Towards a fitting procedure for DVCS at next-to-leading order and beyond

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Collaboration with:

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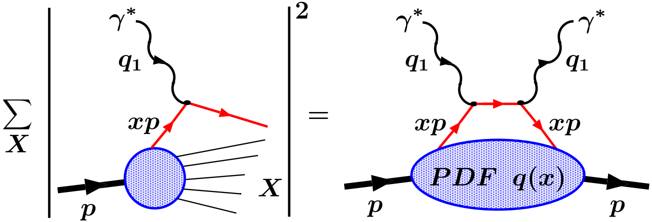
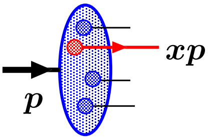
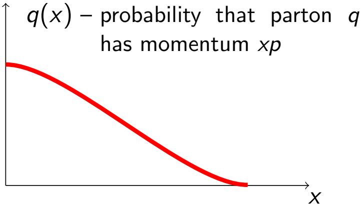
Outline

Parton distribution functions

- Deeply inelastic scattering, $-q_1^2 \to \infty,\ x_{BJ} \equiv \frac{-q_1^2}{2p\cdot q_1} \to \mathrm{const}$







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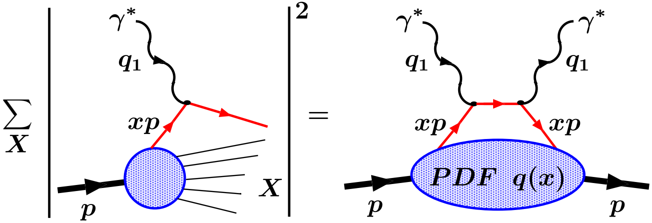
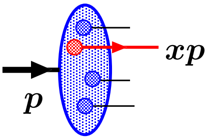
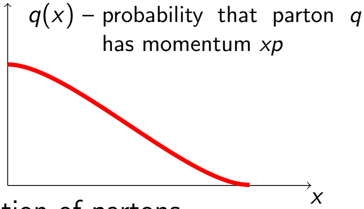
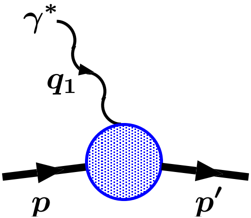






- no information on spatial distribution of partons

Electromagnetic form factors



- Dirac and Pauli form factors:

$$F_{1,2}(t = q_1^2)$$

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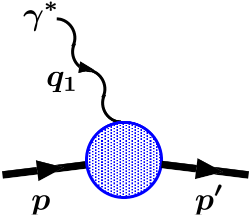

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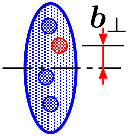

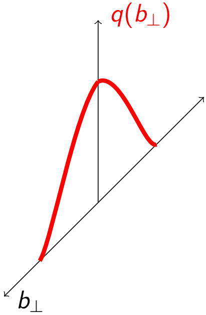

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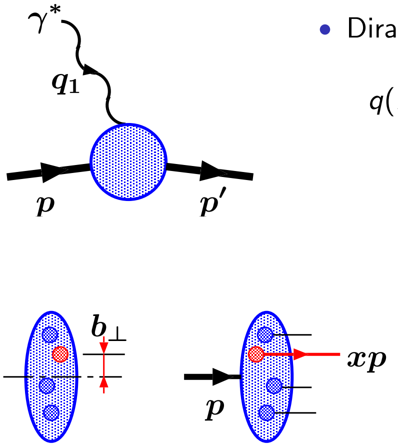

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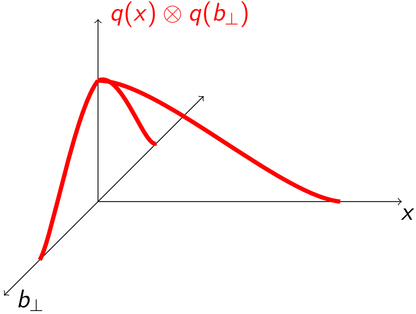

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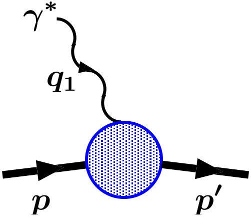

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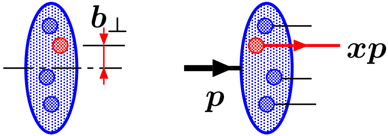

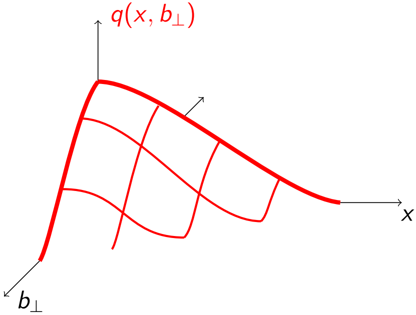

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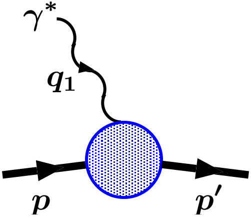

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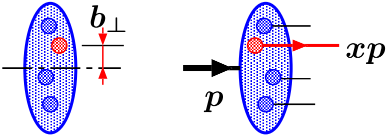
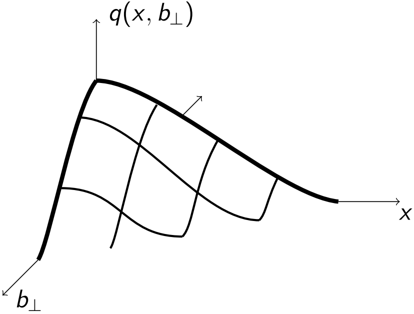
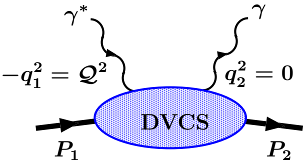




- "skewless" GPD: $H^q(x, 0, t = \Delta^2) = \int \mathrm{d}b_\perp \, e^{i\Delta \cdot b_\perp} q(x, b_\perp)$

Probing the proton with two photons

- Deeply virtual Compton scattering [Müller '92, et al. '94]



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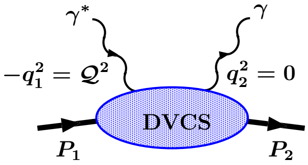


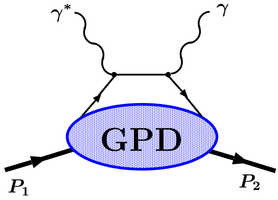
$$P = P_1 + P_2\,, \quad q = (q_1 + q_2)/2\,, \quad \Delta = P_2 - P_1$$

Generalized Bjorken limit:

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const}$$

- QCD: factorization of short- and long-distance physics



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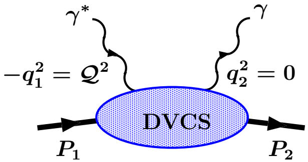

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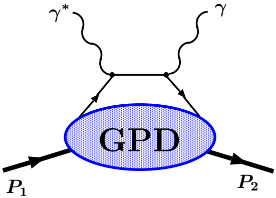
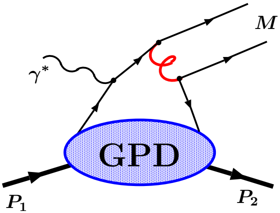




Alternative probe:
meson production

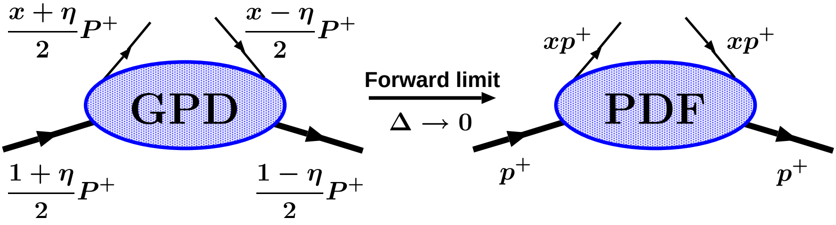
- more difficult, but access to flavours
- see [talk by D. Ivanov]

Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$F^g(x,\eta,\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)G_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$



$$P = P_1 + P_2\ ; \qquad \Delta = P_2 - P_1\ ; \qquad \eta = -\frac{\Delta^+}{P^+}\ \text{(skewedness)}$$

Properties of GPDs

- Decomposing into helicity conserving and non-conserving part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

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- Sum rules:

$$\sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l} H^q(x,\eta,\Delta^2) \\ E^q(x,\eta,\Delta^2) \end{array} \right. = \left\{ \begin{array}{l} F_1(\Delta^2) \\ F_2(\Delta^2) \end{array} \right.$$

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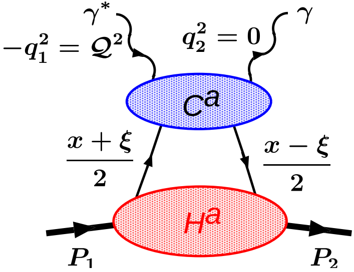
- Possibility of solution of proton spin problem

$$\frac{1}{2}\int_{-1}^{1} dx\, x \left[H^q(x,\eta,\Delta^2) + E^q(x,\eta,\Delta^2) \right] = J^q(\Delta^2) \qquad \text{[Ji '96]}$$

Deeply virtual Compton scattering



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$(\Delta \to 0,\ \xi \to x_{BJ}/2)$

(Dominant) Compton form factor (CFF):

$$^a\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = \int \mathrm{d}x\ C^a(x, \xi, \mathcal{Q}^2/\mu^2)\ H^a(x, \eta = \xi, \Delta^2, \mu^2)$$

a=NS,S(Σ,G)

Conformal moment series representation

- Experiment: measurements at DESY, JLab, CERN
- Theory: LO, NLO (1st order in α_s)

  [Ji et al, Belitsky et al, Mankiewicz et al, '97]

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- singlet DVCS CFF in terms of conformal moments:

$$^{\mathrm{S}}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = 2 \sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi = \eta, \Delta^2, \mu^2)$$

$$H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots)$$

$$\rightarrow \langle P_2 | O_{jj}^q | P_1 \rangle$$

...analogous to Mellin moments in DIS: $x^n \rightarrow C_n^{3/2}(x)$

Mellin-Barnes representation of CFFs

- Series is summed using Mellin-Barnes integral

$$^{\mathrm{S}}\mathcal{H}(\xi,\Delta^2,\mathcal{Q}^2) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\left[i+\tan\left(\frac{\pi j}{2}\right)\right]$$

$$\times \mathbf{C}_j(\mathcal{Q}^2/\mu^2,\alpha_s(\mu))\mathbf{H}_j(\xi,\Delta^2,\mu^2)$$

- Advantages of conformal moments i.e. Mellin-Barnes representation
    - possible efficient and stable numerical treatment
    - enables easier inclusion of evolution effects
    - opens the door for alternative modelling of GPDs
    - by making use of conformal OPE and known NNLO DIS results, NNLO predictions obtained

OPE

- DVCS belongs to a class of two-photon processes (DIS, DVCS, two-photon production of hadronic states ...) calculable by means of OPE

$$
\begin{aligned}
T_{\mu\nu}(q,P,\Delta) &= \frac{i}{e^2}\int d^4x\, e^{ix\cdot q}\langle P_2,S_2|\,Tj_\mu(x/2)j_\nu(-x/2)|P_1,S_1\rangle \\
&\to \sum_j C_j\, O_j
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$$\downarrow$$

$$\left.\begin{array}{l}\text{generalized Bjorken kinematics}\\ \text{conformal symmetry}\end{array}\right\} \to \text{unified description}$$

Conformal OPE (COPE)

- COPE prediction for general kinematics reads

$$C_j(\eta/\xi, Q^2/\mu^2, \alpha_s^* = fixed)$$

$$= c_j(\alpha_s^*)\, {}_2F_1\!\left({(2+2j+\gamma_j(\alpha_s^*))/4, (4+2j+\gamma_j(\alpha_s^*))/4 \atop (5+2j+\gamma_j(\alpha_s^*))/2} \middle| \frac{\eta^2}{\xi^2} \right) \left(\frac{\mu^2}{Q^2}\right)^{\frac{\gamma_j(\alpha_s^*)}{2}},$$

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- $\eta = 0$: DIS

$$\lim_{\eta\to 0} C_j(\eta/\xi, Q^2/\mu^2, \alpha_s^*) = c_j^{DIS}(\alpha_s^*)|_{\beta=0} \left(\frac{\mu^2}{Q^2}\right)^{\gamma_j(\alpha_s^*)/2}$$

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- $\eta = \xi$: DVCS
- $\eta = 1$: photon-to-pion transition form factor

Breaking of conformal symmetry

- massless QCD is conformally symmetric at the tree level
- conformal symmetry broken at the loop level (renormalization introduces mass scale)
  - running of the coupling constant $\Rightarrow \beta \neq 0$
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$$\mu \frac{d}{d\mu} O_j(\ldots, \mu^2) = - \sum_{k=0}^{j} \gamma_{jk}(\alpha_s(\mu))\, \eta^{j-k}\, O_k\,(\ldots, \mu^2)\,,$$

$$\mu \frac{d}{d\mu} C_j(\ldots, Q^2/\mu^2, \alpha_s(\mu))] = \sum_{k=j}^{\infty} C_k(\ldots, Q^2/\mu^2, \alpha_s(\mu))\gamma_{kj}(\alpha_s(\mu)) \left(\frac{\eta}{\xi}\right)^{k-j}$$

Conformal scheme

- non-diagonal terms of anomalous dimensions ($\overline{\text{MS}}$ scheme) can be removed by finite renormalization, i.e, specific choice of factorization scheme → conformal subtraction ($\overline{\text{CS}}$) scheme:

$$C^{\overline{\text{MS}}}\, O^{\overline{\text{MS}}} = C^{\overline{\text{MS}}}\, B\, B^{-1}\, O^{\overline{\text{MS}}} = C^{\overline{\text{CS}}} O^{\overline{\text{CS}}}$$

$$\gamma_{jk}^{\overline{\text{CS}}} = \delta_{jk}\gamma_k + \frac{\beta}{g}\Delta_{jk}$$

- however, there is ambiguity in $\overline{\text{MS}}$ → CS rotation matrix:

$$B = B^{(\beta=0)} + \frac{\beta}{g}\delta B$$

and by judicious choice of δB one can "push" mixing to NNLO ($\overline{\text{CS}}$ scheme, [Melić et al. '02]) → Δ_{jk} — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected

NNLO DVCS

- Finally

$$C_j^{\overline{\mathrm{CS}},\mathrm{DVCS}}(Q^2/\mu^2, \alpha_s(\mu))$$

$$= C_k(1, \alpha_s(Q))\, \mathcal{P} \exp\left\{ \int_Q^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_j(\alpha_s(\mu'))\delta_{kj} + \frac{\beta}{g}\Delta_{kj}(\ldots) \right] \right\}$$

with

$$C_j(1, \alpha_s(Q)) = \frac{2^{1+j+\gamma_j(\alpha_s)/2}\Gamma\left(\frac{5}{2} + j + \gamma_j(\alpha_s)/2\right)}{\Gamma(3/2)\Gamma\left(3 + j + \gamma_j(\alpha_s)/2\right)}\; c_j^{\overline{\mathrm{MS}},\mathrm{DIS}}(\alpha_s)$$

- we take

$c_j^{\overline{\mathrm{MS}},\mathrm{DIS}}(\alpha_s)$ from [Zijlstra, v. Neerven '92,'94, v. Neerven and Vogt '00]
γ_j from [Vogt, Moch and Vermaseren '04]

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  2. compare the NLO predictions to complete (non-diagonal evolution included) $\overline{\mathrm{MS}}$ NLO predictions [Kumerički, Müller and K. P-K. '07]
  3. perform fits (in both schemes) to DVCS (and DIS) data and extract information about GPDs [Kumerički, Müller and K. P-K. '07]

Choice of GPD Ansatz

- We use simple Regge-inspired ansatz for GPDs ...

$$\mathbf{H}_j(\xi, \Delta^2, \mu_0^2) = \begin{pmatrix} N'_\Sigma F_\Sigma(\Delta^2)\, \mathrm{B}\big(1+j-\alpha_\Sigma(0), 8\big) \\ N'_G F_G(\Delta^2)\, \mathrm{B}\big(1+j-\alpha_G(0), 6\big) \end{pmatrix}$$

$$\alpha_a(\Delta^2) = \alpha_a(0) + 0.15\Delta^2 \qquad F_a(\Delta^2) = \frac{j+1-\alpha(0)}{j+1-\alpha(\Delta^2)} \left(1 - \frac{\Delta^2}{M_0^{a2}}\right)^{-p_a}$$

... corresponding in forward case ($\Delta = 0$) to PDFs of form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)} (1-x)^7 \;; \qquad G(x) = N'_G\, x^{-\alpha_G(0)} (1-x)^5$$

Choice of GPD Ansatz

- We use simple Regge-inspired ansatz for GPDs ...

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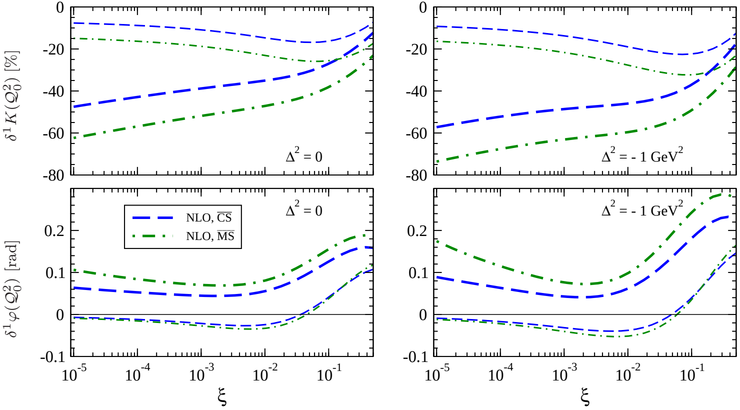
$$\Downarrow$$

- analysis of radiative corrections (with generic parameters)
- fit of N_Σ, $\alpha_\Sigma(0)$, M_0^Σ, N_G, $\alpha_G(0)$, M_0^G

for small ξ (small x) valence quarks less important: $\Sigma \approx$ sea

NLO corrections



Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)=\alpha_\Sigma(0) + 0.05$

Thin lines:
"soft" gluon
$N_G = 0.3$
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$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

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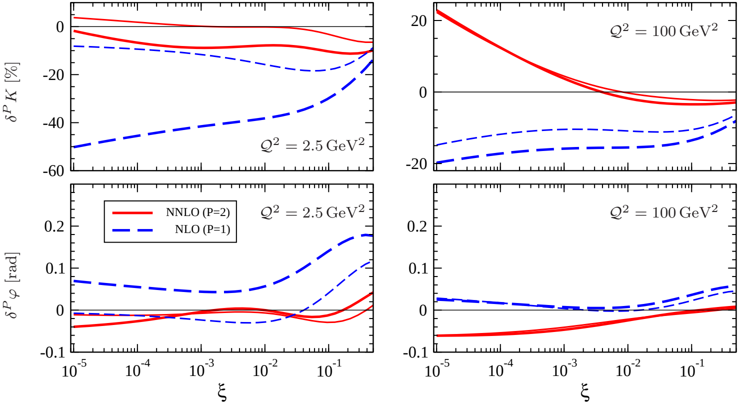

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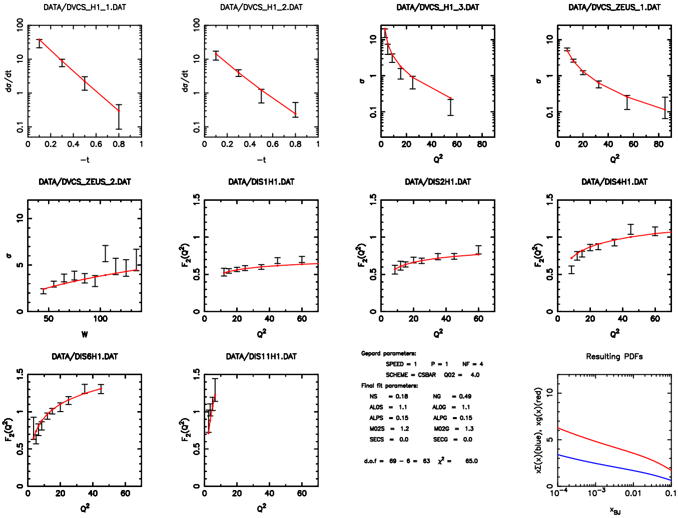
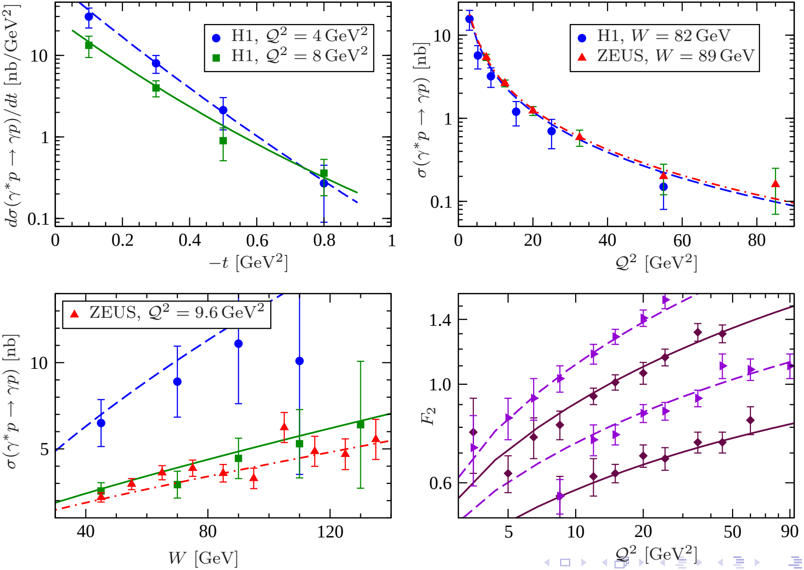
Fast fitting routine (GeParD)



6-parameter fit $\longrightarrow \chi^2/\mathrm{d.o.f} = 65/63$

Example of final fit result



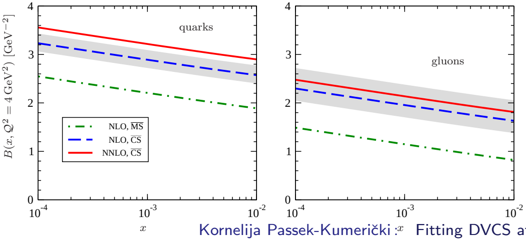
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on x and transversal distance b [Burkardt '00, '02]

$$H(x, \vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2)\,,$$

- Average transversal distance :

$$\langle \vec{b}^2 \rangle(x, \mathcal{Q}^2) = \frac{\int d\vec{b}\, \vec{b}^2 H(x, \vec{b}, \mathcal{Q}^2)}{\int d\vec{b}\, H(x, \vec{b}, \mathcal{Q}^2)} = 4B(x, \mathcal{Q}^2)\,,$$

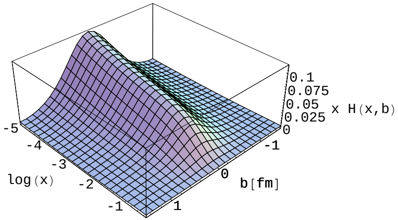


(at $\mathcal{Q}^2 = 4\,\mathrm{GeV}^2$)

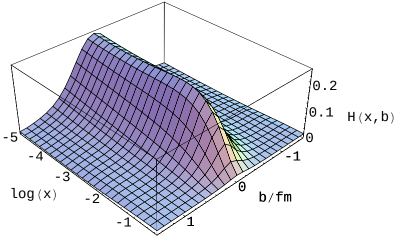
$$\langle \vec{b}^2 \rangle_{\mathrm{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04}\,\mathrm{fm}^2$$

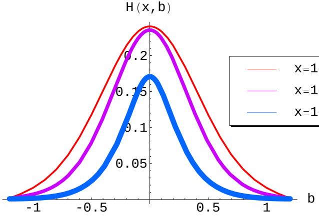
Three-dimensional image of a proton

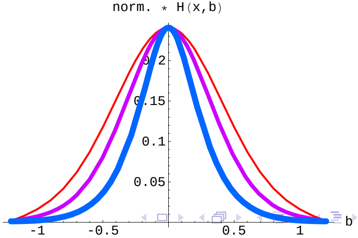
Quarks:



Gluons:







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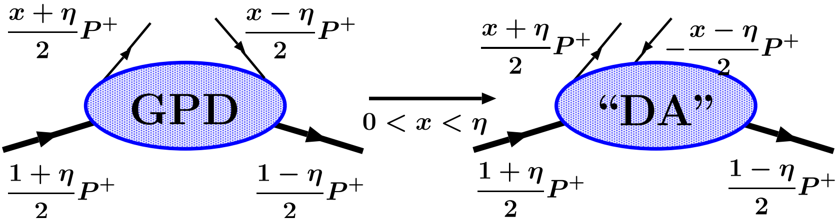
The End

Relation to distribution amplitudes

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

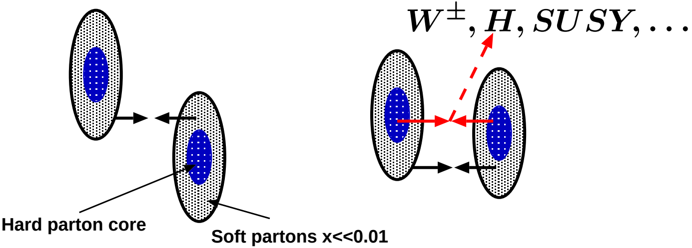
$$F^q(x,\eta,\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$F^g(x,\eta,\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)G_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\ldots}$$



$$P = P_1 + P_2\,; \qquad \Delta = P_2 - P_1\,; \qquad \eta = -\frac{\Delta^+}{P^+} \quad \text{(skewedness)}$$

Relevance of GPDs for collider physics



- heavy particle production $\Rightarrow$ collision is more central $\Rightarrow$ larger probability for multiple parton collisions
- [Frankfurt, Strikman and Weiss '04]

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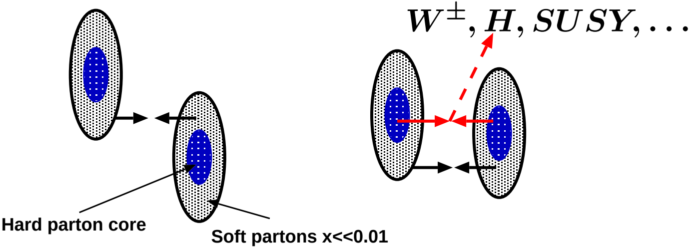

- heavy particle production $\Rightarrow$ collision is more central
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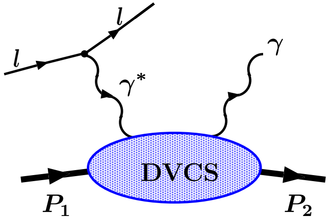
- [Frankfurt, Strikman and Weiss '04]
- No influence on total inclusive cross sections, but event structure is sensitive to transversal parton distributions.
- Relevant for LHC?

Deeply virtual Compton scattering (I)

- Measured in leptoproduction of a real photon:



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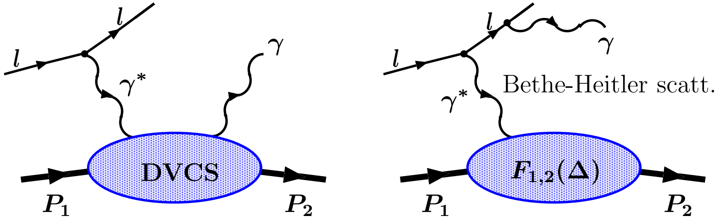

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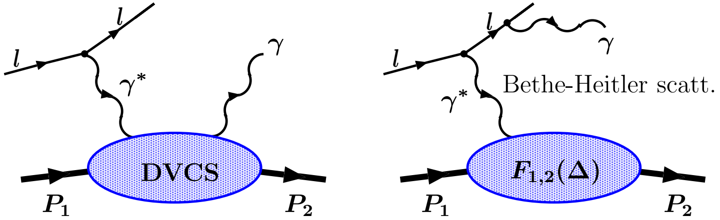

- There is a background process but it can be used to our advantage:

$$\sigma \propto |\mathcal{T}_{\rm DVCS}|^2 + |\mathcal{T}_{\rm BH}|^2 + \mathcal{T}^*_{\rm DVCS}\mathcal{T}_{\rm BH} + \mathcal{T}_{\rm DVCS}\mathcal{T}^*_{\rm BH}$$

- Using $\mathcal{T}_{\rm BH}$ as a referent "source" enables measurement of the phase of $\mathcal{T}_{\rm DVCS}$ $\rightarrow$ proton "holography" [Belitsky and Müller '02]

Conformal algebra

- Conformal group restricted to light-cone $\sim O(2,1)$

$$L_+ = -iP_+ \qquad [L_0, L_\mp] = \mp L_\mp$$

$$L_- = \frac{i}{2}K_- \qquad [L_-, L_+] = -2L_0$$

$$L_0 = \frac{i}{2}(D + M_{-+})$$

Casimir:

$$L^2 = L_0^2 - L_0 + L_- L_+$$

conf.spin j:

$$[L^2, \mathbb{O}_{n,n+k}] = j(j-1)\mathbb{O}_{n,k}$$

(D — dilatations, K_- — special conformal transformation (SCT))

Operator Product Expansion

$$J_{\rm em}(x)J_{\rm em}(0) \longrightarrow \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{1}{x^2}\right)^2 x_-^{n+k+1}\, C_{n,k}\, {\color{red}O_{n,k}}$$

$${\color{red}O_{n,k}} \equiv (i\partial_+)^k\, \bar\psi\, \gamma^+ (i \overleftrightarrow{D}_+)^n \psi$$

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- (At least) to LO answer is: use conformal operators.

Conformal operators

$$\mathbb{O}_{n,n+k} = (i\partial^+)^{n+k}\, \bar{\psi}\, \gamma^+\, C_n^{3/2}\left(\frac{\overleftrightarrow{D}^+}{\partial^+}\right) \psi$$

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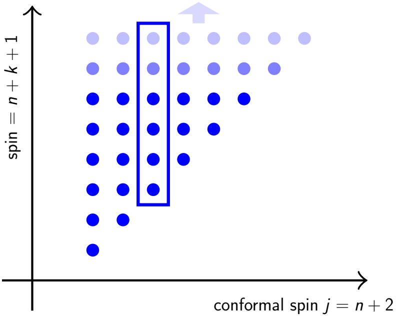
$\Rightarrow$ $\mathbb{O}_{n,n+k}$ start to mix at NLO

Conformal Towers



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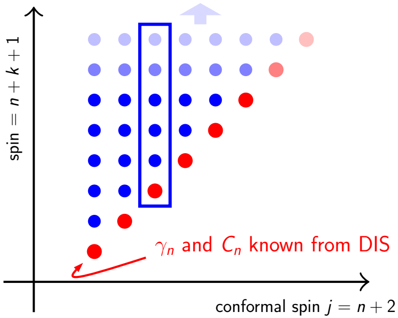

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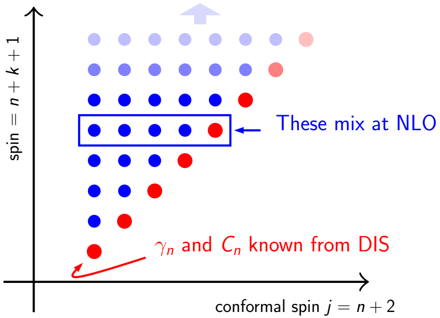

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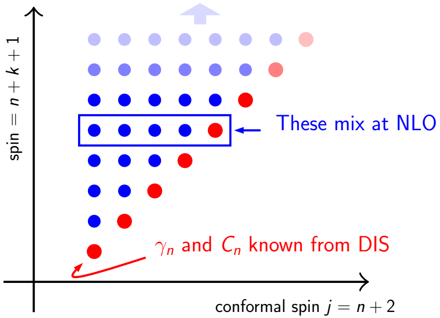

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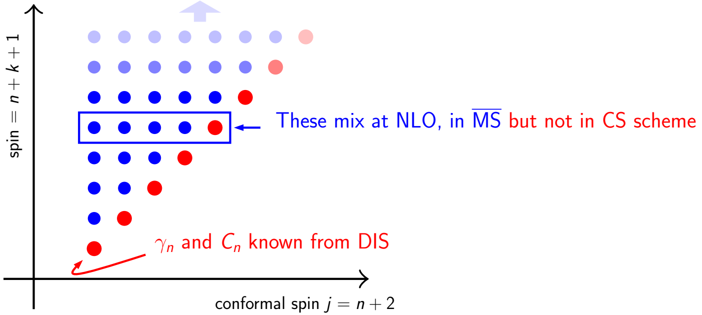

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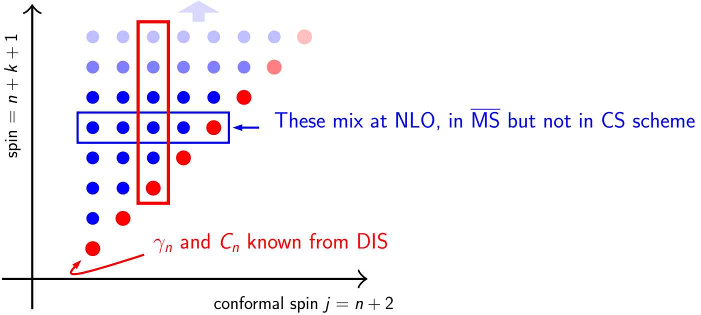

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- $C_{n,k} = (-1)^k \frac{(n+2)_k}{k!(2n+4)_k} C_{n,0} \qquad \Rightarrow$ summing complete tower

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- But how to calculate rotation matrix B? This is problem equivalent to calculation of $\gamma_{j,k}$.

$\beta \neq 0$ (II)

- The $B^{(\beta=0)}$ is constrained by conformal Ward identities ...

$$B_{jk}^{(\beta=0)\mathrm{NLO}} = \delta_{jk} - \frac{\alpha_s}{2\pi}\theta(j>k)\frac{\gamma_{jk}^{\mathrm{SCT,\ LO}}}{a_{jk}}$$

(a_{jk} — known matrix)
[Müller '93]

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- ... and, as a consequence

$$\overline{\mathrm{MS}}\,\gamma_{jk}^{\mathrm{ND},(1)} = \frac{\left[\gamma^{\mathrm{SCT,\ (0)}} - \beta_0 \frac{b}{g}, \gamma^{(0)}\right]_{jk}}{a_{jk}}$$

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SCT $\equiv$ special conformal transformation

- ... and, as a consequence

$$^{\overline{\mathrm{MS}}}\gamma_{jk}^{\mathrm{ND},(1)} = \frac{\left[\gamma^{\mathrm{SCT,\ }(0)} - \beta_0 \frac{b}{g}, \gamma^{(0)}\right]_{jk}}{a_{jk}}$$

- Final result:
  n-loop DIS (diagonal) result + $(n-1)$-loop SCT anomaly =
  n-loop non-diagonal prediction

NNLO DVCS (I)

- DVCS amplitude in terms of conformal moments:

$$ {}^{\mathrm{S}}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = 2 \sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi = \eta, \Delta^2, \mu^2) $$

$$ H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots) $$

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- Here, Wilson coefficients C_j read ...

$\Rightarrow$

NNLO DVCS (II)

$$C_j(Q^2/\mu^2, Q^2/\mu^{*2}, \alpha_s(\mu)) = \sum_{k=j}^{\infty} C_k(1, \alpha_s(Q))\, \mathcal{P}\exp\left\{\int_Q^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_j(\alpha_s(\mu'))\delta_{kj} + \frac{\beta}{g}\Delta_{kj}(\alpha_s(\mu'), \mu'/\mu^*)\right]\right\}$$

with

$$C_j(1, \alpha_s(Q)) = \frac{2^{1+j+\gamma_j(\alpha_s)/2}\Gamma\left(\frac{5}{2} + j + \gamma_j(\alpha_s)/2\right)}{\Gamma(3/2)\Gamma\left(3 + j + \gamma_j(\alpha_s)/2\right)}\, c_j^{\overline{\mathrm{MS}},\mathrm{DIS}}(\alpha_s)$$

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- $c_j^{\overline{\mathrm{MS}},\mathrm{DIS}}(\alpha_s)$ from [Zijlstra, v. Neerven '92,'94, v. Neerven and Vogt '00]
- Finally, evolution of conformal moments is given by . . . $\Rightarrow$

NNLO DVCS (III)

$$\mu\frac{d}{d\mu}H_j(\cdots,\mu^2) = -\gamma_j(\alpha_s(\mu))\,H_j\,(\cdots,\mu^2) - \frac{\beta(\alpha_s(\mu))}{g(\mu)}\sum_{k=0}^{j-2}\eta^{j-k}\Delta_{jk}\Big(\alpha_s(\mu),\frac{\mu}{\mu^*}\Big)\,H_k(\cdots,\mu^2)$$

- Δ_{jk} — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected
- γ_j from [Vogt, Moch and Vermaseren '04]

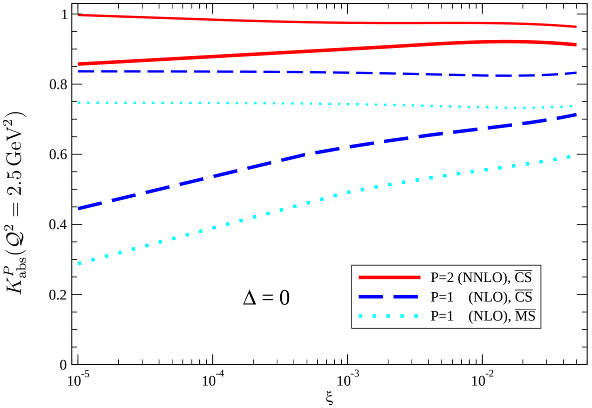
NNLO DVCS (III)

$$\mu \frac{d}{d\mu} H_j(\cdots, \mu^2) = -\gamma_j(\alpha_s(\mu))\, H_j\,(\cdots, \mu^2)$$
$$- \frac{\beta(\alpha_s(\mu))}{g(\mu)} \sum_{k=0}^{j-2} \eta^{j-k} \Delta_{jk}\Big(\alpha_s(\mu), \frac{\mu}{\mu^*}\Big)\, H_k(\cdots, \mu^2)$$

- Δ_{jk} — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected
- γ_j from [Vogt, Moch and Vermaseren '04]

- We have used these expressions to
  1. investigate size of NNLO corrections to non-singlet [Müller '05] and singlet [K.K., Müller, Passek-Kumerički and Schäfer '06] Compton form factors
  2. perform fits to DVCS (and DIS) data and extract information about GPDs [K.K., Müller and Passek-Kumerički '07]

Size of Radiative Corrections - Modulus



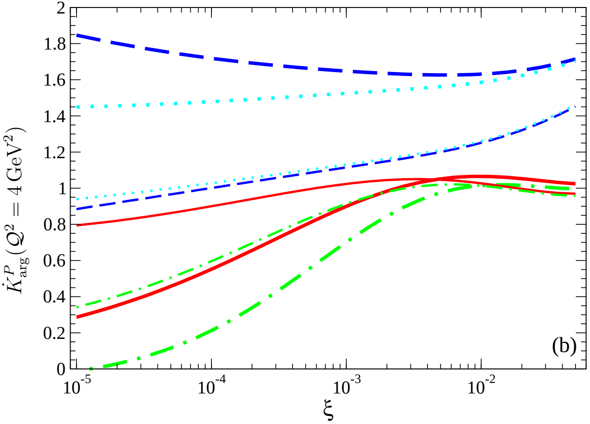
Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)=\alpha_\Sigma(0)+0.1$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0)=\alpha_\Sigma(0)$

- NLO: up to 40–70% ($\overline{\rm MS}$); up to 30–55% ($\overline{\rm CS}$) ["hard"]
- NNLO: 8–14% ("hard"); 1-4% ("soft") [$\overline{\rm CS}$]

Scale Dependence

Same K-factors, but with $\mathcal{H} \to \mathrm{d}\mathcal{H}/\mathrm{d}\ln \mathcal{Q}^2$

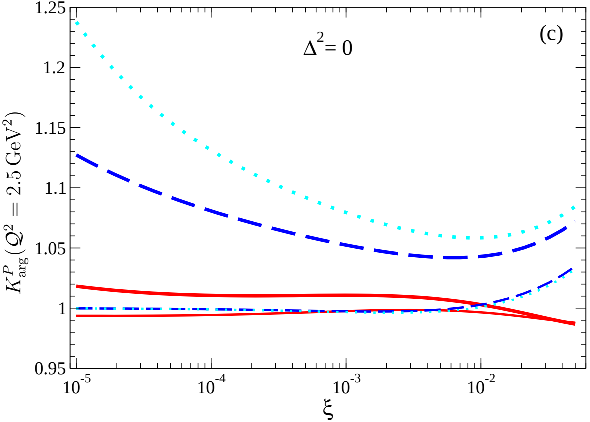


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0) = \alpha_\Sigma(0) + 0.1$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0) = \alpha_\Sigma(0)$

- NLO: even 100%
- NNLO: somewhat smaller, but acceptable only for larger ξ

Size of Radiative Corrections - phase

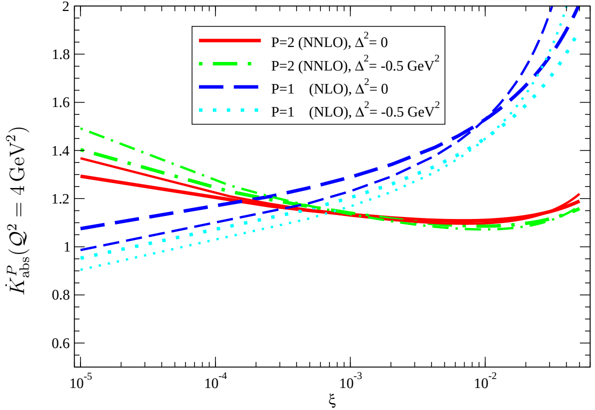


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)=\alpha_\Sigma(0)+0.1$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0)=\alpha_\Sigma(0)$

- NLO: up to 24% ($\overline{\rm MS}$); up to 13% ($\overline{\rm CS}$) ["hard"]
- NNLO and "soft" NLO — less than 5%

Scale Dependence - Modulus



Thick lines:
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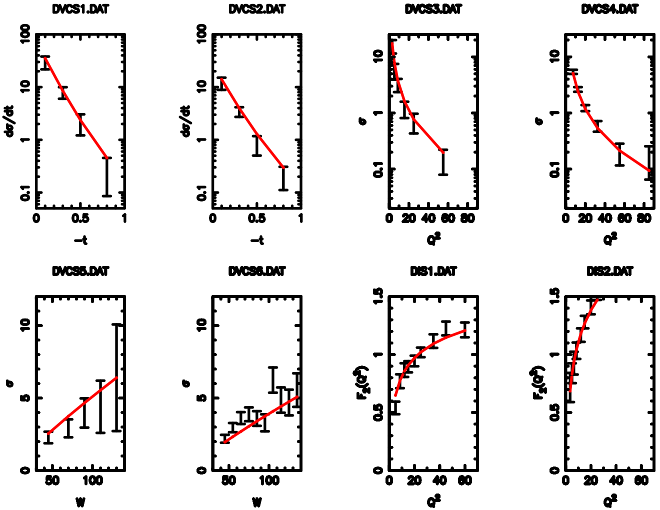
$N_G = 0.4$
$\alpha_G(0) = \alpha_\Sigma(0) + 0.1$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0) = \alpha_\Sigma(0)$

- NLO: even 100%
- NNLO: smaller (largest for "hard" gluons)

Fast fitting routine (GeParD)



- $N_\Sigma = 0.143,\ \alpha_\Sigma(0) = 1.10,\ m_\Sigma = 1.26,\ N_G = 0.549,\ \alpha_G(0) = 0.915,\ m_G = 1.66,\ \mathcal{Q}_0^2 = 2.5\ \mathrm{GeV}^2$
- $\chi^2/(\text{number of degrees of freedom}) = 54/64$