Multiple interactions and AGK rules in pQCD

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Abstract
We review some aspects of multiple interactions in High Energy QCD; we discuss in particular AGK rules and present some results concerning multiple interactions in the context of jet production.

1 Introduction

Many years ago Abramovsky, Gribov and Kancheli in their pioneering paper [1] have pointed out that, for high energy hadron-hadron scattering, multiple exchange of pomerons leads to observable effects in multiparticle final states. Multi pomeron exchange induces indeed fluctuations in the rapidity densities of the produced particles; concerning the multiple inclusive production of jets, they cause long range rapidity correlations. Nowdays it has become evident that multiple interactions play a substantial role in determining the behaviour of high energy scattering. As in-depth studies of DIS at HERA in the small x region and jet physics at Tevatron have shown, diffractive events represent a substantial fraction of the total cross section.

The advent of LHC will open up a much wider kinematical window with respect to any other hadron collider which as been available so far. Needless to say, the challenging measurements aimed at the discovery of physics beyond the Standard Model require an extremely precise understanding of the background physics. In particular, it is needed to assess the effects introduced by multiple interactions. There are kinematical regions where the power suppression due to the “higher twist” nature of these effect is expected to be compensated. If a jet is produced close to the forward direction for example, one of the colliding hadrons PDF is probed in the region of very small longitudinal fraction, where the dominant gluon density undergo a step rise of the type $\propto (1/x)^{\lambda}$, $x \to 0$, $\lambda > 0$. Here is where the mellow concepts developed in the pre QCD era becomes topical again.

As far as the theoretical motivations for considering multiple interactions are concerned, it is well known that they are expected to unitarize cross sections. The resummation of Leading Logarithms (LL) $\log(1/x)$ in pQCD results in the perturbative BFKL pomeron [2–4], which violated the unitarity constraint expressed by the Froissart bound, $\sigma_{\text{tot}} \leq \log^2(s)$, $s \to \infty$. Finding a systematic way of including a minimal subset of subleading corrections in order to restore unitarity has been subject to intensive research in the past decade.

In this talk we discuss AGK rules in the context of pQCD and some recent results regarding multiple interaction effects in inclusive jet production. The relevant papers are [5, 6].

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2 Review of AGK rules in Regge theory

In their simplest version, AGK cutting rules are nothing but a statement about how the s-channel unitarity is encoded in reggeon diagrams. In standard perturbation theory the corresponding tool is the set of Cutkosky rules [7], which tells us how to build up the discontinuity of an amplitude at a particular order in the coupling, summing up all the possible cut diagrams contributing to the amplitude. Symbolically, we could write

$$2\mathrm{Im}\mathcal{A} = \sum_{\text{cuts}} \not{\mathcal{A}}, \tag{1}$$

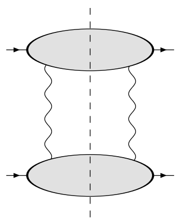
where $\not{\mathcal{A}}$ indicates generically any cut version of the amplitude $\mathcal{A}$. Such an approach is clearly unfeasible when considering reggeon diagrams, since their being already the sum of a full series in the coupling implies that we should consider an infinite set of cut amplitudes already at the simplest level beyond a single ladder. Each term in the sum of multi-ladder diagrams contains a number of possible cuts quickly growing with the number of the rungs in each ladder.

The fundamental observation of AGK was that cut reggeon diagrams where the cut cross a side line of the ladder are strongly suppressed compared to diagrams where the reggeons are cut or uncut completely. The natural question arising from this observation is whether the latter set of diagrams is alone sufficient to reconstruct the full discontinuity; the striking result pointed out in the AGK papers is the affirmative answer to this question. Let us then review their arguments.

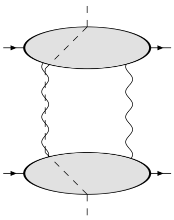
The starting point of the AGK analysis is the Sommerfeld-Watson representation of the elastic scattering amplitude $\mathcal{A}$:

$$\mathcal{A}(s,t) = \int \frac{d\omega}{2i} \xi(\omega) s^{1+\omega} \mathcal{F}(\omega, t), \qquad \xi(\omega) = \frac{\tau - e^{-i\pi\omega}}{\sin \pi\omega}. \tag{2}$$

The signature function can be found in [5]. The (real-valued) partial wave $\mathcal{F}(\omega, t)$ has singularities in the complex ω-plane, and the multi-Regge exchange corresponds to a particular branch cut. As we have already pointed out, the central goal of the AGK analysis is the decomposition of the contribution of the n-reggeon cut in terms of s-channel intermediate states. The absorptive part of the amplitude will consist of several different contributions: each piece belongs to a particular energy cut line, and there are several different ways of drawing such energy-cutting lines. Each of them belongs to a particular set of s-channel intermediate states. For example, a cutting line between reggeons,



belongs to double diffractive production on both sides of the cut: there is a rapidity gap between what is inside the upper blob and the lower blob. When relating this contribution with the full diagram, one requires a 'cut version' of the reggeon particle couplings N_n (represented as grey blobs in the figures). Similarly, the cut through a reggeon,



corresponds to a so-called multiperipheral intermediate state, and another cut version of the particle-reggeon coupling appears. The basis of the AGK analysis is the observation that, under very general assumptions for the underlying dynamical theory, these couplings are fully symmetric under the exchange of reggeons, and all their cut versions are identical. This property then allows to find simple relations between the different cut contributions, and to derive a set of counting rules.

Due to the analytic structure of the partial wave $\mathcal{F}(\omega, t)$ in the s-channel physical region, which is given by a set of cuts and poles on the real axis on the right side of a fixed point $\omega = \omega_0$, the amplitude $\mathcal{A}$ can be decomposed into the sum of the different contributions due to the various singularities,

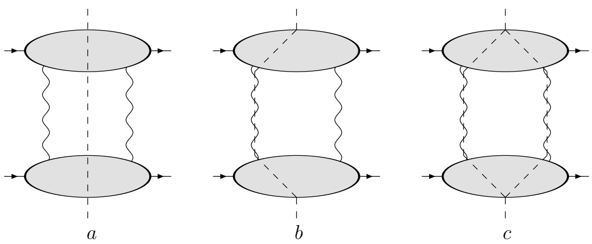
$$\mathcal{A}(s,t) = \sum_{n=1}^{\infty} \mathcal{A}_n(s,t), \tag{3}$$

where $\mathcal{A}_n(s,t)$ is the contribution due to the n-reggeon branch point. A further decomposition of $\mathrm{Im}\mathcal{A}_n$ was found by AGK by observing that *the complete result is obtained by summing up just the diagrams where the reggeons are cut or uncut completely*, therefore neglecting all the (multitude of) diagrams where the cutting line breaks up at least one of the reggeons. For the n-reggeon cut there are $n+1$ of such contributions (any number of cut reggeon from 0 to n), and one ends up with

$$\boxed{2\mathrm{Im}\mathcal{A}(s,t) = \sum_{n=1}^{\infty} \sum_{k=0}^{n} \not{\mathcal{A}}_{nk}(s,t).} \tag{4}$$

Comparing this last equation with (1), it is immediately clear that we have struck a big deal: in building up the imaginary part of the amplitude, we have got rid of most of the cuts on the r.h.s. of (1), only those in (4) being left.

The simplest case, the two-Pomeron exchange, has the three contributions:

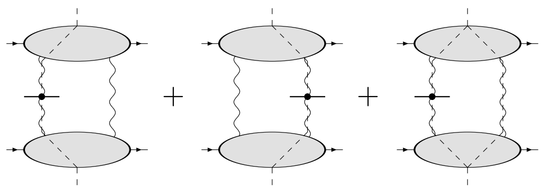


(a) in the diffractive cut all the pomerons are left uncut, and there is a rapidity gap between the fragmentation regions of the two particles; (b) in the single multiplicity cut only one pomeron has been cut; (c) when both pomerons are cut the multiplicity of particles is doubled with respect to the previous case. In this case one obtains the well known result that the different contributions are in the proportion

$$\not{A}_{20} : \not{A}_{21} : \not{A}_{22} = 1 : -4 : 2. \tag{5}$$

Unfortunately AGK constraints can be formulated only for a very restricted class of interaction vertices (in particular, for the $1 \to n$ Pomeron vertex). For the general case (for example, for the $2 \to 2$ vertex) this is not the case; only explicit models, e.g. calculations in pQCD as the one discussed in section 4, can provide further information.

Another remarkable result stems from the AGK analysis: for the n-particle inclusive cross section, large classes of multi-pomeron corrections cancel. For the single inclusive case all multi-Pomeron exchanges across the produced particle cancel,



$$= 0 \tag{6}$$

We are now ready to discuss the novel features arising in QCD.

3 AGK rules in pQCD

From this brief review it follows that the central task of performing the AGK analysis in pQCD requires the computation and study of the coupling functions N_n. The simplest task is the study of the two-Pomeron exchange. Since the BFKL Pomeron is a composite state of two reggeized gluons, we have to start from the exchange of four reggeized gluons.

In the simple perturbative situation where the external particles are photons, the computation of these couplings (which are denote D_n) was performed in [8]. The two pomerons exchange contribution is encoded in the amplitude D_4. The most peculiar fact about D_4 is its decomposition into two pieces:

$$D_4 = D_4^I + D_4^R. \tag{7}$$

The first term, D_4^I, is completely symmetric under the exchange of any two gluons, whereas the second one, D_4^R, is a sum of antisymmetric terms which, as a result of bootstrap properties, can be expressed in terms of two-gluon amplitudes, D_2. Under the exchange of the two reggeized gluons, D_2 is symmetric. It is only after this decomposition has been performed, and we have arrived at reggeon particle couplings with 'good' properties, that we can start with the AGK analysis.

Starting from these functions D_4^I and D_4^R, the investigation in [9] has shown in some detail how the AGK counting rules work in pQCD: the analysis has to be done seperately for D_4^I and D_4^R. For the former piece, we obtain the counting arguments for the Pomerons (which is even-signatured) given by AGK; here the essential ingredient is the complete symmetry of D_4^I under the permutation of reggeized gluons. In the latter piece, the odd-signature reggeizing gluons lead to counting rules which are slightly different from those of the even signature Pomeron:

once the bootstrap properties have been invoked and D_4^R is expressed in terms of D_2 functions, cutting lines through the reggeized gluon appear. Since it carries negative signature, the relative weight between cut and uncut reggeon is different from the Pomeron.

In the light of these facts, we will now attempt to use of the pQCD cutting rules in a nonperturbative environment (e.g.multi-ladder exchanges in pp scattering). Basic ingredients are the nonperturbative couplings of n reggeized gluons to the proton. In order to justify the use of pQCD we need a hard scattering subprocess: we will assume that all reggeized gluons are connected to some hard scattering subprocess; consequently, each gluon line will have its transverse momentum in the kinematic region where the use of pQCD can be justified. Since AGK applies to the high energy limit (i.e. the small-x region), all t-channel gluons are reggeized. Based upon the analysis in pQCD, we now formulate a few general conditions which the nonperturbative couplings N_n have to satisfy in order to get the usual AGK counting rules:

(i) they are symmetric under the simultaneous exchange of momenta and color;
(ii) cut and uncut vertices are identical, independently where the cut line enters.

Whenever these two properties are satisfied, it can be proved that the n-reggeized gluon cut satisfies a similar set of counting rules as the original ones found by AGK, but note that, in contrast to the discussion above, in the case of reggeized gluons we do not need to consider cutting lines inside the reggeized gluons: compared to an uncut gluon, a cut gluon line is suppressed in order α_s. The multiplicity of the final state arises due to the s-channel gluons mediating the interactions between the reggeized ones.

A simple (oversimplified) model for the coupling N_n correspond to eikonal couplings:

$$N_{2n} = \Phi(1,2)\Phi(3,4)\ldots\Phi(2n-1,2n) + \text{permutations} \tag{8}$$

Squaring two of these couplings and taking the large N_c limit one obtains that the multiplicity-k contribution to the total cross section is

$$\sigma_k = \int d^2\boldsymbol{k}\; e^{i\boldsymbol{b}\cdot\boldsymbol{q}} P_k(s,\boldsymbol{b})\,, \tag{9}$$

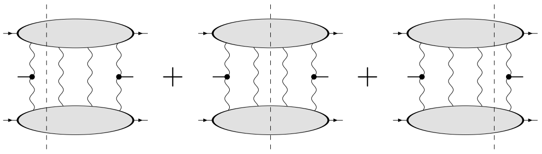
where P_k is a Poissonian distribution,

$$P_k(s,\boldsymbol{b}) = \frac{\Omega(s,\boldsymbol{b})^k}{k!} e^{-\Omega(s,\boldsymbol{b})}\,, \tag{10}$$

which is interpreted as the probability to have k cut pomerons at fixed impact parameter $\boldsymbol{b}$ and total energy $\sqrt{s}$.

4 Multiple interactions in jet production

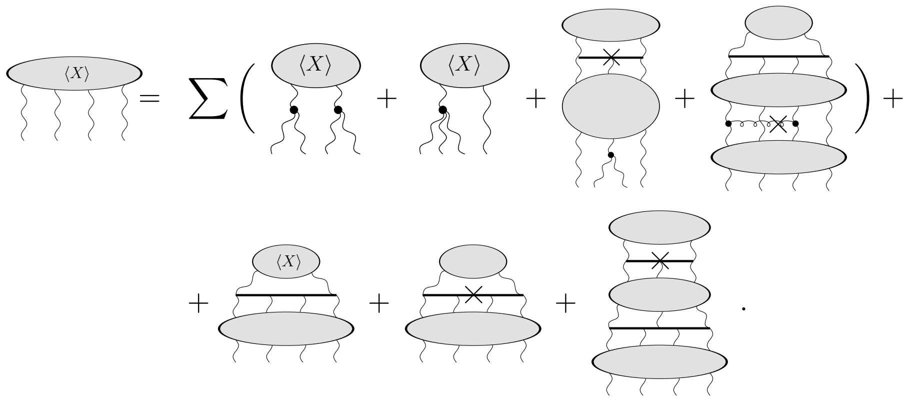
The other remarkable result of AGK, the destructive interference leading to the cancellations of diagrams for jet production as the one represented in eq. (6), has also a counterpart in QCD. The analogous result in QCD reads

$\sum$  $= 0\,,$ (11)

where the sum is over all the possible ways to attach the jet (gluon) on each side of the cut. Such interference take place only due to the summation over all possible final states and integration over the phase space. Any given final state (underlying event), would provide by itself a non-vanishing contribution. The result expressed by eq. (11) is quite general; it holds indeed for an arbitrary number of reggeized gluons and jets (see [5]).

What is left after these cancellation have been exploited are production vertices for the reggeized gluon interactions. Such vertices are model dependent, and must be computed from the underlying theory. The first step in this direction has been taken in [6], where the three cuts of the two-to-four reggeized gluon vertex have been computed. The techniques used are similar to those exploited in [8] for the computation of the two-to-four inclusive vertex (triple pomeron vertex). One writes down a set of coupled evolution equations for the particle-reggeized gluon couplings, where the evolution is given by the exchange of s-channel gluons, and the virtual corrections are taken into account by properly including the gluon Regge trajectories. In the case of single jet production, the kinematics of a gluon is kept fixed. Reshuffling such equation and using the bootstrap property, one obtains the factorization expressed by eq. (7) for the inclusive case.

In the single jet production, the coherence leading to the simple factorization (7) is partially broken by the missing integration over the phase space of the produced jet. In [6] was observed that is possible to obtain gauge invariant objects by identifying antipodal jets: one gives up the distinction between jets emitted in opposite directions in the transverse plane. If doing so, it is possible to factorize the amplitude as a sum of gauge invariant pieces. Explicitely one gets in a pictorial form



(12)

The grey blob on the l.h.s. represents the full particle-reggeized gluons coupling with a gluon fixed. On the r.h.s. one observes the various gauge invariant terms contributing to such coupling. The first three represent “reggeized” terms: they describe the exchange of less then four gluons, some of which are “composite” (they contain corrections beyond the LL reggeized gluon). The fourth term does not contribute to the cross section thanks to the AGK argument expressed by equation (11). The first term in the last line is very simple: the jet is emitted inside the pomeron attached to the external particles, and subsequently the pomeron decays into a four reggeized gluon state via the standard two-to-four vertex. The last two pieces contain new ingredients. In the first appear for the first time the production vertices for the two-to-four transition with the emission of one jet. There are three variants of such a vertex, depending where the s-channel cut passes. In the last piece the new objects are a universal two-to-three production vertex, and three versions of a three-to-four inclusive vertex. More details and explicit expression can be found in [6].

5 Concluding remarks

We have reviewed some aspects of multiple interactions in pQCD, in the context of the total cross section and associated multiplicity distributions (AGK), and of inclusive jet production. We have shown that similar cutting rules as those first obtained in the framework of soft pomeron Regge theory emerge in pQCD as well. We have also presented new vertices for the inclusive production of a jet across the transition between two and three or four reggeized gluons. The full particle-four-gluons coupling has been decomposed in a sum of gauge invariant pieces, and each of them has been computed explicitely. Explicit expressions for all the new vertices are computed and presented in [6].

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