# Low-*x* and Diffractive Physics at Future Electron-Proton/Ion Colliders

*Henri Kowalski* DESY Notkestrasse 85, 22607 Hamburg, Germany

### Abstract

In this talk we discuss the present and future low-x and diffractive physics. The main topics are nuclear tomography and the investigation of the different QCD evolution dynamics in the transverse plane.

# **1** HERA measurements

# 1.1 Introduction

Exclusive diffractive processes at HERA, such as exclusive vector meson production or deeply virtual Compton scattering (DVCS), are excellent probes of the evolution and the proton structure in the gluon dominated regime. Several investigations have already shown that these processes can be well described within a QCD dipole approach with the vector meson wave functions determined by educated guesses and the photon wave function computed within QED. For an overview and the complete set of references see [1].

We start with a short description of the HERA data in terms of the dipole model. The vector meson and DVCS processes are measured at HERA in the small-x regime where the behaviour of the inclusive deep-inelastic scattering (DIS) cross section, or the structure function  $F_2$ , is driven by the gluon density. The dipole model allows these processes to be calculated, through the optical theorem, from the gluon density determined by a fit to the total inclusive DIS cross sections.

We base this talk on the impact parameter dipole model developed by Kowalski, Teaney, Motyka and Watt [2], [1], since it takes the effects of the proton shape into account in a complete way. The observed t-distributions in the vector meson and DVCS processes show clearly that the proton has a Gaussian-like shape in the transverse plane. The gluon density is high in the center and low on the outskirts of the proton. Usually, it is assumed that the evolution of the gluon density is independent of the proton shape. The investigations of Ref. [2] and [1] show, however, that the transverse variation of the gluon density has implications on the emerging pattern of QCD evolution and saturation effects. The interplay of saturation and evolution effects was first investigated by Bartels, Golec-Biernat and Kowalski [3], leaving out the effects of transverse density variation. In this case the  $F_2$  data can be described, as a function of x and  $Q^2$ , either by strong saturation and weak evolution or by strong evolution and weak saturation effects. The investigation of Ref. [1,2], which took into account also the proton shape in the transverse plane, concluded that only the second scenario is in agreement with data. The Gaussian form implies that a large contribution to the cross section has to come from the outskirts of the proton, where the gluon density is diluted. Hence, the evolution effects (DGLAP-like) are dominating the overall behaviour of the  $F_2$  data whereas the saturation effects are limited to the center of the proton. In the center, however, saturation effects are substantial.



Fig. 1: Total cross section for  $\gamma^* p \to V p$  for different vector mesons compared to predictions from the b-Sat model.

## 1.2 Description of the HERA data

Various cross sections measured as a function of  $Q^2$ , W and t can be described by a model with a minimal number of free parameters, namely the parameters  $\mu_0^2$ ,  $A_g$  and  $\lambda_g$  of the initial gluon distribution,  $xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1-x)^{5.6}$ , and the proton width  $B_G$ . The wave functions of the virtual photon are known from QED, while the vector meson wave functions are obtained by educated guesses. The long distance behaviour of the wave functions is also determined by the assumed values of the quark masses, see Ref. [1].

The observed cross sections are obtained from the overlap integral of the wave functions with the dipole cross section. The dipole cross section is assumed to be of the Glauber-Mueller form:

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c}r^2\alpha_S(\mu^2)xg(x,\mu^2)T(b)\right)\right].$$
(1)

Here, the scale  $\mu^2$  is related to the dipole size r by  $\mu^2 = 4/r^2 + \mu_0^2$ . The gluon density,  $xg(x, \mu^2)$ , is evolved from a scale  $\mu_0^2$  up to  $\mu^2$  using LO DGLAP evolution, b denotes here the impact parameter. T(b) is the proton shape. This assumption, together with the form of the dipole cross section defines the so called b-Sat model (called also KMW model on some plots below), described in detailed in Ref. [1]. Note, that in this type of model the dipole cross section determines the (unintegrated) gluon density [4]. The function  $xg(x, \mu^2)$  coincides with the gluon density when the argument of the G-M exponent is small. The analysis of data in terms of the b-Sat model shows that this is not always the case. If the argument of the G-M exponent is substantial the gluon density saturates, with a strong impact parameter dependence.

The model parameters, which were fixed by the fit to the total inclusive DIS cross section and the vector meson t-distributions, describe the measured  $Q^2$  and W dependence of vector meson production and DVCS very well, together with the absolute normalization, as shown in Figures 1 and 2. The data are compared to the results of the b-Sat model. Figures 3 shows the



Fig. 2: Total DVCS cross sections  $\sigma$  vs.  $Q^2$  (left) and  $\sigma$  vs. W (right) compared to predictions from the b-Sat model.



Fig. 3: The two left plots show the *t*-slope parameter  $B_D$  vs.  $(Q^2 + M_V^2)$ , where  $B_D$  is defined by fitting  $d\sigma/dt \propto \exp(-B_D|t|)$ , for the processes  $\gamma^* p \to J/\Psi p$  and  $\gamma^* p \to \phi p$ . The right plot shows the same same parameter, called here b, determined in the process  $\gamma^* p \to \rho p$ .



Fig. 4: The power  $\delta$  vs.  $(Q^2 + M_V^2)$  or  $Q^2$ , where  $\delta$  is defined by fitting  $\sigma \propto W^{\delta}$ , compared to predictions from the dipole models. The left two plots show the  $\gamma^* p \to J/\Psi p$  and  $\gamma^* p \to \phi p$  processes, the right one shows the  $\gamma^* p \to \rho p$  production.

t-slope parameter  $B_D$  vs.  $(Q^2 + M_V^2)$ . The plot for  $\rho$  mesons is taken from the recent ZEUS publication [5]. The parameter  $B_D$  is obtained by making a fit to the t-distributions of the form  $d\sigma/dt \propto \exp(-B_D|t|)$ . It has a physical interpretation as the size of the interaction region. For scattering of very small dipoles  $B_D$  is connected to the proton radius  $R_p$  via  $B_D = R_p^2/3$ . However, for larger dipoles the size of the interaction area depends not only on the proton radius but also on the size of the produced vector meson or real photon, which were taken into account following the work of Bartels, Golec-Biernat and Peters [4]. This allows the data for all vector mesons and DVCS to be described using a unique Gaussian proton shape, independent of the produced final state. The measured t-distributions agrees well with the model expectations. The slight underestimation of the parameter  $B_D$  by the model predictions for the  $\rho$  meson production.

Although the vector meson wave functions are just guessed, the observed distributions for the  $J/\Psi$  and  $\phi$  mesons are fairly insensitive to the particular assumptions and agrees well in all aspects with data. For the  $\rho$  meson the ratio  $\sigma_L/\sigma_T$  is not well described, see Ref. [5] for more details. This is presumably due to the lack of knowledge of the proper  $\rho$  meson wave function. A more precise measurement of the ratio  $\sigma_L/\sigma_T$  and of the spin density matrix elements would allow better constraints to be made on the form of the  $\rho$  wave function, as discussed in Ref. [1].

Figure 4 shows the power  $\delta$  compared to the predictions from the b-Sat model. This power is a measure of the rate of rise of the cross sections with increasing W or diminishing x. The overall behaviour of data is well described by the b-Sat model which uses the Glauber-Mueller dipole cross section with DGLAP evolution of the gluon density. The gluon density is obtained from a fit to  $F_2$ . This plot is an important test on the universality of the gluon density and its evolution. The rate of rises are expected to depend on the sizes of the vector mesons, smaller size corresponds to the larger rate of rise,  $\delta$ . The most precisely measured points, for  $J/\Psi$ photoproduction (the most left points with very small errors on the upper left plot), are very well reproduced by the dipole model. The  $\phi$  data are also well reproduced but are not very precise.



Fig. 5: The *t*-slope parameter  $B_D$  vs. W compared to predictions from the b-Sat and b-CGC models using the "boosted Gaussian" vector meson wave function.

In the  $\rho$  case we observe a small systematic deviation between data and model predictions which can be attributed to an underestimation of the size of  $\rho$  meson by the assumed wave function, as also seen, independently, in the plots of  $B_D$ , Fig. 3.

An important finding of this investigation is that the *t*-dependences of all three vector mesons and the DVCS process can be simultaneously described with one universal shape of the proton, see Figure 3. The parameter characterizing the size of the proton,  $B_G = 4 \text{ GeV}^{-2}$ , determined in this investigation, corresponds to the proton radius of  $R_p = \sqrt{3B_G} = 0.67$  fm. This is smaller than the proton charge radius of  $0.870 \pm 0.008$  fm [6]. This leads to a rather surprising result that gluons are more concentrated in the center of the proton than quarks.

The b-Sat model, which gives the best description of data, uses the Glauber–Mueller dipole cross section with DGLAP evolution of the gluon density. Although the overall description of exclusive processes is very good, this approach has limitations, seen most clearly in the lack of Wdependence of  $B_D$  in  $J/\psi$  photoproduction, see Fig.5. The measurement precision is sufficient to conclude that there is a coupling between the transverse and longitudinal evolution variables, that is,  $\alpha'_{\mathbb{P}} \neq 0$ . This indicates that DGLAP cannot be the only evolution schema in the low-xregion. Such a coupling is more natural in the BFKL evolution which is a basis of the CGC evolution. The effect can be described by the impact parameter dependent CGC model. The "b-CGC" model gives a better description of the  $\alpha'_{\mathbb{P}}$  effect but it provides a considerably poorer fit of  $F_2$  than the b-Sat model and a worse overall description of exclusive processes.

The strong enhancement of gluon bremsstrahlung at small x leads to the specific nature of universal small-x dynamics in QCD. As x decreases, the occupation number of a transverse momentum mode  $k_{\perp}$  in the hadron or nuclear wave-function grows rapidly. However, it can maximally be of order  $1/\alpha_s$  in QCD and it is saturated by the competing dynamics of bremsstrahlung and multi-parton recombination and screening contributions which deplete the gluon density at small x. In particular, the occupation number is maximal for modes with  $k_{\perp} \leq Q_s$ , where  $Q_s(x)$ , called the saturation scale, is a scale generated by the multi-parton dynamics. For a probe with transverse resolution  $1/Q^2$ , this scale is manifest in a universal scaling form of observables as a



Fig. 6: The saturation scale  $Q_S^2 \equiv 2/r_S^2$ , where  $r_S$  is the dipole size and the scattering amplitude has the value  $1 - \exp(-1/2) \simeq 0.4$ .

function of  $Q/Q_{\rm S}$  in a wide kinematical range in x and  $Q^2$ .

The saturation effects are best quantified by the value of the saturation scale  $Q_S^2 \equiv 2/r_S^2$ , where the saturation radius  $r_S$  is the dipole size and the scattering amplitude has the value  $1 - \exp(-1/2) \simeq 0.4$ . Figure 6 shows the saturation scale for the impact parameter dependent, b-Sat and b-CGC, models. The saturation scale is strongly dependent on the impact parameter b; in the center of the proton ( $b \approx 0$ ), the b-Sat and b-CGC models have a similar saturation scale, comparable to the value in the GBW model. As b increases the value of the saturation scale drops quickly in both models. This is understandable since, in the b-Sat model with a Gaussian proton shape, at larger values of b the gluon density is diluted by the factor T(b) and so the smaller gluon density leads to smaller saturation scales, as discussed in detail in [1].

Figure 7 shows the *b*-dependence of the total cross section, it gives a feeling for the relative contributions from the different impact parameters. The median value of this distribution is around  $b = 2.6 \text{ GeV}^{-1}$ , that is, the majority of the cross section is determined by the dilute gluon region, where the saturation scale is small.

The investigation presented here demonstrates that a wide class of high-energy scattering processes measured at HERA may be understood within a simple and unified framework. The key ingredient is the gluon density which is probed in the longitudinal and transverse directions. The success of the description indicates the universality of the emerging gluon distribution.

#### 2 Future possibilities

The vector meson and DVCS processes may be used to probe the properties of nuclear matter in a new way. As in the optical electromagnetic investigations, diffractive processes should allow the



Fig. 7: The *b*-dependence of the total cross section,  $\sigma_{\text{tot}}^{\gamma^* p}$ , for  $Q^2 = 0.4$ , 4 and 40 GeV<sup>2</sup> with  $x = 10^{-4}$ ,  $10^{-3}$  and  $10^{-2}$  respectively, using a Gaussian T(b) of width  $B_G = 4 \text{ GeV}^{-2}$ .

detailed investigation of target properties. In measurements with polarized beams it is possible to achieve precision which would allow a tomographic picture of protons and nuclei to be obtained. The prerequisite of this type of measurement are detectors controlling particle production in the whole rapidity range [7].

The new facilities recently proposed are the ep and eA collider EIC and LHeC. EIC should have roughly a half of the HERA center-of-mass energy and a luminosity of a factor 100 to 1000 higher than HERA. The high luminosity should allow to improve substantially the measurement precision for low-x and diffractive processes. This would allow to reduce the errors on the measurement of the rate of rise of the exclusive diffractive process, shown in Fig. 4, by a large factor. This would allow to determine precisely the gluon density evolution and saturation effects also in the non-forward region,  $t \neq 0$ .

An important advantage of EIC is the possibility of electron-ion collisions. Because of the Lorentz contraction of the nuclear parton density, in the probe rest frame, the saturation scale  $Q_{\rm S}$  has a strong A dependence,  $\sim A^{1/3}$ . In addition, since the density profile in a nucleus is more uniform than that of the proton, the saturation scale in nuclei decreases more slowly with b than in the proton. The dependence of the saturation scale on the impact parameter is plotted in Fig. 8. The saturation scale in Au nuclei at the median impact parameter for the total cross section  $b_{\rm med.}$  is about 70% of the value at b = 0; in contrast, for proton,  $Q_{\rm s,p}(b_{\rm med.})$  is only  $\sim 35\%$  of the value at b = 0. Thus while the saturation scale in the center of the proton is a third of that in the center of the gold nucleus, the saturation scale at the median impact parameter is about a factor of six smaller. This leads to a significant enhancement of the saturation effects in nuclei as discussed in details in ref. [8].

The LHeC electron proton/ion collider would achieve a CMS energy of a factor 5 higher than HERA and can extend the x range by about a factor 25 over HERA. The largely extended x-range should give a new insight into QCD evolution dynamics. It is generally accepted that the evolution dynamics seen in HERA data is of the DGLAP type. The investigation of Ref. [1,2] show that this is mainly due to a large contribution of the outskirts of the proton, in which gluon density is fairly diluted. The observed Gaussian shape of the proton implies a high, saturated



Fig. 8: Impact parameter dependence of the saturation scale for p, Ca and Au. The values are scaled by the median impact parameter  $b_{\text{med}}$  for the total cross section at x = 0.001 and  $Q^2 = 1 \text{ GeV}^2$ .

gluon density in the center of the proton, irrespective of the particular assumption about the evolution dynamics. Therefore, the evolution dynamics depends on the impact parameter, and  $F_2$  is determined by a mixture of different contributions. The vector meson diffractive measurement at LHeC (and also EIC) should have a potential to disentangle it. The main tools should be the dedicated detectors allowing the measurement of the *t*-dependence in the largely extended *x* range. A better understanding of the evolution is a fascinating theoretical and experimental problem. Somewhere in the mixture of the different evolution schema should be a region dominated by the properties determined by the pomeron-graviton correspondence [9], [10]. The largely extended *x* range of LHeC could be of crucial importance for this task.

Finally, let us note that in the pp scattering at LHC it is possible to measure processes with very small x values. In exclusive diffractive production of charmed or bottom mesons it is possible to reach x values which are three to four orders of magnitude lower than at HERA.

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