Non-linear QCD at high energies

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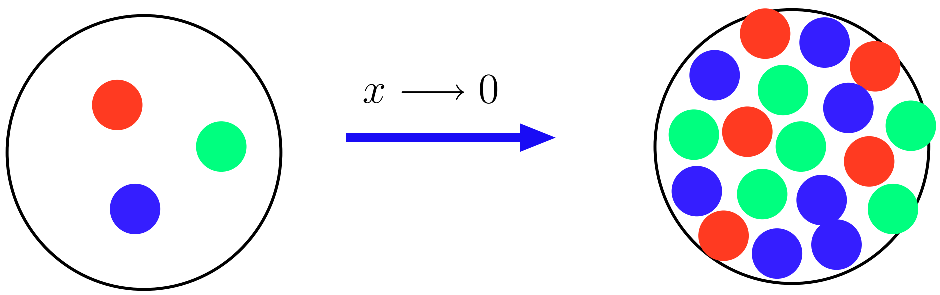
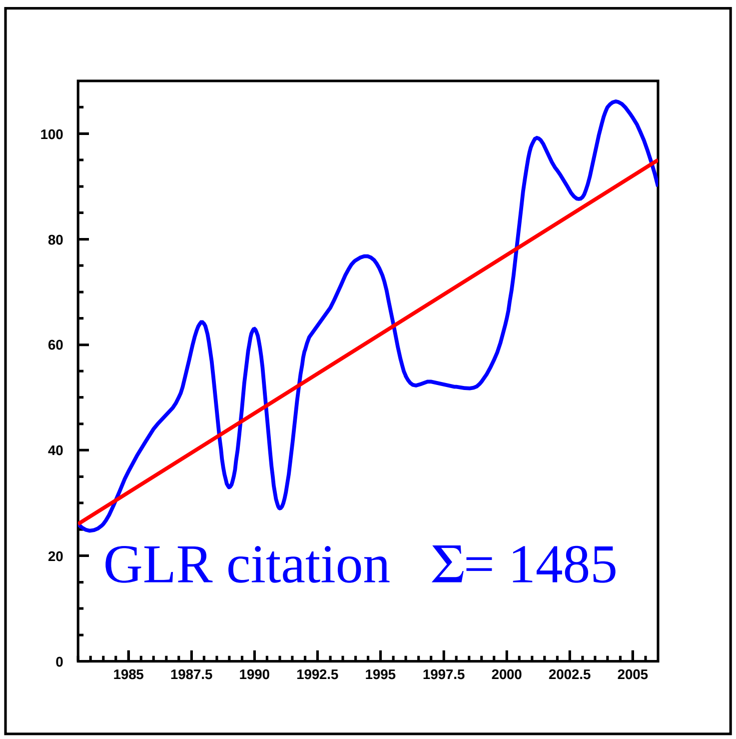
A mini-review with personal opinion and attitude which can be wrong

The state of the art





The problem of high density QCD

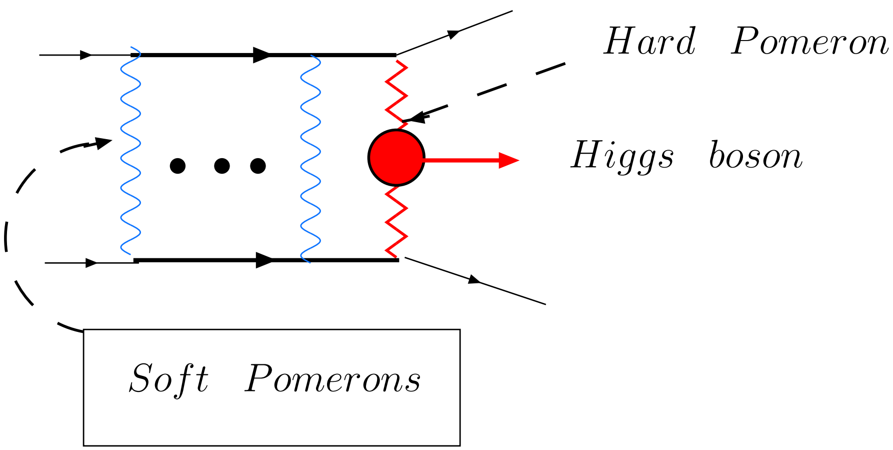
- **has not been solved;**
- **is needed to be solved;**

Outline:

- Practical impact on the LHC physics;
- The BFKL Pomeron calculus: ups and downs;
- Statistical analogy approach: Langevin equation and generating functional;
- Summing Pomeron loops in the BFKL Pomeron calculus;
- The high energy asymptotic behaviour:
- Problems, ideas, solutions ... $\equiv$ $\boxed{\text{bright future}}$;

Practical impact on the LHC physics

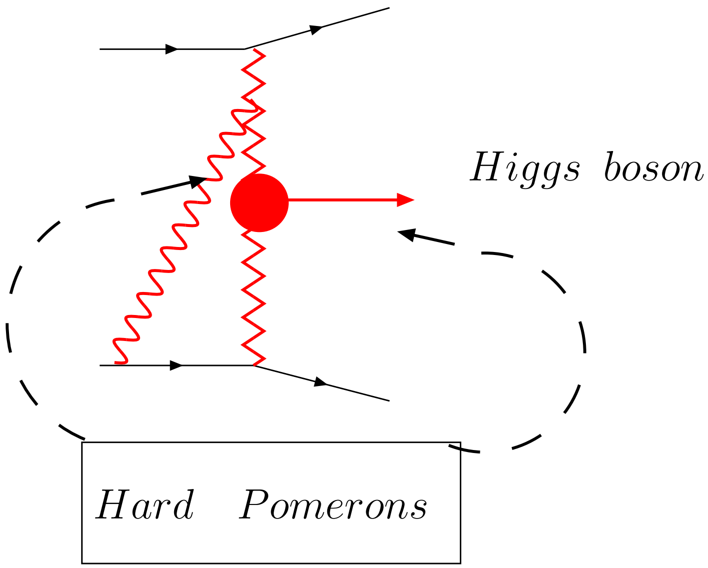
Survival Probability for diffractive Higgs production



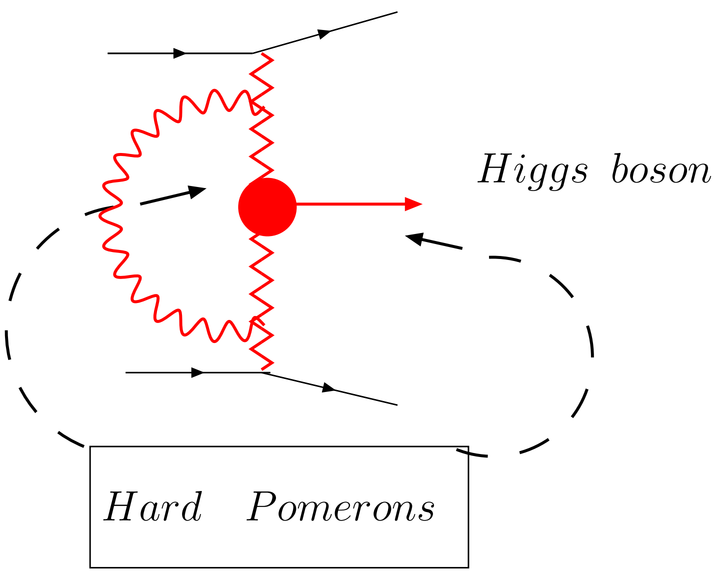
Tel Aviv Group (1994 - present)

Durham Group (1998 - present)

$$< |S^2| > \approx 0.02$$



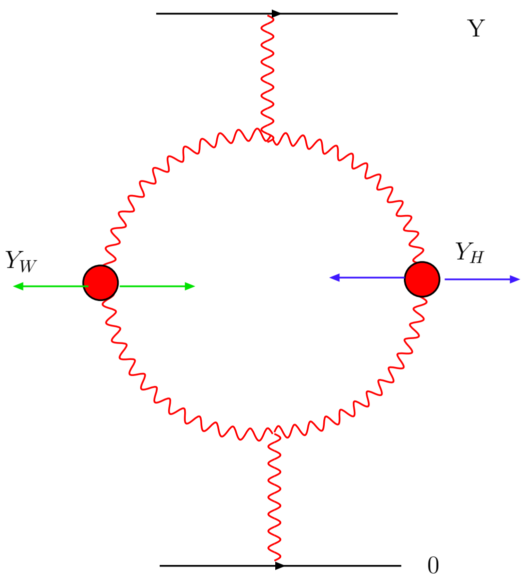
Bartels, Bondarenko and Kutak (2006)



Miller (2006)

$$< |S^2| > \approx \; \mathbf{0.004}$$

W - Higgs boson correlations in inclusive Higgs production



$Y_H \approx Y_W$

CDF (1997)

$$\sigma_{DP} = m\,\frac{\sigma(2\,jets)\,\sigma(2\,jets)}{2\,\sigma_{eff}}$$

$$\sigma_{eff} = 14.5 \pm 1.7 \pm 2.3\ mb$$

$$\frac{1}{\sigma_{eff}} = \frac{1}{2\pi R^2_H}$$

$$R^2_H = 5 \div 7 GeV^{-2}$$

- Hannes Jung at GGI WS on high density QCD asked to estimate such correlation for the LHC energy

- J. Miller (preliminary)

$$\sigma_{DP}(LHC) = \frac{\sigma(W)\,\sigma(H)}{2\,\sigma_{eff}} \approx 3\sigma_{DP}(CDF)$$

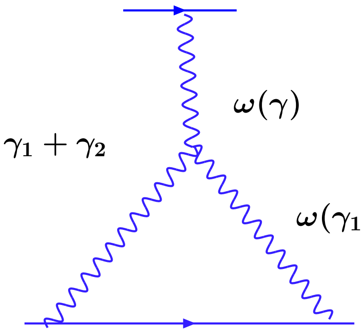
The BFKL Pomeron calculus

News:

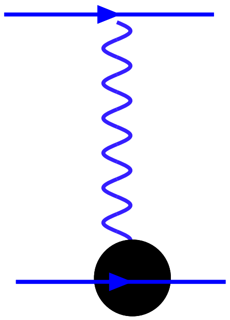
- **Everything that has been done during the past three years is nothing more than understanding of the BFKL Pomeron calculus** (Kozlov,E.L. & Prygarin; Bondarenko);

- **The feedback from the probabilistic interpretation: four Pomeron interactions** (Lublinsky & E.L) ;

- **The news: The Pomeron interaction generates a new state with the intercept large than intercept of two BFKL Pomerons** (Hatta & Mueller; E.L, Miller & Prygarin);

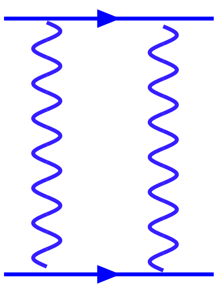


- $$A \propto \frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty} d\omega\; e^{\omega\,Y}\,\frac{1}{\omega-\omega(\gamma)}\,\frac{1}{\omega-\omega(\gamma_1)-\omega(\gamma_2)}$$



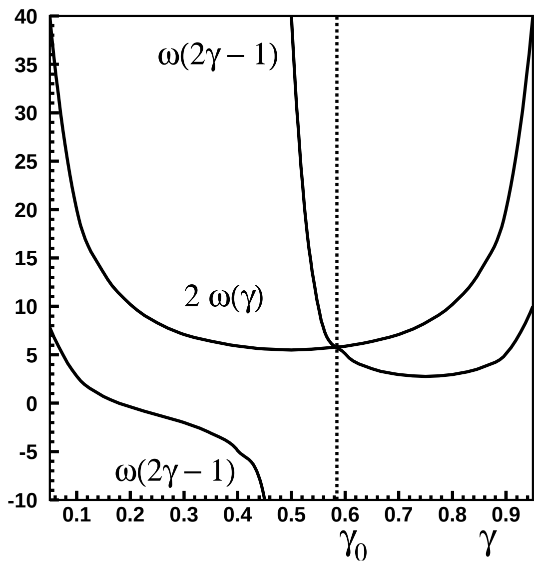
- $$\omega = \omega(\gamma)$$

$$A \propto \frac{e^{\omega(\gamma)\,Y}}{\omega(\gamma)-\omega(\gamma_1)-\omega(\gamma_2)}$$



- $$\omega = \omega(\gamma_1)\,+\,\omega(\gamma_2)$$

$$A \propto \frac{e^{(\omega(\gamma_1)+\omega(\gamma_2))\,Y}}{\omega(\gamma_1)+\omega(\gamma_2)-\omega(\gamma)}$$



- $\omega(2\gamma_0 - 1) = 2\,\omega(\gamma_0)$
- $2\,\omega(\gamma_0) > \omega(\gamma = 1/2)$
- $A \propto Y\, e^{2\,\omega(\gamma_0)\,Y}$

The sad truth: we have to start from the very beginning not only in summing Pomeron loops but also in MFA ? !

Statistical approach: its beauty and problems

We know from the old good days of Reggeon Field Theory (Pomeron calculus) that this theory is the field theory for directed percolation

(Grassberger & Sudermeyer (1978), Obukhov (1980), Cardy & Sugar (1980))

but because of the advent of QCD we did not investigate this idea in the full strength.

TIME HAS COME

- **People:** **Blaizot, Brunet, Derrida, Enberg, Golec-Biernat, Hatta, Iancu, Itakura, E.L., Lublinsky, McLerran, Marquet, Mueller, Munier, Peshanski, Shoshi, Soyez, Triantafyllopoulos + ... nearly everybody**

The BEAUTY: we can model our high energy amplitude by creating some statistical system (glass???) and make the real experiment, may be, just on the desk

Langevin equation:

- $$\frac{\partial \Phi}{\partial Y} = \frac{\bar{\alpha}_S}{2\pi} K \bigotimes \Phi - \frac{2\pi \bar{\alpha}_S^2}{N_c} \Phi^2 + \zeta$$
- $$< |\zeta| > = 0; \qquad < |\zeta\,\zeta| > \neq 0$$

Langevin equation for Einstein diffusion:

- $$\frac{d\vec{v}}{dt} = -\lambda \vec{v} + \zeta$$

The main prediction:

(Iancu, Mueller & Munier (2004))

- violation of the geometrical scaling behaviour ;
- appearance of new saturation scale;

- $A(z,Y) \; = \; A\left(\frac{\ln(r^2\,Q^2_{new,s})}{\sigma}\right) \; = \; \frac{1}{\sigma\sqrt{2\,\pi}}\int dz\,T\,(z)\;\; e^{-\frac{(z-<z>)^2}{2\,\sigma^2}}$
- $\sigma^2 \; \propto \; Y \;; \quad z = \; \ln(r^2\,Q^2_s)$ where r is the dipole size;
- $< z > \; = \; \ln(r^2\,Q^2_{new,s}) \;;\;\; Q_{new,s} =$new saturation (diffusion) scale
- $T\,(z) \;\; = \;$ solution in the MFA;

The BFKL Pomeron calculus leads to (Kozlov,E.L. & Prygarin)

- $$< \zeta(x_1, x_2; Y)\, \zeta(x'_1, x'_2; Y') >= B\ \delta(Y - Y')\ \delta(x_1 - x'_1)\ \delta(x_2 - x'_2)$$

with

- $$B \quad \equiv \quad 2\frac{2\pi\bar{\alpha}_S}{N_c}\left(\frac{1}{p_1^2 p_2^2 \Phi^+(x_1, x_2; Y)}\right)^2$$
$$\times \quad \int \frac{d^2 x_3}{x_{12}^2 x_{13}^2 x_{23}^2} (L_{12}\Phi(x_1, x_2; Y))\Phi^+(x_1, x_3; Y)\Phi^+(x_3, x_2; Y)$$

Price for simplification :

1. assumption: $L_{12}\,\Phi(x_1, x_2; Y) = \Phi(x_1, x_2; Y)$;
2. simplification: using the momentum represntaion and looosing the connection to correct D.O.F.;
3. assumption: $b \gg \{x_{12}; x_{13}; x_{32}\}$ and, therefore, we are looosing a possibily to calculate the Pomeron loops;

Finally

- $$< \zeta(k, b; Y)\zeta(k', b'; Y) > = \frac{4\pi\bar{\alpha}_S^2}{N_c}\,\Phi(k, b; Y)\,\delta^{(2)}(\vec{b} - \vec{b'})\,\delta^{(2)}(\vec{k} - \vec{k'})\,\delta(Y - Y')$$

Questions and Surprises:

- **The Langevin equation for the amplitude looks as follows:**

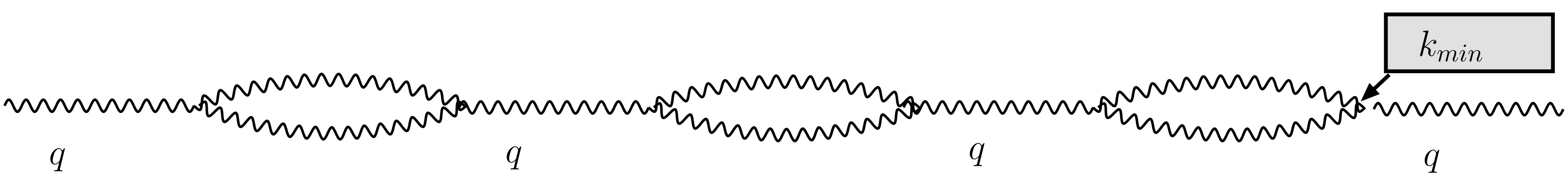
$$\frac{\partial N}{\partial Y} = \bar{\alpha}_S \left(N \; - \; N^2 \; + \; \bar{\alpha}_S \sqrt{2\,N}\,\zeta \right)$$

Only at $N < \bar{\alpha}_S^2$ the third term is essential . On the other hand it sums the enhanced diagrams, how is it possible?

- **In the toy-model equation does not lead to decreasing amplitude at high energy** (Munier(talk at GGI WS); Naftali (private communication)) **in contradiction with the exact solution?!**

- **For $z < 0$ the solution of the BFKL equation leads to the geometrical scaling behaviour** (Iancu, Itakura & McLerran (2002). **How to match this solution with the scaling violating one?!**

- **At first sight the b correlation depend on future. What is wrong?**

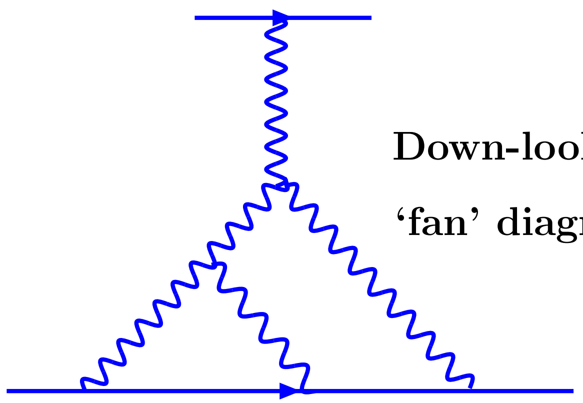


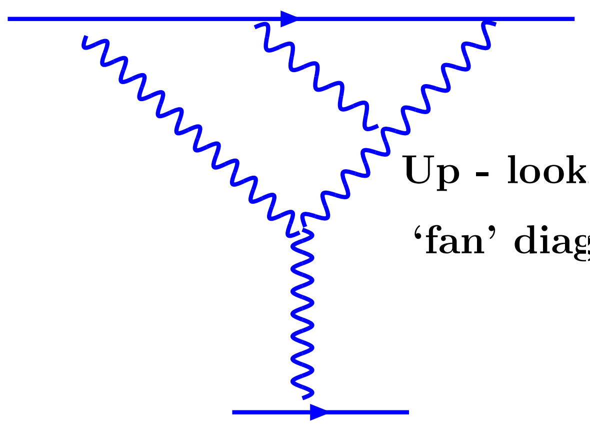
- **For up-looking fan diagrams:**

$$\frac{\partial N}{\partial Y} = \bar{\alpha}_S \left\{ K \bigotimes N \;+\; \sqrt{\bar{\alpha}_S^2 N} \right\}$$

(i) what is the initial condition to get MFA?

(ii) why there is no scaling violation?



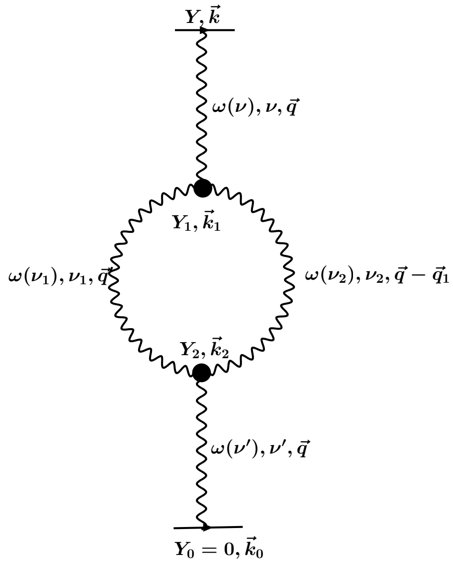


Summinmg Pomeron loops in the BFKL Pomeron calculus

(E.L. , Miller & Prygarin)

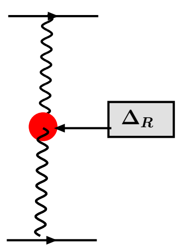
$$1 \approx \bar{\alpha}_S Y \leq \ln 1/\bar{\alpha}_S^2 \leq \bar{\alpha}_S Y \leq \bar{\alpha}_S Y \leq 1/\bar{\alpha}_S$$

- $1 \approx \bar{\alpha}_S Y \leq \ln 1/\bar{\alpha}_S^2 \longrightarrow$ **LO BFKL Pomeron**
- $\ln 1/\bar{\alpha}_S^2 \leq \bar{\alpha}_S Y \leq 1/\bar{\alpha}_S \longrightarrow$ **BFKL Pomeron calculus**
- $1/\bar{\alpha}_S \leq \bar{\alpha}_S Y \longrightarrow$ **NLO BFKL Pomeron and non-linear QCD**



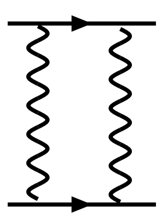
- $$A \propto \int \frac{d\omega}{2\pi i} \frac{1}{\omega - \omega(\gamma)} \frac{1}{\omega - \omega(\gamma_1) - \omega(\gamma_2)} \frac{1}{\omega - \omega(\gamma')}$$

- $$\omega = \omega(\gamma) = \omega(\gamma')$$



$$A \propto \frac{Y\, e^{\omega(\gamma)\, Y}}{\omega(\gamma) - \omega(\gamma_1) - \omega(\gamma_2)}$$

- $$\omega = \omega(\gamma_1) + \omega(\gamma_2)$$



$$A \propto \frac{Y\, e^{(\omega(\gamma) + \omega(\gamma_2) Y}}{(\omega(\gamma) + \omega(\gamma_2 - \omega(\gamma))^2}$$

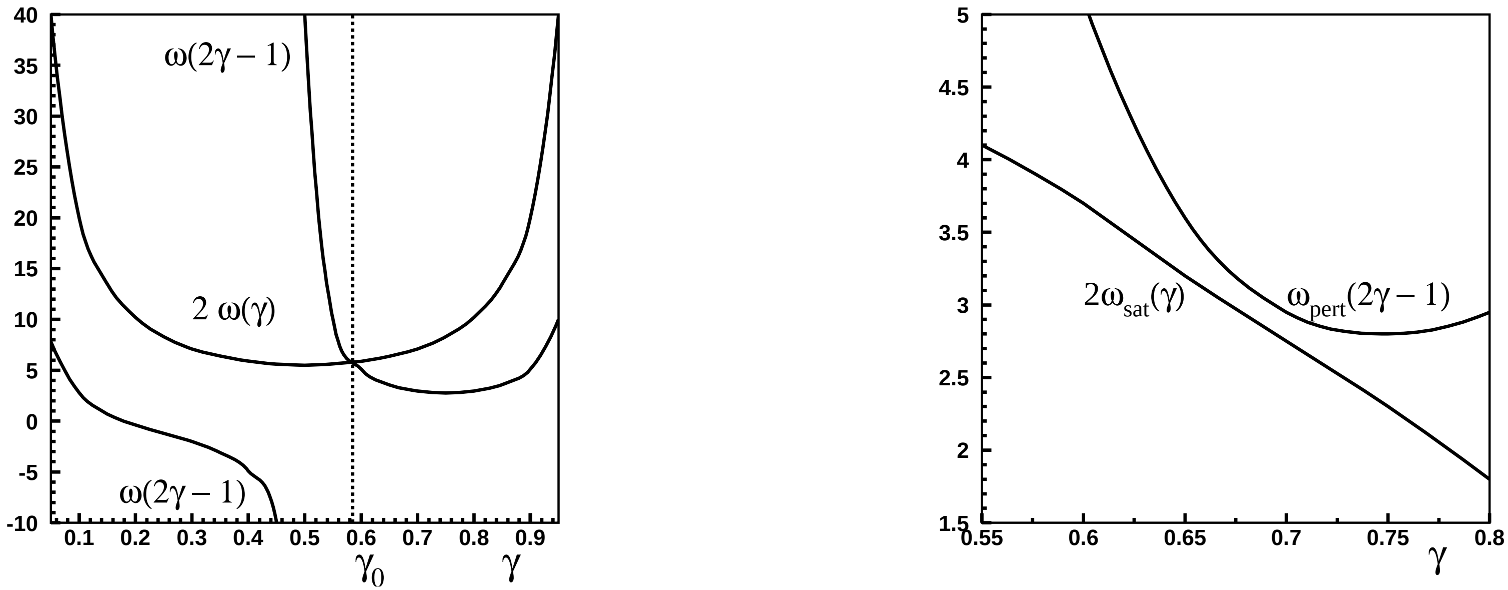
Overlapping singularity:

- $$\omega \;=\; \omega(2\,\gamma_0 - 1) \;=\; 2\,\omega(\gamma_0)$$

- $$A \;\propto\; Y^2\,e^{2\omega(\gamma_0)Y} \;>\; e^{2\omega_{BFKL}(\gamma=1/2)Y}$$

Scenario :

- $\gamma_0 > \gamma_{cr}$ therefore, two Pomerons (γ_1 and γ_2) are inside the saturation region;
- Inside the saturation region $\omega_{sat}(\gamma) \;=\; \frac{\omega(\gamma_{cr})}{1-\gamma_{cr}}\,(1-\gamma)$ (Bartels & E.L. (1992)) ;
- equation $2\omega_{sat}(\gamma_0) \;=\; \omega_{pert}(2\gamma_0 - 1)$ has no solution;

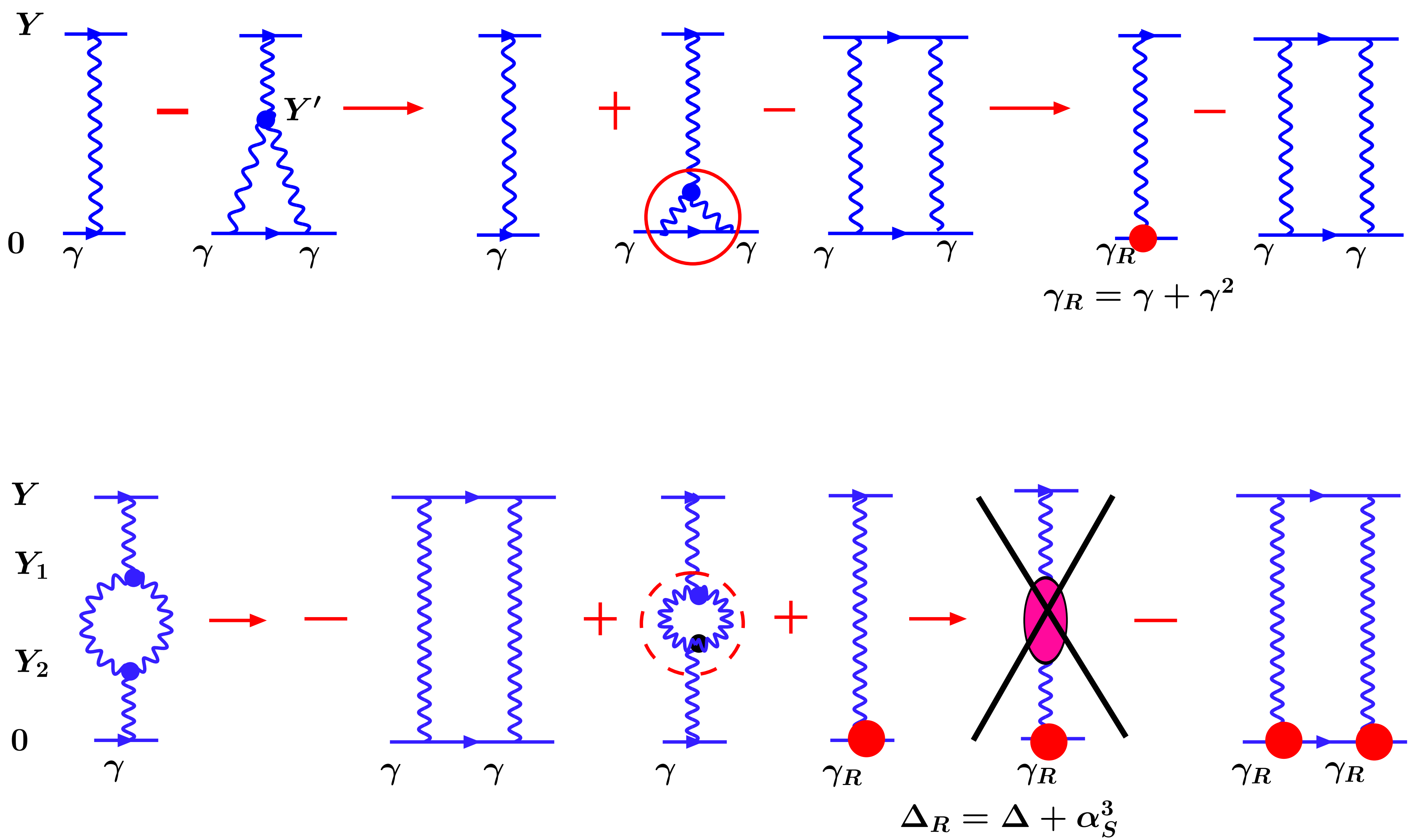


40
35
30
25
20
15
10
5
0
-5
-10
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
ω(2γ – 1)
2 ω(γ)
ω(2γ – 1)
γ0
γ
5
4.5
4
3.5
3
2.5
2
1.5
0.55 0.6 0.65 0.7 0.75 0.8
2ωsat(γ)
ωpert(2γ – 1)
γ

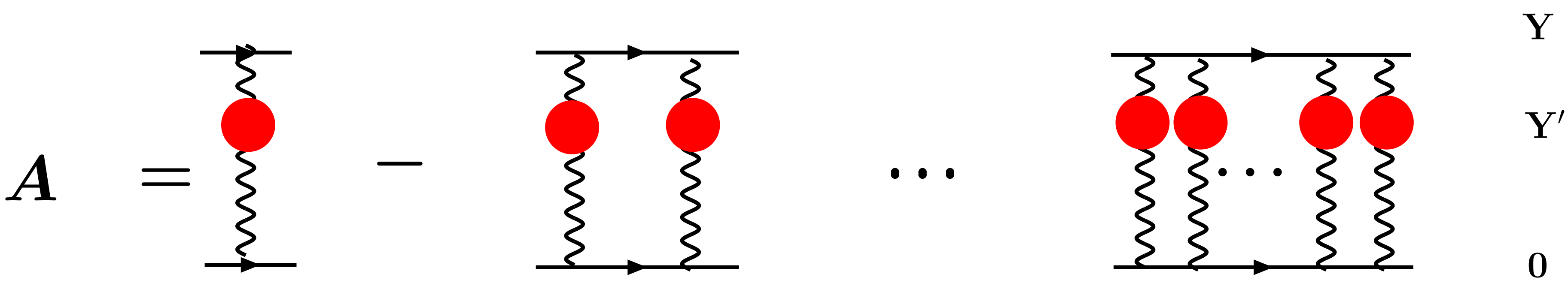
The scenario:

- We can neglect the overlapping singularities;
- We are dealing with the system of the non-interacting BFKL Pomerons;
- For summing Pomeron loops we can use the Iancu-Mueller-Patel -Salam approximation, improved by the renormalization of the scattering amplitude at low energies;

An example:



Y
Y'
0
γ
γ_R
$\gamma_R = \gamma + \gamma^2$
Y_1
Y_2
$\Delta_R = \Delta + \alpha_S^3$



γ^{BA} low energy amplitude $\gamma_R \longrightarrow \gamma^{BA}$

Solution:

For model BFKL kernel

$$\omega(\gamma) \;=\; \bar{\alpha}_S \begin{cases} \dfrac{1}{\gamma} & \text{for } r^2\,Q_s^2 \ll 1 - \text{ summing } \;\; (\bar{\alpha}_S \ln(1/(r^2\,Q_s^2)))^n; \\[2ex] \dfrac{1}{1-\gamma} & \text{for } r^2\,Q_s^2 \gg 1 - \text{ summing } \;\; (\bar{\alpha}_S \ln(r^2\,Q_s^2))^n; \end{cases}$$

we obtain:

- **geometrical scaling behaviour;**
- **rather slow approaching the asymptotic value, namely** $1 - N \propto \exp(-z)$ **where** $z = \ln(r^2 Q_s^2)$;

Conclusions

" Once you eleminate the impossible what remains is the solution - no matter how improbable it may seem"