# The Reggeon $\to$ 2 Reggeons + Particle vertex in the Lipatov effective action formalism

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**Abstract**
The Reggeon$\to$ 2 Reggeons+Particle vertex is constructed in the framework of Lipatov effective action formalism. Its reduction to a pure transverse form for the diffractive amplitude gives the standard Bartels vertex plus an additional contribution given by a longitudinal integral divergent both in the ultraviolet and infrared. A certain specific recipe for this part, including the principal value prescription for the integration, allows to eliminate this unwanted contribution.

## 1 Introduction

Particle interaction in the Regge kinematics and QCD perturbative region , $\Lambda^2_{QCD} << |t| << s$, is described by exchange of reggeized gluons ('reggeons') accompanied by emission of real gluons ('particles'). To automatically calculate all relevant diagrams in a systematic and self-consistent way a potentially powerful formalism of an effective action has been proposed by L.N.Lipatov [1], in which the longidudinal and transverse variables are separated from the start and one arrives at a theory of interaction of reggeized gluons and particles described by independent fields. However the resulting vertices are 4-dimensional and reduction of them to the 2-dimensional transverse form still has to be done.

Up to the present, several application of this formalism have been done and a number of interaction vertices have been calculated [2]. In this paper we study a vertex for the transition of a reggeon into a pair of reggeons and a particle (the RRRP vertex). The 2-dimensional form of this vertex (the 'Bartels' vertex) is well-known [3, 4]. The found 4-dimensional vertex resembles the Bartels vertex, although it contains a new structure absent in the latter and of course longitudinal variables. Unlike the vertices obtained so far in the effective action formalism, reduction of the RRRP vertex to the 2-dimensional form involves a non-trivial integration in the loop formed by the two reggeons and the target. For a simple diffractive diagram, in which both the projectile and target are quarks, a literal integration over the longitudinal variables proves to be impossible (divergent). The Bartels vertex is obtained only if certain *ad hoc* rules are followed, which reduce to neglecting all small longitudinal momenta in the target factor and subsequent integration over the minus component of the loop momentum according to the principal value prescription.

## 2 Calculation of the vertex

In the effective action formalism, calculation of the RRRP vertex is straightforward. The relevant Feyman rules were presented in [2] and we only recapitulate them here for convenience and to

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fix our notations. We write $D_\mu = \partial_\mu + gV_\mu$ where $V_\mu = -iV_\mu^a T^a$ is the gluon field and $T^a$ are the $SU(N)$ generators in the adjoint representation. The reggeon field $A_\pm = -iA_\pm^a T^a$ satisfies the kinematical condition

$$\partial_+ A_- = \partial_- A_+ = 0. \tag{1}$$

The field $A_+$ comes from the region with a higher rapidity, its momentum $q_-$ is small, the field $A_-$ comes from the region with a smaller rapidity, its $q_+$ is small. The QCD Lagrangian for the particle (gluon) field $V$ is standard and so are the Feynman rules. The effective action is

$$\mathcal{L}_{eff} = \mathcal{L}_{QCD}(V_\mu + A_\mu) + \mathrm{Tr}\,\Big(\mathcal{A}_+(V_+ + A_+) - A_+)\partial_\perp^2 A_- + \mathcal{A}_-(V_- + A_-) - A_-)\partial_\perp^2 A_+\Big), \tag{2}$$

where

$$\mathcal{A}_\pm(V_\pm) = -\frac{1}{g}\partial_\pm \frac{1}{D_\pm}\partial_\pm * 1 = \sum_{n=0}(-g)^n V_\pm(\partial_\pm^{-1} V_\pm)^n$$
$$= V_\pm - gV_\pm \partial_\pm^{-1} V_\pm + g^2 V_\pm \partial_\pm^{-1} V_\pm \partial_\pm^{-1} V_\pm + - ... \tag{3}$$

The reggeon propagator in the momentum space is

$$< A_+^a A_-^b >= -i\frac{2\delta_{ab}}{q_\perp^2}. \tag{4}$$

The well-known R→RP ('Lipatov') vertex is

$$\frac{gf^{ab_2d}}{2}\Big[q_\sigma + q_{2\sigma} + \Big(\frac{q_2^2}{q_+} - q_{2-}\Big)n_\sigma^+ + \Big(\frac{q^2}{q_{2-}} - q_+\Big)n_\sigma^-\Big]. \tag{5}$$

In the convenient gauge $V_+ = 0$ the polarization vector has the form

$$\epsilon_\mu(k) = \epsilon_\mu^\perp(k) - \frac{k\epsilon^\perp}{k_+}n_\mu^+. \tag{6}$$

Convolutiong the vertex (5) with this polarization vector we find

$$gf^{ab_2d}q_\perp^2\Big(\frac{q\epsilon_\perp}{q_\perp^2} - \frac{k\epsilon_\perp}{k_\perp^2}\Big), \tag{7}$$

which form of the Lipatov vertex is widely used in literature.

Our aim is to calculate the vertex R→RRP. The total contribution to it is represented by four diagrams shown in Fig. 1, in which particles are shown by solid lines and reggeons by wavy lines.

Of these diagrams the one described by Fig. 1.3 vanishes due to condition (1). Diagram 1.4 gives

$$V_4 = \frac{ig^2}{4}\Big[f^{b_1cd}f^{ab_2d}\Big(2\frac{q_\perp^2 n_\mu^-}{p_- q_{2-}} - n_\mu^+\Big) + f^{ab_1d}f^{b_2cd}\Big(2\frac{q_\perp^2 n_\mu^-}{p_- q_{1-}} - n_\mu^+\Big)\Big]. \tag{8}$$

Diagram Fig. 1.1 gives

$$V_{1.1} = \frac{-ig^2 f^{b_1cd}f^{ab_2d}}{4k^2}\Big( - 2p_+ g_{\mu\sigma} + (p + 2q_1)_\mu n_\sigma^+ + (p - q_1)_\sigma n_\mu^+ + \frac{q_1^2}{p_+}n_\mu^+ n_\sigma^+\Big)$$

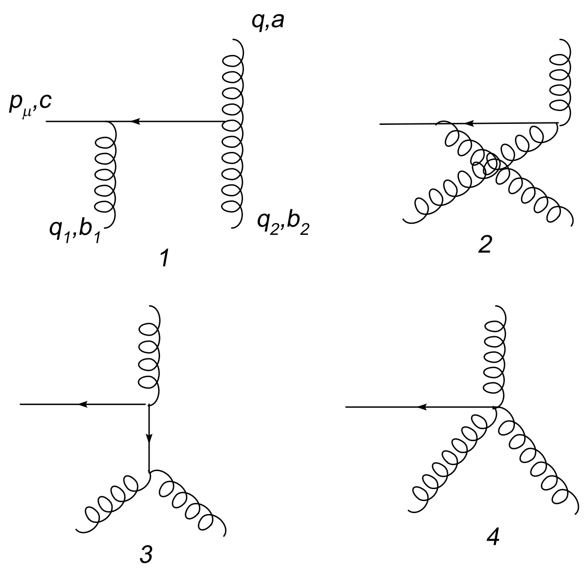


Fig. 1: Reggeon diagrams for the vertex R→RRP

$$\Big[q_\sigma + q_{2\sigma} + \Big(\frac{q_2^2}{q_+} - q_{2-}\Big)n_\sigma^+ + \Big(\frac{q^2}{q_{2-}} - q_+\Big)n_\sigma^-\Big]. \tag{9}$$

The contribution from diagram Fig. 1.2 is obtained by the interchange of the outgoing gluons 1 and 2.

The total vertex obtained as a sum of the contributions from diagrams shown in Fig. 1 is found to be transversal (convolution with $p_\mu$ is zero) as it should be.

To find the transition amplitude one has to convolute the sum of diagrams Fig. 2.1+2+4 with the polarization vector $\epsilon^*_\mu(p)$. In the gauge $V_+ = 0$ the result of the convolution is

$$g^2\frac{f^{b_1cd}f^{ab_2d}}{(q-q_2)^2}\Big[q_+(q\epsilon^*_\perp) + \frac{q^2}{q_{2-}}\Big(-(q-q_2)\epsilon^*_\perp + \frac{(q-q_2)^2}{p_\perp^2}(p\epsilon^*_\perp)\Big)\Big]$$

$$+g^2\frac{f^{b_2cd}f^{ab_1d}}{(q-q_1)^2}\Big[q_+(q\epsilon^*_\perp) + \frac{q^2}{q_{1-}}\Big(-(q-q_1)\epsilon^*_\perp + \frac{(q-q_1)^2}{p_\perp^2}(p\epsilon^*_\perp)\Big)\Big]. \tag{10}$$

## 3 The diffractive amplitude

The obtained R→RRP vertex (10) has a 4-dimensional form. To reduce it to the transverse vertex we have to study a concrete amplitude which involves this vertex. We choose the simplest amplitude possible: production of a real gluon in collision of two quarks, the target quark interacting with the two final reggeized gluons in the colourless state (Fig. 2). Note that there are additional diagrams for this process which involve two reggeon exchange between projectile and target with the gluon emitted by the Lipatov vertex. However, as is known, for them integration over longitudinal variables in the loop presents no difficulties, so that all the problem is concentrated in the diagram with the found R→RRP vertex.

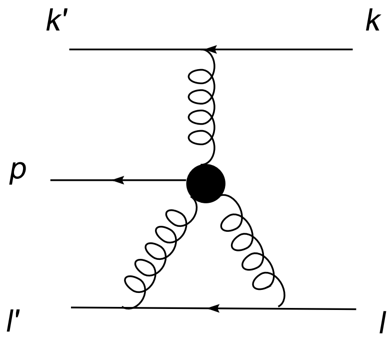


Fig. 2: The difractive diagram with a R→RPP vertex

The two terms in the vertex (10), which differ by the permutation $1 \leftrightarrow 2$, evidently give the same contribution after integration over the loop momentum and summation over colour indices. So it is sufficient to study only one of them. We choose the second term.

Reduction to the 2-dimensional form requires integration over the longitudinal variables. We first integrate over $q_{1+}$ and then over $q_{1-}$.

The first term in the second part of (10), proportional to $(q\epsilon)_\perp$ has two denominators depending on the longitudinal variables:

$$[2q_{1-}(q_{1+}-q_+)+(q-q_1)\perp^2+i0][2q_{1+}(l_-+q_{1-})+q_{1\perp}^2+i0] \tag{11}$$

in the direct term and

$$[2q_{1-}(q_{1+}-q_+)+(q-q_1)\perp^2+i0][-2(q_{1+}-l'_+)(l'_- - q_{1-})+(l'-q_1)_\perp^2+i0]. \tag{12}$$

in the crossed term. A non-zero result is obtained only when the two poles in $q_{1+}$ are on the opposite sides from the real axis. This limits the integration over $q_{1-}$: $-l_- < q_{1-} < 0$ and $0 < q_{1-} < l'_- \simeq l_-$ in the direct and crossed terms respectively. Calculation of integrals is straightforward, meets no trouble and can be done separately in the direct and crossed term. In their sum the real parts cancel and only the imaginary part remains, given exclusively by the direct term.

Taking into account the colour factor and momentum factors coming from the two impact factors we find the contribution from the first term in the second part of (10) as

$$\int \frac{dq_{1+}dq_{1-}}{2\pi i}T_1 = 4\pi i(pl)\delta_{ac}u't^a u \cdot \frac{1}{q_{1\perp}^2 q_{2\perp}^2}\frac{q\epsilon_\perp}{q_\perp^2}. \tag{13}$$

It coincides with the first term in the contribution from the Bartels vertex to the diffractive amplitude.

Note that the same result is obtained if one introduces energies squared $s_1 = (q-q_1)^2$ and $s_2 = (l+q_1)^2$ for the amplitudes R+R→ P+R and q+R→ q+R and then deforms the Feynman integration contour to close on the unitarity cuts of both amplitudes. The contribution from the intermediate quark+gluon state will immediately give (13).

The situation is a bit more complicated with the second term in the second part of (10), proportional to $(q-q_1)\epsilon_\perp$. In both the direct and crossed term an extra factor $1/q_{1-}$ appears. The integration over $q_{1+}$ is done exactly as before. However the subsequent integration over $q_{1-}$ cannot be done separately for the direct and crossed term because of the singularity at $q_{1-}=0$. However in their sum this singularity cancels and integration becomes trivial. Adding all the rest factors we get for the second term in the second part of (10)

$$\int \frac{dq_{1+}dq_{1-}}{2\pi i} T_2 = -4\pi i(pl)\delta_{ac}u' t^a u \cdot \frac{1}{q_{1\perp}^2 q_{2\perp}^2} \frac{(q-q_1)\epsilon_\perp}{(q-q_1)_\perp^2}. \tag{14}$$

This coincides with the second part of the contribution corresponding to the Bartels vertex.

Note that if one tries to use here the method of integration over energies of the R+R→R+P and q+R→ q+P amplitudes by closing the contour around the unitarity cuts, then one encounters the singularity at $q_{1-}=0$ with an unknown way of integration around it. If one just neglects this singularity, that is takes into account only the unitarity contribution to the dicontinuities of the amplitudes, then one gets a result which is twice larger than (14) and hence incorrect.

We are left with the third term in the second part of (10) with a structure which has no counterpart in the Bartels vertex. For the 4 dimensional and 2-dimensional pictures to coincide this contribution has to disappear.

The only denominator in the direct term is

$$(l+q_1)^2 + i0 = 2q_{1+}(l_- + q_{1\_}) + q_{1\perp}^2 + i0 \tag{15}$$

and in the crossed term

$$(l'-q_1)^2 + i0 = -2(q_{1+} - l'_+)(l'_- - q_{1-}) + (l'-q_1)_\perp^2 + i0. \tag{16}$$

Integration over $q_{1+}$ gives

$$-\frac{i\pi}{2}\Big(\frac{\mathrm{sign}\,(l_- + q_{1-})}{l_- + q_{1-}} + \frac{\mathrm{sign}\,(l'_- - q_{1-})}{l'_- - q_{1-}}\Big). \tag{17}$$

This expression does not vanish at $q_{1-}=0$, so that the subsequent integration with the denominator $q_{1-}$ is meaningless.

To give some sense to this integration we may assume that in the target denominators one may neglect minus components of all momenta except for the incoming target momentum $l_-$. This means that in (15) and (16) we take $l'_- = l_-$ and $q_{1-}=0$. Then the result of integration over $q_{1+}$ becomes independent of $q_{1-}$. After that we find the integral over $q_{1-}$ of the form

$$\int_{-\infty}^{\infty} \frac{dq_{1-}}{q_{1-}}. \tag{18}$$

If we assume that this integral should be taken according to the principal value prescription then it vanishes, the third term in (10) disappears and the obtained transverse vertex coincides with the Bartels one.

## 4 Conclusions

We have found that the longitudinal integration of the 4-dimensional vertex constructed by the effective action technique requires certain caution. One finds a piece, for which a strict integration is divergent. To overcome this diffculty one has to neglect the small minus components in the target impact factor (and then do the $q_{1+}$ integration in the trival manner closing the contour around the unitarity cut of the reggeon-target amplitude) and afterwards do the remaining $q_{1-}$ integration by the principal value recipe. This result has been derived only for the diffractive amplitude. It remains an open question if it has a wider validity and applies also to other cases, which correspond to double and single cuts of the general triple pomeron amplitude.

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