GLUON SATURATION: FROM pp TO AA

or what would my grandma say?

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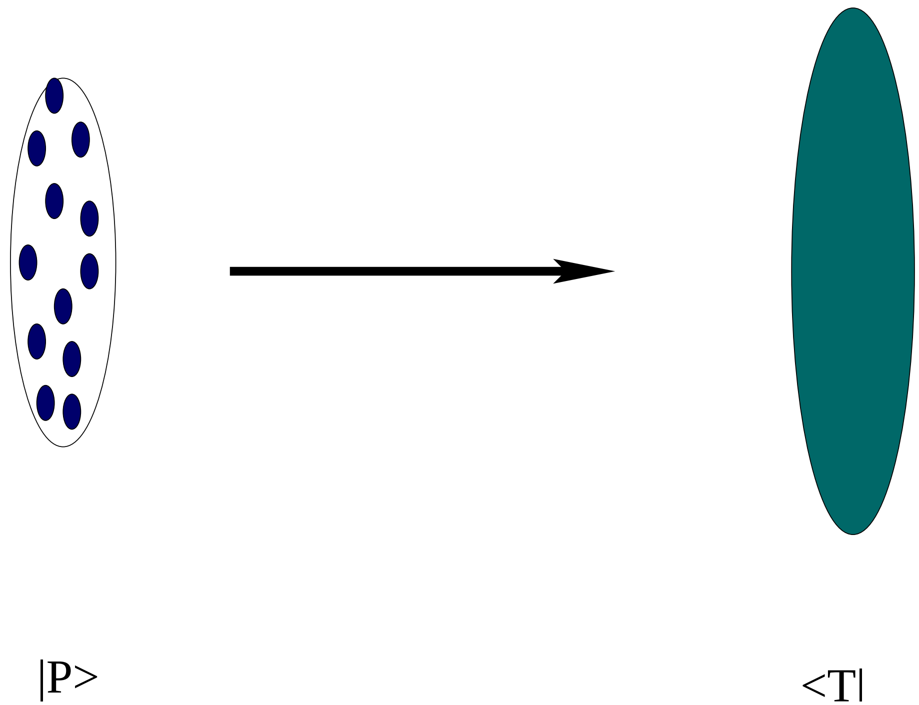
PERTURBATIVE SATURATION - THE BASIC APPROACH



The "projectile light cone gauge" $A^+ = 0$.

Projectile - a bunch (may be a pretty big bunch) of color charges - partons. Color charge density $\rho^a(x)$

Target - a (dark) stain of color fields. Distribution of color fields $\alpha^a(x) \equiv A^-(x)$.

COLOR CHARGES SCATTER EIKONALLY ON COLOR FIELDS.

$$\hat{S} = e^{i\int_x \rho^a(x)\alpha^a(x)} = \Pi_i \mathcal{P}\exp\{i\int dx^+ T^a A^-(x^+, x_i)\}$$

The forward scattering amplitude:

$$\text{S}=\langle T|\langle P|\, e^{i\int_x \rho^a(x)\alpha^a(x)}\, |P\rangle|T\rangle$$

PROJECTILE WAVE FUNCTION CHANGES WHEN BOOSTED

Boost the projectile wave function - softer gluons materialize from the "longitudinal field".

$$|P\rangle \;\rightarrow\; \Omega_Y[\rho, a, a^\dagger]|P\rangle$$

Now there are more color charges - the density increases

$$\rho^a \rightarrow \rho^a + \int_Y d\eta\, a^\dagger(\eta, x) T^a a(\eta, x)$$

Two faces that are important to stress.

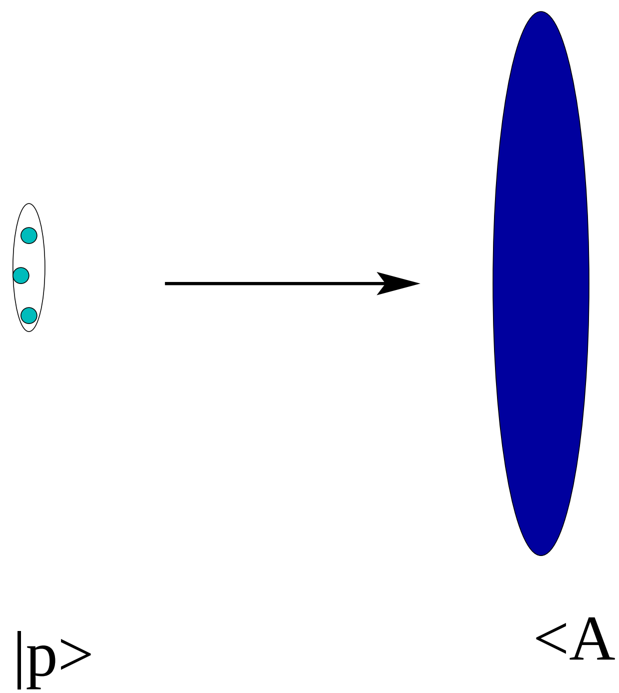
A. $\Omega[\rho]$ - DEPENDS ON ρ. **Probability of emission of soft gluons in the boosted wave function depends nonlinearly on the valence color charge density.**

B. $e^{i\int(\rho^a + a^\dagger T^a a)\alpha^a}$ - accounts for all multiple scatterings. **All gluons are allowed to scatter independently.**

THE EIKONAL HOLY GRAIL: EQUATION THEAT TAKES BOTH A and B INTO ACCOUNT WITHOUT APPROXIMATIONS.

WHERE ARE WE: TO HAVE AND HAVE NOT.

SMALL ON LARGE: **p-A scattering**.



Balitsky(1995), Kovchegov(1999) , "KLWMIJ" evolution (A.K. and Misha Lublinsky, 2005)

Ω - in the small density limit is a coherent operator. Creates "classical" Weizsacker-Williams field

$$\Omega = \exp\{\frac{i}{2\pi}\int_{x,z}\left[\rho^a(x)\frac{(x-z)_i}{(x-z)^2}\left(a_i^a(z)+a_i^{\dagger a}(z)\right)\right]\}$$

All multiple scattering corrections are taken into account: **ALL** soft gluons are allowed to scatter.

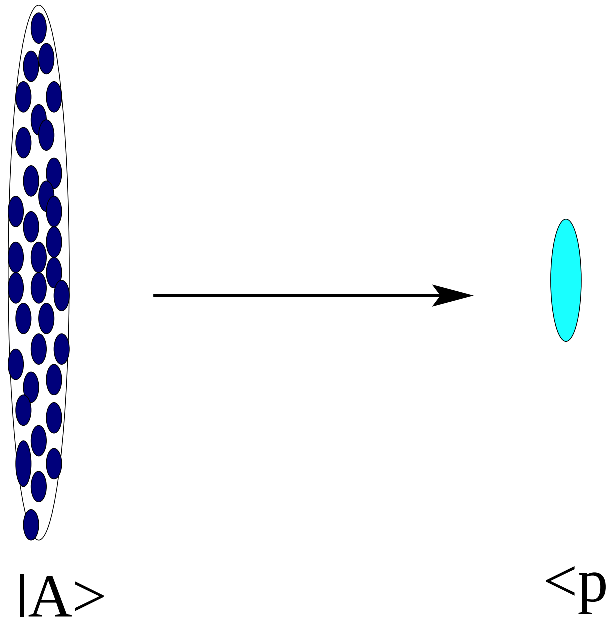
IMPORTANT: color charge grows exponentially according to BFKL

$$\rho_Y^2 \propto \rho_0^2 e^{\omega Y}$$

After $Y \sim \frac{1}{\omega}$ all gluons are soft (rapidity closest to the target). Simultaneous scattering of soft gluons is the leading effect.

The scattering amplitude unitarizes (does not exceed unity) due to multiple scattering effects even though the color charge density in the wave function grows exponentially (BFKL).

LARGE ON SMALL : **A - p scattering**.



Physically of course the same situation as before - but the evolution now resides in the wave function of the large dense object.

So we need to know more about the wave function, but less about multiple scatterings.

JIMWLK equation [A.K, Jalilian Marian, Leonidov, Weigert (1997), Iancu, Leonidov, McLerran (2001)]

$$" \left[\frac{d}{dY} S = \alpha H^{JIMWLK}[S] S \right] "$$

Evolution of $|P\rangle$ is fully nonlinear - good!

[A.K, M.Lublinksy, U. Wiedemann arXiv:0705.1713]

Target is small - multiple scatterings are less important. Many soft gluons per unit rapidity are produced in the evolution but their totality scatter only via two gluon exchange.

"Long range" multiple scatterings only - only particles at different rapidities scatter simultaneously.

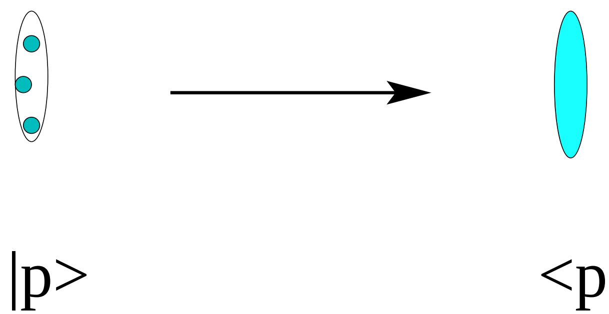
And that's fine;

In the saturated wave function color charge density grows slowly

$$\rho_Y^2 \sim \rho_0^2 + \#Y$$

Gluons are distributed homogeneously in rapidity. Two gluons are likely to scatter simultaneously if they sit far away from each other in rapidity.

SMALL ON SMALL : **p - p scattering**.



Simple - but NOT. At $Y < \frac{1}{\omega} \ln \frac{1}{\alpha_s}$ - BFKL.

For larger rapidity neither JIMWLK nor KLWMIJ work.

KLWMIJ obviously. At $Y \sim \frac{1}{\omega} \ln \alpha_s$ the density becomes large - the "dilute" wave function evolution is not appropriate.

JIMWLK a little less obvious. Still there is a range of rapidities (just before the saturation is reached) where ρ is not small but the evolution is still exponential - so "short range multiple scatterings" are important.

"STATISTICAL MODELS" APPROACH Mueller, Munier,Iancu (2004)

[Brunet, Derrida, Mueller, Munier: cond-mat/0512021 - inspiring analysis of front propagation in stochastic FKKP]

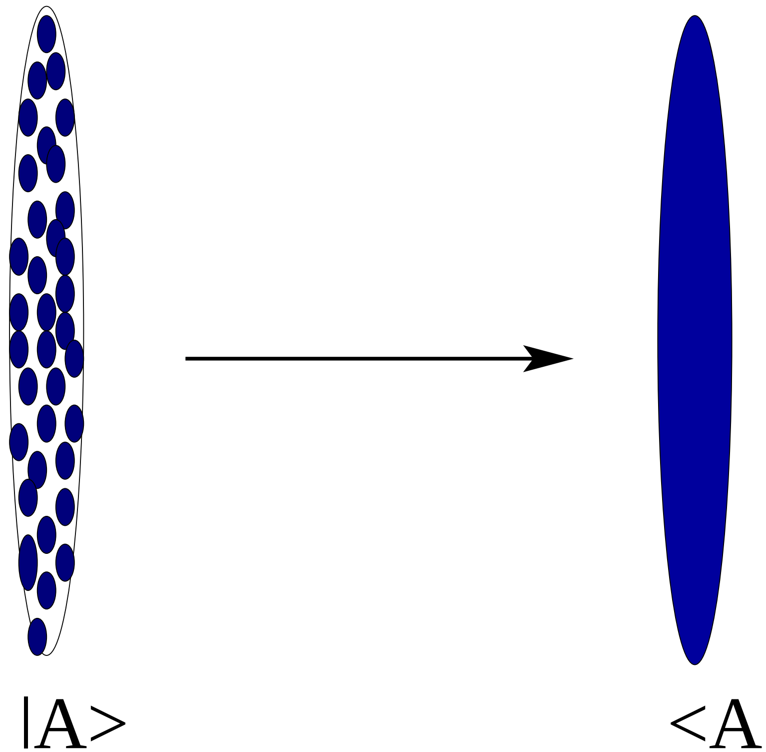
BUT: Connection to QCD tenuous.

If taken literally, BFKL does not hold for small density when two scattering objects are very different in size (for example two dipoles of different sizes).

Physics seems to be very different from the "finite density+multiple scatterings". BUT WHAT ELSE IS THERE?

Time will tell, but for now I am sceptical.

LARGE ON LARGE : A - A scattering.



Clearly both aspects are important: emission is nonlinear and scattering is multiple.

On the other hand calculating S is kinda silly: it's all saturated anyway, so $S = 0$.

WHAT SHOULD (AND WHAT CAN) WE CALCULATE

NOT total cross section: should not and cannot gluons a perturbatively massless - Froissart bound is asymptotically violated.

Perturbation Theory does not work for peripheral collisions - periphery is never saturated.

We need to find more exclusive observables insensitive to uncontrollable IR effects.

RECENTLY: theoretical framework of calculation of semi inclusive observables: diffractive, transverse momentum transfer (Kovchegov, Levin (dipole model 1999), Hentschinski, Weigert and Schafer 2005; A.K., Lublinsky and Weigert, 2006)

Inclusive gluon production (two gluons: Kovchegov and Jalilian-Marian (2004), two and any: A.K. and Lublinsky (2006))

All theoretical developments are for "p - A". Within the dipole model approximation - although complicated, but does not look intractable numerically. Awaits numerical implementation.

TOWARDS A - A

Wave function evolution - A.K., Lublinsky and Wiedemann (2007).

$$\Omega = \exp\{i \int_z \left[b^i[\rho, z] \left(a_i^a(z) + a_i^{\dagger a}(z)\right)\right]\}\times$$

$$\exp\{\int_{z,u} (a_i^a(z) + a_i^{a\dagger}(z)) M_{ij}^{ab}[\rho, z, u](a_j^b(u) + a_j^{b\dagger}(u))\}$$

"Pomeron loops" must be very important for the final state structure. Saturation strongly affects multiplicity of gluons below Q_s. Expect the number of gluons produced below Q_s to grow only as

$$\frac{dN}{dY} \propto Y$$

Also - and very interesting: Gelis, Venugopalan, 2006 - and in progress inclusive gluon porduction solving classical equations pluc leading logarithmic corrections.

HIGHER ORDER CORRECTIONS

NLO BFKL has very large effect - slows down the growth of the cross section. Expect the same kind of effect in the saturation regime.

Next to leading corrections to JIMWLK. Balitsky, Belitsky (2001), Balitsky (2006), Kovchegov and Weigert (2006), Gardi, Rummukainen, Weigert (2006), Kovchegov, Albacete (2007)

Quark-antiquark contribution to the final states and the running of the coupling constant.

Gluon contribution is still not finished, although some elements are known.

Indeed has a very large effect on the saturation physics. Everything grows much slower: both due to the running of the coupling and to “genuine” NLO effects.(Kovchegov, Albacete, (2007)

$Q_s^2 \sim \exp\{0.15Y\}$ as opposed to $Q_s^2 \sim \exp\{0.4Y\}$ for fixed coupling $\alpha_s = .25$.

More complete understanding including gluon contribution (presumably the most important piece) is crucial for sensible applications to phenomenology.