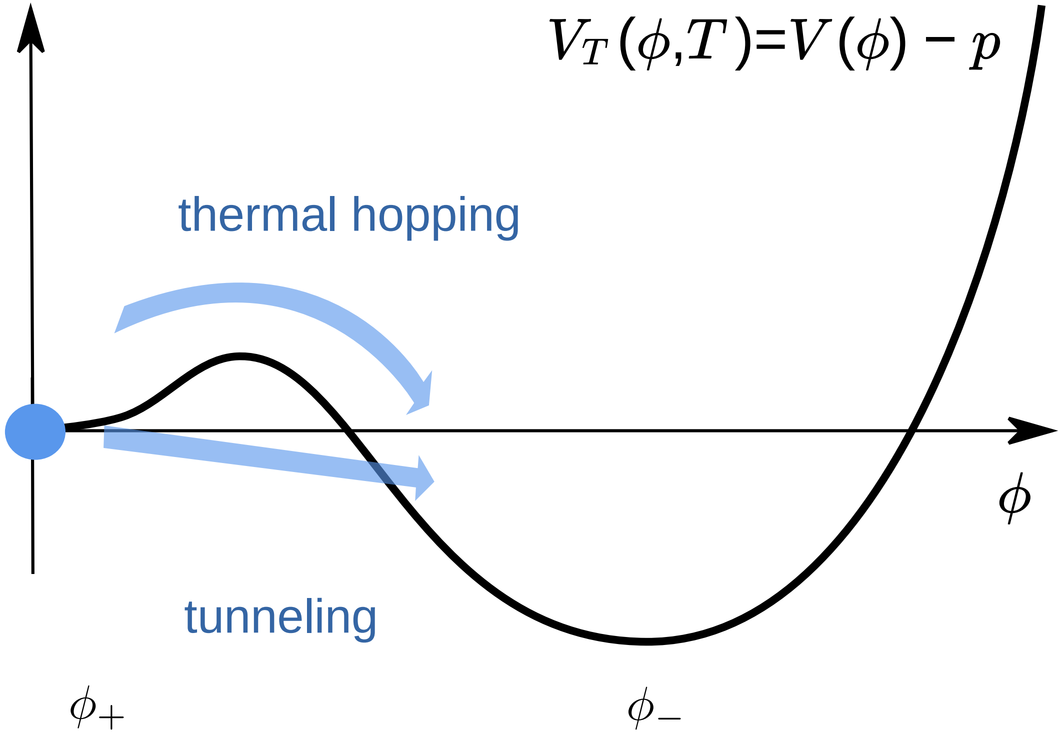
- Finite T field theory relates free-energy in background $\bar{\phi}$ to **effective potential**

$$Z[\bar{\phi}, T] = e^{-\beta V f} = \int_{\delta\phi(\tau)=\delta\phi(\tau+\beta)} \mathcal{D}\delta\phi e^{-S_{E,0\leq\tau<\beta}[\phi]}[\bar{\phi}+\delta\phi] = e^{-\beta V V_T(\bar{\phi},T)}$$

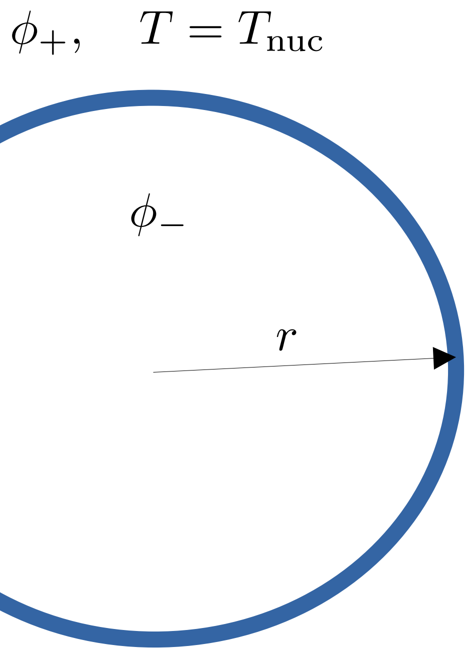
- Thermodynamics relates free-energy density to **pressure**

$$\left.\begin{array}{r} dU = TdS - pdV \\ F = U - TS \end{array}\right\} dF = SdT - pdV \Rightarrow f \equiv \left.\frac{\partial F}{\partial V}\right|_T = -p = V_T(\bar{\phi}, T)$$

Bubbles in a plasma



Bubbles in a plasma



$$V_T(\phi_+, T) - V_T(\phi_-, T) \sim -\Delta p > 0$$ → **Accelerated expansion in equilibrium?**

Friction from beyond-equilibrium effects

- The usual treatment is based on the **scalar equation** of motion, **averaged** in plasma

$$\Box\phi + \frac{\partial V(\phi)}{\partial \phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i} f_i(\mathbf{p}, x) = 0$$

[Prokopec-Moore ’95]

- For particles in equilibrium one recovers the finite T effective potential

$$f_i(\mathbf{p}, x) = f_i^{\mathrm{eq}}(\mathbf{p}) + \delta f_i(\mathbf{p}, x)$$

$$\Box\phi + \frac{\partial V_T(\phi, T)}{\partial \phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i} \boldsymbol{\delta}\mathbf{f_i(p, x)} = 0$$

Friction effect from deviations of equilibrium

Where could friction come from?

- In a **finite-temperature** medium in **equilibrium**, the scalar equation of motion is:

$$\Box\phi + \frac{\partial V_T(\phi, T)}{\partial\phi} = 0$$

Still looks **Lorentz invariant** at first sight → **no friction naively expected**

- For friction to appear, one generally expects terms of the form:

$$\Box\phi + \frac{\partial V_T(\phi, T)}{\partial\phi} + \eta\, u^\mu \partial_\mu \phi = 0$$

“background” u^μ **breaks Lorentz invariance** → **friction possible**

Friction force per unit area

$$\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

Friction force per unit area

$$\int dz \frac{d\phi}{dz}\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

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(In the static wall frame)

$$\Delta V_T = -\sum_i \int d\phi \frac{dm_i^2}{d\phi}\int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)$$

Driving force

Friction per unit area, out of eq.
[Bödeker-Moore]

Friction force per unit area

$$\int dz \frac{d\phi}{dz}\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

(In the static wall frame)

$$\Delta V_T = -\sum_i \int d\phi \frac{dm_i^2}{d\phi}\int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)$$

Driving force

Friction per unit area, out of eq.
[Bödeker-Moore]

- Alternatively, assuming an **ultrarrelativistic wall**, *f* does not change (no reflection)

$$\Delta V_{\rm vac} = -\sum_i \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_i(\mathbf{p},z)\Delta p_z$$

Vacuum driving force

Total force from plasma, incl. friction
[Bödeker-Moore]

Friction in local equilibrium?

- It would seem that **constant v_w< c** for $T < T_c$ requires **non-equilibrium effects.** For **relativistic bubbles:**
  - **Leading order** friction v_w -independent: allows **runaways** [Bödeker-Moore]
  - Higher order effects v_w-dependent: ultrarelativistic but **subluminal** speeds

    [Bödeker-Moore] [Höche, Kozaczuk, Long, Turner, Wang
    [Gouttenoire, Jinno, Sala]
- It has been **commonly assumed** that there is **no friction in local equilibrium**

Hydrodynamic effects

- The previous reasoning focused only on the scalar equation of motion, and did not account for **hydrodynamic effects**

- These can be incorporated by modelling the plasma as a perfect fluid and demanding **stress-energy conservation**

$$T^{\mu\nu}_{\text{plasma}} = (\rho + p)u^\mu u^\nu - p\eta^{\mu\nu} \equiv \omega u^\mu u^\nu - p\eta^{\mu\nu}$$

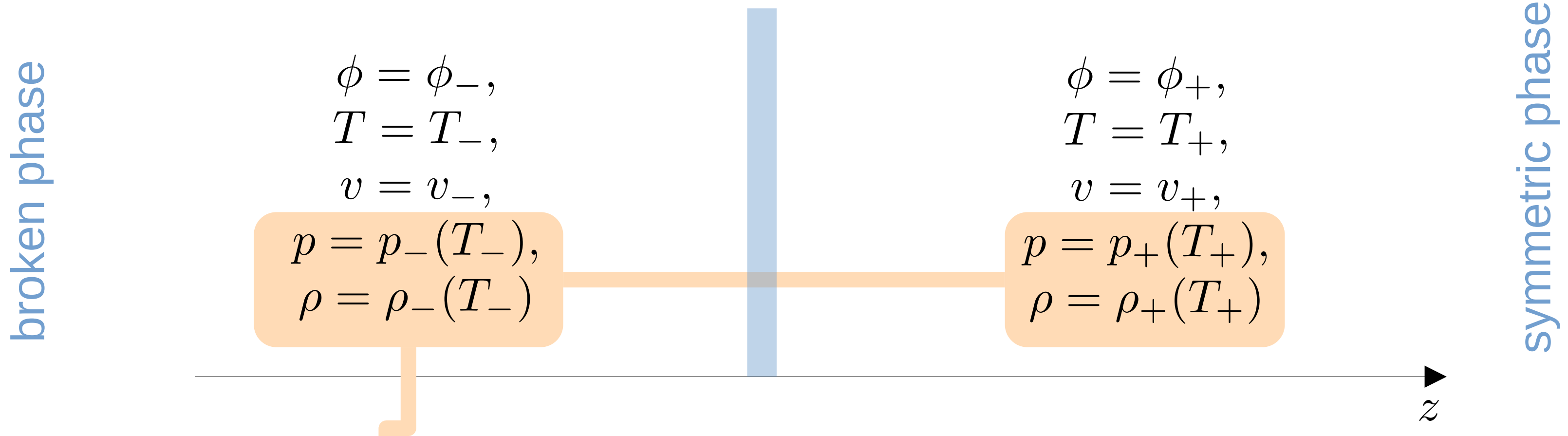
enthalpy

- Away from the bubble front, the scalar field settles to a constant (and so does $T^{\mu\nu}_\phi$)

$$\nabla_\mu T^{\mu\nu}_{\text{plasma}} = 0$$

Hydrodynamic effects without scalar

- Planar wall, static frame with bubble propagating in z+ direction, $v \equiv v^z$.



$$\phi = \phi_-, \quad T = T_-, \quad v = v_-, \quad p = p_-(T_-), \quad \rho = \rho_-(T_-)$$

$$\phi = \phi_+, \quad T = T_+, \quad v = v_+, \quad p = p_+(T_+), \quad \rho = \rho_+(T_+)$$

- Assuming an equation of state in each phase relates p, ρ, ω to T

Hydrodynamic effects without scalar

- 5 plasma unknowns: v_w, v_+, v_-, T_+, T_-

- 2 constraints from boundary conditions:

  fluid at rest far from the bubble: fixes v_w from v_+, v_-

  T matches T$_{\text{nuc}}$ far from the bubble

- 2 matching conditions from stress-energy conservation

$$\nabla_\mu T^{\mu\nu}_{\text{plasma}} = 0 \qquad \begin{array}{c}\nu = z,\\ \nu = 0\end{array}$$

Unconstrained system! Need to **solve scalar e.o.m.**

Hydrodynamic + scalar effects

- Equation of state fixed on a scalar background
- 7 plasma/scalar unknowns: $v_w, v_+, v_-, T_+, T_-, \phi_+, \phi_-$
- 4 constraints from boundary conditions:

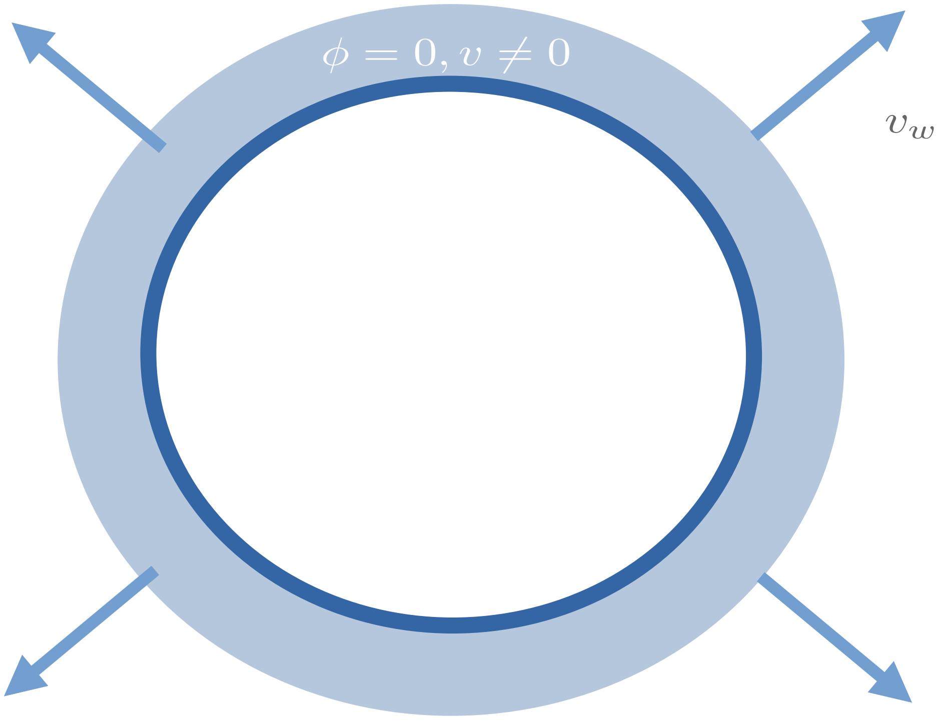
  fluid at rest far from the bubble: fixes v_w from v_+, v_-

  T matches T_{nuc} far from the bubble

  ϕ **settles to minima** of potential far in front or far behind bubble
- 2 matching conditions from stress-energy conservation
- 1 e.om. scalar field

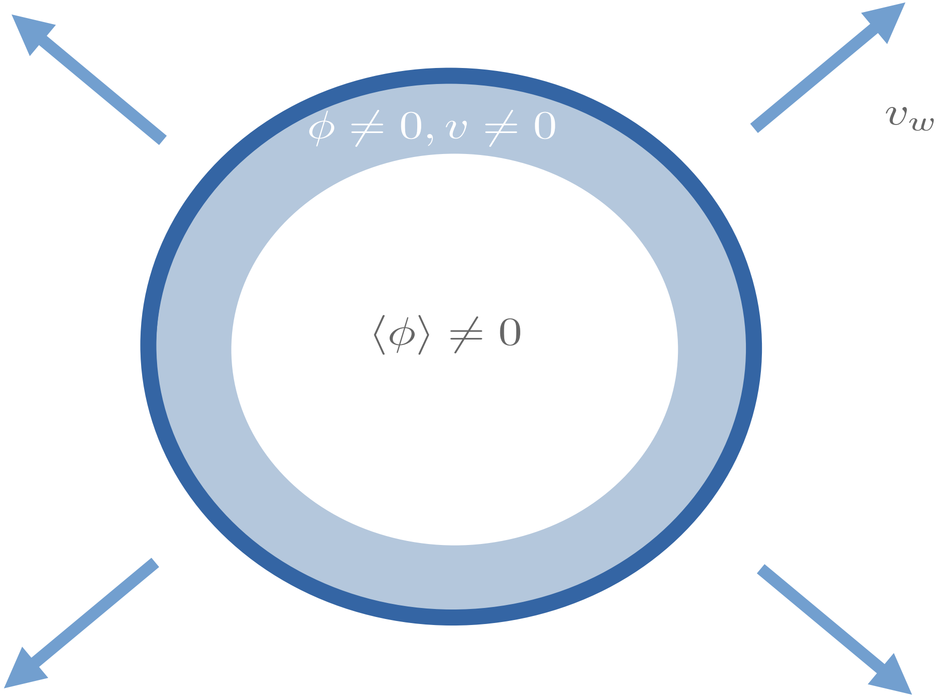
Solvable system!

Bubble hydrodynamics: deflagration



$$\langle\phi\rangle = 0, \quad T = T_{\rm nuc}$$

Bubble hydrodynamics: detonation



$\phi \neq 0, v \neq 0$

$\langle\phi\rangle \neq 0$

$v_w > c_s$

$\langle\phi\rangle = 0, \quad T = T_{\rm nuc}$

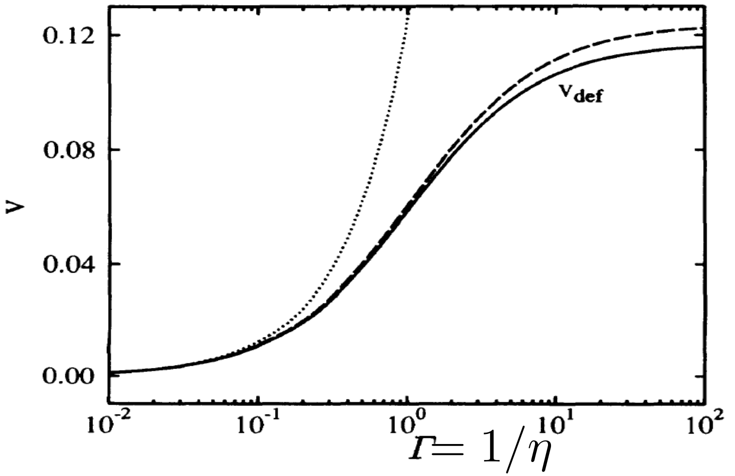
Local equilibrium: previous literature

Friction in local equilibrium?

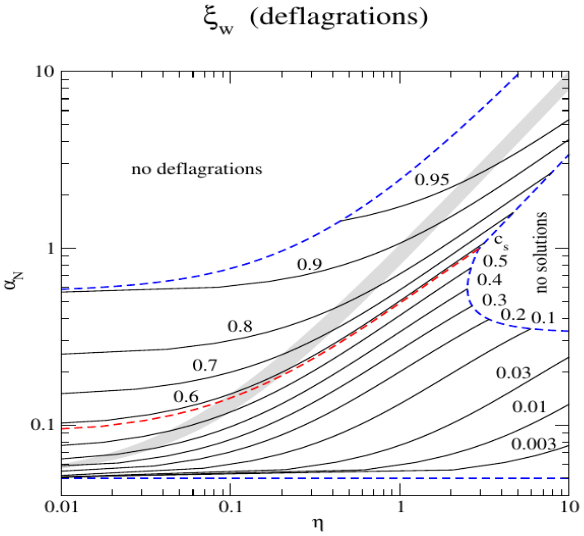
Phenomenological friction term

$$\Box\phi + \frac{\partial V_T(\phi, T)}{\partial\phi} + \eta\, u^\mu \partial_\mu \phi = 0$$

[Ignatius, Kajantie, Kurki-Suonio, Laine ‘93]



[Espinosa, Konstandin, No, Servant ‘10]



Subluminal velocity for **deflagrations without friction**?

Friction in local equilibrium

[Konstandin, No ‘10]

- First direct study of **bubble velocity** in **local equilibrium**
- Subluminal velocities as a result of hydrodynamic equations causing the **fluid** to **heat up** in front of the bubbles, which **reduces driving force**
- Effect thought to happen **only** in **deflagrations**

Friction in local equilibrium

[Barroso Mancha, Prokopec, Świeżewska ‘20]

- **Stress-energy conservation** plus **Lorentz invariance**, away from bubble wall

$$\begin{aligned} T^{\mu\nu}_{\text{plasma}} &= (\rho + p)u^\mu u^\nu - p\eta^{\mu\nu} \\ &= Tsu^\mu u^\nu - p\eta^{\mu\nu} \end{aligned}$$

$$T^{\mu\nu}_\phi = \eta^{\mu\nu} V(\phi)$$

$$\langle \Delta T^{zz}_\phi \rangle + \langle \Delta T^{zz}_{\text{plasma}} \rangle = 0$$

$$\underbrace{-\Delta p + \Delta V_\phi}_{\text{Driving force}} = \underbrace{(\gamma^2 - 1)T\Delta s = \frac{F_{\text{fr}}}{A}}_{\text{Friction}}$$

- **No distinction** between **detonations** and **deflagrations**

- **Friction grows with** v_w: **no runaway** behaviour

- Emphasized that bath of d.o.f. in local equilibrium lead to larger friction

Questions addressed in this talk

- Is the **hydrodynamic obstruction** of [Konstandin, No] the **same** effect as the **equilibrium friction force** of [Barroso Mancha, Prokopec, Świeżewska] ?

- If so, can one **extend results** of [Konstandin, No] to **detonations**?

- **Where is friction encoded** in the time-dependent, **differential equations** for the scalar and plasma?

- Does the **equilibrium friction force** prevent runaways?

Friction in equilibrium from local stress-energy conservation

Local stress-energy conservation

$$T^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{p}$$

$$T^{\mu\nu}_{\phi} = \partial^\mu\phi\partial^\nu\phi - \eta^{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V(\phi)\right)$$

$$T^{\mu\nu}_{p} = (\rho + p)u^\mu u^\mu - \eta^{\mu\nu}p = \omega\, u^\mu u^\mu - \eta^{\mu\nu}p$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\Box\phi + \frac{\partial}{\partial\phi}(V(\phi) - p) = 0, \Leftrightarrow \Box\phi + \frac{\partial V_T(\phi, T)}{\partial\phi} = 0$$

$$\partial_\mu(\omega u^\mu u^\nu - \eta^{\mu\nu}p) + \frac{\partial p}{\partial\phi}\partial^\nu\phi = 0.$$

[Ignatius, Kajantie, Kurki-Suonio, Laine ‘93]

Friction-like behaviour comes from **field-dependence** of $\omega = Ts$

It is all about pressure

- **Pressure** ⟷ free-energy density ⟷ **finite T corrections to potential**

$$p = -\Delta_T V \equiv -(V_T(\phi, T) - V(\phi))$$

- Calculable in arbitrary model from finite T field theory

$$V_T(\phi, T) = \frac{1}{2\,\pi^2} T^4 \left[\sum_B n_B\, J_B\left(\frac{m_B^2(\phi)}{T^2}\right) - \sum_F n_F\, J_F\left(\frac{m_F^2(\phi)}{T^2}\right)\right].$$

Everything follows from thermal potential

- Standard **thermodynamical identities relate entropy/enthalpy to pressure**

$$dF = SdT - pdV \Longrightarrow p = -\frac{\partial F}{\partial V}, \quad S = -\frac{\partial F}{dT}$$

$$s \equiv \left.\frac{\partial S}{\partial V}\right|_T = -\frac{\partial^2 F}{\partial V \partial T} = -\frac{\partial^2 F}{\partial T \partial V} = \frac{\partial p}{\partial T}$$

$$dU = TdS - pdV \Longrightarrow \rho = \left.\frac{\partial U}{\partial V}\right|_T = Ts - p = T\frac{\partial p}{\partial T} - p$$

$$\omega = \rho + p = T\,s = T\frac{\partial p}{\partial T}$$

- **Matches direct computations** of $\langle T^{\mu\nu}\rangle$ [Barroso Mancha, Prokopec, Świeżewska]

Conservation of entropy current

$$\left(\partial_\mu(\omega u^\mu u^\nu - \eta^{\mu\nu} p) + \frac{\partial p}{\partial\phi}\partial^\nu\phi\right) = 0$$

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$$u_\nu \left(\partial_\mu (\omega u^\mu u^\nu - \eta^{\mu\nu} p) + \frac{\partial p}{\partial \phi} \partial^\nu \phi \right) = 0$$

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$$u_\nu u^\nu = 1, \quad u_\nu \partial_\mu u^\nu = 0$$

$$\partial_\mu (\omega u^\mu) - u_\nu \partial^\nu p + u_\nu \frac{\partial p}{\partial \phi} \partial^\nu \phi = 0$$

Conservation of entropy current

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$$\begin{aligned} u_\nu u^\nu &= 1, \\ u_\nu \partial_\mu u^\nu &= 0 \end{aligned}$$

$$\partial_\mu(\omega u^\mu) - u_\nu\partial^\nu p + u_\nu\frac{\partial p}{\partial\phi}\,\partial^\nu\phi = 0$$

$$u_\nu\frac{\partial p}{\partial\phi}\partial^\nu\phi + u_\nu\frac{\partial p}{\partial T}\partial^\nu T$$

Conservation of entropy current

$$u_\nu \left(\partial_\mu (\omega u^\mu u^\nu - \eta^{\mu\nu} p) + \frac{\partial p}{\partial \phi} \partial^\nu \phi \right) = 0$$

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$$u_\nu\frac{\partial p}{\partial\phi}\partial^\nu\phi + u_\nu\frac{\partial p}{\partial T}\partial^\nu T$$

$$\partial_\mu(\omega u^\mu) - u_\nu\frac{\partial p}{\partial T}\,\partial^\nu T = 0$$

$$\omega = Ts,$$

$$s = \frac{\partial p}{\partial T}$$

$$\partial_\mu(Tsu^\mu) - u_\mu s\,\partial^\mu T = 0 \Rightarrow T\partial_\mu(u^\mu s) = 0$$

entropy current

Total entropy conservation

- Integrate over spatial volume with fluid at rest at the boundary:

$$\frac{d}{dt}S = \frac{d}{dt}\int d^3x\gamma s = \int d^3x\,\partial_t(\gamma s) = -\int d^3x\partial_i(u^i s) = 0$$

- **Entropy density** dominated by **relativistic degrees of freedom**

$$s = \frac{2\pi^2}{45}g_{\star s}T^4$$

- **Phase transition** makes some d.o.f heavy: **local decrease in entropy density**
- This has to be **compensated by a heating effect** in front or behind the bubble wall
- **Heating reduces net driving force** leading to an **effective friction**

Connection to [Konstandin, No], but **should also apply to detonations**

Planar wall frame

Assuming stationary regime in the wall frame $v^z \equiv v$

$$-\phi''(z) + \frac{\partial}{\partial\phi}(V_T(\phi,T)) = 0,$$

$$\omega\gamma^2 v^2 + \frac{1}{2}\left(\phi'(z)\right)^2 - V_T(\phi,T) = c_1,$$

$$\omega\gamma^2 v = c_2,$$

Also solved in [Konstandin, No]
cf [Espinosa, Konstandin, No, Servant]

From the second equation, comparing 2 sides of the wall where $\phi' = 0$

$$\Delta V_T(\phi,T) = -\Delta p + \Delta V(\phi) = \Delta(\omega\gamma^2 v^2) = \Delta((\gamma^2 - 1)Ts) = \frac{F_{\text{fr}}}{A}$$

Friction force of [Barroso Mancha, Prokopec, Świeżewska] **recovered** when **assuming constant** v, T **across wall**

→ Same effect as hydrodynamic obstruction of [Konstandin, No]

Reduction to single scalar equation

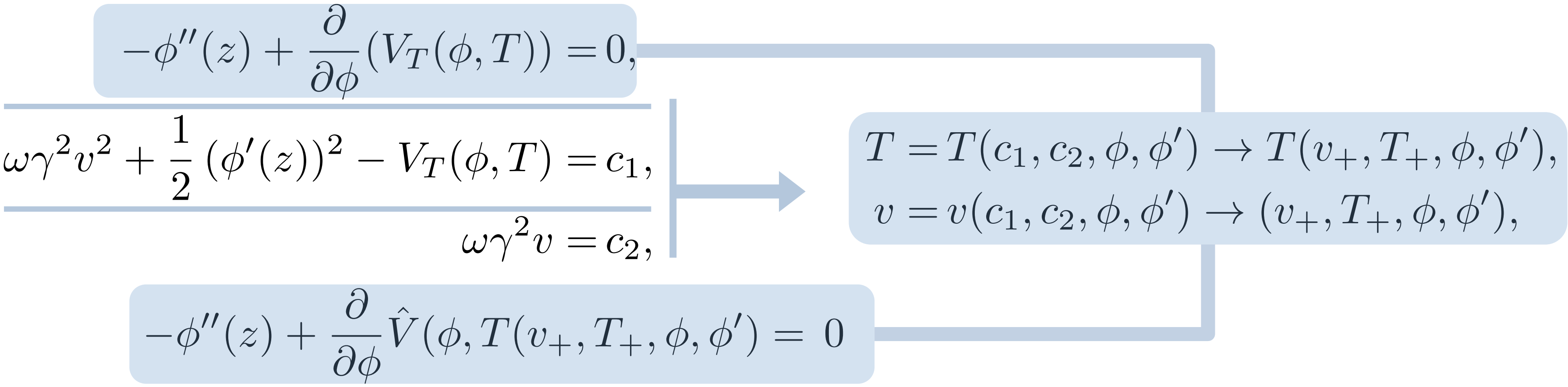
$$-\phi''(z) + \frac{\partial}{\partial\phi}(V_T(\phi,T)) = 0,$$

$$\omega\gamma^2 v^2 + \frac{1}{2}\,(\phi'(z))^2 - V_T(\phi,T) = c_1,$$

$$\omega\gamma^2 v = c_2,$$

$$\begin{aligned} T &= T(c_1,c_2,\phi,\phi') \to T(v_+,T_+,\phi,\phi'),\\ v &= v(c_1,c_2,\phi,\phi') \to (v_+,T_+,\phi,\phi'), \end{aligned}$$

Reduction to single scalar equation



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$$\omega\gamma^2 v^2 + \frac{1}{2}(\phi'(z))^2 - V_T(\phi,T) = c_1,$$

$$\omega\gamma^2 v = c_2,$$

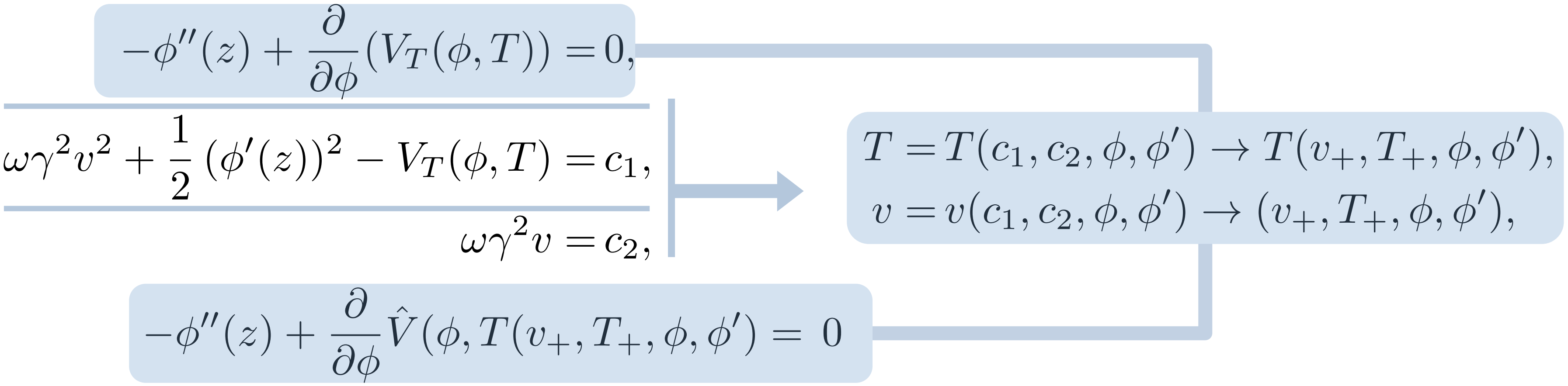
$$T = T(c_1,c_2,\phi,\phi') \to T(v_+,T_+,\phi,\phi'),$$

$$v = v(c_1,c_2,\phi,\phi') \to (v_+,T_+,\phi,\phi'),$$

$$-\phi''(z) + \frac{\partial}{\partial\phi}\hat{V}(\phi,T(v_+,T_+,\phi,\phi') = 0$$

[Ignatius, Kajantie, Kurki-Suonio, Laine]

Reduction to single scalar equation



$$-\phi''(z) + \frac{\partial}{\partial \phi}(V_T(\phi, T)) = 0,$$

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$$T = T(c_1, c_2, \phi, \phi') \to T(v_+, T_+, \phi, \phi'),$$

$$v = v(c_1, c_2, \phi, \phi') \to (v_+, T_+, \phi, \phi'),$$

$$-\phi''(z) + \frac{\partial}{\partial \phi}\hat{V}(\phi, T(v_+, T_+, \phi, \phi') = 0$$

[Ignatius, Kajantie, Kurki-Suonio, Laine]

Boundary conditions $\phi(z) \to \phi_+, \quad z \to \infty, \quad \phi'(z) \to 0, \quad |z| \to \infty,$

In **(-) phase,** field goes to a **minimum** [Konstandin, No] $\phi''(z) \to 0, \quad z \to -\infty,$

These conditions fix v_+ in terms of T_+. Latter fixed by nucleation temperature away from wall (accounting from extra hydrodynamic profile for deflagrations)

Example model

SM extension by *N* additional **complex singlets** allowing for **first order phase transition** for the Higgs

$$\mathcal{L} \supset -m_H^2 \Phi^\dagger \underbrace{\Phi}_{\text{Higgs}} - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 - m_\chi^2 \chi^\dagger \underbrace{\chi}_{\text{Extra scalars}} - \frac{\lambda_\chi}{2}(\chi^\dagger\chi)^2 - \lambda_{H\chi}\Phi^\dagger\Phi\chi^\dagger\chi\,.$$

Pressure from **thermal corrections to potential** in high-*T* expansion

$$p(h,T) =$$

$$\frac{\pi^2 T^4}{90}(g_{*,\mathrm{SM}} + 2N) - T^2\left(h^2\left(\frac{y_b^2}{8} + \frac{3g_1^2}{160} + \frac{3g_2^2}{32} + \frac{\lambda}{8} + \frac{N\lambda_{H\chi}}{24} + \frac{y_t^2}{8}\right) + \frac{m_H^2}{6} + \frac{Nm_\chi^2}{12}\right)$$

$$-\frac{T}{12\pi}\left(-\frac{3}{4}(g_2 h)^3 - \frac{3h^3}{8}\left(\frac{3g_1^2}{5} + g_2^2\right)^{3/2} - 3\left(\frac{h^2\lambda}{2} + m_H^2\right)^{3/2} - \left(\frac{3h^2\lambda}{2} + m_H^2\right)^{3/2}\right.$$

$$\left.-2N\left(\frac{h^2\lambda_{H\chi}}{2} + m_\chi^2\right)^{3/2}\right)$$

Time-dependent solutions

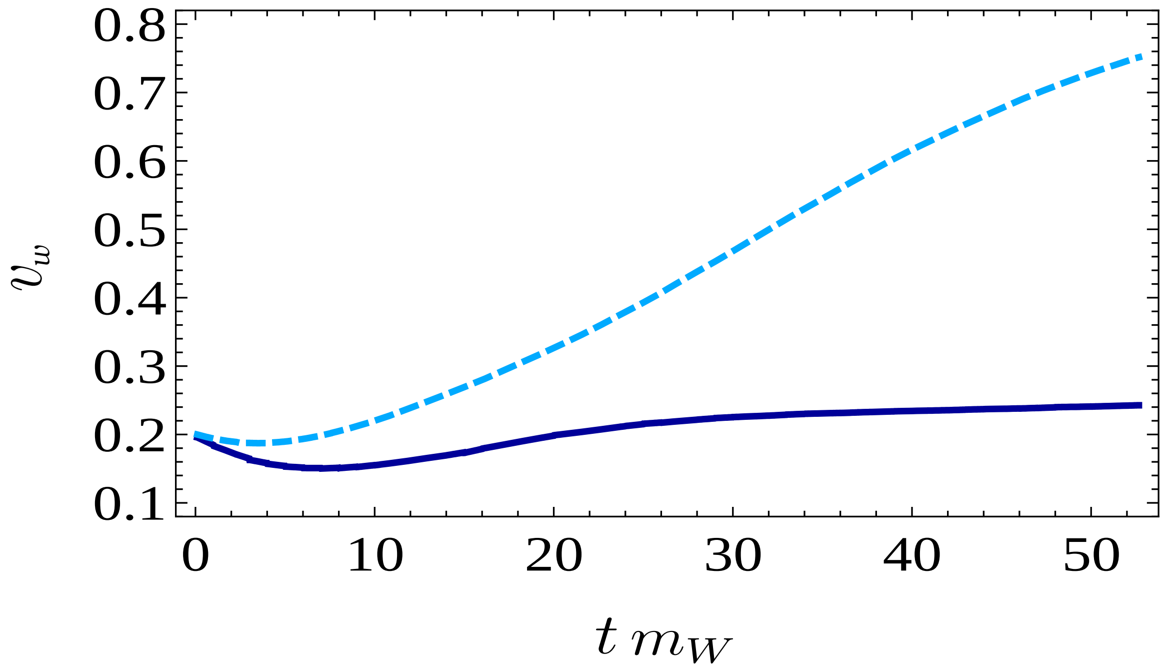
- We want to understand which terms in the differential equations lead to a friction-like behaviour.

- For this we solve **time-dependent** equations assuming **spherical symmetry**

$$
\begin{aligned}
\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} &= 0, \\
\partial_\mu(\omega u^\mu u^\nu - \eta^{\mu\nu} p) + \frac{\partial p}{\partial\phi}\partial^\nu\phi &= 0.
\end{aligned}
$$

$$
\begin{aligned}
\partial_t^2\phi - \frac{1}{r^2}\partial_r(r^2\partial_r\phi) + \frac{\partial V_T(\phi,T)}{\partial\phi} &= 0, \\
\partial_t(\omega\gamma^2) + \frac{1}{r^2}\partial_r(r^2\omega\gamma^2 v) - \frac{\partial p}{\partial T}\partial_t T &= 0, \\
\partial_t(\omega\gamma^2 v) + \frac{1}{r^2}\partial_r(r^2\omega\gamma^2 v^2) + \frac{\partial p}{\partial T}\partial_r T &= 0.
\end{aligned}
$$

Time-dependent deflagrations



Ignoring $\frac{\partial \omega}{\partial \phi} = \frac{\partial (Ts)}{\partial \phi}$

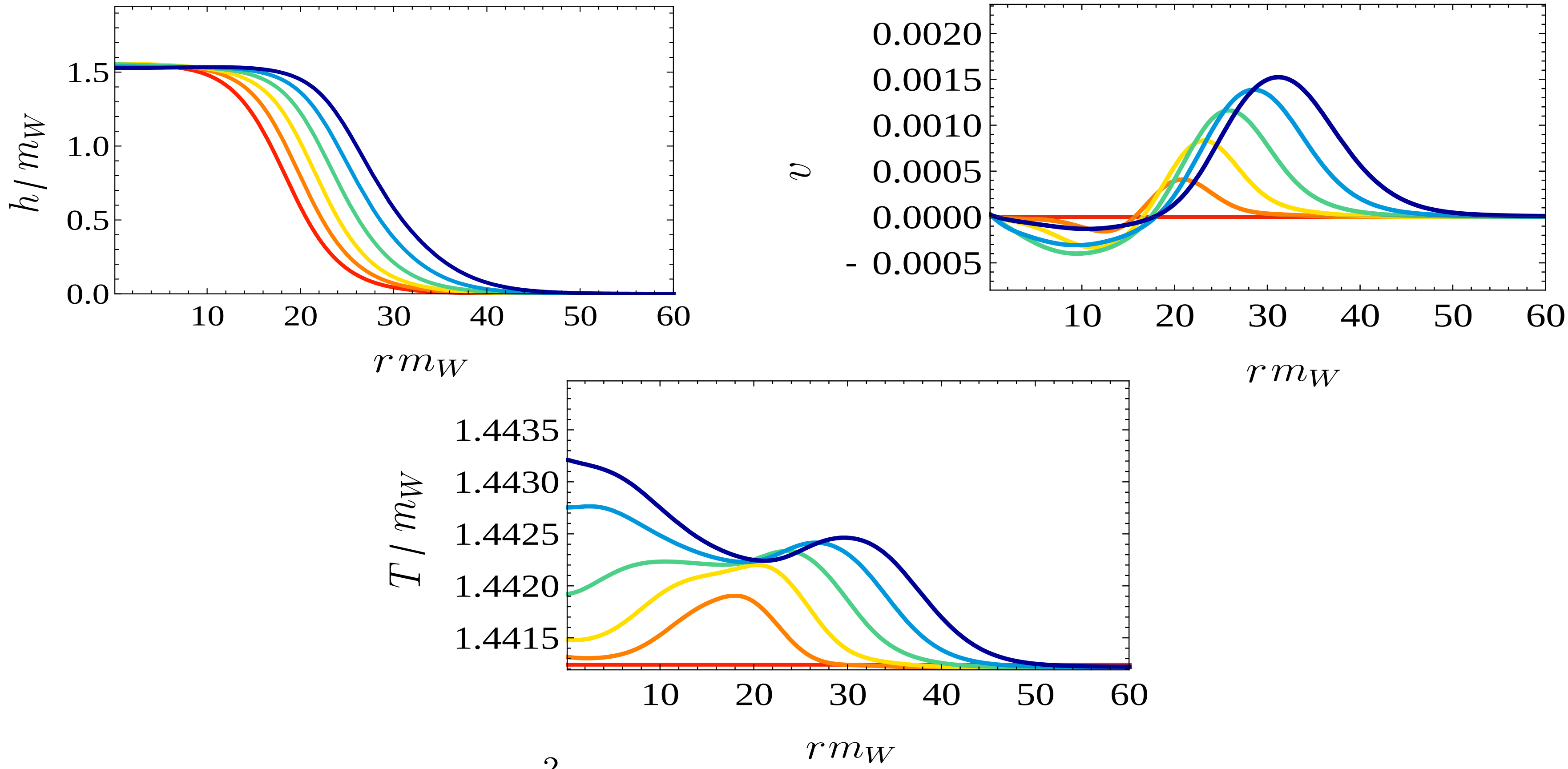
Accounting for $\frac{\partial \omega}{\partial \phi} = \frac{\partial (Ts)}{\partial \phi}$

Friction-like behaviour!

$$N = 4, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

- Obtained with neural network pre-trained with Mathematica solution

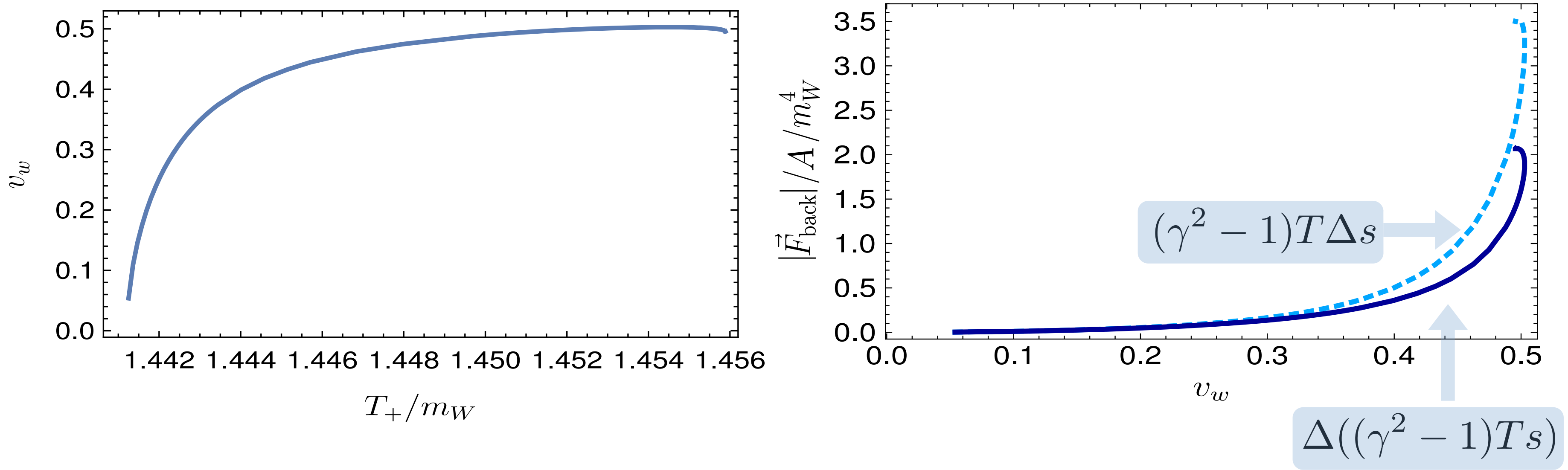
Time-dependent deflagrations



$$N = 4, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

Static deflagrations in wall frame

Family of solutions without necessarily imposing $\phi''(z) \to 0, \quad z \to -\infty$



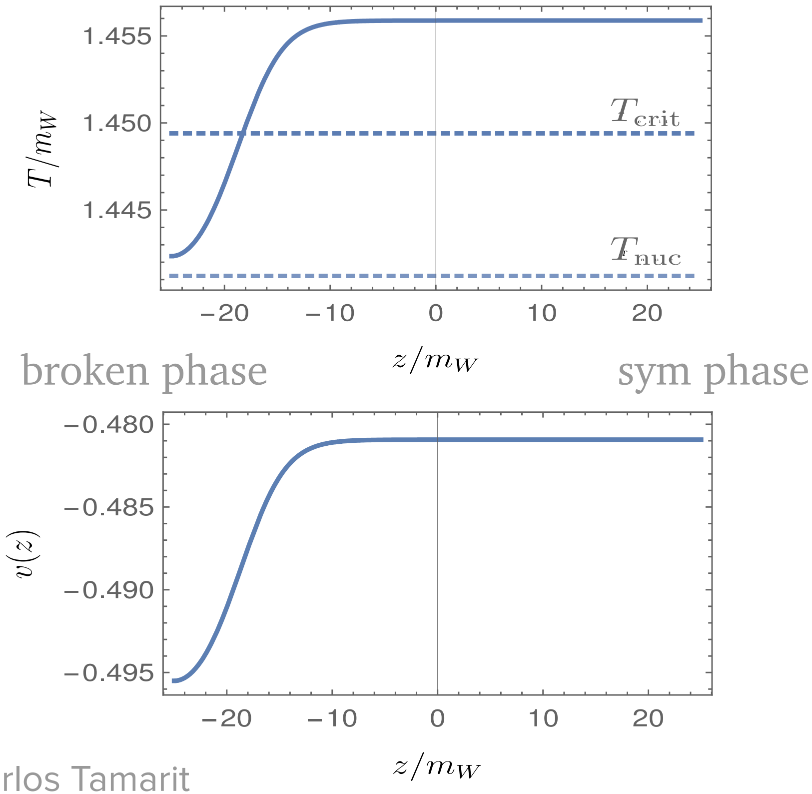
Friction force grows with velocity!

Physical case with $\phi''(-\infty) \to 0$ corresponds to right endpoint of curves

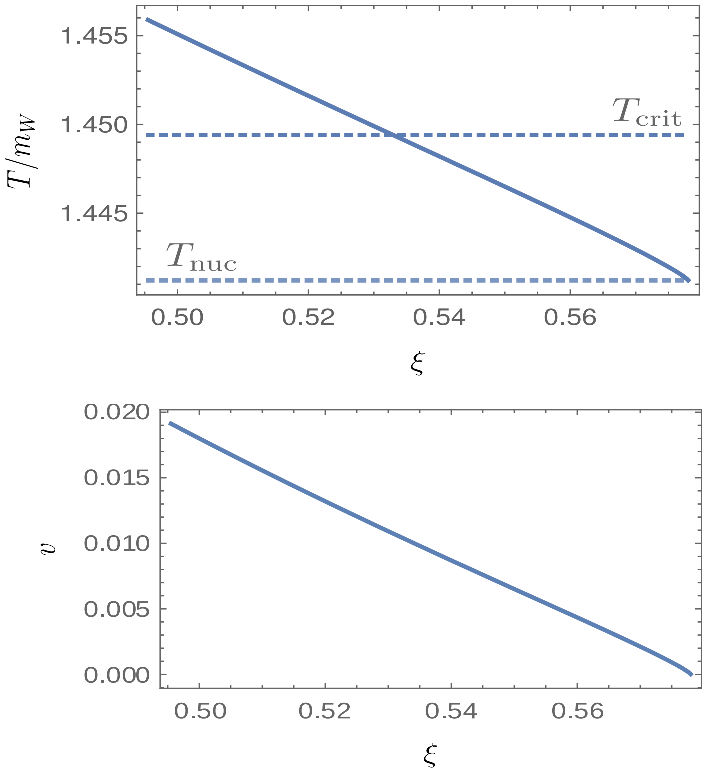
Static deflagrations in wall frame

Physical solution:

Static solution near wall



Self-similar hydrodynamic profile $(\xi = r/t)$



Novel static detonations in wall frame

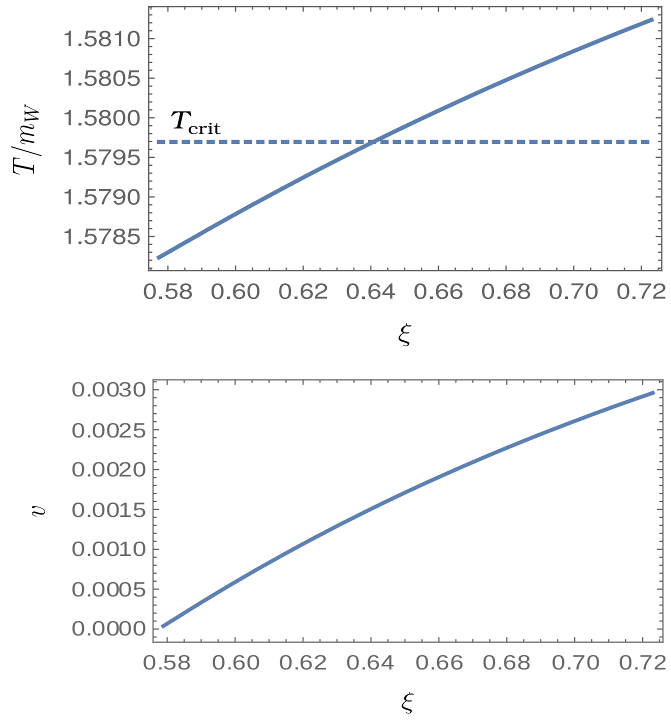
- The solutions $T(v_+, T_+, \phi, \phi'), v(v_+, T_+, \phi, \phi')$ are actually **multivalued,** and so is the "pseudopotential" $\hat{V}(\phi, T(v_+, T_+, \phi, \phi'))$

- We find that a branch of solutions with larger fluid velocities supports **static detonation solutions**

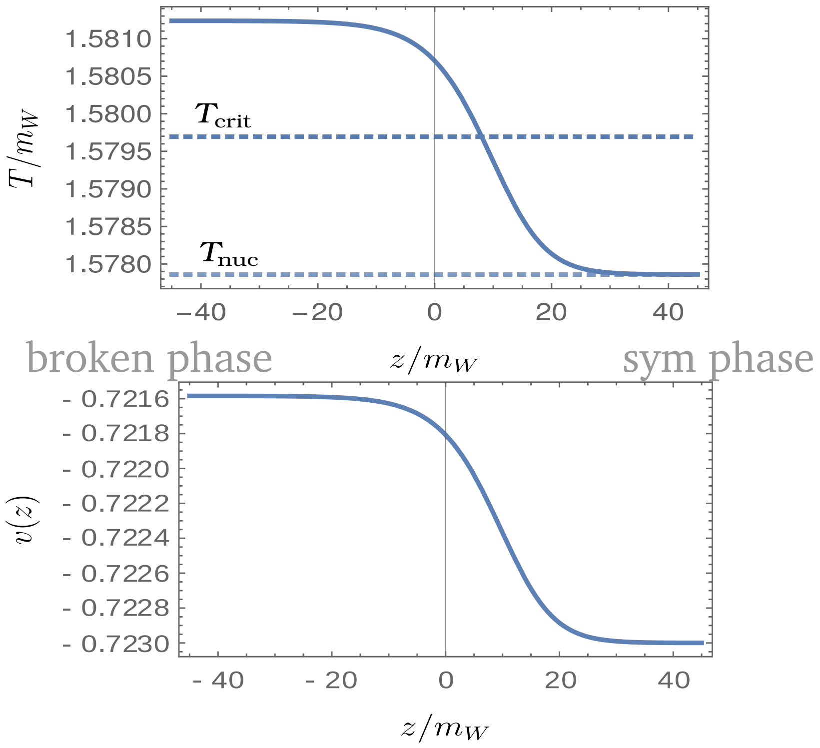
- We have found that the friction force can deviate from [Barroso Mancha et al] by a large factor

Static detonation solutions in wall frame

Self-similar hydrodynamic profile

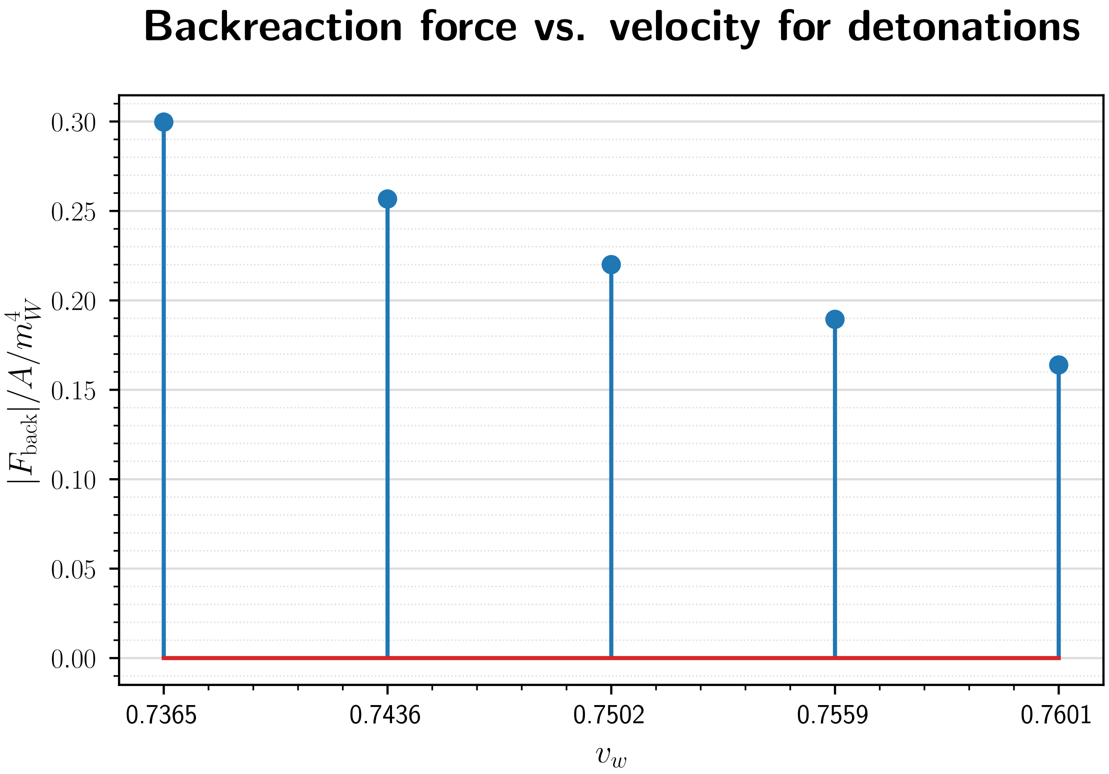


Static solution near wall



$$N = 2, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

No growth of friction force with velocity



$$N = 2, \quad \frac{m_S^2}{m_W^2} = 25, \quad \lambda_\chi = 0.085, \quad \text{decreasing} \quad \lambda_{H\chi}, 0.95 \geq \lambda_{H\chi} \geq 0.75$$

Friction force per unit area revisited

$$\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

Friction force per unit area revisited

$$\int dz \frac{d\phi}{dz}\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

Friction force per unit area revisited

$$\int dz\,\frac{d\phi}{dz}\left(\Box\phi+\frac{\partial V_T(\phi,T)}{\partial\phi}+\sum_i\frac{dm_i^2}{d\phi}\int\frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right)=0$$

$$\frac{dV_T(\phi,z)}{dz}-\frac{\partial V_T(\phi,T)}{\partial T}\,\frac{dT}{dz}$$

(omitted earllier)

Friction force per unit area revisited

$$\int dz \frac{d\phi}{dz}\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

$$\frac{dV_T(\phi,z)}{dz} - \frac{\partial V_T(\phi,T)}{\partial T}\frac{dT}{dz}$$

$$\Delta V_T = \int dz \frac{\partial V_T(\phi,T)}{\partial T}\frac{dT}{dz} - \sum_i \int d\phi \frac{dm_i^2}{d\phi}\int \frac{d^3\mathbf{p}}{(s\pi)^3 2E_i}\delta f_i(\mathbf{p},x)$$

Driving force

Hydrodynamic backreaction

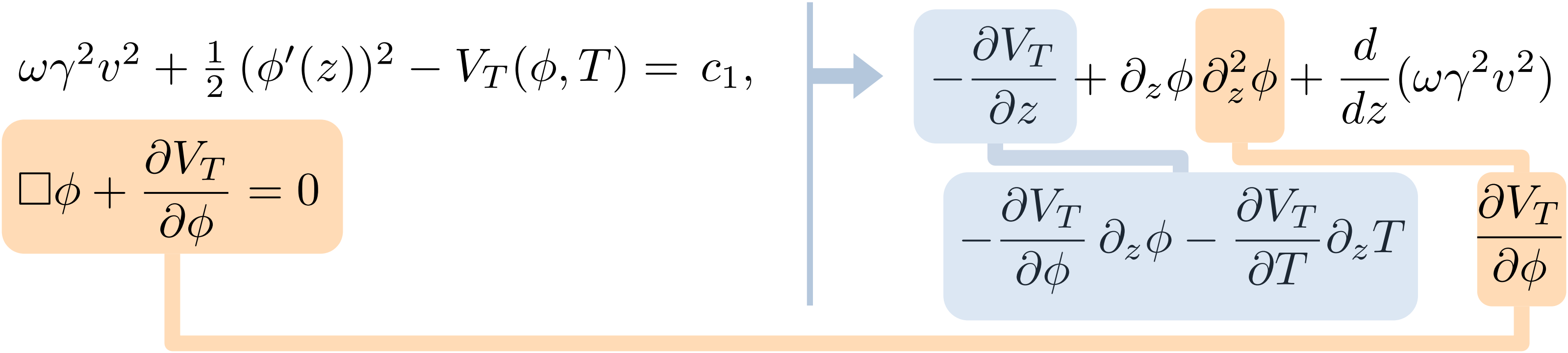
Friction per unit area, out of eq.
[Bödeker-Moore]

[Ignatius et al] [Espinosa et al] [Konstandin-No]

Absorbed hydro backreaction into driving force

Friction force per unit area revisited

The hydrodynamic backreaction coincides with our previous expression $\Delta\omega\gamma^2 v^2$



$$\omega\gamma^2 v^2 + \tfrac{1}{2}\left(\phi'(z)\right)^2 - V_T(\phi, T) = c_1,$$

$$\Box\phi + \frac{\partial V_T}{\partial \phi} = 0$$

$$\longmapsto$$

$$-\frac{\partial V_T}{\partial z} + \partial_z\phi\,\partial_z^2\phi + \frac{d}{dz}(\omega\gamma^2 v^2) = 0$$

$$-\frac{\partial V_T}{\partial \phi}\,\partial_z\phi - \frac{\partial V_T}{\partial T}\partial_z T$$

$$\frac{\partial V_T}{\partial \phi}$$

$$\frac{\partial V_T}{\partial T}\,\partial_z T = \frac{d}{dz}(\omega\gamma^2 v^2)$$

$$\frac{F_{\text{back}}}{A} = \int dz \frac{\partial V_T(\phi, T)}{\partial T}\,\frac{dT}{dz} = \int dz \frac{d}{dz}(\omega\gamma^2 v^2) = \Delta(\omega\gamma^2 v^2) = \Delta(Ts(\gamma^2 - 1))$$

Friction force per unit area revisited

Balance of forces in local equilibrium → **T gradient across wall**

Stress-energy + entropy conservation + T gradiendt → **v gradient across wall**

[Wen-Yuan's talk]

$$\frac{F_{\rm back}}{A} = \Delta(Ts(\gamma^2 - 1)) \neq (\gamma^2 - 1)\Delta(Ts)$$

The γ^2 **growth** of the friction force is **not guaranteed**, as seen in detonations

Conclusions

Even in **local equilibrium**, there is a **non-dissipative**, friction-like **backreaction effect**

This effect is behind the runaway obstruction of [Konstandin, No] and the friction force of [Barroso Mancha, Prokopec, Świeżewska]

We provided an **intuitive understanding** based on **entropy conservation**

By solving the time-dependent equations for bubble propagation, we showed that the **backreaction** is **generated locally** by the **field-derivatives of the enthalpy**

We showed that, as expected from the results of [Barroso Mancha et et al], the **backreaction exists for detonations**

Friction force departs from γ^2 scaling (and decreases with v_w for detonations) due **changes** of v, T across the bubble.

Thank you!