

# On the behaviour of $R_{pA}$ at high energy

Misha Kozlov<sup>1†</sup>, Arif I. Shoshi<sup>1</sup>, Bo-Wen Xiao<sup>2</sup>

<sup>1</sup> Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany

<sup>2</sup> Department of Physics, Columbia University, New York, NY, 10027, USA

## Abstract

We discuss the behaviour of  $R_{pA}$ , the ratio of the unintegrated gluon distribution of a nucleus over the unintegrated gluon distribution of a proton scaled up by  $A^{1/3}$ , at high energy and fixed coupling. We show that  $R_{pA}$  exhibits a rising gluon shadowing with growing rapidity, approaching  $1/A^{1/3}$  at asymptotic rapidity, which means total gluon shadowing due to gluon number fluctuations or Pomeron loops.

## 1 Introduction

We study the ratio of the unintegrated gluon distribution of a nucleus  $\varphi_A(k_\perp, Y)$  over the unintegrated gluon distribution of a proton  $\varphi_p(k_\perp, Y)$  scaled up by  $A^{1/3}$

$$R_{pA} = \frac{\varphi_A(k_\perp, Y)}{A^{1/3} \varphi_p(k_\perp, Y)}. \quad (1)$$

This ratio is a measure of the number of particles produced in a proton-nucleus collision versus the number of particles in proton-proton collisions times the number of collisions. The transverse momentum of gluons is denoted by  $k_\perp$  and the rapidity variable by  $Y = \ln(1/x)$ .

The ratio  $R_{pA}$  has been widely studied [1] in the framework of the BK-equation [2] which describes the small- $x$  physics in the *mean field approximation*. Using the BK-equation one finds in the geometric scaling regime (transition from high to low gluon density, see Fig.1) in the fixed coupling case that the shape of the unintegrated gluon distribution of the nucleus and proton as a function of  $k_\perp$  is preserved with increasing  $Y$ , see Fig.2(a), because of the geometric scaling behaviour  $\varphi_{p,A}(k_\perp, Y) = \varphi_{p,A}(k_\perp^2/Q_s^2(Y))$ , and therefore the leading contribution to the ratio  $R_{pA}$  is basically  $k_\perp$  and  $Y$  independent, scaling with the atomic number  $A$  as [3,4]

$$R_{pA} \simeq \frac{1}{A^{1/3(1-\gamma_0)}}, \quad (2)$$

where  $\gamma_0 = 0.6275$ . This means that gluons inside the nucleus and proton are somewhat shadowed since  $\varphi_A/\varphi_p = A^{\gamma_0/3}$  lies between total ( $\varphi_A/\varphi_p = 1$ ) and zero ( $\varphi_A/\varphi_p = A^{1/3}$ ) gluon shadowing. The *partial gluon shadowing* comes from the anomalous behaviour of the unintegrated gluon distributions which stems from the BFKL evolution. The partial gluon shadowing may explain why particle production in heavy ion collisions scales, roughly, like  $N_{part}$  [5].

Over the last few years, it has been understood how to deal with small- $x$  physics at high energy *beyond the mean field approximation*, i.e., beyond the BK [2] and JIMWLK [6] equations. We have learned how to account for the elements missed in the mean field evolution,

---

<sup>†</sup> speaker

such as the discreteness and fluctuations of gluon numbers [7, 8] or the Pomeron loops [9]. The main result as a consequence of the above is the emerging of a new scaling behaviour for the dipole-hadron/nucleus scattering amplitude at high rapidities [7, 8], the so-called diffusive scaling. This is different from the geometric scaling behaviour which is the hallmark of the "mean-field" evolution equations (JIMWLK and BK equations). The effects of fluctuations on the scattering amplitude [10], the diffractive scattering processes [11, 12] and forward gluon production in hadronic scattering processes [13, 14] has been studied so far. In this work we show how the behaviour of  $R_{pA}$  as a function of  $k_\perp$  and  $Y$  in the fixed coupling case is completely changed due the effects of gluon number fluctuations or Pomeron loops at high rapidity [15].

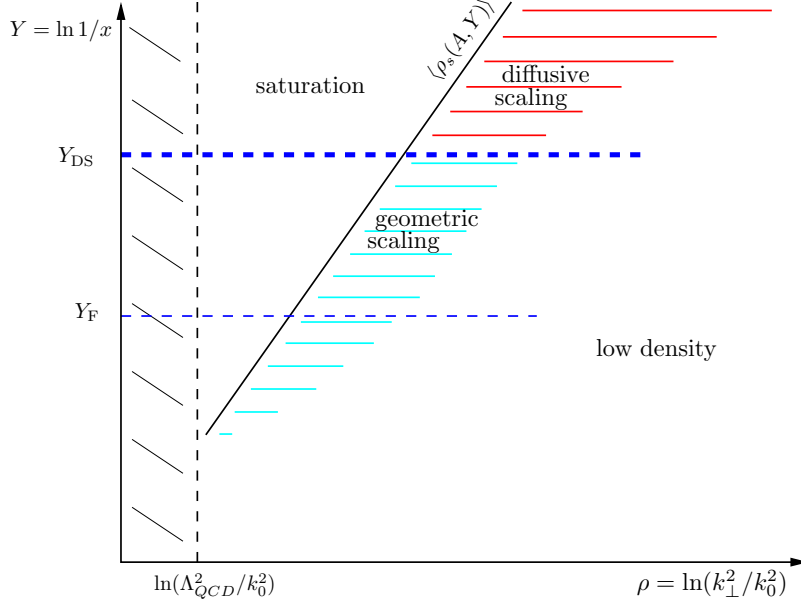


Fig. 1: Phase diagram of a highly evolved nucleus/proton.

## 2 $R_{pA}$ ratio in the diffusive scaling regime

According to the statistical physics/high energy QCD correspondence [8] the influence of fluctuations on the unintegrated gluon distribution of a nucleus/proton is as follows: Starting with an initial gluon distribution of the nucleus/proton at zero rapidity, the stochastic evolution generates an ensemble of distributions at rapidity  $Y$ , where the individual distributions seen by a probe typically have different saturation momenta and correspond to different events in an experiment. To include gluon number fluctuations one has to average over all individual events,

$$\langle \varphi_{p,A}(\rho - \rho_s) \rangle = \int d\rho_s \varphi_{p,A}(\rho - \rho_s) P(\rho_s - \langle \rho_s \rangle), \quad (3)$$

where  $\varphi_{p,A}(\rho - \rho_s)$  is the distribution for a single event with  $\rho = \ln(k_\perp^2/k_0^2)$  and  $P(\rho_s - \langle \rho_s \rangle)$  the probability distribution of the logarithm of the saturation momentum,  $\rho_s(Y) = \ln(Q_s^2(Y)/k_0^2)$ ,

which is argued to have a Gaussian form [16],

$$P(\rho_s) \simeq \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{(\rho_s - \langle \rho_s \rangle)^2}{2\sigma^2} \right] \quad \text{for } \rho - \rho_s \ll \gamma_c^2 \sigma^2, \quad (4)$$

with the dispersion

$$\sigma^2 = \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = DY. \quad (5)$$

The main consequence of fluctuations is the replacement of the geometric scaling,  $\varphi_{p,A}(k_\perp, Y) = \varphi_{p,A}(k_\perp^2/Q_s^2(Y))$ , by a new scaling, the diffusive scaling [7, 8], namely,  $\langle \varphi_{p,A}(k_\perp, Y) \rangle$  is a function of another scaling variable ( $\langle Q_s \rangle$  is the average saturation momentum),

$$\langle \varphi_{p,A}(k_\perp, Y) \rangle = F_{p,A} \left( \frac{\ln(k_\perp^2/\langle Q_s(Y) \rangle^2)}{DY} \right). \quad (6)$$

The diffusive scaling, see Fig. 1, sets in when the dispersion of the different events is large,  $\sigma^2 = \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = DY \gg 1$ , i.e.,  $Y \gg Y_{DS} = 1/D$  where  $D$  is the diffusion coefficient, and is valid in the region  $\sigma \ll \ln(k_\perp^2/\langle Q_s(Y) \rangle^2) \ll \gamma_0 \sigma^2$ .

The diffusive scaling means that the shape of the unintegrated gluon distribution of the nucleus/proton changes with increasing  $Y$  because of the additional  $DY$  dependence as compared with the geometric scaling. The shape becomes flatter and flatter with increasing rapidity  $Y$ , as shown in Fig.2(b), in contrast to the preserved shape in the geometric scaling regime shown in Fig.2(a). This flattening will lead to a new phenomenon for  $R_{pA}$  as discussed below.

Using Eq.(3) for the averaging over all events and the result from the BK-equation for the single event distribution one obtains [15] for the ratio

$$R_{pA} \simeq \frac{1}{A^{\frac{1}{3}(1-\frac{\Delta\rho_s}{2\sigma^2})}} \left[ \frac{k_\perp^2}{\langle Q_s^2(A, y) \rangle} \right]^{\frac{\Delta\rho_s}{\sigma^2}} \quad (7)$$

with the difference between the average saturation lines of the nucleus and the proton

$$\Delta\rho_s \equiv \langle \rho_s(A, Y) \rangle - \langle \rho_s(p, Y) \rangle = \ln \frac{\langle Q_s(A, Y) \rangle^2}{\langle Q_s(p, Y) \rangle^2} \quad (8)$$

where  $\langle Q_s(A, Y) \rangle$  ( $\langle Q_s(p, Y) \rangle$ ) is the average saturation momentum of the nucleus (proton). The difference  $\Delta\rho_s$  is fixed by the initial conditions for the average saturation momenta of the nucleus and proton and is  $Y$ -independent in the fixed coupling case. For example, using the known assumption  $\langle Q_s(A) \rangle^2 = A^{1/3} \langle Q_s(p) \rangle^2$  one obtains  $\Delta\rho_s = \ln A^{1/3}$ .

The ratio  $R_{pA}$  in Eq. (7) shows the following very different features as compared with the ratio in the geometric scaling regime given in Eq. (2):

- In the diffusive scaling regime where  $k_\perp^2$  is close to  $\langle Q_s^2(A, Y) \rangle$ , the gluon shadowing characterized by  $A^{\frac{1}{3}(\frac{\Delta\rho_s}{2\sigma^2}-1)}$  increases as the rapidity grows (at fixed  $A$  or  $\Delta\rho_s$ ) because of  $\sigma^2 = DY$ . At asymptotic rapidity one obtains *total gluon shadowing*,  $R_{pA} = A^{1/3}$ , which means that the unintegrated gluon distribution of the nucleus and that of the proton

become the same in the diffusive scaling regime at  $Y \rightarrow \infty$ . The phenomenon of total gluon shadowing is universal since it does not depend on the initial conditions ( $\Delta\rho_s$ ).

Total gluon shadowing is an effect of gluon number fluctuations (or Pomeron loops) since fluctuations make the unintegrated gluon distributions of the nucleus and of the proton flatter and flatter [8] and their ratio closer and closer to 1 (at fixed  $\Delta\rho_s$ ) with rising rapidity, as shown in Fig.2(b). Total gluon shadowing is not possible in the geometric scaling regime in the fixed coupling case since the shapes of the gluon distributions of the nucleus and of the proton remain the same with increasing  $Y$  giving a constant ratio unequal one, as shown in Fig.2(a). In the absence of fluctuations one can expect only partial gluon shadowing, see Eq. (2), in the fixed coupling case.

- The ratio  $R_{pA}$  increases with rising  $k_\perp^2$  within the diffusive scaling region. Since the exponent  $\Delta\rho_s/\sigma^2$  decreases with rapidity, the slope of  $R_{pA}$  as a function of  $k_\perp^2$  becomes smaller with growing  $Y$ . The result for  $R_{pA}$  in the diffusive scaling regime in Eq.(3) is very different from the result obtained in the mean field approximation given in Eq. (2), where gluon number fluctuations are not included, which is basically  $k_\perp$  and  $Y$ -independent.

The qualitative behaviour of  $R_{pA}$  as a function of  $k_\perp$  at four different rapidities,  $Y_1 \leq Y_2 \leq Y_3 \leq Y_4$ , in the diffusive scaling regime and for a fixed coupling is shown in Fig. 3. Note that  $R_{pA}$  is always smaller than one for values of  $k_\perp$  in the diffusive scaling regime.

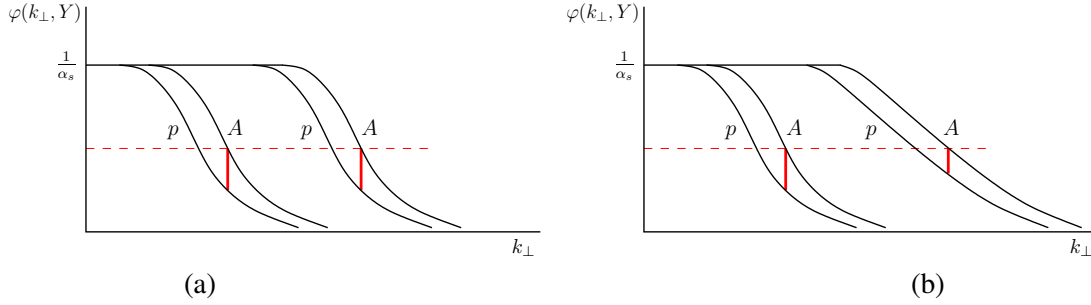


Fig. 2: The qualitative behaviour of the unintegrated gluon distribution of a nucleus (A) and a proton (p) at two different rapidities in the geometric scaling regime (a) and diffusive scaling regime (b).

The above effects of fluctuations on  $R_{pA}$  are valid in the fixed coupling case and at very large energy. It isn't clear yet whether the energy at LHC is high enough for them to become important. Recently, while in Ref. [17] a possible evidence of gluon number fluctuations in the HERA data has been found, in Ref. [18], using a toy model, it has been argued that in case of a running coupling fluctuations can be neglected in the range of HERA and LHC energies. See also Refs. [7, 19] for more studies on running coupling plus fluctuation effects.

Moreover, the running of the coupling [20] may become more important than the effect of gluon number fluctuations [18]. In case of a running coupling, the gluon shadowing increases with rising rapidity in the geometric scaling regime [1], as opposed to the (roughly) fixed value (partial shadowing) in the fixed-coupling case, and would lead to total gluon shadowing [4]

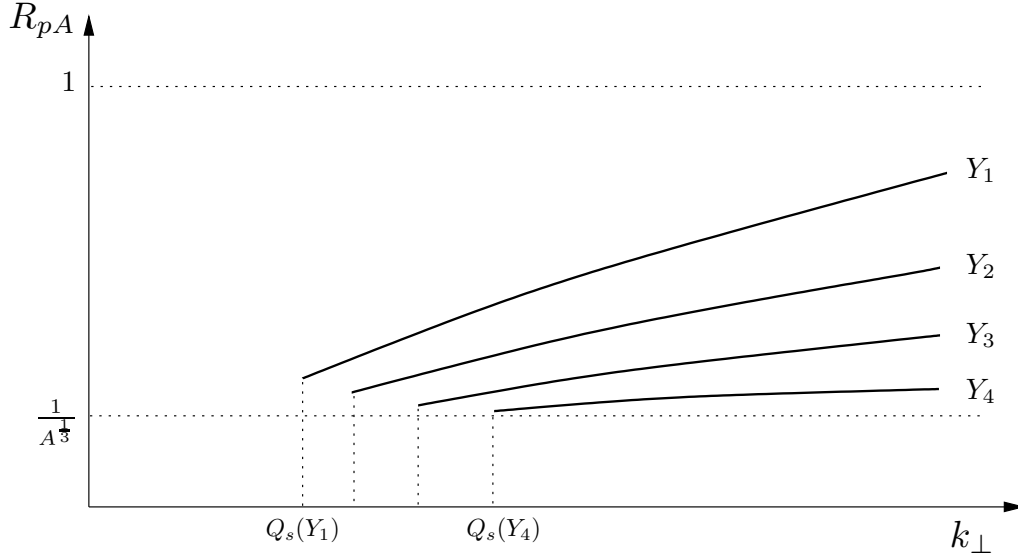


Fig. 3: The qualitative behaviour of the ratio  $R_{pA}$  as a function of  $k_{\perp}$  at four different rapidities,  $Y_1 \leq Y_2 \leq Y_3 \leq Y_4$ , in the diffusive scaling regime.  $R_{pA}$  is always smaller than one for values of  $k_{\perp}$  in the diffusive scaling regime.

at very high rapidities even if fluctuations were absent. In case fluctuations are important at LHC energy, in addition to the theoretically interesting consequences of fluctuations on  $R_{pA}$ , the features of  $R_{pA}$  worked out here, as the increase of the gluon shadowing and the decrease as a function of the gluon momentum with rising rapidity, may be viewed as signatures for fluctuation effects in the LHC data. More work remains to be done in order to clarify how important fluctuation or running coupling effects are at given energy, e.g., at LHC energy. An extension of this work by the running coupling may help to clarify some of the open questions.

### Acknowledgments

A. Sh. and M. K. acknowledge financial support by the Deutsche Forschungsgemeinschaft under contract Sh 92/2-1.

### References

- [1] E. Iancu, K. Itakura, and D. N. Triantafyllopoulos, Nucl. Phys. **A742**, 182 (2004). hep-ph/0403103;  
A. H. Mueller, Nucl. Phys. **A724**, 223 (2003). hep-ph/0301109;  
J. L. Albacete, N. Armesto, A. Kovner, C. A. Salgado, and U. A. Wiedemann, Phys. Rev. Lett. **92**, 082001 (2004). hep-ph/0307179;  
R. Baier, A. H. Mueller, and D. Schiff, Nucl. Phys. **A741**, 358 (2004). hep-ph/0403201;  
R. Baier, A. Kovner, and U. A. Wiedemann, Phys. Rev. **D68**, 054009 (2003). hep-ph/0305265;  
D. Kharzeev, Y. V. Kovchegov, and K. Tuchin, Phys. Rev. **D68**, 094013 (2003). hep-ph/0307037;  
J. Jalilian-Marian, Y. Nara, and R. Venugopalan, Phys. Lett. **B577**, 54 (2003). nucl-th/0307022;  
F. Gelis, A. M. Stasto, and R. Venugopalan, Eur. Phys. J. **C48**, 489 (2006). hep-ph/0605087.
- [2] I. Balitsky, Nucl. Phys. **B463**, 99 (1996). hep-ph/9509348;  
I. Balitsky, Phys. Lett. **B518**, 235 (2001). hep-ph/0105334;

- Y. V. Kovchegov, Phys. Rev. **D60**, 034008 (1999). hep-ph/9901281;  
Y. V. Kovchegov, Phys. Rev. **D61**, 074018 (2000). hep-ph/9905214.
- [3] A. H. Mueller, Nucl. Phys. **A724**, 223 (2003). hep-ph/0301109.
- [4] E. Iancu, K. Itakura, and D. N. Triantafyllopoulos, Nucl. Phys. **A742**, 182 (2004). hep-ph/0403103.
- [5] D. Kharzeev, E. Levin, and L. McLerran, Phys. Lett. **B561**, 93 (2003). hep-ph/0210332.
- [6] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Nucl. Phys. **B504**, 415 (1997). hep-ph/9701284;  
J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Phys. Rev. **D59**, 014014 (1999). hep-ph/9706377;  
J. Jalilian-Marian, A. Kovner, and H. Weigert, Phys. Rev. **D59**, 014015 (1999). hep-ph/9709432;  
E. Iancu, A. Leonidov, and L. D. McLerran, Phys. Lett. **B510**, 133 (2001). hep-ph/0102009;  
E. Ferreira, E. Iancu, A. Leonidov, and L. McLerran, Nucl. Phys. **A703**, 489 (2002). hep-ph/0109115.
- [7] A. H. Mueller and A. I. Shoshi, Nucl. Phys. **B692**, 175 (2004). hep-ph/0402193.
- [8] E. Iancu, A. H. Mueller, and S. Munier, Phys. Lett. **B606**, 342 (2005). hep-ph/0410018.
- [9] A. H. Mueller, A. I. Shoshi, and S. M. H. Wong, Nucl. Phys. **B715**, 440 (2005). hep-ph/0501088;  
E. Iancu and D. N. Triantafyllopoulos, Phys. Lett. **B610**, 253 (2005). hep-ph/0501193;  
A. Kovner and M. Lublinsky, Phys. Rev. **D71**, 085004 (2005). hep-ph/0501198;  
E. Levin and M. Lublinsky, Nucl. Phys. **A763**, 172 (2005). hep-ph/0501173;  
Y. Hatta, E. Iancu, L. McLerran, A. Stasto, and D. N. Triantafyllopoulos, Nucl. Phys. **A764**, 423 (2006). hep-ph/0504182.
- [10] E. Brunet, B. Derrida, A. H. Mueller, and S. Munier, Phys. Rev. **E73**, 056126 (2006). cond-mat/0512021;  
S. Munier, Phys. Rev. **D75**, 034009 (2007). hep-ph/0608036;  
Y. Hatta and A. H. Mueller, Nucl. Phys. **A789**, 285 (2007). hep-ph/0702023;  
M. Kozlov, E. Levin, and A. Prygarin (2006). hep-ph/0606260;  
M. Kozlov, E. Levin, and A. Prygarin, Nucl. Phys. **A792**, 122 (2007). arXiv:0704.2124 [hep-ph];  
E. Levin, J. Miller, and A. Prygarin (2007). arXiv:0706.2944;  
A. I. Shoshi and B.-W. Xiao, Phys. Rev. **D73**, 094014 (2006). hep-ph/0512206;  
M. Kozlov and E. Levin, Nucl. Phys. **A779**, 142 (2006). hep-ph/0604039;  
S. Bondarenko, L. Motyka, A. H. Mueller, A. I. Shoshi, and B. W. Xiao, Eur. Phys. J. **C50**, 593 (2007). hep-ph/0609213;  
E. Levin and A. Prygarin (2007). hep-ph/0701178;  
M. A. Braun and G. P. Vacca, Eur. Phys. J. **C50**, 857 (2007). hep-ph/0612162;  
J. P. Blaizot, E. Iancu, and D. N. Triantafyllopoulos, Nucl. Phys. **A784**, 227 (2007). hep-ph/0606253;  
E. Iancu, J. T. de Santana Amaral, G. Soyez, and D. N. Triantafyllopoulos, Nucl. Phys. **A786**, 131 (2007). hep-ph/0611105;  
G. Soyez, Phys. Rev. **D72**, 016007 (2005). hep-ph/0504129.
- [11] Y. Hatta, E. Iancu, C. Marquet, G. Soyez, and D. N. Triantafyllopoulos, Nucl. Phys. **A773**, 95 (2006). hep-ph/0601150.
- [12] A. Kovner, M. Lublinsky, and H. Weigert, Phys. Rev. **D74**, 114023 (2006). hep-ph/0608258;  
A. I. Shoshi and B.-W. Xiao, Phys. Rev. **D75**, 054002 (2007). hep-ph/0605282;  
M. Kozlov, E. Levin, V. Khachatryan, and J. Miller, Nucl. Phys. **A791**, 382 (2007). hep-ph/0610084.
- [13] E. Iancu, C. Marquet, and G. Soyez, Nucl. Phys. **A780**, 52 (2006). hep-ph/0605174.
- [14] A. Kovner and M. Lublinsky, JHEP **11**, 083 (2006). hep-ph/0609227.
- [15] M. Kozlov, A. I. Shoshi, and B.-W. Xiao, Nucl. Phys. **A792**, 170 (2007). hep-ph/0612053;  
M. Kozlov, A. I. Shoshi, and B.-W. Xiao (2007). arXiv:0706.3998 [hep-ph].
- [16] C. Marquet, G. Soyez, and B.-W. Xiao, Phys. Lett. **B639**, 635 (2006). hep-ph/0606233.
- [17] M. Kozlov, A. Shoshi, and W. Xiang (2007). arXiv:0707.4142.
- [18] A. Dumitru, E. Iancu, L. Portugal, G. Soyez, and D. N. Triantafyllopoulos (2007). arXiv:0706.2540.

- [19] G. Beuf (2007). [arXiv:0708.3659](#) [hep-ph].
- [20] J. L. Albacete and Y. V. Kovchegov, *Phys. Rev.* **D75**, 125021 (2007). [arXiv:0704.0612](#) [hep-ph];  
Y. V. Kovchegov and H. Weigert, *Nucl. Phys.* **A789**, 260 (2007). [hep-ph/0612071](#);  
I. Balitsky, *Phys. Rev.* **D75**, 014001 (2007). [hep-ph/0609105](#);  
M. A. Braun, *Eur. Phys. J.* **C51**, 625 (2007). [hep-ph/0703006](#);  
M. A. Braun (2007). [arXiv:0707.2314](#);  
J. L. Albacete (2007). [arXiv:0707.2545](#) [hep-ph].