

Gluon Saturation and Black Hole Criticality

L. Álvarez-Gaumé¹, C. Gómez^{2†}, A. Sabio Vera¹, A. Tavanfar^{1,3}, M. A. Vázquez-Mozo⁴

¹Theory Group, Physics Department, CERN, CH-1211 Geneva 23, Switzerland,

²Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid, E-28049 Madrid, Spain,

³Institute for Studies in Theoretical Physics and Mathematics (IPM) P.O. Box 19395-5531, Tehran, Iran,

⁴Departamento de Física Fundamental, Universidad de Salamanca, Plaza de la Merced s/n, E-37008 Salamanca, Spain and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, Spain.

Abstract

We discuss the recent proposal [1] where it was shown that the critical anomalous dimension associated to the onset of non-linear effects in the high energy limit of QCD coincides with the critical exponent governing the radius of the black hole formed in the spherically symmetric collapse of a massless scalar field. We argue that a new essential ingredient in this mapping between gauge theory and gravity is continuous self-similarity, not present in the scalar field case but in the spherical collapse of a perfect fluid with barotropic equation of state. We identify this property with geometric scaling, present in DIS data at small values of Bjorken x . We also show that the Choptuik exponent in dimension five tends to the QCD critical value in the traceless limit of the energy momentum tensor.

1 Criticality in high energy QCD and black hole formation

One of the major insights of string theory is the unexpected connection between black hole physics and confinement in QCD. This connection is realized on the basis of the deep holographic [2] duality between gravity and gauge theories [3, 4]. A particularly interesting connection between black holes and gauge theories is the dual interpretation [4] of the Hawking–Page phase transition [5] in gravity as the confinement / deconfinement transition in gauge theory at finite temperature. Holography is based on a very concrete set of rules for computing quantum

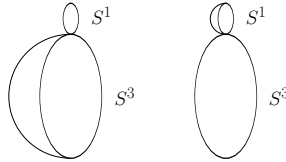


Fig. 1: The two relevant bulk geometries for confinement / deconfinement transition.

[†] speaker

field theory expectation values in terms of classical solutions to SUGRA equations with concrete boundary conditions. A further important ingredient of holography is the generalization of the gauge–gravity correspondence to non local observables such as Wilson or Polyakov loops. In this ‘non local’ version of the correspondence expectation values of Wilson loops are defined as sums of string world sheets in the bulk geometry with boundary determined by the loop.

Wilson loops are natural candidates to define the order parameter of QCD phases. This is the case not only for confinement but also for the transition from a dilute gas of partons to the so-called ‘color glass condensate’ [6] of hadronic parton distributions. In the same way as there exists a geometrical qualitative picture of the transition to confinement, illustrated in Fig. 1, it is natural to search for the corresponding holographic description of the saturation phenomena present in the high energy limit of QCD [7]. A very important experimental discovery in HERA data was the rise of the gluon distribution function $x G(x, Q^2)$ (which is related to the number of gluons in the target proton wave function with effective transverse size of order $1/Q^2$ carrying a fraction x of the hadron longitudinal momentum) when, for fixed Q^2 , the value of x becomes small. This can be seen, *e.g.*, in Fig. 2. In the ‘dipole frame’ the total cross section for

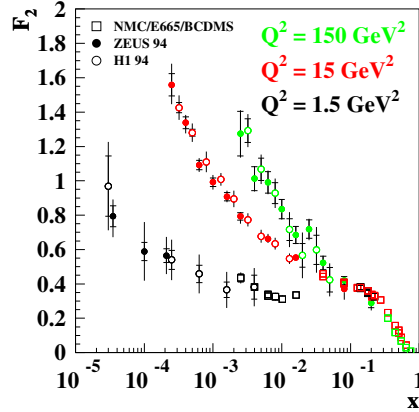


Fig. 2: The dependence on x of the proton structure function F_2 for different values of Q^2 .

the scattering of the virtual photon off the hadron can be expressed in terms of the probability amplitude for the photon to decay into a quark–antiquark pair, creating a colour dipole of size $r = 1/Q$, which then scatters off the proton’s effective colour field. The forward scattering amplitude for the dipole depends on r and the rapidity variable $Y = \log(1/x)$. In the leading order approximation this scattering amplitude depends linearly on $x G(x, Q^2)$. Since unitarity requires the forward scattering amplitude not to be larger than one this indicates that the rise of the gluon distribution should reach a saturation point, leading to the kinematic diagram shown in Fig. 3, where the saturation line indicates the critical value $x_c(Q^2)$ such that for $x < x_c$ and fixed Q^2 the gluon distribution function becomes effectively constant in x .

The two main theoretical problems associated with the previous picture are to identify *i)* the dynamical origin of the rise of $x G(x, Q^2)$ with decreasing x and *ii)* the nature of the non–

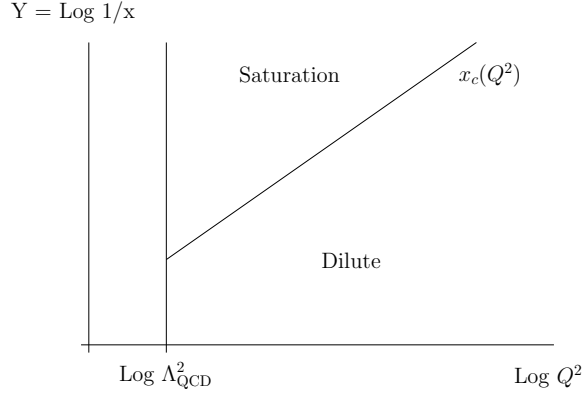


Fig. 3: QCD kinematic space in DIS

linear dynamics responsible for saturation and restoration of unitarity. The current understanding is that the dynamical origin of the rise of the gluon distribution function is due to the dominance of BFKL dynamics [8] while the non-linear BK equation [9] is responsible for the onset of saturation effects in the high energy limit of scattering amplitudes. In the eikonal approximation the exponential rise in Y for the forward scattering amplitude can be computed as the Wilson loop for the quark–antiquark pair propagating in the effective colour field of the proton which, in the proton infinite momentum frame and large center-of-mass energies, is dominated by soft gluon emissions in multi-Regge kinematics, with strong ordering in longitudinal components but not in transverse ones. These configurations, shown in Fig. 4, build up the BFKL hard pomeron.

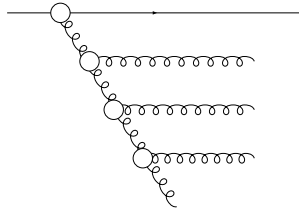


Fig. 4: BFKL gluon cascade in multi-Regge kinematics

In this framework the rapidity Y acts as a cutoff in the effective integration over longitudinal momenta and the BFKL equation controlling the evolution in Y plays the conceptual rôle of a renormalization group equation with a ‘fixed point’, generated by non-linear effects, at the saturation line. This line, of the form $Q_s(x) \sim x^{-\lambda}$, is characterized by the ‘saturation exponent’, which, in the limit of a very small coupling, reads $\lambda \simeq \alpha_s N_c 2.44/\pi$. A direct consequence of the onset of non-linear effects is that asymptotic amplitudes only depend on the variable $\sqrt{\tau} \simeq Qx^\lambda$ nearby the saturation region, *i.e.*, in the $(Y, \log Q^2)$ plane this implies that physical observables only depend on lines of constant τ as it is shown in Fig. 5. Along those lines any continuous boost in longitudinal components can be compensated by an equivalent one in the transverse directions to leave physical quantities invariant. The ‘geometric scaling’ on this variable has been

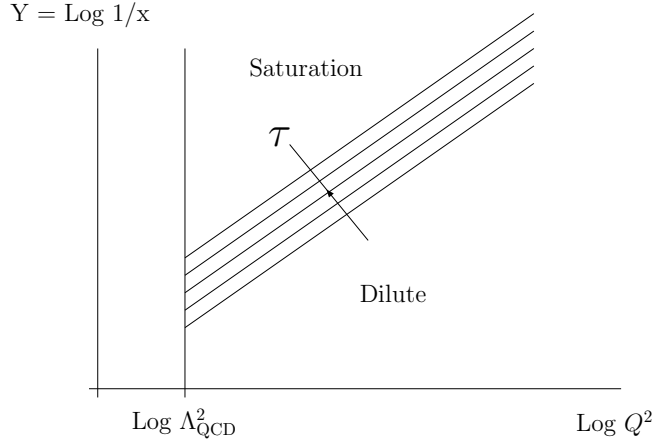


Fig. 5: Continuous self-similarity on the $(Y, \log Q^2)$ plane.

experimentally observed in HERA data [7] for the γ^*p cross sections in the region $x < 0.01$ over a large range in Q^2 , see Fig.6.

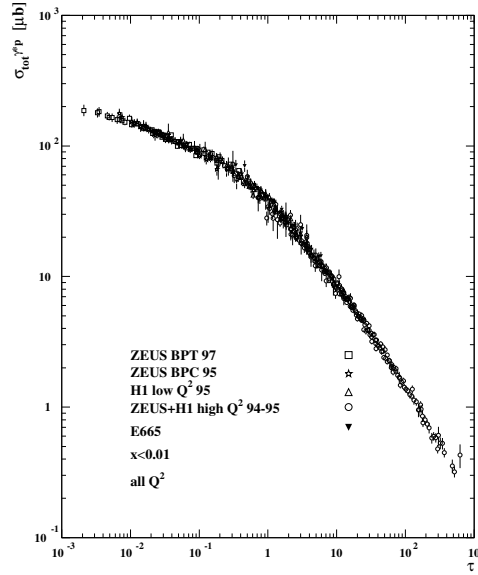


Fig. 6: HERA data for σ_{γ^*p} with $x < 0.01$ versus the variable τ .

Our target is to find the holographic dual of the saturation line [10]. Taking into account the Wilson loop picture of the forward scattering amplitude for the dipole, the holographic representation of this quantity leads us to sum over world sheet amplitudes for a certain bulk geometry [11]. Since the dynamics we want to describe in gravity dual terms is the rapidity dependence of the amplitude, we will formally consider a background metric depending on the dual variable to Y in such a way that in the transition from the dilute to the dense or saturated regime it man-

ifests some sort of ‘geometric scaling’ in terms of properly chosen holographic variables. To establish the correspondence correctly this scaling must be characterized by a critical quantity related to the saturation exponent λ .

A first hint in this direction was shown in [1]. There it was argued that in the numerical studies of black hole formation for the spherically symmetric collapse of a massless scalar field carried out by Choptuik [12] (see [13] for a review) there appears a critical exponent very similar to λ . In particular, if we denote by p a generic parameter describing the initial radial density for imploding scalar waves, Choptuik found that there are critical lines in p , $p = p^*$, such that, if $p < p^*$ the scalar wave packet implodes through $r = 0$ and then disperses into flat space–time. But if $p > p^*$, *i.e.*, the ‘supercritical’ case, then after the implosion there is a fraction of field which forms a small black hole. The interesting point is that its radius scales as $r_{\text{BH}} \simeq |p - p^*|^{\frac{1}{\lambda_c}}$, and it turns out that precisely in dimension five $\lambda_c \simeq 2.44$.

Nonetheless, there is a difficulty to map the collapse of a scalar field with QCD, namely, the metric and field components obtained in this case manifest ‘discrete self–similarity’. This means that a similar variable as the above–mentioned τ leaves physical observables invariant under the transformation $\tau \rightarrow \tau + \Delta$, with Δ a constant which has no analogue in four dimensional high energy scattering. We have investigated in detail a different type of gravitational collapse which has self–similarity, in this case ‘continuous self–similarity’ (CSS): the spherical collapse of a perfect fluid with a barotropic equation of state. The Einstein’s equations together with matter’s equations of motion in d dimensions can be solved assuming a unique dependence on the variable $\tau = -r/t$. As an example we discuss the function $y(r, t)$ which is proportional to the ratio of the mean density inside the sphere of radius r to the local density at r . In Fig. 7 it can be seen how $y(r, t)$ maintains a constant r –profile for different values of t . This implies that the solution is CSS since any change in the time coordinate can be compensated by a change in r keeping y unchanged. This CSS property is what we associate with ‘geometric scaling’ in QCD, with t and r being, respectively, the holographic duals of $\alpha_s N_c Y$ and $\log Q^2$ in QCD. The Choptuik exponent characterizing the black hole radius can be obtained by searching for

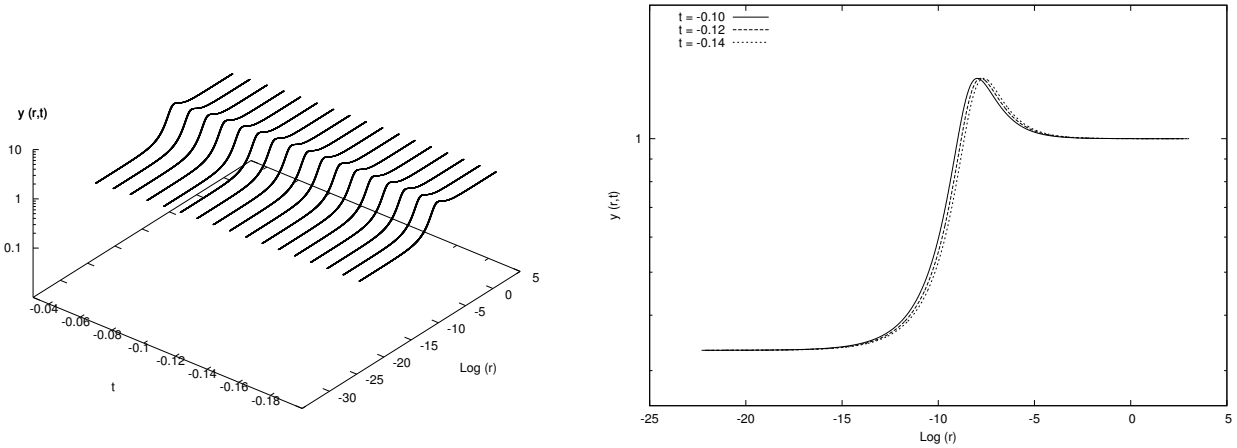


Fig. 7: A typical solution to gravitational collapse of a perfect fluid with continuous self–similarity.

Liapunov modes of instability of the CSS solution. In Fig. 8 we show how the exponentially

growing mode removes CSS from the solution to the collapse. The rate of growth of this mode is given by a coefficient which coincides with Choptuik's exponent. We have numerically extracted this coefficient in the five dimensional case and proven that it is very close to the QCD saturation exponent in the limit of traceless energy–momentum tensor for the fluid [10].

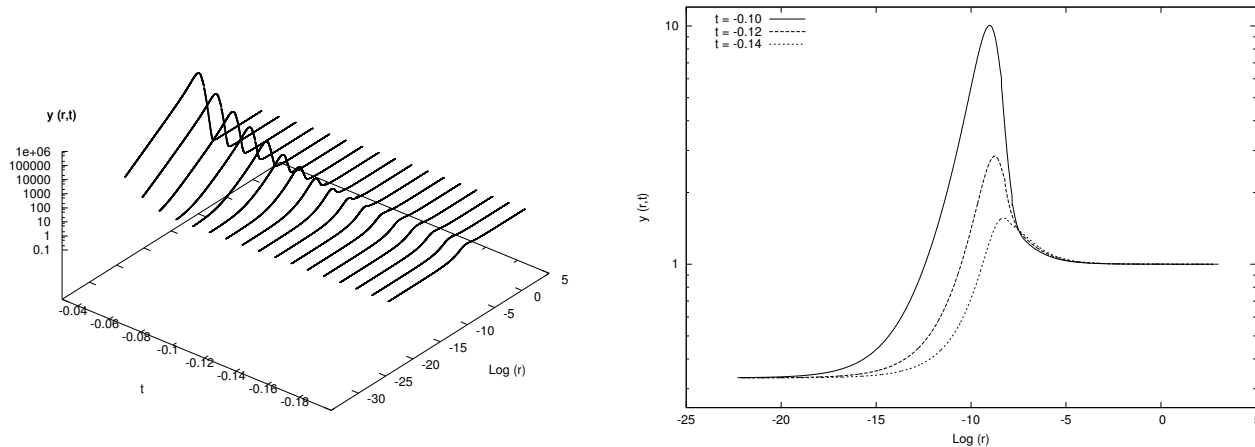


Fig. 8: Continuous self-similarity is lost by an exponentially growing mode in t .

References

- [1] L. Álvarez-Gaumé, C. Gómez, and M. A. Vázquez-Mozo, Phys. Lett. **B649**, 478 (2007). hep-th/0611312.
- [2] G. 't Hooft (1993). gr-qc/9310026;
L. Susskind, J. Math. Phys. **36**, 6377 (1995). hep-th/9409089.
- [3] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998). hep-th/9711200;
S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. **B428**, 105 (1998). hep-th/9802109;
O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, Phys. Rept. **323**, 183 (2000).
hep-th/9905111.
- [4] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998). hep-th/9802150.
- [5] S. W. Hawking and D. N. Page, Commun. Math. Phys. **87**, 577 (1983).
- [6] E. Iancu, A. Leonidov, and L. D. McLerran, Nucl. Phys. **A692**, 583 (2001). hep-ph/0011241;
E. Iancu, A. Leonidov, and L. D. McLerran, Phys. Lett. **B510**, 133 (2001). hep-ph/0102009.
- [7] A. M. Stasto, K. J. Golec-Biernat, and J. Kwiecinski, Phys. Rev. Lett. **86**, 596 (2001). hep-ph/0007192.
- [8] V. S. Fadin, E. A. Kuraev, and L. N. Lipatov, Phys. Lett. **B60**, 50 (1975);
E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP **44**, 443 (1976);
E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP **45**, 199 (1977);
I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978).
- [9] I. Balitsky, Nucl. Phys. **B463**, 99 (1996). hep-ph/9509348;
Y. V. Kovchegov, Phys. Rev. **D60**, 034008 (1999). hep-ph/9901281.
- [10] L. Álvarez-Gaumé, C. Gómez, A. Sabio Vera, A. Tavanfar, and M. A. Vázquez-Mozo, work in progress.
- [11] H. Liu, K. Rajagopal, and U. A. Wiedemann, Phys. Rev. Lett. **97**, 182301 (2006). hep-ph/0605178;
H. Liu, K. Rajagopal, and U. A. Wiedemann, Phys. Rev. Lett. **98**, 182301 (2007). hep-ph/0607062;
H. Liu, K. Rajagopal, and U. A. Wiedemann, JHEP **03**, 066 (2007). hep-ph/0612168.
- [12] M. W. Choptuik, Phys. Rev. Lett. **70**, 9 (1993).
- [13] C. Gundlach, Phys. Rept. **376**, 339 (2003). gr-qc/0210101.