Pomeron intercept and slope: the QCD connection

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Abstract
We present a model for the ratio of intercept to slope of the Pomeron trajectory as measured from elastic, diffractive and total cross sections.

1 Regge approach to diffraction

Hadronic diffraction has traditionally been treated in the framework of Regge theory [1–3]. In this approach, the key player mediating diffractive processes is the *Regge trajectory* of the Pomeron, presumed to be formed by a *family* of particles carrying the quantum numbers of the vacuum. Although no particles were known (and have yet to be found!) to belong to this family, the Pomeron trajectory was introduced in the 1970s to account for the observations that the K^+p cross section was found to be increasing with energy at the Serpukov 70 GeV ($\sqrt{s} = 11.5$ GeV for pp collisions) proton synchrontron, and the elastic and total pp cross sections, which at low energies were falling with increasing energy, started to flatten out and then rose with energy as collision energies up to $\sqrt{s}$=60 GeV became available at the Intersecting Storage Rings (ISR) at CERN.

In the Regge approach, high energy cross sections are dominated by Pomeron exchange. For pp interactions, the Pomeron exchange contribution to total, elastic, and single diffractive cross sections is given by

$$\sigma^{tot}(s) = \beta^2_{I\!Ppp}(0)\left(\frac{s}{s_0}\right)^{\alpha_{I\!P}(0)-1} \tag{1}$$

$$\frac{d\sigma^{el}(s,t)}{dt} = \frac{\beta^4_{I\!Ppp}(t)}{16\pi}\left(\frac{s}{s_0}\right)^{2[\alpha_{I\!P}(t)-1]} \tag{2}$$

$$\frac{d^2\sigma_{sd}(s,\xi,t)}{d\xi dt} = \underbrace{\frac{\beta^2_{I\!Ppp}(t)}{16\pi}\,\xi^{1-2\alpha_{I\!P}(t)}}_{f_{I\!P/p}(\xi,t)}\;\underbrace{\beta_{I\!Ppp}(0)\,g(t)\,(\frac{s}{s_\circ})^{\alpha_{I\!P}(0)-1}}_{\sigma^{I\!Pp}(s\xi,t)}, \tag{3}$$

where $\alpha_{I\!P}(t) = \alpha_{I\!P}(0) + \alpha' t = (1+\epsilon) + \alpha' t$ is the Pomeron trajectory, $\beta_{I\!Ppp}(t)$ the coupling of the Pomeron to the proton, $g(t)$ the $I\!PI\!PI\!P$ coupling, $s' = M^2$ the $I\!P$-p center of mass energy squared, $\xi = 1 - x_F = s'/s \approx M^2/s$ the fraction of the momentum of the proton carried by the Pomeron, and s_0 an energy scale parameter traditionally set to the hadron mass scale of 1 GeV^2. The single diffractive cross section, Eq. (3), factorizes into two terms, the one on the right which can be viewed as the $I\!P$-p total cross section, and the other labeled $f_{I\!P/p}(\xi,t)$, which may be interpreted as the Pomeron flux emitted by the diffracted proton [4].

Regge theory worked reasonably well in describing elastic, diffractive and total hadronic cross sections at energies of up to $\sqrt{s} \sim 60$ GeV, with all processes accommodated in a simple

Pomeron pole approach, as documented in Ref. [5]. Results from a Rockefeller University experiment on photon dissociation on hydrogen published in 1985 [6] were also well described by this approach.

The early success of Regge theory, however, was precarious. The theory was known to asymptotically violate unitarity, as the $\sim s^{\epsilon}$ power law increase of total cross sections would eventually exceed the Froissart bound of $\sigma_T < \frac{\pi}{m_\pi^2} \cdot \ln^2 s$ based on analyticity and unitarity. But the confrontation with unitarity came at much lower energies than what would be considered asymptopia by Froissart bound considerations. As collision energies climbed upwards in the 1980s to reach $\sqrt{s} = 630$ GeV at the CERN S$\bar{p}p$S collider and $\sqrt{s} = 1800$ GeV at the Fermilab Tevatron $\bar{p}p$ collider, diffraction dissociation could no longer be described by Eq. (3), signaling a breakdown of factorization.

The first clear experimental evidence for a breakdown of factorization in Regge theory was reported by the CDF Collaboration in 1994 [7]. In a measurement of the single diffractive cross section in $\bar{p}p$ collisions at $\sqrt{s}$ =546 and 1800 GeV CDF found a suppression factor of ~ 5 (~ 10) at $\sqrt{s}$ =546 GeV (1800 GeV) relative to predictions based on extrapolations from $\sqrt{s}$ ∼20 GeV.

2 Scaling properties and renormalization of diffraction

The breakdown of factorization in Regge theory was traced back to the expected energy dependence of the single diffractive cross section, $\sigma_{sd}^{tot}(s) \sim s^{2\epsilon}$, which is faster than that of the total cross section, $\sigma^{tot}(s) \sim s^{\epsilon}$, so that as s increased unitarity would have to be violated if factorization held. This is reflected in the $s^{2\epsilon}$ dependence of $d\sigma_{sd}(M^2,t)/dM^2|_{t=0}$:

$$\text{Regge theory: } d\sigma_{sd}(M^2,t)/dM|_{t=0}^2 \sim s^{2\epsilon}/(M^2)^{1+\epsilon}. \tag{4}$$

In a paper first presented by this author in 1995 at the La Thuile [8] and Blois [9] winter conferences and later published in Phys. Lett. B [10], it was shown that unitarization could be achieved, and the factorization breakdown in single diffraction fully accounted for, by interpreting the Pomeron flux of Eq. (3) as a probability density and *renormalizing* its integral over ξ and t to unity,

$$f_{I\!P/p}(\xi,t) \Rightarrow N_s^{-1} \cdot f_{I\!P/p}(\xi,t), \quad N_s \equiv \int_{\xi(min)}^{\xi(max)} d\xi \int_{t=0}^{-\infty} dt\, f_{I\!P/p}(\xi,t) \sim s^{2\epsilon}/\ln s, \tag{5}$$

where $\xi(min) = M_0^2/s$, with $M_0^2 = 1.4$ GeV^2 being the effective threshold for diffraction dissociation, and $\xi(max) = 0.1$. The s-dependence introduced into the Pomeron flux through the renormalization factor N_s^{-1} replaces the power law factor $s^{2\epsilon}$ in Eq. (4) with $\ln s$ ensuring unitarization. In Fig. 1, $\sigma_{sd}^{tot}(s)$ is compared with Regge predictions using the standard or renormalized Pomeron flux. The renormalized prediction is in excellent agreement with the data.

The elastic and total cross sections are not affected by this procedure. Unitarization may be achieved in these cases by using the eikonal approach, e.g. as reported in Ref. [11], where

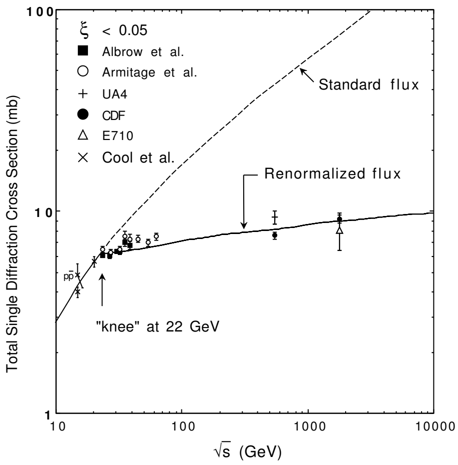

Fig. 1: *Total $pp/\bar{p}p$ single diffraction dissociation cross section data (* both $\bar{p}$ *and* p *sides) for* $\xi < 0.05$ *compared with predictions based on the standard and the renormalized Pomeron flux [10].*

excellent agreement is obtained between elastic and total cross section data and predictions based on Regge theory and eikonalization.

An important aspect of renormalization is that it leads to an approximate scaling behavior, whereby $d\sigma_{sd}(M^2)/dM^2$ has no power law dependence on s. This 'scaling law' holds for the differential soft single diffractive cross section as well, as shown in Fig. 2 [12].

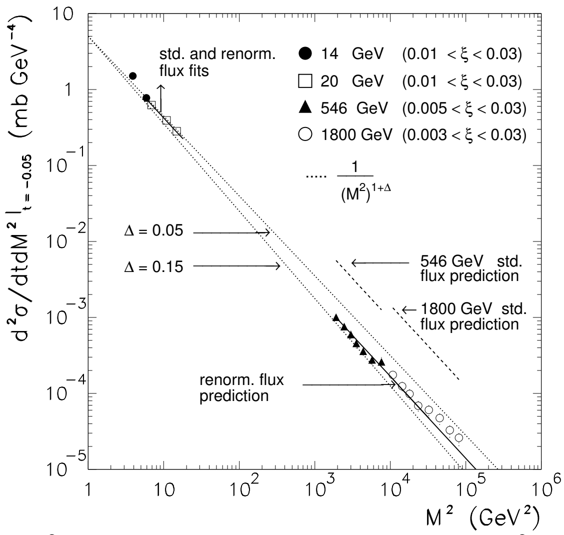

Fig. 2: *Cross sections* $d^2\sigma_{sd}/dM^2dt$ *for* $p + p(\bar{p}) \rightarrow p(\bar{p}) + X$ *at* $t = -0.05$ *GeV*^{2} *and* $\sqrt{s} = 14$*, 20, 546 and 1800 GeV. Standard (renormalized) flux predictions are shown as dashed (solid) lines. At* $\sqrt{s}$*=14 and 20 GeV, the fits using the standard and renormalized fluxes coincide [12].*

3 Parton model approach to diffraction

The Regge theory form of the rise of the total $pp/\bar{p}p$ cross section at high energies, $\sigma^{tot}_{pp/\bar{p}p}(s) = \sigma_0 \cdot s^{\epsilon}$, which requires a Pomeron trajectory with intercept $\alpha(0) = 1 + \epsilon$, is precisely the form expected in a parton model approach, where cross sections are proportional to the number of available "wee" (lowest energy) partons: $\sigma^{tot}_{pp/\bar{p}p} = N \times \sigma_0$, where N is the flux of wee partons and σ_0 the cross section of one wee parton with the target proton (see Ref. [13]). The wee partons originate from emissions of single partons cascading down to lower energy partons in tree-like chains. The average spacing in (pseudo)rapidity[1] between two successive parton emissions is $\sim 1/\alpha_s$. This spacing governs the wee parton density in the region of particle production, $\Delta\eta' = \ln s$, leading to a total pp cross section of the form

$$\sigma^{tot}_{pp/\bar{p}p} = \sigma_0 \cdot e^{\epsilon\Delta\eta'}. \tag{6}$$

Since from the optical theorem $\sigma^{tot}_{pp/\bar{p}p}$ is proportional to the imaginary part of the forward ($t = 0$) elastic scattering amplitude, the full parton model amplitude may be written as

$$\mathrm{Im}\, \mathrm{f}^{\mathrm{el}}_{\mathrm{pp}/\bar{\mathrm{p}}\mathrm{p}}(t, \Delta\eta) \sim e^{(\epsilon+\alpha' t)\Delta\eta}, \tag{7}$$

where $\alpha'(t)$ is added as a simple parameterization of the t-dependence.

The parameter α' reflects the transverse size of the cluster of wee partons in a chain, which is governed by the $\Delta\eta$ spacing between successive chains, and therefore must be related to the parameter ϵ. For the relationship between α' and ϵ, we turn to single diffraction, which through the coherence requirement isolates the cross section from one wee parton interacting with the proton, since all possible interaction of other wee partons are shielded by the formation of the diffractive rapidity gap.

Based on the above amplitude, the single diffractive cross section is expected to have the form

$$\frac{d^2\sigma_{sd}(s, \Delta\eta, t)}{dt\, d\Delta\eta} = \frac{1}{N_{gap}(s)} \cdot \underbrace{C_{gap} \cdot F_p^2(t) \left\{ e^{(\epsilon+\alpha'\, t)\Delta\eta} \right\}^2}_{P_{gap}(\Delta\eta, t)} \cdot \kappa \cdot \left[\sigma_0\, e^{\epsilon\Delta\eta'} \right], \tag{8}$$

where, from right to left, the factor in square brackets represents the cross section due to the partons in the region of particle production, $\Delta\eta' = \ln s$-$\Delta\eta$, κ is a QCD color factor selecting color singlet di-gluon or $q\bar{q}$ exchanges to form the rapidity gap, $P_{gap}(\Delta\eta, t)$ is a gap probability factor representing the scattering between the cluster of dissociation particles and the surviving proton - with $F_p(t)$ being the proton form factor and C_{gap} a constant, and $N_{gap}(s)$ is the integral of the gap probability over all phase space in t and $\Delta\eta$. As $\Delta\eta = -\ln \xi$, the form of Eq. (8) is identical to the Regge form of Eq. (3).

3.1 The ratio $r = \alpha'/\epsilon$

In terms of M^2, Eq. (8) takes the form

$$\frac{d^2\sigma(s, M^2, t)}{dM^2 dt} = \left[\frac{\sigma_0^{pp}}{16\pi} \sigma_0^{I\!Pp} \right] \frac{s^{2\epsilon}}{N(s)} \frac{1}{(M^2)^{1+\epsilon}} e^{bt} \overset{s\to\infty}{\longrightarrow} \left[2\alpha'\, e^{\frac{\epsilon b_0}{\alpha'}} \sigma_0^{I\!Pp} \right] \underbrace{\frac{\ln s^{2\epsilon}}{(M^2)^{1+\epsilon}} e^{bt}}_{b = b_0 + 2\alpha' \ln \frac{s}{M^2}}. \tag{9}$$

[1]We assume $p_T = 1$ GeV, so that $\Delta y' = \Delta\eta'$.

Integrating this expression over M^2 and t yields the total single diffractive cross section,

$$\sigma_{sd} \stackrel{s\to\infty}{\to} 2\sigma_0^{I\!Pp}\, e^{\frac{\epsilon\, b_0}{2\alpha'}} = \sigma_{sd}^{\infty} = \text{constant}. \tag{10}$$

The remarkable property that the total single diffractive cross section becomes constant as $s \to \infty$ is a direct consequence of the coherence required for the recoil proton to escape intact, which results in selecting one out of the many possible wee partons available for interaction, while another parton provides the color shield to form the diffractive rapidity gap. Since diffraction selects the interaction of one of the partons of the outgoing proton, the constant σ_{sd}^{∞} is identified as the σ_0 of Eq. (6), which is specific to the dissociating particle, in this case the proton, and therefore equals σ_0^{pp}. We thus have

$$2\sigma_0^{I\!Pp}\, e^{\frac{\epsilon\, b_0}{2\alpha'}} = \sigma_0^{pp}, \tag{11}$$

which is the sought after relationship between ϵ and α' in terms of constants which can be deduced from fundamental QCD parameters through the relationships

$$\sigma_0^{I\!Pp} = \beta_{I\!Ppp}(0)\cdot g(t) = \kappa\sigma_0^{pp} \tag{12}$$

$$\kappa = \frac{f_g^{\infty}}{N_c^2-1} + \frac{f_q^{\infty}}{N_c} \tag{13}$$

$$b_0 = R_p^2/2 = 1/(2m_\pi^2), \tag{14}$$

where the color factor κ is expressed in terms of the gg and $q\bar{q}$ color factors weighted by the corresponding gluon and quark fractions, and R_p is the radius of the proton expressed in terms of the pion mass, m_π. Inserting these parameters in Eq. (10) yields

$$r = \frac{\alpha'}{\epsilon} = -[16\, m_\pi^2 \ln(2k)]^{-1}. \tag{15}$$

Numerically, using $m_\pi = 0.14$ GeV and $\kappa = 0.18$, as respectively obtained for gluon and quark fractions of $f_g^{\infty} = 0.75$ and $f_q^{\infty} = 0.25$ (see Ref. [14]), yields $r = 3.14$ - which coincidentally is equal to π! This result is in excellent agreement with the ratio calculated from the values of $\epsilon = 0.08$ and $\alpha' = 0.25$ GeV^{-2} for the soft Pomeron trajectory obtained from fits to experimental data of total and elastic pp and $\bar{p}p$ cross sections for collision energies up to $\sqrt{s}$ =540 GeV, $r_{exp} = 0.25/0.08 = 3.13$ [15]. The smaller value of r obtained from the global fit of Ref. [11] to pp, $\bar{p}p$, $\pi^{\pm}p$, and $K^{\pm}p$ cross sections, $r = 0.26/0.104 = 2.5$, could be attributed to the increase of the intercept due to the additional radiation from hard partonic exchanges at higher energies, as for example in the two-Pomeron model of Ref. [16].

4 Summary

In a QCD based parton model approach to elastic, diffractive, and total cross sections, interactions occur through the emission of partons, which cascade down to wee partons in chains of tree-like configurations. As the spacing between successive emissions is controlled by the strong coupling constant, the total cross section, which is proportional to the number of wee partons produced, assumes a power law behavior similar to that of Regge theory, thereby relating the Pomeron intercept to the underlying parton distribution function. The transverse size of the cluster of wee partons in a chain originating from one emission, which is the source of the parameter α', depends on the distance in rapidity of the next emission and thereby on the parameter ϵ. Exploiting single diffraction, which through the coherence requirement isolates a chain from a single parton emission, a relationship between ϵ and α' is obtained, which is in excellent agreement with experimental values.

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