

# Pomeron loops/Gluon number fluctuations in high density QCD evolution

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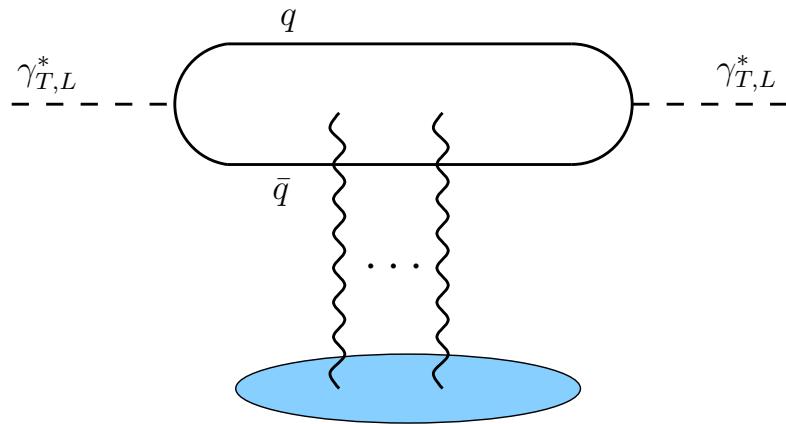
DESY, Hamburg, May 21-25, 2007

# Outline

- “Mean field equations”:
  - Kovchegov equation, B-JIMWLK equation
  - Results: Geometric scaling of  $T$ , energy dependence of  $Q_s(s)$
- Beyond mean field:
  - Pomeron loops/gluon number fluctuations
  - Pomeron loop equations
  - Results: Diffusive scaling of  $T$ , energy dependence of  $Q_s(s)$
  - Possible phenomenological consequences

# Deep Inelastic Scattering

- A proton (nucleus) probed with a dipole:



- Total cross section:

$$\sigma^{\gamma_{T,L}^* p}(Q, x_{Bj}) = \int d^2\mathbf{r} \int dz |\psi_{T,L}(z, Q)|^2 2 \int d^2\mathbf{b} T(\mathbf{r}, x_{Bj}, \mathbf{b})$$

- How does  $T(\mathbf{r}, Y)$  change if  $Y \rightarrow Y + dY$ ?

- Kinematics:

$$Q^2 = -q^2$$

– photon virtuality

$$s = (p + q)^2$$

– total collision energy

$$x_{Bj} \approx \frac{Q^2}{s} \ll 1$$

– Bjorken variable (small- $x_{Bj}$ )

$$Y = \ln(1/x_{Bj}) \sim \ln(s)$$

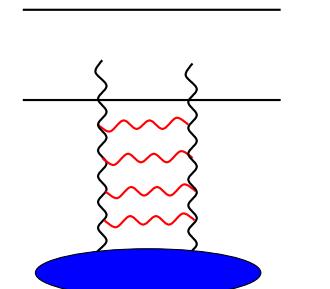
– total rapidity

# “Mean Field Equations”

- BFKL equation:

$$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \bar{\alpha}_s \langle T \rangle_Y$$

\*  $\langle T \rangle_Y \sim \exp[c\bar{\alpha}_s Y] \rightarrow$  unitarity violation!

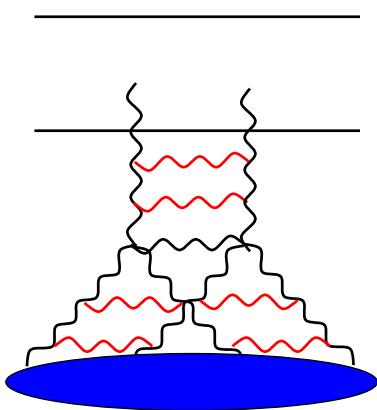


- Kovchegov equation:

$$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \bar{\alpha}_s [\langle T \rangle_Y - \langle T \rangle_Y \langle T \rangle_Y]$$

\*  $\langle T \rangle_Y \langle T \rangle_Y$ ; non-linear evolution,  $\langle T \rangle_Y \leq 1$

\*  $\langle T \rangle_Y \ll 1$ ; linear BFKL evolution



- B-JIMWLK equations:

$$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \bar{\alpha}_s [\langle T \rangle_Y - \langle T T \rangle_Y] ,$$

$$\frac{\partial}{\partial Y} \langle T T \rangle_Y \propto \bar{\alpha}_s [\langle T T \rangle_Y - \langle T T T \rangle_Y] , \dots$$

\* Fluctuations:  $\langle T T \rangle_Y \neq \langle T \rangle_Y \langle T \rangle_Y$ ?

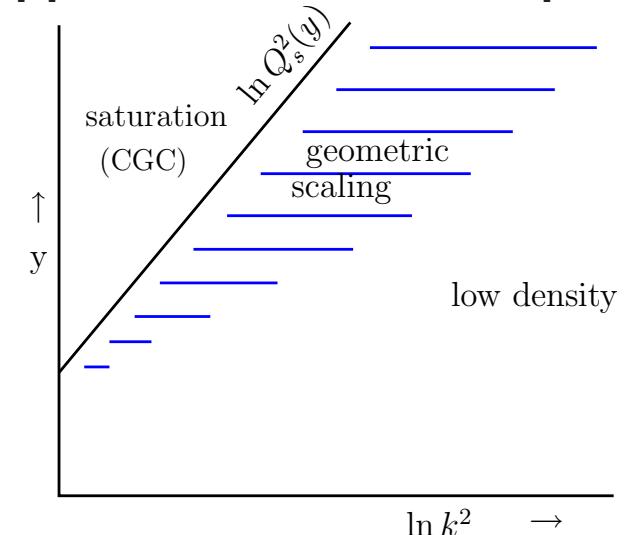
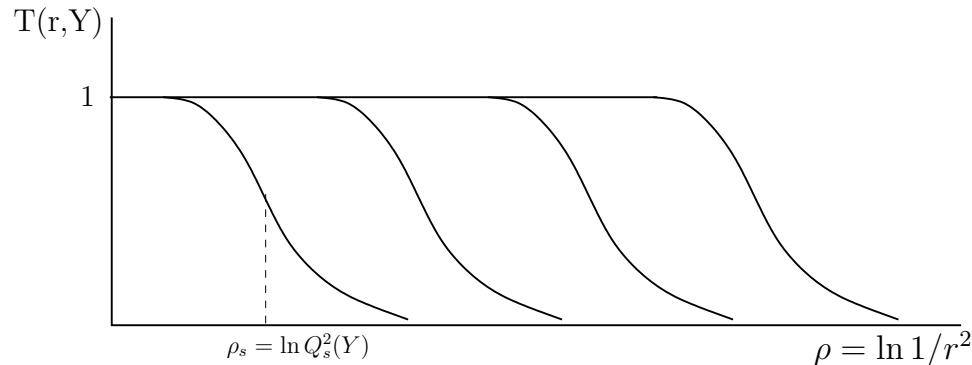
\* Mean field approximation:  $\langle T T \rangle_Y \approx \langle T \rangle_Y \langle T \rangle_Y \rightarrow$  Kovchegov Eq.

• Numerical result [Rummukainen, Weigert 04]:  $\langle T \rangle_Y^{\text{Kovchegov}} \approx \langle T \rangle_Y^{\text{B-JIMWLK}}$

# Results from “Mean Field Equations”

[Levin, Tuchin 2000],[Mueller, Triantafyllopoulos 2002],[Munier, Peschanski 2003]

- Physical picture:



- Geometric scaling:

$$T(r, Y) = T(Q_s^2(Y) r^2)$$

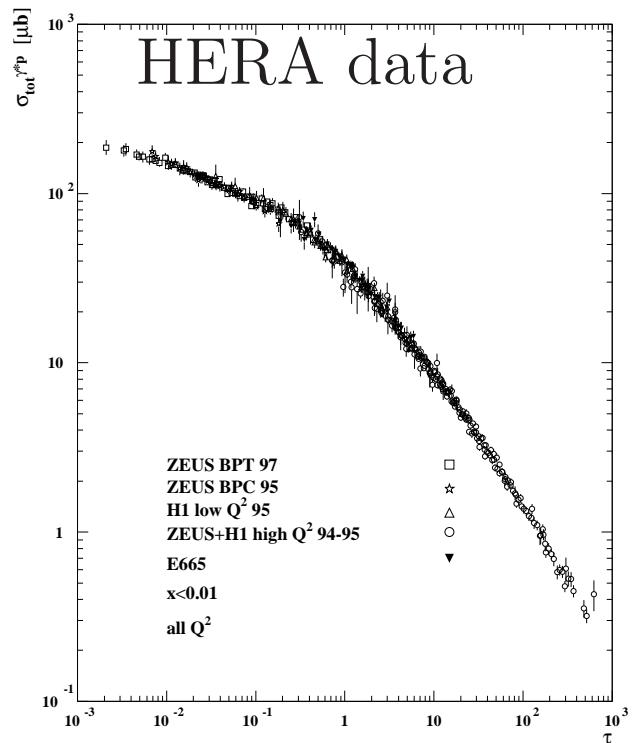
$$\Rightarrow \sigma^{\gamma^* p}(Q, x_{Bj}) = \sigma^{\gamma^* p}(Q/Q_s(Y))!$$

- Saturation momentum:

$$Q_s(x_{Bj}) = Q_0^2 (1/x_{Bj})^\lambda,$$

Experiment:  $\lambda \approx 0.3$

Theory [Triantafyllopoulos 02]:  $\lambda$  close to 0.3

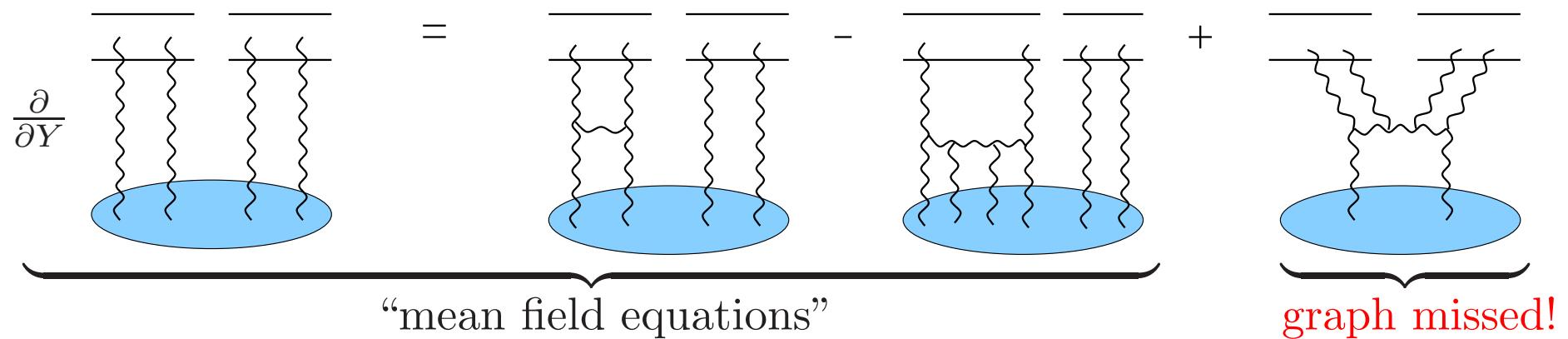


[Stasto, Golec-Biernat, Kwiecinski 2001]

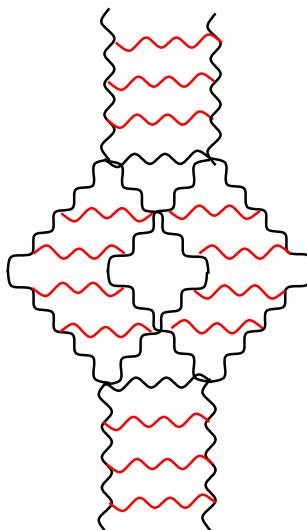
# Shortcomings of “mean field equations”: Pomeron loops

[Iancu, Triantafyllopoulos 2005]

Two dipoles scattering off a target:



- Pomeron loops missed!



# “Pomeron Loop Equations”

- “JIMWLK language” [Mueller,Shoshi,Wong 2005]:

$$\frac{\partial}{\partial Y} W_Y[\alpha] \propto \left( \alpha^2 \left( \frac{\delta}{\delta \alpha} \right)^2 + \sum_{n>1} \alpha^{2n} \left( \frac{\delta}{\delta \alpha} \right)^2 + \left( \frac{\delta}{\delta \alpha} \right)^4 \alpha^2 \right) W_Y[\alpha]$$

- Langevin-type version:

$$\frac{\delta}{\delta Y} \alpha^a(\mathbf{u}) = \sigma^a(\mathbf{u}) + \int_{\mathbf{z}} \epsilon^{ab,i}(\mathbf{u}, \mathbf{z}) \nu^{b,i}(\mathbf{z}) + \int_{\mathbf{x}, \mathbf{w}, \mathbf{z}} \rho(\mathbf{u}, \mathbf{x}, \mathbf{w}, \mathbf{z}) \bar{\nu}^a(\mathbf{x}, \mathbf{w}) \zeta(\mathbf{z}) \sqrt{\xi(\mathbf{x}, \mathbf{w})}$$

- “Balitsky language” [Iancu,Triantafyllopoulos 2005]:

$$\begin{aligned} \frac{\partial}{\partial Y} \langle T \rangle_Y &\propto \alpha_s [\langle T \rangle_Y - \langle T \ T \rangle_Y] \\ \frac{\partial}{\partial Y} \langle T \ T \rangle_Y &\propto \alpha_s [\langle T \ T \rangle_Y - \langle T \ T \ T \rangle_Y + \alpha_s^2 \langle T \rangle_Y] \end{aligned}$$

- Langevin-type version:

$$\frac{\partial}{\partial Y} T_Y \propto \alpha_s [T_Y - T_Y T_Y + \sqrt{\alpha_s^2 T} \nu]$$

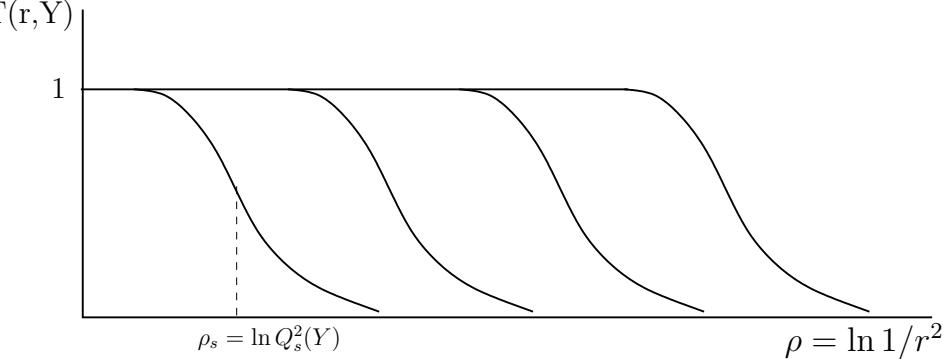
⇒ Gluon number fluctuations from event to event!

- (See also [Kovner, Lublinsky 2005], [Levin, Lublinsky 2005])

# Gluon number fluctuations $\Rightarrow$ Diffusive Scaling

[Mueller, Shoshi 2004], [Iancu, Mueller, Munier 2004]

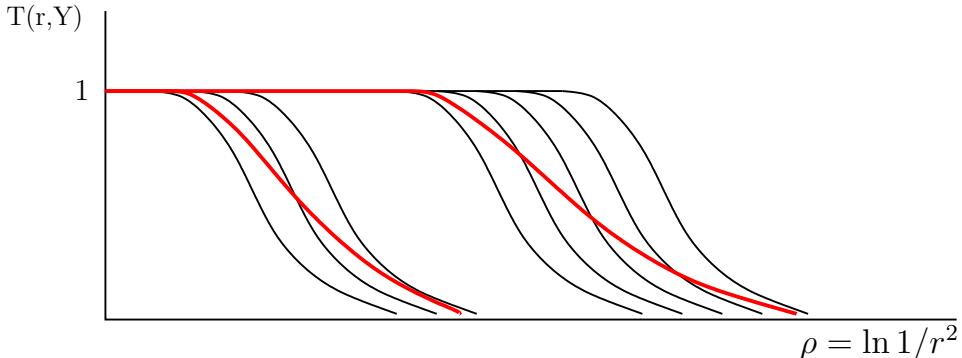
- Single events



Geometric scaling

$$T(r, Y) = T(r^2 Q_s^2(Y))$$

- Average over events



Diffusive scaling

$$\langle T(r, Y) \rangle = T \left( \frac{\ln(\bar{Q}_s^2(Y) r^2)}{\sqrt{\alpha_s Y / \ln^3(1/\alpha_s^2)}} \right)$$

$$\text{for } Y \gg \frac{1}{\alpha_s} \ln^3(1/\alpha_s^2)$$

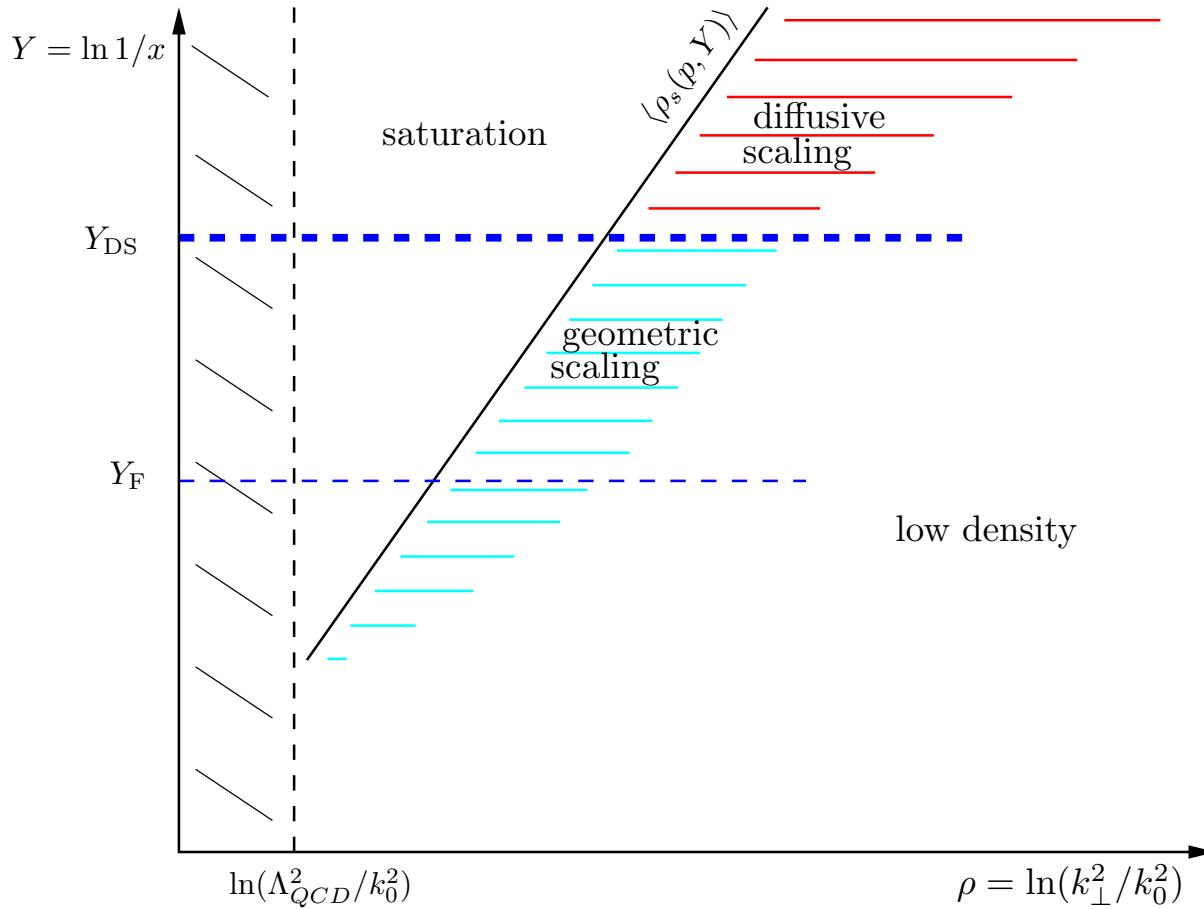
Averaging:

$$\langle T(\rho - \rho_s) \rangle = \int d\rho_s P(\rho_s) T(\rho - \rho_s)$$

where  $P(\rho_s) \simeq \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{(\rho_s - \langle \rho_s \rangle)^2}{2\sigma^2}\right]$ ,  $\sigma^2 = \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 \propto \frac{\alpha_s Y}{\ln^3(1/\alpha_s^2)}$

- saturation momentum:  $\bar{Q}_s^2(Y) = Q_0^2 \left(\frac{1}{x_{Bj}}\right)^\lambda$ ,  $\lambda = \lambda - \frac{C}{\ln^2(1/\alpha_s^2)}$

# “Phase diagram”



# Phenomenological consequences at large energies

- nuclear modification factor  $R_{pA}$  [Kozlov, Shoshi, Xiao 2006]:  
total shadowing due to Pomeron loops/fluctuations,  $R_{pA} \rightarrow \frac{1}{A^{1/3}}$   
 $\Rightarrow$  See talk by M. Kozlov.
- forward gluon production cross section [Iancu, Marquet, Soyez 2006]:  
scales with diffusive scaling variable
- total photon-proton cross section [Mueller,Shoshi 2004],[Iancu,Mueller,Munier 2004]:  
scales with diffusive scaling variable
- diffractive cross section [Hatta,Iancu,Marquet,Soyez,Triantafyllopoulos 2006]:  
different as compared to mean-field results

# Conclusions

- Large energy:
  - geometric scaling  $\Rightarrow$  diffusive scaling
  - energy dependence of saturation momentum
- QCD evolution equations including fluctuations
- fluctuations  $\Rightarrow$  strong phenomenological effects

# Beyond the Mean Field Approximation

[Mueller, Shoshi 2004]

- Shortcomings of the “mean field equation”:

Completeness relation:

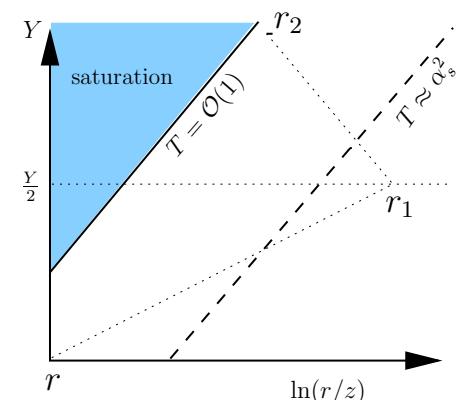
$$\underbrace{\frac{1}{r_2^2} T(r, r_2, Y)}_{\approx \mathcal{O}(1)} \approx \frac{1}{\alpha^2} \int d^2\rho \underbrace{\left( \frac{1}{r_1^2} T(r, r_1, Y/2) \right)}_{< \alpha^2} \underbrace{\left( \frac{1}{r_2^2} T(r_1, r_2, Y/2) \right)}_{> 1}$$

- Extension: Kovchegov equation  
+ absorptive boundary

- Results at very large  $Y$ :

No geometric scaling:  $T \left( \frac{\ln(Q_s^2(Y) r^2)}{[\alpha_s Y / \ln^3(1/\alpha_s^2)]} \right)$

saturation momentum:  $Q_s^2(x_{Bj}) = Q_0^2 \left( \frac{1}{x_{Bj}} \right)^\lambda, \quad \lambda = \lambda_0 - \frac{C}{\ln^2(1/\alpha_s^2)}$



# Extended Balitsky equations

[Iancu, Triantafyllopoulos 2005]

- Extended Balitsky hierarchy of equations:

$$\begin{aligned}\frac{\partial}{\partial Y} \langle T_Y \rangle &\propto \langle T_Y \rangle - \langle T_Y T_Y \rangle \\ \frac{\partial}{\partial Y} \langle T_Y T_Y \rangle &\propto \langle T_Y T_Y \rangle - \langle T_Y T_Y T_Y \rangle + \langle \textcolor{red}{T}_Y \rangle\end{aligned}$$

- Langevin-type version:

$$\begin{aligned}\frac{\partial}{\partial Y} T_Y(\mathbf{x}, \mathbf{y}) &= \frac{\alpha_s N_c}{2\pi^2} \int_z \mathcal{M}(\mathbf{x}, \mathbf{y}, \mathbf{z}) [-T_Y(\mathbf{x}, \mathbf{y}) + T_Y(\mathbf{x}, \mathbf{z}) - T_Y(\mathbf{z}, \mathbf{y}) - T_Y(\mathbf{x}, \mathbf{z}) T_Y(\mathbf{z}, \mathbf{y})] \\ &+ \left( \frac{\alpha_s}{2\pi} \right)^{3/2} \int_{\mathbf{u}, \mathbf{v}, \mathbf{z}} \mathcal{A}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v}, \mathbf{z}) \sqrt{\nabla_{\mathbf{u}} \nabla_{\mathbf{v}} T_Y(\mathbf{u}, \mathbf{v})} \nu(\mathbf{u}, \mathbf{v}, \mathbf{z}, Y)\end{aligned}$$

with the off-diagonal noise:

$$\langle \nu(\mathbf{u}_1, \mathbf{v}_1, \mathbf{z}_1, Y) \nu(\mathbf{u}_2, \mathbf{v}_2, \mathbf{z}_2, Y') \rangle = \delta^{(2)}(\mathbf{u}_1 - \mathbf{v}_2) \delta^{(2)}(\mathbf{v}_1 - \mathbf{u}_2) \delta^{(2)}(\mathbf{z}_1 - \mathbf{z}_2) \delta(Y - Y')$$

# Extended JIMWLK equation

[Mueller,Shoshi,Wong 2005]

- Extended JIMWLK equation:

$$\frac{\partial}{\partial Y} W_Y[\alpha] \propto \left( \underbrace{\alpha^2 \left( \frac{\delta}{\delta \alpha} \right)^2}_{\text{BKFL}} + \underbrace{\sum_{n>1} \alpha^{2n} \left( \frac{\delta}{\delta \alpha} \right)^2}_{\text{gluon merging}} + \underbrace{\left( \frac{\delta}{\delta \alpha} \right)^4 \alpha^2}_{\text{gluon splitting}} \right) W_Y[\alpha]$$

(Dense-Dilute Duality [Kovner,Lublinsky 2005]:  $\sum_{n>1} \left( \frac{\delta}{\delta \alpha} \right)^{2n} \alpha^2$ )

- “Langevin-type” version:

$$\frac{\delta}{\delta Y} \alpha^a(\mathbf{u}) = \sigma^a(\mathbf{u}) + \int_z \epsilon^{ab,i}(\mathbf{u}, \mathbf{z}) \nu^{b,i}(\mathbf{z}) + \int_{\mathbf{x}, \mathbf{w}, \mathbf{z}} \rho(\mathbf{u}, \mathbf{x}, \mathbf{w}, \mathbf{z}) \bar{\nu}^a(\mathbf{x}, \mathbf{w}) \zeta(\mathbf{z}) \sqrt{\xi(\mathbf{x}, \mathbf{w})}$$

Fourth order noise:  $\langle \zeta \rangle = \langle \zeta^2 \rangle = \langle \zeta^3 \rangle = 0, \quad \langle \zeta^4 \rangle = c$

Off-diagonal noise:  $\langle \xi(\mathbf{x}, \mathbf{w}) \xi(\mathbf{y}, \mathbf{w}') \rangle = \delta^{(2)}(\mathbf{x} - \mathbf{w}') \delta^{(2)}(\mathbf{w} - \mathbf{y})$

# “Pomeron Loop Equations”

[Mueller,Shoshi,Wong 2005], [Iancu,Triantafyllopoulos 2005]

- Hierarchy of equations:

$$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \alpha_s [\langle T \rangle_Y - \langle T T \rangle_Y]$$
$$\frac{\partial}{\partial Y} \langle T T \rangle_Y \propto \alpha_s [\langle T T \rangle_Y - \langle T T T \rangle_Y + \alpha_s^2 \langle T \rangle_Y]$$

- Langevin-type version:

$$\frac{\partial}{\partial Y} T_Y \propto \alpha_s \left[ T_Y - T_Y T_Y + \sqrt{\alpha_s^2 T} \nu \right]$$

⇒ Gluon number fluctuations from event to event!

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