Reggeon$\rightarrow$ 2 Reggeons+Particle Vertex in the Lipatov Effective Action Formalism

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1. Lipatov's effective action

Regge kinematics: $s >> t >> \Lambda^2$.

LL approx.: $\alpha_s \ln s \sim 1$, $\alpha_s \ln k^2 << 1$.

Fixed $\alpha_s \Rightarrow$ a parameter!

Assumed valid at $k^2 >> \Lambda^2$.

Two fields: gluon $V_\mu = -iV_\mu^a T^a$,

Reggeon $A_\pm = -iA_\pm^a T^A$

$$\partial_+ A_- = \partial_- A_+ = 0$$

The effective Lagrangian density

$$\mathcal{L}_{eff} = \mathcal{L}_{QCD}(V + A) + \mathcal{L}_{ind}$$

The induced term is

$$\mathcal{L}_{ind} = \text{Tr}\left\{\mathcal{A}_+(V_+ + A_+) - A_+)\partial^2 A_- \right.$$
$$\left. + \mathcal{A}_-(V_+ + A_+) - A_+)\partial^2 A_+\right\}$$

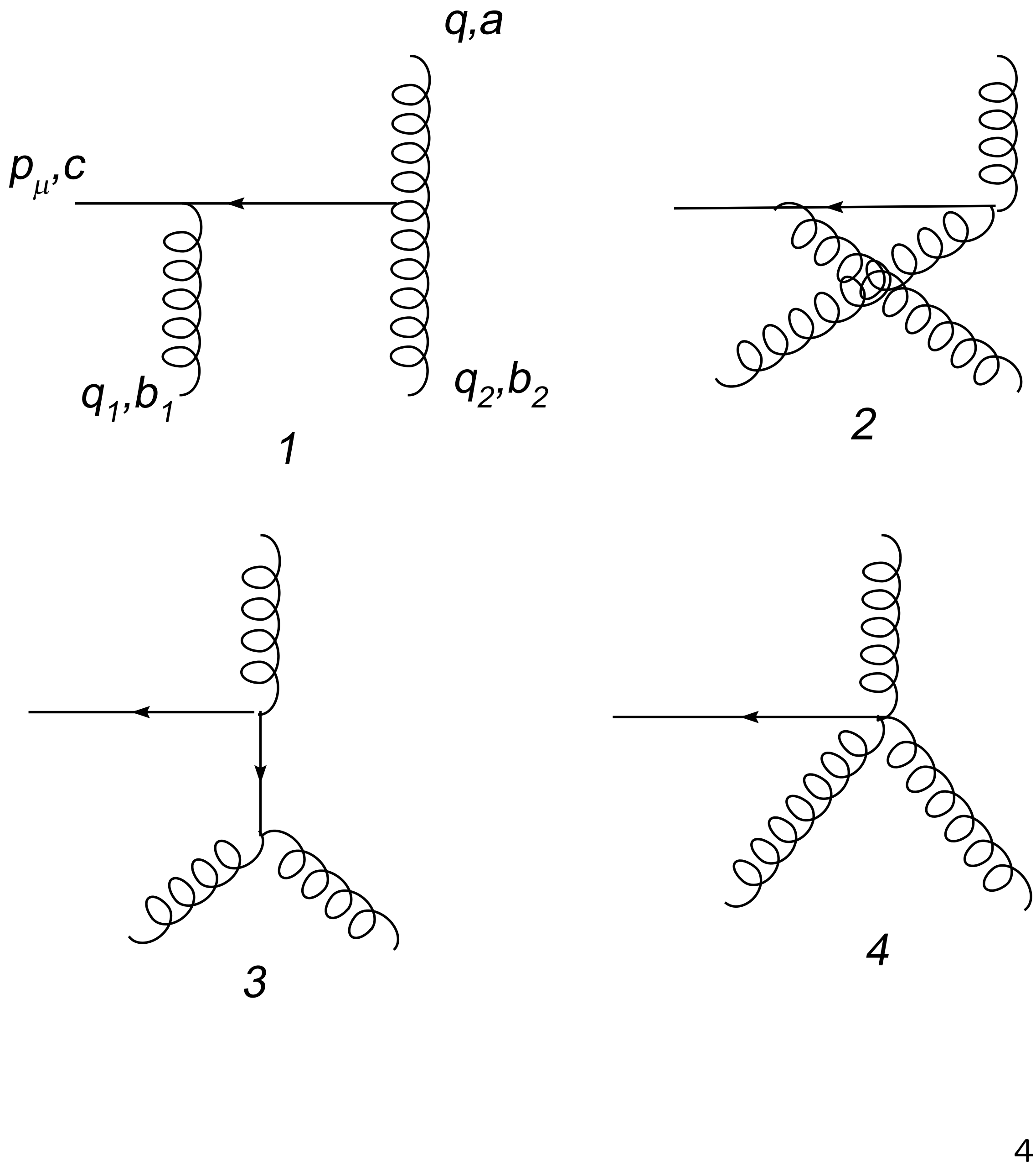
$$\mathcal{A}_\pm(V_\pm) = \sum_{n=0}(-g)^n V_\pm\left(\partial_\pm^{-1} V_\pm\right)^n$$

Propagators come from the quadratic part in V and A

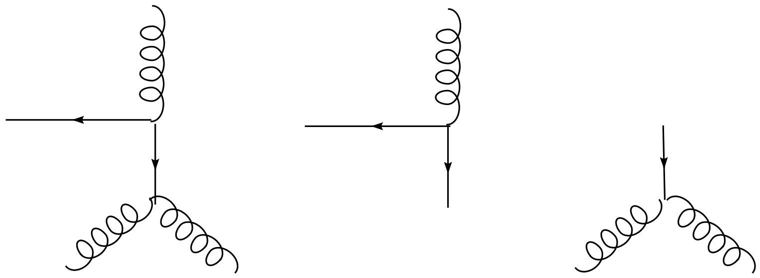
$$\frac{1}{2}V_\nu^a \partial^2 V_\nu^a + \frac{1}{4}(A_-^a \partial_\perp^2 A_+^a + A_+^a \partial_\perp^2 A_-^a)$$

so that $< A_+^a A_-^b >= -i\frac{2\delta_{ab}}{q_\perp^2}$.

2. Diagrams for the vertex and their calculation.



2.1 Diagram 3



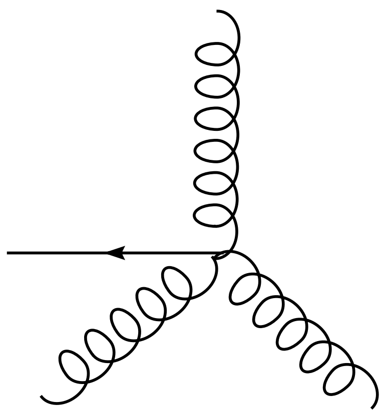
Vertex $< VA_-A_- >$ comes from the part of $\mathcal{L}$ quadratic in A_- fields

$$-\frac{1}{4}\mathrm{Tr}\left(\partial_+ A_-^b(-iT^b) + gV_+^a A_-^b(-if^{abc}T^c)\right)^2$$

However $\partial_+ A_- = 0$ so that vertex $< VA_-A_- >$ vanishes.

$\text{Diagram}_3 = 0.$

2.2. Diagram 4



Vertex $< A_+ V A_- A_- >$ comes from part of $\mathcal{L}$

$$-\frac{g}{2}\mathrm{Tr}\Big(-g[V_+, A_-][A_-, A_+] - 2g(V_- \frac{1}{\partial_-} A_- \frac{1}{\partial_-} A_- \partial_\perp^2 A_+$$

$$+ A_- \frac{1}{\partial_-} V_- \frac{1}{\partial_-} A_- \partial_\perp^2 A_+ + A_- \frac{1}{\partial_-} A_- \frac{1}{\partial_-} V_- \partial_\perp^2 A_+)\Big)$$

The first term is the standard quartic interaction, others give the induced interaction.

The standard interaction gives

$$-i\frac{g^2}{4}n^+_\mu(f^{b_1cd}f^{ab_2d}+f^{ab_1d}f^{b_2cd})$$

The induced interaction gives

$$i\frac{g^2}{2}n^-_\mu q_\perp^2\left(\frac{1}{p_- q_{2-}}f^{b_1cd}f^{ab_2d}+\frac{1}{p_- q_{1-}}f^{ab_1d}f^{b_2cd}\right)$$

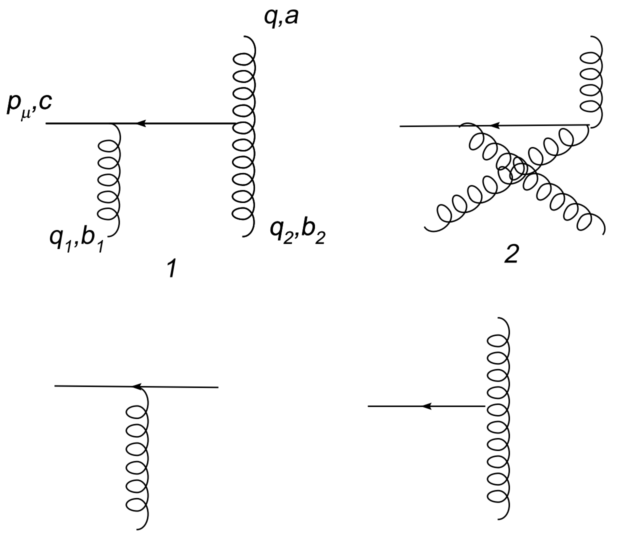
The total contribution in arbitrary gauge is

$$V_4=\frac{ig^2}{4}\Big[f^{b_1cd}f^{ab_2d}\left(2\frac{q_\perp^2 n^-_\mu}{p_- q_{2-}}-n^+_\mu\right)$$
$$+f^{ab_1d}f^{b_2cd}\left(2\frac{q_\perp^2 n^-_\mu}{p_- q_{1-}}-n^+_\mu\right)\Big]$$

In gauge $V_+=0$, convoluted with the gluon polarization vector

$$ig^2\frac{p\epsilon_\perp}{p_\perp^2}q_\perp^2\left(\frac{1}{q_{2-}}f^{b_1cd}f^{ab_2d}+\frac{1}{q_{1-}}f^{b_1ad}f^{cb_2d}\right)$$

2.3 Diagrams 1 and 2



Two verteces $< A_- V A_- >$ and $< V_+ V_+ A_- >$. The first is the standard Lipatov vertex

$$< A_- V A_- >= \frac{g f^{ab_2 d}}{2}\Big[q_\sigma + q_{2\sigma} + \Big(\frac{q_2^2}{q_+} - q_{2-}\Big) n_\sigma^+ + \Big(\frac{q^2}{q_{2-}} - q_+\Big) n_\sigma^-\Big].$$

In gauge $V_+ = 0$, convoluted with the gluon polarization vector

$$g f^{ab_2 d} q_\perp^2 \Big(\frac{q\epsilon_\perp}{q_\perp^2} - \frac{k\epsilon_\perp}{k_\perp^2}\Big)$$

Vertex $< VA_-A_- >$ comes from the part of $\mathcal{L}$

$$\mathcal{L}_{AVV} = g\mathrm{Tr}\left[\Big([V_\nu,\partial_+ V_\nu]-[\partial_\nu V_\nu, V_+]-2[V_\nu,\partial_\nu V_+]- V_+\partial_+^{-1}V_+\partial_\perp^2\Big)A_-\right].$$

One gets

$$< VA_-A_- >= \tfrac{gf^{b_1cd}}{2}\Big(-2p_+g_{\mu\sigma}+(p+2q_1)_\mu n_\sigma^+ + (p-q_1)_\sigma n_\mu^+ + \tfrac{q_1^2}{p_+}n_\mu^+ n_\sigma^+\Big)$$

Convoluting $< A_-VA_- >$, $< VA_-A_- >$ and the gluon propagator we get the total Diagram$_1$

$$\frac{-ig^2f^{b_1cd}f^{ab_2d}}{4k^2}\Big(-2p_+g_{\mu\sigma}+(p+2q_1)_\mu n_\sigma^+ +(p-q_1)_\sigma n_\mu^+ + \frac{q_1^2}{p_+}n_\mu^+ n_\sigma^+\Big)\left[q_\sigma+q_{2\sigma}+\Big(\frac{q_2^2}{q_+}-q_{2-}\Big)n_\sigma^+ +\Big(\frac{q^2}{q_{2-}}-q_+\Big)n_\sigma^-\right]$$

2.4 The total vertex

Diagram 2=Diagram 1(1$\leftrightarrow$2).

The total RRRP vertex $=\sum_{i=1}^{4}$Diagram (i). One checks that it is indeed transverse.

In gauge $V_+ = 0$, convoluted with the gluon polarization vector the total vertex is

$$g^2 \frac{f^{b_1cd}f^{ab_2d}}{(q-q_2)^2}\Big[q_+(q\epsilon^*_\perp)+$$

$$\frac{q^2}{q_{2-}}\Big(-(q-q_2)\epsilon^*_\perp + \frac{(q-q_2)^2}{p^2_\perp}(p\epsilon^*_\perp)\Big)\Big]$$

$$+g^2 \frac{f^{b_2cd}f^{ab_1d}}{(q-q_1)^2}\Big[q_+(q\epsilon^*_\perp)+$$

$$\frac{q^2}{q_{1-}}\Big(-(q-q_1)\epsilon^*_\perp + \frac{(q-q_1)^2}{p^2_\perp}(p\epsilon^*_\perp)\Big)\Big]$$

3.Diffractive amplitude

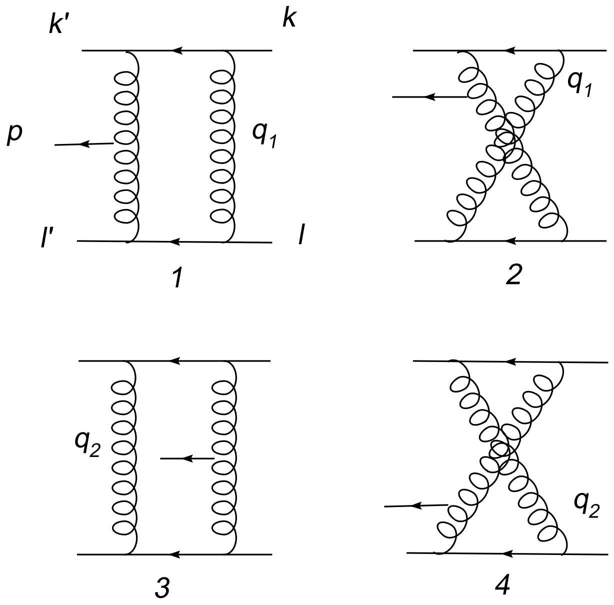
The obtained R$\to$RRP vertex has a 4-dimensional form. To reduce it to the transverse vertex we have to study a concrete amplitude which involves this vertex.

The simplest amplitude : production of a real gluon in collision of two quarks, the target quark interacting with the two final reggeized gluons in the colourless state.

Additional diagrams for this process involve two-Reggeon exchange between projectile and target with the gluon emitted by the Lipatov vertex.

However as we shall see (and as is known) for them integration over longitudinal variables in the loop presents no difficulties, so that all the problem is concentrated in the diagram with the found R$\to$RRP vertex.

3.1 Two-Reggeon exchange



Impact projectile $\otimes$ target factors.

For Diagrams 1 and 2, respectively

$$\frac{16(kl)^2}{(k-q_1)^2(l+q_1)^2} u' t^{a_2} t^{a_1} u \cdot w' t^{b_2} t^{b_1} w$$

$$\frac{16(kl)^2}{(k-q_1)^2(l'-q_1)^2} u' t^{a_2} t^{a_1} u \cdot w' t^{b_1} t^{b_2} w$$

u, u' and w, w' are initial and final colour wave functions of the projectile and target.

Longitudinal integrations over $q_{1\pm}$.

For Diagrams 1 and 2 one obtains, respectively

$$\int_{-l_-}^{0} dq_{1-} \int \frac{dq_{1+}}{2\pi i} D_1 = \frac{1}{2(kl)}\left(\ln \frac{|q_{1\perp}^2|}{2(kl)} + \pi i \right)$$

$$\int_{0}^{l_-} dq_{1-} \int \frac{dq_{1+}}{2\pi i} D_2 = -\frac{1}{2(kl)} \ln \frac{|q_{1\perp}^2|}{2(kl)}$$

D_1 and D_2 are the longitudinal factors in Diagrams 1 and 2 (product of quark denominators).

In the sum the real parts cancel and the final result is

$$\int \frac{dq_{1+} dq_{1-}}{2\pi i} (D_1 + D_2) = \frac{\pi i}{2(kl)}$$

Adding color and transverse momenta factors, the amplitude

$$f^{a_2 c a_1} u' t^{a_2} t^{a_1} u \cdot 4(kl) \frac{1}{q_{1\perp}^2 q_{2\perp}^2} \Big(\frac{(q\epsilon)_\perp}{q_\perp^2} - \frac{(p\epsilon)_\perp}{p_\perp^2} \Big)$$

$$q_2 = q - p - q_1$$

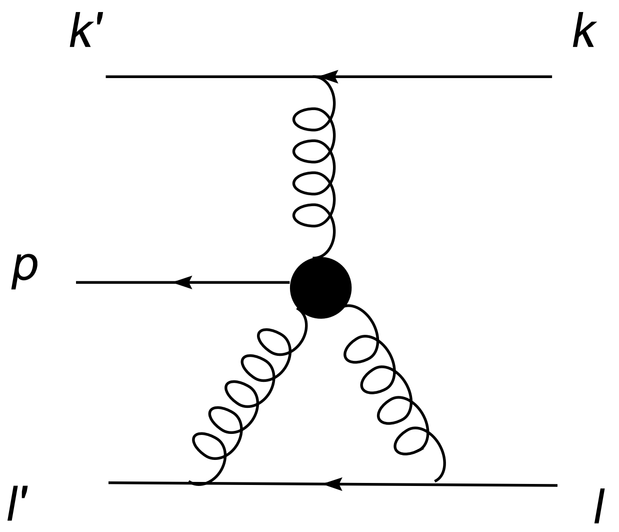
.

Coincides with the known transverse momentum space result (J.Bartels and M.Wuesthoff, 1995)

The same result is obtained if one passes variables $s_1 = (k - q_1)^2$ and $s_2 = (l + q_1)^2$ for the upper and lower quark-reggeon amplitudes deforms the Feynman contours of integration to pass around the unitarity cuts at $s_{1,2} \geq 0$ and takes into account the intermediate quark states at $s_1 = s_2 = 0$.

Diagrams 3 and 4 are calculated in the same manner and with similar results.

3.2 Contribution from R$\to$RRP vertex



Contribution from

$$+g^2 \frac{f^{b_2cd} f^{ab_1d}}{(q-q_1)^2} \Big[q_+ (q\epsilon^*_\perp) +$$

$$\frac{q^2}{q_{1-}} \Big(-(q-q_1)\epsilon^*_\perp + \frac{(q-q_1)^2}{p_\perp^2} (p\epsilon^*_\perp) \Big) \Big]$$

$+\Big(1 \leftrightarrow 2 \Big).$

3 terms.

1. 1st term $\propto (q\epsilon^*_\perp)$. Longitudinal integrations are done exactly as for the 2-Reggeon exchange and give

$$\int \frac{dq_{1+}dq_{1-}}{2\pi i}(D_1 + D_2)) = \frac{\pi i}{2(ql)}$$

Together with colour and transverse momentum factors we get the 1st part of the amplitude

$$(1) = 4\pi i(pl)\delta_{ac}u' t^a u \cdot \frac{1}{q_{1\perp}^2 q_{2\perp}^2}\frac{q\epsilon_\perp}{q_\perp^2}$$

This coincides with the 1st term in the contribution from the Bartels transverse vertex.

2. 2nd term $\propto (q-q_1)\epsilon^*_\perp$.

Here one cannot integrate direct and crossed term separately due to singular factor $1/q_{1-}$. From the direct term one gets integral

$$-\int_0^{l_-}\frac{dq_{1-}}{q_{1-}}\,\frac{1}{4k_+q_{1-}^2-4k_+l_-q_{1-}-2l_+(q-q_1)_\perp^2}$$

and from the crossed

$$-\int_0^{l_-}\frac{dq_{1-}}{q_{1-}}\,\frac{1}{4k_+q_{1-}^2-4k_+l_-q_{1-}+2l_-(q-q_1)_\perp^2}$$

Their sum is regular at $q_{1-}=0$. One obtains

$$\int\frac{dq_{1+}dq_{1-}}{2\pi i q_{1-}}(D_1+D_2)=-\frac{\pi i}{2l_-(q-q_1)_\perp^2}$$

Adding all the rest factors, the 2nd part of the amplitude

$$(2)=-4\pi i(pl)\delta_{ac}u' t^a u\cdot\frac{1}{q_{1\perp}^2q_{2\perp}^2}\frac{(q-q_1)\epsilon_\perp}{(q-q_1)_\perp^2}$$

Coincides with the 2nd part of the contribution from the Bartels transverse vertex.

3. 3d term $\propto (p\epsilon^*_\perp)$.

No counterpart in the Bartels vertex.

The longitudinal integral is divergent. In the sum of direct and crossed term one finds denominator

$$\frac{1}{2q_{1+}(l_-+q_{1\_})} - \frac{1}{2q_{1+}(l'_--q_{1\_})}$$

Here $l'_- - q_{1-} =\simeq l_- - p_- - q_{1-}$ and is NOT EQUAL to $l_- + q_{1-}$.

Therefore at $q_{1+} \to \infty$ the sum does not go to zero faster than $1/q_{1+}$, so that integration over q_{1+} remains divergent even after summing the direct and crossed terms.

The only possibility to give some sense to this integration is to assume that in the target denominator one may neglect minus components of all momenta except for the incoming target momentum l_-. This means that one takes

$$q_{1-} = 0 \quad \text{and} \quad l'_- = l_-$$

Then integration over q_{1+} becomes convergent and since the poles of the two terms lie at opposite sides from the real axis, gives a non-zero result, which does not depend on the value of q_{1-}. After that one finds integral over q_{1-}

$$\int_{-\infty}^{\infty} \frac{dq_{1-}}{q_{1-}}$$

If one further assumes that it should be taken as a principal value integral then it vanishes the 3d term indeed gives no contribution.

Conclusions

1. The effective action allows to build the R$\to$RRP vertex almost automatically, in contrast to earlier derivations, in which this required a considerable effort.

2. However the obtained vertex is found in the 4-dimensional form and formally contains singularities in the longitudinal variables.

3. In the diagram with an intermediate gluon this singularity requires to do the longitudinal integration preserving the dependence of the target impact factor on small longitudinal momenta. As a result the mentioned singularity disappears.

4. In the diagram with pointlike vertex, on the contrary, small longitudinal momenta are to be neglected in the target impact factor. Then the remaining singularity has to be integrated over by the principal value recipe. After that one finally obtains the same transverse vertex, as found earlier by direct methods.

5. The fact that one has to apply different rules to treat different contributions certainly leaves a certain feeling of uneasiness. We hope that the only contributions which require special treatment are those with pointlike emission, which seem to have been dropped from the start in previous derivations using either dispersion approach or the dominance of large nuclear distances. If this is so then the use of the effective action preserves its universality except for these exceptional cases. To verify this one has to study more complicated processes involving the found vertex, such as double scattering off a colorless target. This study is now in progress.