The coherent inelastic processes on nuclei at ultrarelativistic energies

V.L. Lyuboshitz , V.V. Lyuboshitz [1] †
Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia
[1] *E-mail: Valery.Lyuboshitz@jinr.ru*

Abstract
The coherent inelastic processes of the type $a \to b$, which may take place in the collisions of hadrons and γ-quanta with nuclei at very high energies (the nucleus remains the same), are theoretically investigated. The influence of matter inside the nucleus is taken into account by using the optical model based on the concept of refraction index. Analytical formulas for the effective cross-section $\sigma_{\rm coh}(a \to b)$ are obtained, taking into account that at ultrarelativistic energies the main contribution into $\sigma_{\rm coh}(a \to b)$ is provided by very small transferred momenta in the vicinity of the minimum longitudinal momentum transferred to the nucleus.

1 Momentum transfer at ultrarelativistic energies and coherent reactions on nuclei

In the present work we will investigate theoretically the processes of inelastic coherent scattering at collisions of particles with nuclei at very high energies. It is essential that at ultrarelativistic energies the minimum longitudinal momentum transferred to a nucleus tends to zero, and in connection with this the role of coherent processes increases.

Let $f_{a+N\to b+N}(\mathbf{q}) = [Z f_{a+p\to b+p}(\mathbf{q}) + (A-Z) f_{a+n\to b+n}(\mathbf{q})]/A$ be the average amplitude of an inelastic process $a + N \to b + N$ on a separate nucleon in the rest frame of the nucleus (laboratory frame). Here Z is the number of protons in the target nucleus, $(A - Z)$ is the number of neutrons in the target nucleus, $\mathbf{q} = \mathbf{k}_b - \mathbf{k}_a$ is the momentum transferred to the nucleon, $\mathbf{k}_a$ and $\mathbf{k}_b$ are the momenta of the particles a and b, respectively. In the framework of the impulse approximation [1], taking into account the interference phase shifts at the inelastic scattering of a particle a on the system of nucleons, the expression for the effective cross-section of the coherent inelastic process $a \to b$ on a nucleus can be presented in the following form:

$$\sigma_{\rm coh}(a \to b) = \int |f_{a+N\to b+N}(\mathbf{q})|^2 P(\mathbf{q}) d\Omega_b, \tag{1}$$

where $d\Omega_b$ is the element of the solid angle of flight of the particle b in the laboratory frame, and the magnitude $P(\mathbf{q})$ has the meaning of the probability of the event that at the collision with the particle a all the nucleons will remain in the nucleus and the quantum state of the nucleus will not change. Let us introduce the nucleon density $n(\mathbf{r})$ normalized by the total number of nucleons in the nucleus: $\int_V n(\mathbf{r}) d^3\mathbf{r} = A$, where the integration is performed over the volume of the nucleus. Then

$$P(\mathbf{q}) = \left| \int_V n(\boldsymbol{\rho}, z) \exp(-i\mathbf{q}_\perp \boldsymbol{\rho}) \exp(-i q_\parallel z) d^2\boldsymbol{\rho}\, dz \right|^2. \tag{2}$$

† speaker

Here the axis z is parallel to the initial momentum $\mathbf{k}_a$, $\mathbf{q}_\perp$ and $q_\parallel$ are the transverse and longitudinal components of the transferred momentum, respectively.

It is easy to see that the momenta $|\mathbf{q}| \lesssim 1/R$, transferred to a nucleon (R is the radius of a nucleus), give the main contribution to the effective cross-section of the coherent inelastic process $a \to b$ on the nucleus. At ultrarelativistic energies, when $E_a \gg 1/R$, $E_b \gg 1/R$, the recoil energy of the nucleon $E_{\rm rec} \approx |\mathbf{q}|^2/m_N \lesssim (m_N R^2)^{-1}$ and the much smaller recoil energy of the nucleus can be neglected. In doing so, the effective flight angles for the particle b are very small: $\theta \lesssim 1/kR \ll 1$, where $k = E_a \approx E_b$. Then it is possible to assume in Eqs. (1) and (2) that the transverse and longitudinal transferred momenta are as follows:

$$|\mathbf{q}_\perp| = k\theta, \qquad q_\parallel = q_{\rm min} = \frac{m_a^2 - m_b^2}{2k}, \tag{3}$$

where m_a and m_b are the masses of the particles a and b, respectively. Here $q_{\rm min}$ is the *minimum* transferred momentum corresponding to the "forward" direction.

In most cases the characteristic momentum transferred to the nucleus at the inelastic coherent scattering ($|\mathbf{q}| \sim 1/R$) is small as compared with the characteristic momentum transferred to the nucleon in the process $a+N \to b+N$. In connection with this, the amplitude $f_{a+N\to b+N}(\mathbf{q})$ in Eq.(1) can be replaced by its value $f_{a+N\to b+N}(0)$ corresponding to the flight of the particle b in the "forward" direction. Taking into account that at small angles θ the solid angle in Eq. (1) is $d\Omega_b = \sin\theta d\theta d\phi \approx d^2\mathbf{q}_\perp/k^2$ and using the properties of the two-dimensional δ-function, we obtain, as a result of integrating the expression (1) over the transverse transferred momenta and over the volume of the nucleus, the following equation:

$$\sigma_{\rm coh}(a \to b) = \frac{4\pi^2}{k^2}|f_{a+N\to b+N}(0)|^2 \times \int \left(\left| \int_{-\infty}^{\infty} n(\boldsymbol{\rho}, z) \exp(-iq_{\rm min}\, z)\, dz \right|^2 \right) d^2\boldsymbol{\rho}, \tag{4}$$

where $q_{\rm min}$ is determined by Eq. (3).

In the case of a spherical nucleus with the radius R and the constant density of nucleons $n_0 = 3A/4\pi R^3$, Eq. (4) gives at sufficiently high energies, when $q_{\rm min}R \ll 1$:

$$\sigma_{\rm coh}(a \to b) = \frac{8\pi^3}{k^2} n_0^2\, |f_{a+N\to b+N}(0)|^2 R^4 = \frac{9\pi}{2k^2R^2} A^2\, |f_{a+N\to b+N}(0)|^2. \tag{5}$$

In so doing, the magnitude $\Delta\Omega_b = 9\pi/2k^2R^2$ has the meaning of the "effective" solid angle of flight of the final particle b in the vicinity of the "forward" direction.

It should be noted that our consideration relates not only to binary reactions but also to multiparticle coherent processes $a \to b_1 + b_2 + ...b_i$ on nuclei at very high energies. In the general case vector $\mathbf{k}_b$ has the meaning of the total momentum of the system $b = \{b_1, b_2...b_i\}$ with the *effective mass* m_b. In so doing, the magnitude $|f_{a+N\to b+N}(0)|^2$ determines the cross section of the production of the system b, moving as a whole in the "forward" direction, at the collision of particle a with the separate nucleon.

2 Effect of matter inside the nucleus on coherent processes

In the relations obtained above the multiple scattering of the initial and final particles on nucleons of the nucleus was neglected. This is possible when the mean free paths of particles a and b inside the nucleus are much greater than the nuclear radius R. Actually, the role of matter inside the nucleus may be essential,- especially in the case of medium and heavy nuclei. For the analysis of the effects of matter inside the nucleus we will apply the optical model of the nucleus at high energy based on the concept of refraction index [1, 2].

Further we will consider the influence of matter inside the nucleus for *binary* reactions. According to the known formula for the refraction index, being close to unity, the renormalized momenta of ultrarelativistic particles a and b inside the nucleus can be presented in the form:

$$\widetilde{\mathbf{k}_a} = \mathbf{k}_a + \frac{\mathbf{k}_a}{|\mathbf{k}_a|}\chi_a(\mathbf{r}), \quad \widetilde{\mathbf{k}_b} = \mathbf{k}_b + \frac{\mathbf{k}_b}{|\mathbf{k}_b|}\chi_b(\mathbf{r}),$$

where

$$\chi_a(\mathbf{r}) = \frac{2\pi n(\mathbf{r})}{k} f_{a+N\to a+N}(0), \quad \chi_b(\mathbf{r}) = \frac{2\pi n(\mathbf{r})}{k} f_{b+N\to b+N}(0). \tag{6}$$

Here, as before, $n(\mathbf{r})$ is the density of nucleons inside the nucleus, $k = E_a$ is the initial energy in the rest frame of the nucleus (laboratory frame); $f_{a+N\to a+N}(0)$ and $f_{b+N\to b+N}(0)$ are the average amplitudes of elastic scattering of the particles a and b on a nucleon at the zero angle in the laboratory frame; the complex magnitudes χ_a and χ_b describe the phase shifts and the absorption of the particles a and b at their passage through the matter inside the nucleus, connected with the difference of the refraction indices from unity. The relations (6) hold at $|\chi_a|/k \ll 1$, $|\chi_b|/k \ll 1$.

Taking into account the refraction indices of the particles a and b, the influence of matter inside the nucleus on the coherent inelastic processes implies the introduction of the additional complex phase shift into Eq. (4): the exponential factor $\exp(-iq_{\min}z)$ is replaced by $Q = \exp[-iq_{\min}z + i\delta(\boldsymbol{\rho},\, z)]$. In the case of the spherical nucleus with the constant density $n(\rho,\, z) = n_0$ inside the interval $0 \le |z| \le \sqrt{R^2 - \rho^2}$ ($\rho = |\boldsymbol{\rho}|$) and $n(\rho,\, z) = 0$ outside this interval, the additional phase inside the considered interval is described by the equation:

$$\begin{aligned}\delta(\rho, z) &= \chi_a\,(z + \sqrt{R^2-\rho^2}) + \chi_b\,(\sqrt{R^2-\rho^2} - z) = \\ &= (\chi_a - \chi_b)\,z\, + (\chi_a + \chi_b)\,\sqrt{R^2-\rho^2}\,,\end{aligned} \tag{7}$$

where the magnitudes χ_a and χ_b are determined by Eq. (6) at $n(\mathbf{r}) = n_0$.

Using the optical theorem [3], we can rewrite the relations for χ_a, χ_b in the form:

$$\chi_a = i\,n_0(1 - i\,\alpha_a)\sigma_{aN}/2, \quad \chi_b = i\,n_0(1 - i\,\alpha_b)\sigma_{bN}/2,$$

where σ_{an} and σ_{bn} are the total cross-sections of interaction of the particles a and b with nucleons, averaged over the protons and neutrons of the nucleus, α_a and α_b are the ratios of the real parts of the amplitudes $f_{a+N\to a+N}(0)$ and $f_{b+N\to b+N}(0)$, respectively, to their imaginary parts. Let us note that the quantity $\mathrm{Re}\,(\chi_b - \chi_a)$ determines the additional longitudinal transferred momentum connected with the presence of the matter.

Taking into account Eq. (7), after the replacement $q_{\min} z \rightarrow q_{\min} z - \delta(\boldsymbol{\rho},\, z)$ in Eq. (4) and the integration over z, we obtain the following expression for the cross-section of the coherent reaction $a \rightarrow b$ on a nucleus:

$$\sigma_{\mathrm{coh}}(a \rightarrow b) = \frac{8\pi^3}{k^2} n_0^2 \, \frac{|f_{a+N\rightarrow b+N}(0)|^2}{|q_{\min} + \Delta\chi|^2} \times$$
$$\times \int_0^R \left| \exp\left[-2i\,(q_{\min} - \chi_a)\sqrt{R^2 - \rho^2}\,\right] - \exp\left[2i\chi_b\sqrt{R^2 - \rho^2}\,\right] \right|^2 \rho\, d\rho, \qquad (8)$$

where $\Delta\chi = \chi_b - \chi_a$.

3 Dependence of cross-sections of inelastic coherent processes on the nuclear radius

The results of the section 1 are valid when all effects connected with the rescattering of particles in the matter inside the nucleus are practically absent. In this situation the probabilities of absorption of the particles a and b and the additional phase shifts at their passage through the nucleus are close to zero. In the case of a spherical nucleus with the constant density of nucleons, this leads to the restrictions: $|\chi_a|R \ll 1$, $|\chi_b|R \ll 1$ or $L_a \gg R$, $L_b \gg R$, where

$$L_a = \frac{1}{n_0 \sigma_{aN}}, \qquad L_b = \frac{1}{n_0 \sigma_{bN}} \qquad (9)$$

are the mean free paths inside the nucleus.

In the case of medium and heavy nuclei the radius of the nucleus $R \approx 1.1 \cdot 10^{-13}\, A^{1/3}$ cm then the density of nucleons, incorporated in Eq. (8), amounts to $n_0 \approx 0.28 \cdot 10^{39}$ cm^{-3}.

It follows from Eq. (8) that when both the mean free paths are small as compared with the nuclear radius ($L_a \ll R$, $L_b \ll R$), the coherent processes are conditioned only by the peripheral collisions of the initial particle a with the nucleons located in the surface layer of the nucleus. In the considered case, neglecting in Eq. (8) the particle masses ($|q_{\min}| \ll |\Delta\chi|$), we obtain at $f_{b+N\rightarrow b+N}(0) \neq f_{a+N\rightarrow a+N}(0)$:

$$\sigma_{\mathrm{coh}}(a \rightarrow b) = \pi \, \frac{|f_{a+N\rightarrow b+N}(0)|^2}{|f_{b+N\rightarrow b+N}(0) - f_{a+N\rightarrow a+N}(0)|^2} \times$$
$$\times \left[\frac{L_a^2}{2} + \frac{L_b^2}{2} + 4\, L_a^2 L_b^2 \, \mathrm{Re} \left(\frac{1}{L_a + L_b + i(L_a\alpha_b - L_b\alpha_a)} \right)^2 \right]. \qquad (10)$$

Let us consider now the situation when the total cross-section of the interaction of the initial particle a with nucleons is small, so that $\sigma_{aN} \ll \sigma_{bN}$, $L_a \gg R$, $L_b \lesssim R$; in doing so, the relation $|f_{a+N\rightarrow b+N}(0)| \ll |f_{b+N\rightarrow b+N}(0)|$ should hold. In particular, we can deal with the coherent production of vector mesons ρ^0, ω, ϕ at the interaction of very high energy photons with nuclei.

In the considered case Eq. (8) (without the terms, depending on the masses m_a and m_b) gives:

$$\sigma_{\rm coh}(a\to b)=\pi R^2\left|\frac{f_{a+N\to b+N}(0)}{f_{b+N\to b+N}(0)}\right|^2\times\left\{1+\frac{1}{x^2}\left[\frac{1}{2}(1-e^{-2x})-4\frac{1-\alpha^2}{(1+\alpha^2)^2}\,(1-e^{-x}\cos\alpha x)-\right.\right.$$
$$\left.\left.-\frac{8\alpha}{(1+\alpha^2)^2}\,e^{-x}\sin\alpha x\right]+\frac{1}{x}\left[\frac{4}{1+\alpha^2}\,e^{-x}\cos\alpha x\,-\,\frac{4\alpha}{1+\alpha^2}\,e^{-x}\sin\alpha x-e^{-2x}\right]\right\}, \quad (11)$$

where $\alpha\equiv\alpha_b$, $x=n_0\sigma_{bN}R=R/L_b$. At $x\gg 1$ (large cross-sections σ_{bN}, heavy nuclei) we obtain the simple expression

$$\sigma_{\rm coh}(a\to b)=\pi R^2\left|\frac{f_{a+N\to b+N}(0)}{f_{b+N\to b+N}(0)}\right|^2. \quad (12)$$

Let us emphasize that, according to Eq. (12), the effective cross-section of the coherent process $a\to b$ on a nucleus at very high energies has the same dependence on the number of nucleons (proportional to $A^{2/3}$) as the cross-section of scattering of the final particle b on the "black" nucleus, despite the smallness of the cross-section of interaction of the initial particle a (for example, γ-quantum) with a separate nucleon (in connection with this, see [4,5]).

For the coherent process $\gamma\to\rho^0$ on the lead nucleus ($R=1.1\cdot 10^{-13}\,A^{1/3}$ cm ≈ 6.5 Fm, $L_\rho\sim 1.5$ Fm, $|f_{\gamma+N\to\rho+N}(0)/f_{\rho+N\to\rho+N}(0)|^2\sim 10^{-3}$), the formula (11) is applicable at the energies of γ-quanta above several tens of GeV in the nucleus rest frame ($k\gg m_\rho^2 L_\rho\sim 4.5$ GeV). In doing so, $\sigma_{\rm coh}(\gamma+Pb\to\rho^0+Pb)\sim 1.3$ mb.

When, on the contrary, $\sigma_{aN}\gg\sigma_{bN},\;\; L_b\gg R,\;\; L_a\sim R,\;\; |f_{a+N\to b+N}(0)|\ll|f_{a+N\to a+N}(0)|$, then the effective cross-section of the coherent production of the particle b is described by the same formulas (11), (12), in which one should take $x=R/L_a$, $\alpha\equiv\alpha_a$ and replace the amplitude $f_{b+N\to b+N}(0)$ by $f_{a+N\to a+N}(0)$.

It should be emphasized that at $L_a\gg R$, $L_b\ll R$ the coherent process $a\to b$ is conditioned by the interaction of particle a with nucleons located near the surface of the nucleus in the back hemisphere. On the contrary, at $L_a\ll R$, $L_b\gg R$ this coherent process is conditioned by the interaction of particle a with nucleons located in the vicinity of the nuclear surface in the front hemisphere.

Taking into account that

$$f_{b+N\to b+N}(0)=i\,k\,\sigma_{bN}(1-i\alpha_b)/4\pi, \quad (13)$$

it is easy to verify that the expansion of the expression (11) into the power series over the parameter x leads at $x\ll 1$ to the relation (5), just as one would expect at the conditions $L_a\gg R, L_b\gg R$. In this limit $\sigma_{\rm coh}(a\to b)$ is proportional to R^4 (or to $A^{4/3}$).

Let us note that the ratio of the values of the cross sections calculated according to the formulas (12) and (5), respectively, is the following, taking into account Eqs. (9), (13):

$$\eta_b=\frac{k^2}{8\pi^2|f_{b+N\to b+N}(0)|^2\,n_0^2\,R^2}=2\left(\frac{L_b}{R}\right)^2\frac{1}{1+|\alpha_b|^2}. \quad (14)$$

It is clear that the factor η_b has the magnitude of the order of the squared ratio of the "transparency" volume for particle b in the vicinity of the back hemisphere of the nuclear surface to the total volume of the nucleus. At $L_a \ll R$, $L_b \gg R$ the ratio of the corresponding cross sections $\eta_a \sim (L_a/R)^2$ has the analogous meaning with reference to particle a in the vicinity of the front hemisphere of the nuclear surface.

In the given paper we have performed the concrete calculations for the case of a spherical nucleus with the sharp boundary and the constant nucleon density. However, our general relations contain the nucleon density depending on coordinates (see Eqs. (4), (6)) and make it possible, in principle, to take into account the role of the nuclear surface. It is evident that when the thickness of the boundary layer is very small as compared with the radius of the nucleus core, then expression (5) at $L_a \gg R$, $L_b \gg R$ does not change practically. But, in the case of very small free paths, the "transparency" parameters η_b or η_a and, hence, the cross section of the coherent inelastic process can depend essentially on the concrete structure of the surface of the nucleus.

4 Summary

In the present work the coherent processes at the interaction of ultrarelativistic particles with atomic nuclei are investigated. The role of these processes essentially increases at very high energies due to the fact that the minimum momentum, transferred to a nucleon, tends to zero with increasing energy. For the purpose of the analysis of the influence of matter inside the nucleus on coherent reactions, the concept of refraction index is used. The relations, describing the dependence of the effective cross-sections of the inelastic processes on the nuclear radius and the mean free paths of the initial and final particles in the matter inside the nucleus, are obtained.

We did not consider the reverse transitions at the propagation of final particles in the matter inside the nucleus. In principle, the contribution of these transitions could be studied in the framework of the theory taking into account the distinction of the stationary states in the matter from the stationary states in the vacuum due to the mixing of the vacuum states. One may expect that really, with existing sizes of nuclei, the corresponding effects are relatively small.

This work is supported by Russian Foundation of Basic Research (Grant No. 05-02-16674).

References

[1] M. Goldberger, K. Watson, *Collision theory*. John Wiley & Sons, Inc., New York-London- Sydney, 1964, chapter 11.

[2] R. Jastrov, Phys. Rev. **82**, 261 (1951) .

[3] L.D. Landau, E.M. Lifshitz, *Quantum Mechanics. Nonrelativistic theory.* Moscow, Nauka, 1989 ; §125 [transl. Pergamon Press, New York, 1977].

[4] V.N. Gribov, Zh. Eksp. Teor. Fiz. **57**, 1307 (1969).

[5] L. Frankfurt, V. Guzey, M. McDermott and M. Strikman. Phys. Rev. Lett. **87**, 192301 (2001) .