GBP Main Meeting

Ptarmigan LMA Simulations

K Fleck - 21/12/2022

LMA Simulations
Electron pencil beam

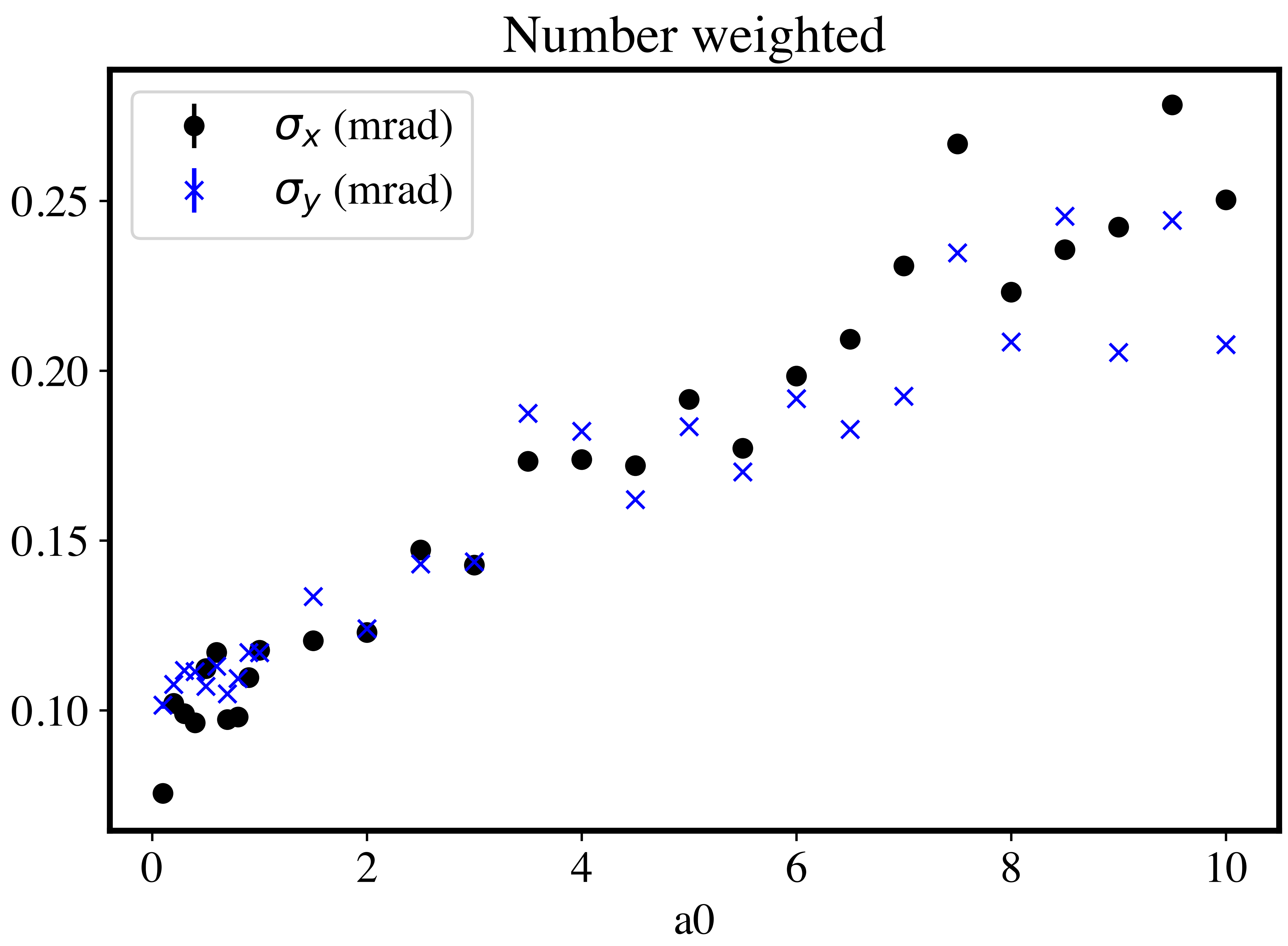
- Pencil electron beam
  - 0.1 μm uniformly distributed in radius (flat-top)
  - 0.1 μm in length along propagation axis
  - 240 pC
  - 16.5 GeV with no energy spread
  - zero divergence
- Laser beam
  - 25.0 μm waist
  - 800 nm wavelength
  - 100 fs FWHM duration
  - linear polarisation
  - zero crossing angle with electron beam
- Physics
  - Rates calculated using LMA
  - Radiation reaction and pair production turned off

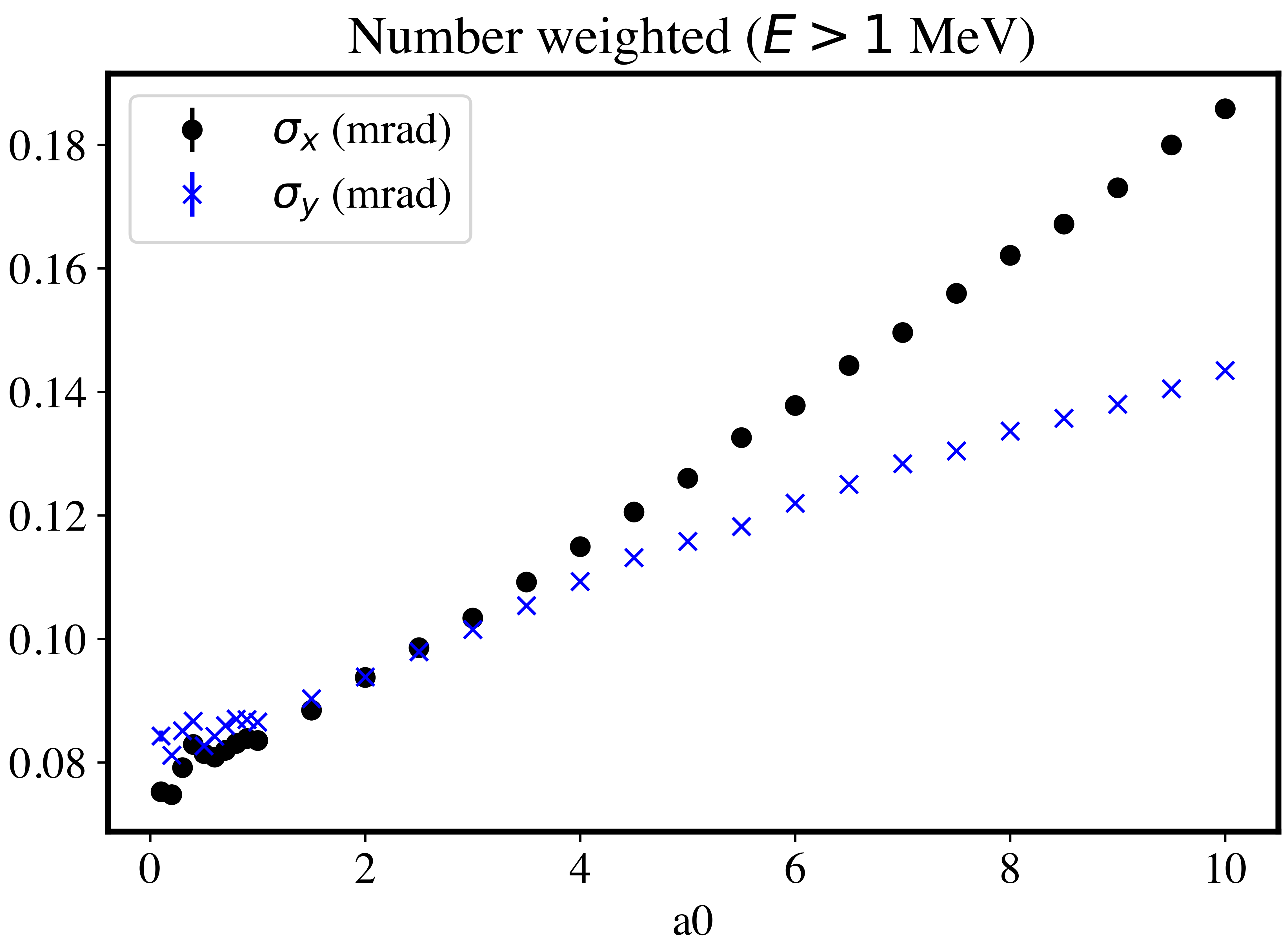
LMA Simulations

Ptarmigan angular profiles - long pulse (100 fs)

LMA Simulations

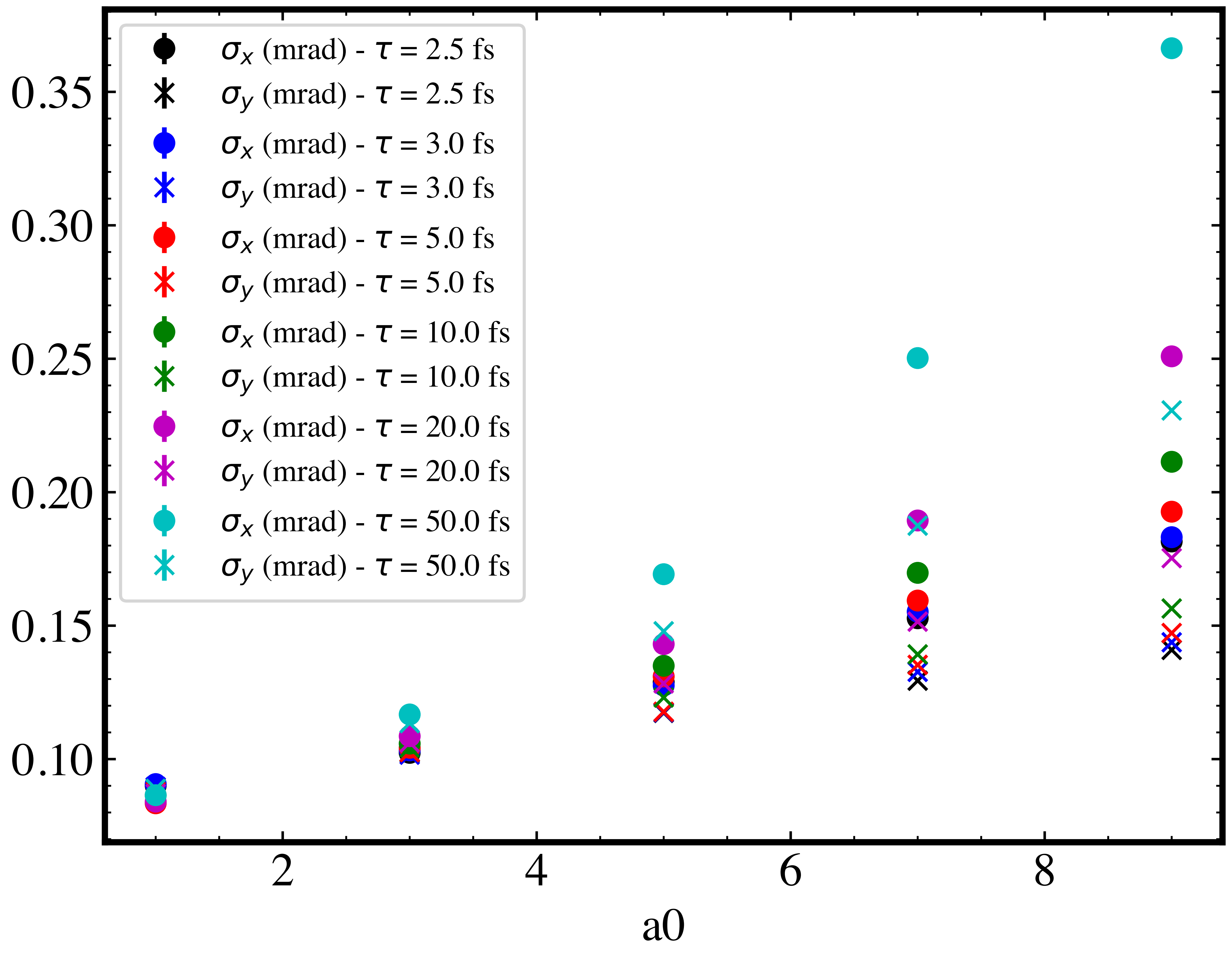
Number weighted width parameter

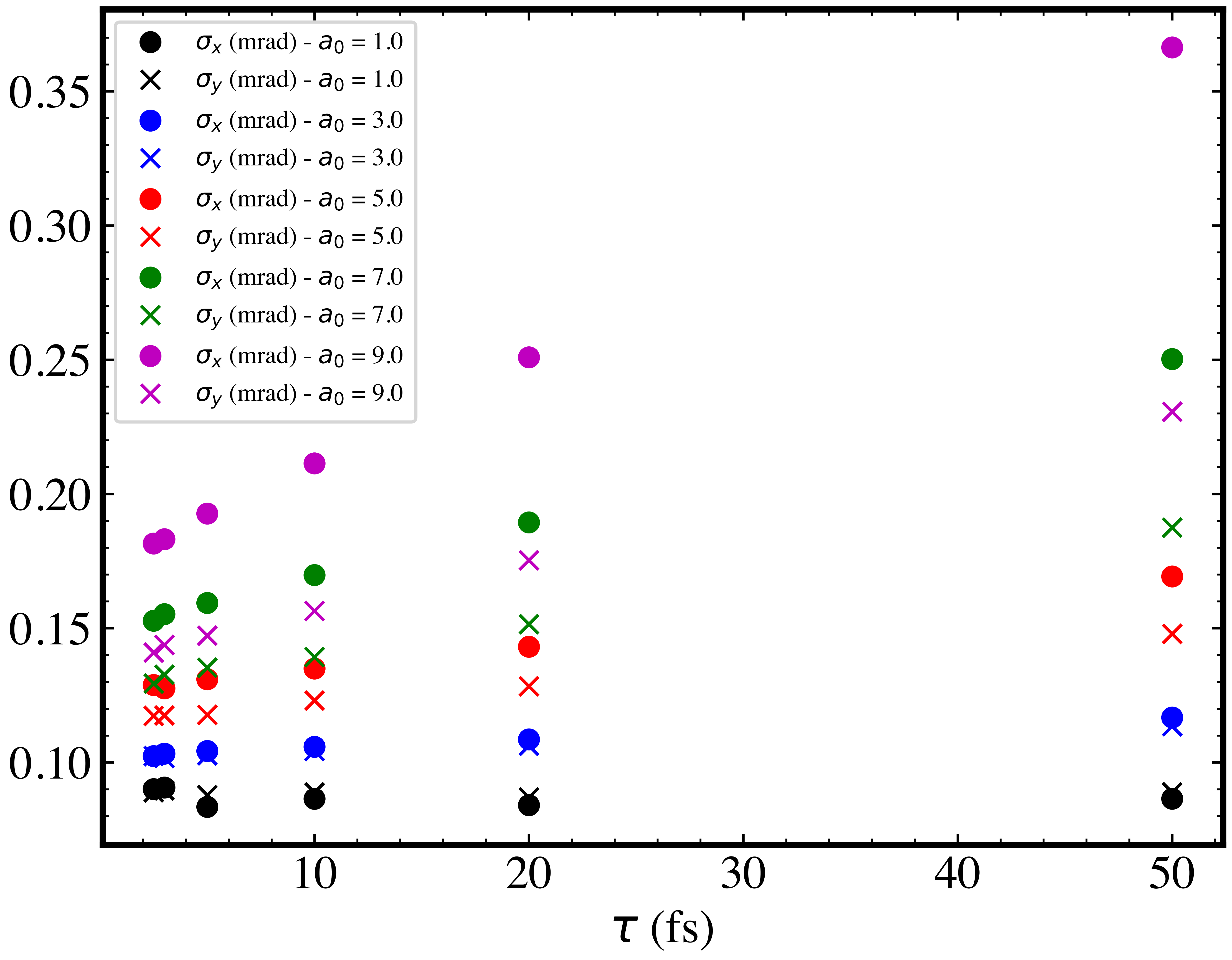




LMA Simulations

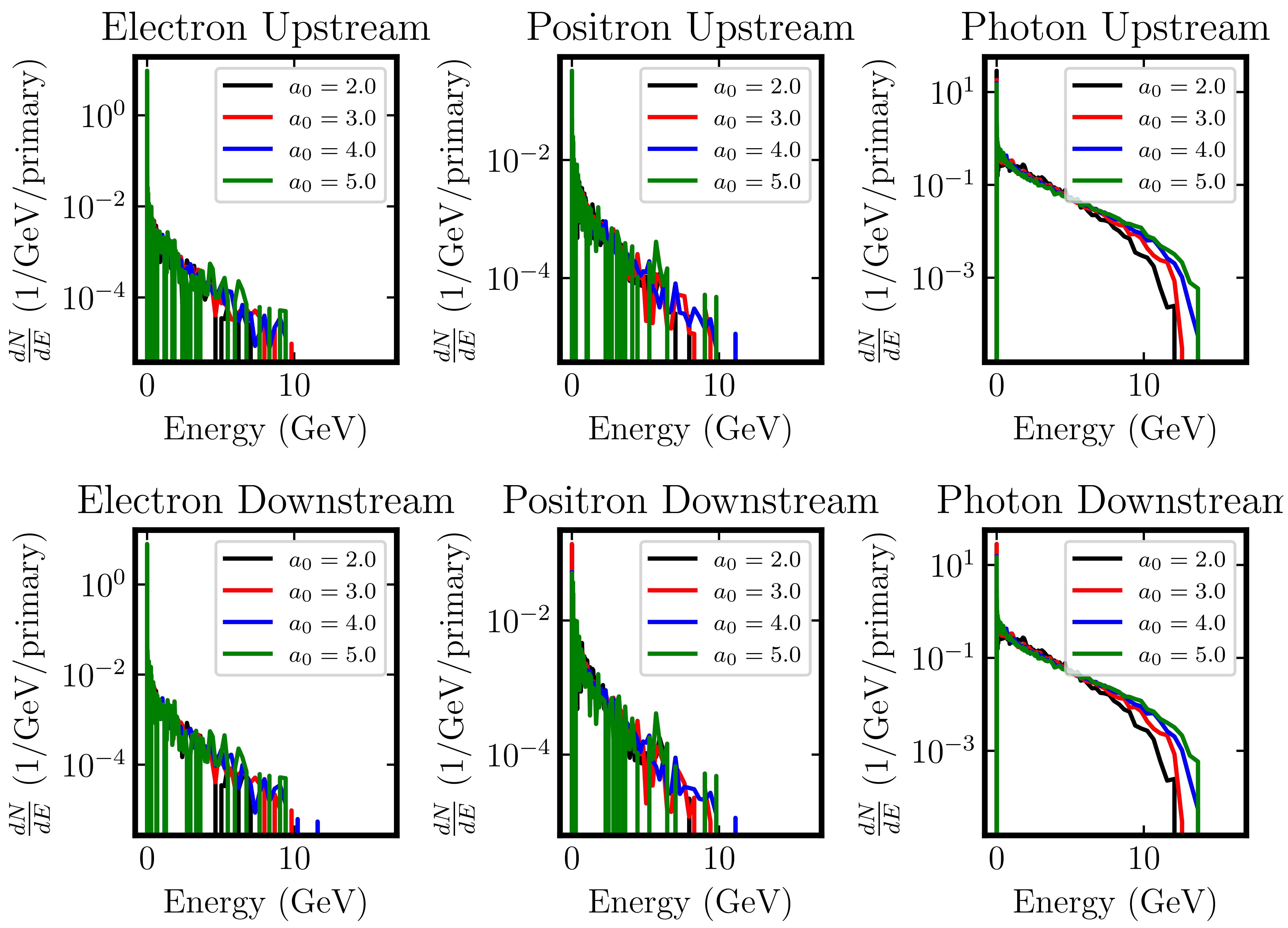
Pulse duration effects





LMA Simulations

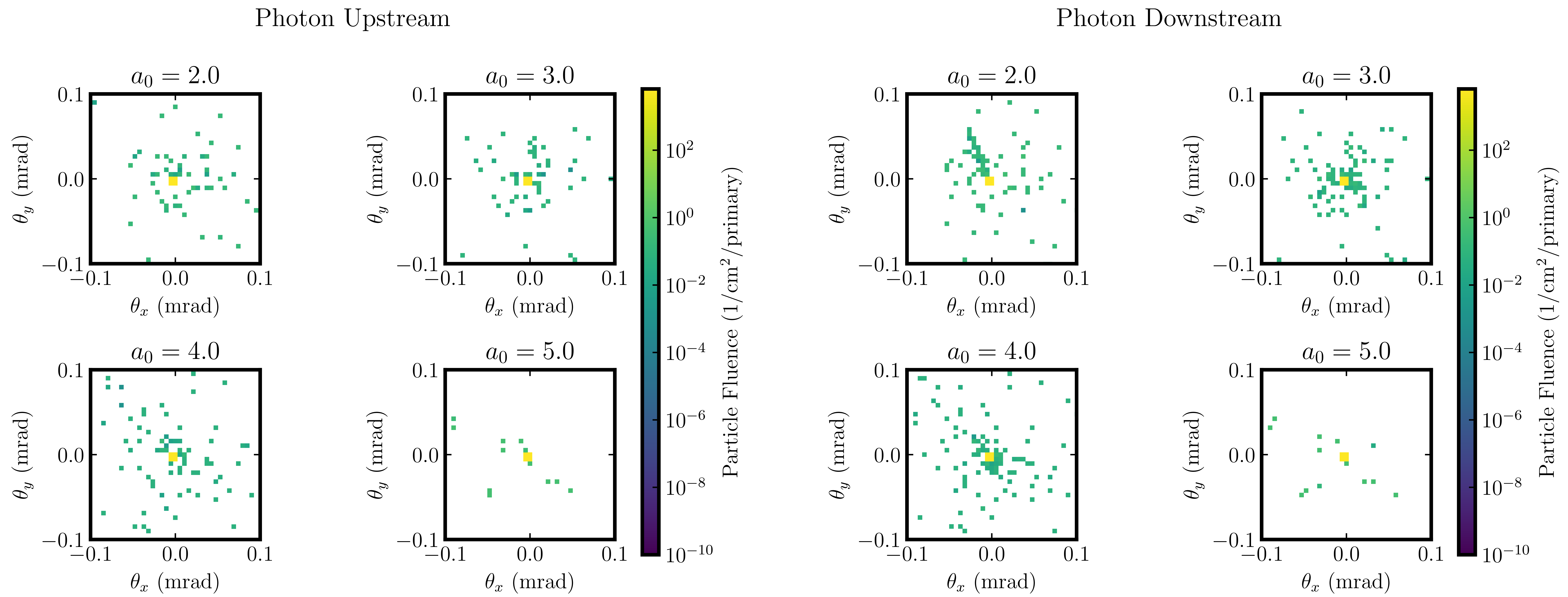
Full scale LUXE sims in FLUKA



- First pass at running LMA data through full scale FLUKA simulation
- Needs more primaries to be able to compare angular distributions
- Geant4 can provide more detailed information than FLUKA

LMA Simulations

Full scale sims in FLUKA



LMA Simulations
Behaviour for low intensities

- For $a_0 < 1$, radiation emission is expected to be semiclassical
- From Jackson, spectral irradiation is given by $$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt\, \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{\beta}\cdot\vec{n})^2} \mathrm{e}^{i\omega(t-\vec{n}\cdot\vec{\mathrm{x}}/c)}\right|^2$$
- Energy radiated is given by $I = N\omega$, the angular distribution is then $\dfrac{dN}{d\Omega} = \displaystyle\int d\omega\, \frac{1}{\omega}\frac{\partial^2 I}{\partial\omega\partial\Omega}$
- Velocity found by solving the Lorentz equation (or Landau-Lifschitz to include RR)

$$\dot{\pi}_\mu = \frac{e}{m}\mathscr{F}_{\mu\nu}\pi^\nu$$

- Alternatively, solve Hamilton-Jacobi equation $(\partial S + e\mathscr{A})^2 = m^2$ for the action S. The kinetic momentum is then

$$\pi_\mu = -\partial_\mu S - e\mathscr{A} = p_\mu - ma_\mu + \left\{\frac{2m(p\cdot a) - e^2m^2a^2}{2(k\cdot p)}\right\}k_\mu$$

- For linearly polarised plane wave, assume $a^\mu(\varphi) = \xi\varepsilon^\mu\cos(\varphi)$ where $\varphi = k\cdot x = \omega x_-$ and $\varepsilon^\mu = (0,1,0,0)$ or $(0,0,1,0)$

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Ptarmigan LMA Simulations

K Fleck - 02/11/2022

Ptarmigan LMA simulations

Simulation parameters

- Laser
    - $\lambda = 0.8$ μm
    - $\tau_{FWHM} = 30$ fs
    - $E_L = 1.2$ J
    - Linear polarisation
- Electrons
    - $1.5 \times 10^9 \, e^-$
    - 16.5 GeV
    - $\Theta_{rms} = 8.672$ μrad
    - $r_b = 5.0$ μm

Radiation reaction = on, pair production = off

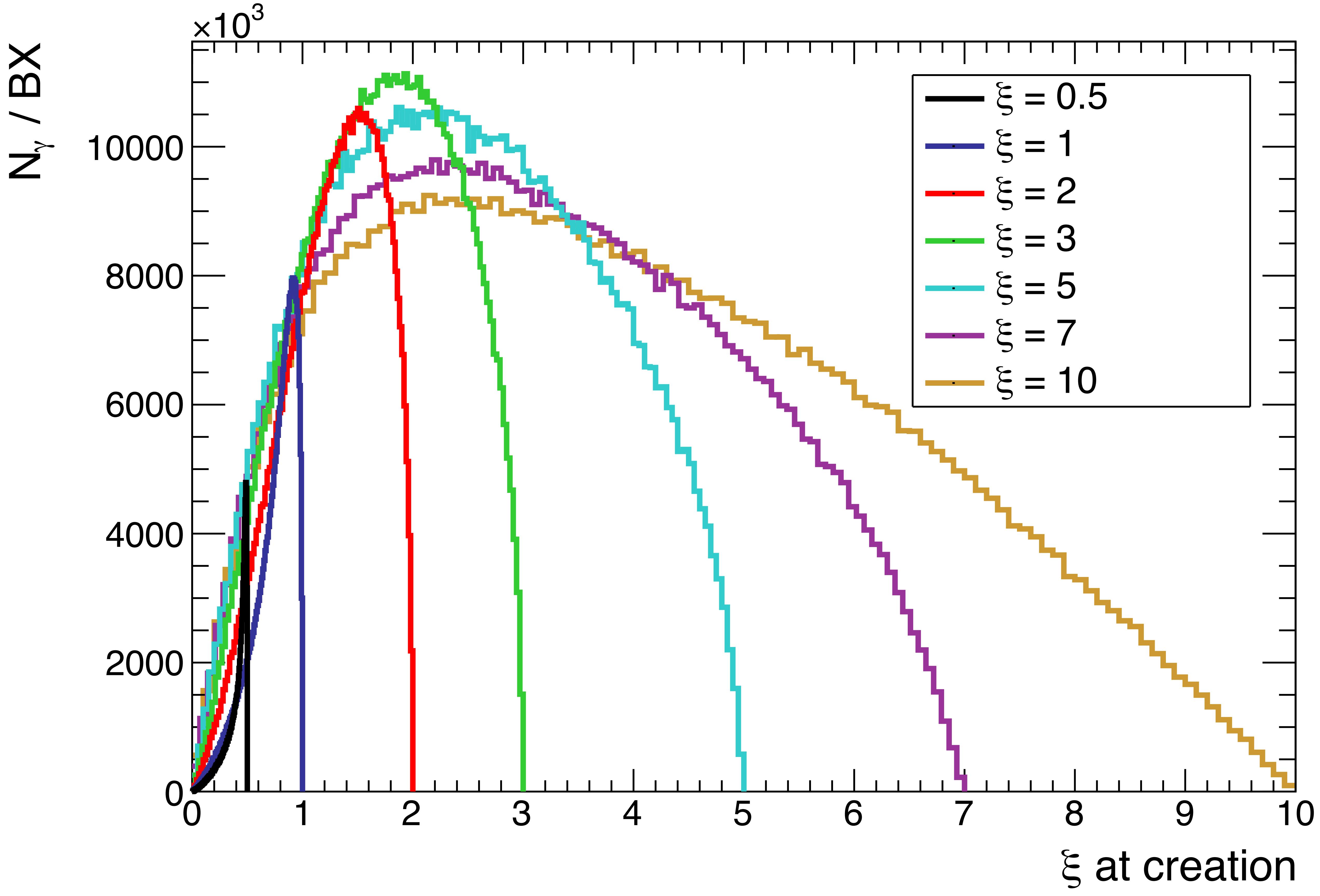
Ptarmigan LMA simulations

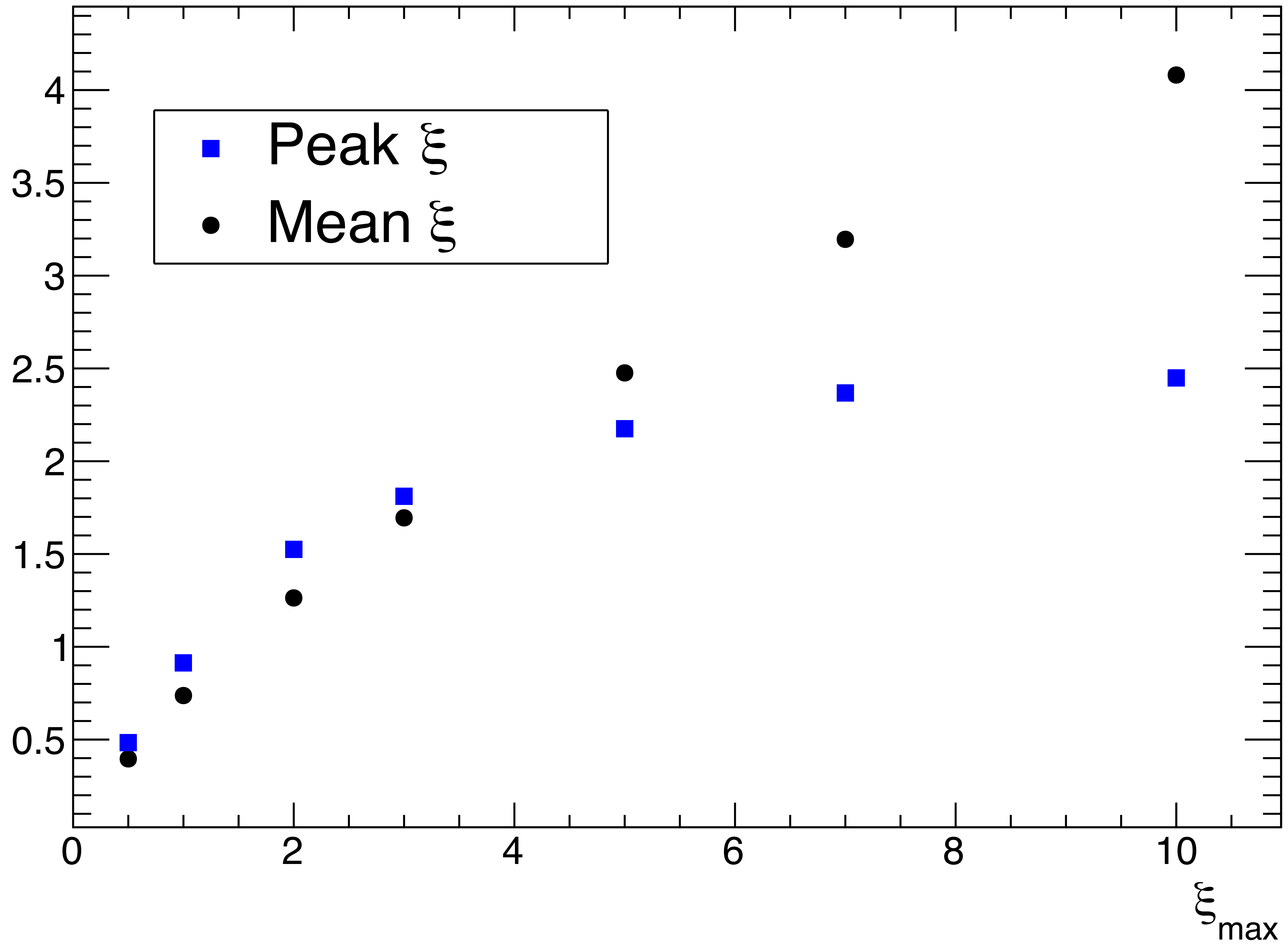
Simulation processing

Nominal ξ	Number of entries processed (1e8)	Number of entries in 10 BX (1e8)	Number of BXs processed
0.5	9.22	9.22	10.0
1.0	28.91	28.91	10.0
2.0	15.35	62.23	2.47
3.0	14.79	75.99	1.95
5.0	5.87	76.01	0.77
7.0	14.05	68.13	2.06
10.0	15.35	58.31	2.63

Ptarmigan LMA simulations

Laser intensity at creation

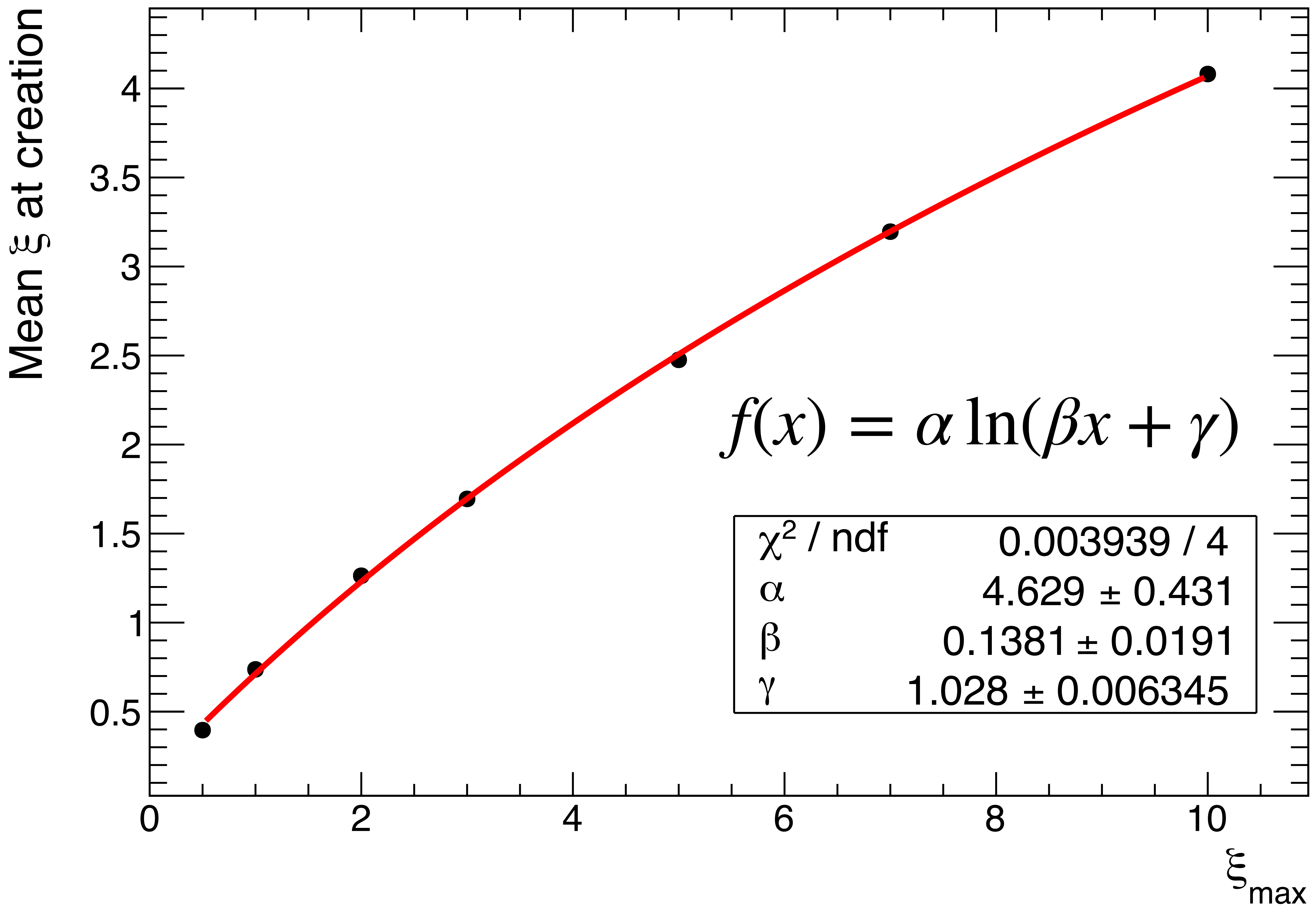


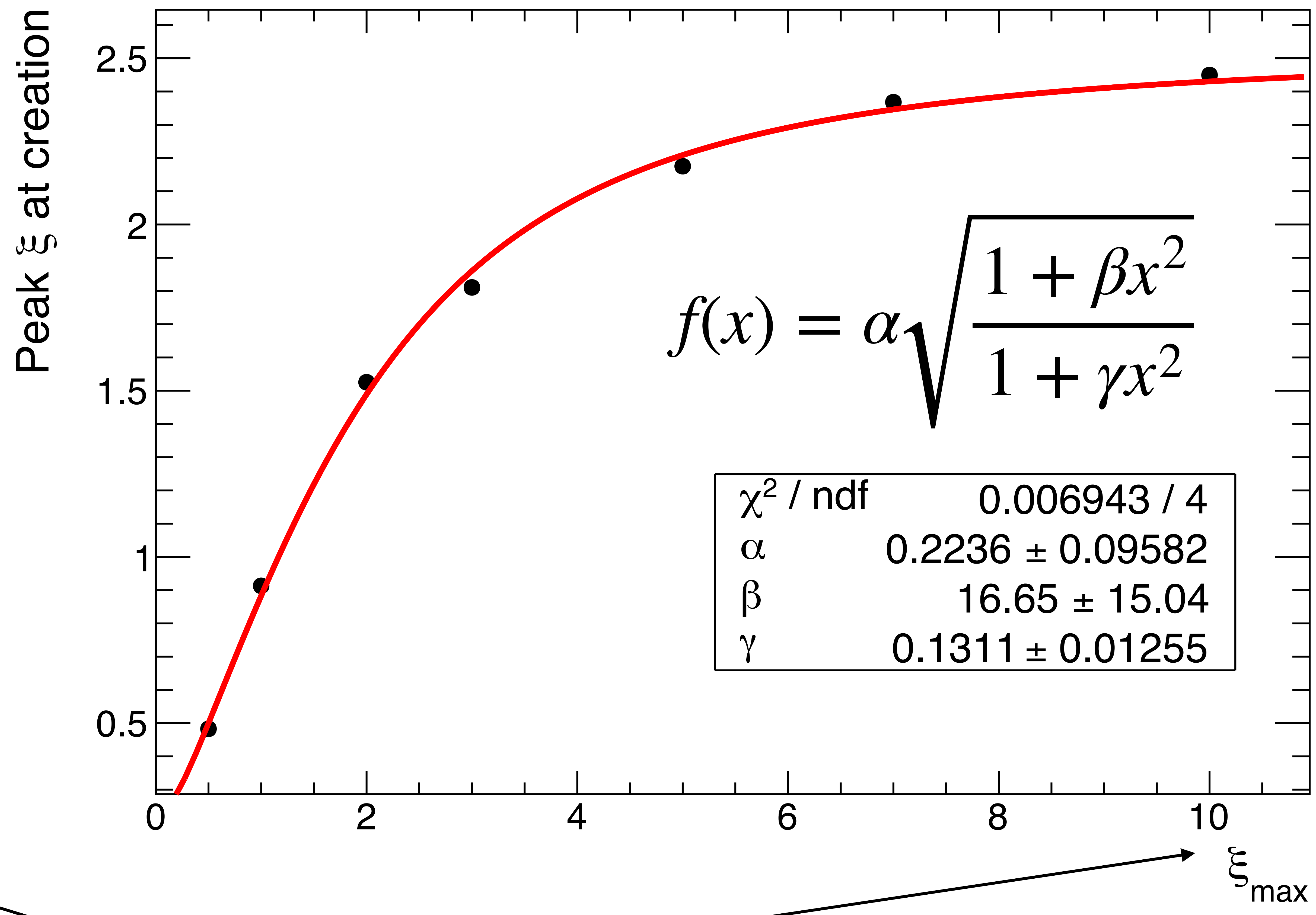


Ptarmigan LMA simulations

Laser intensity at creation

Functional form of fit taken from Blackburn et. al. 2020

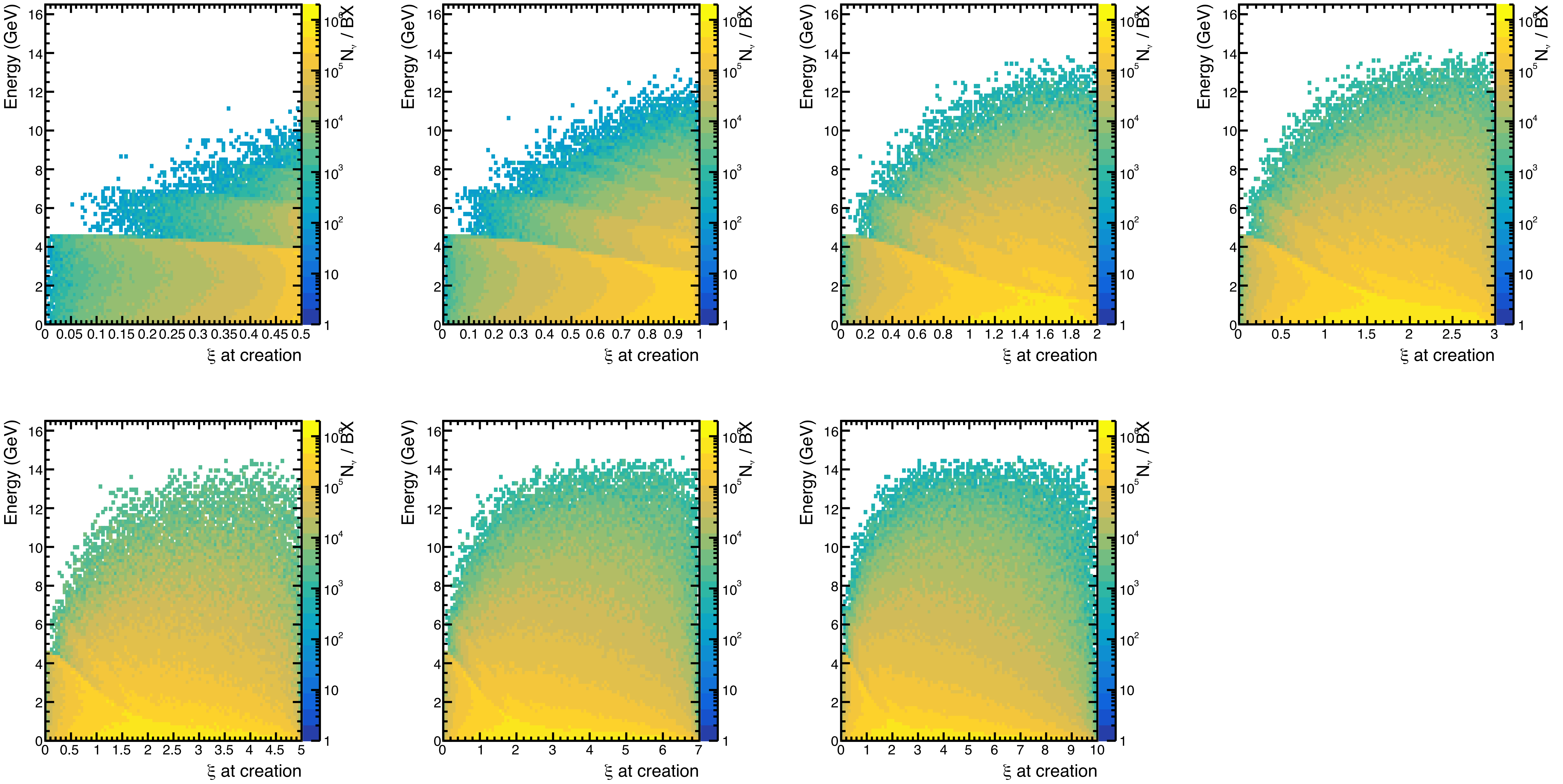




Same as nominal ξ

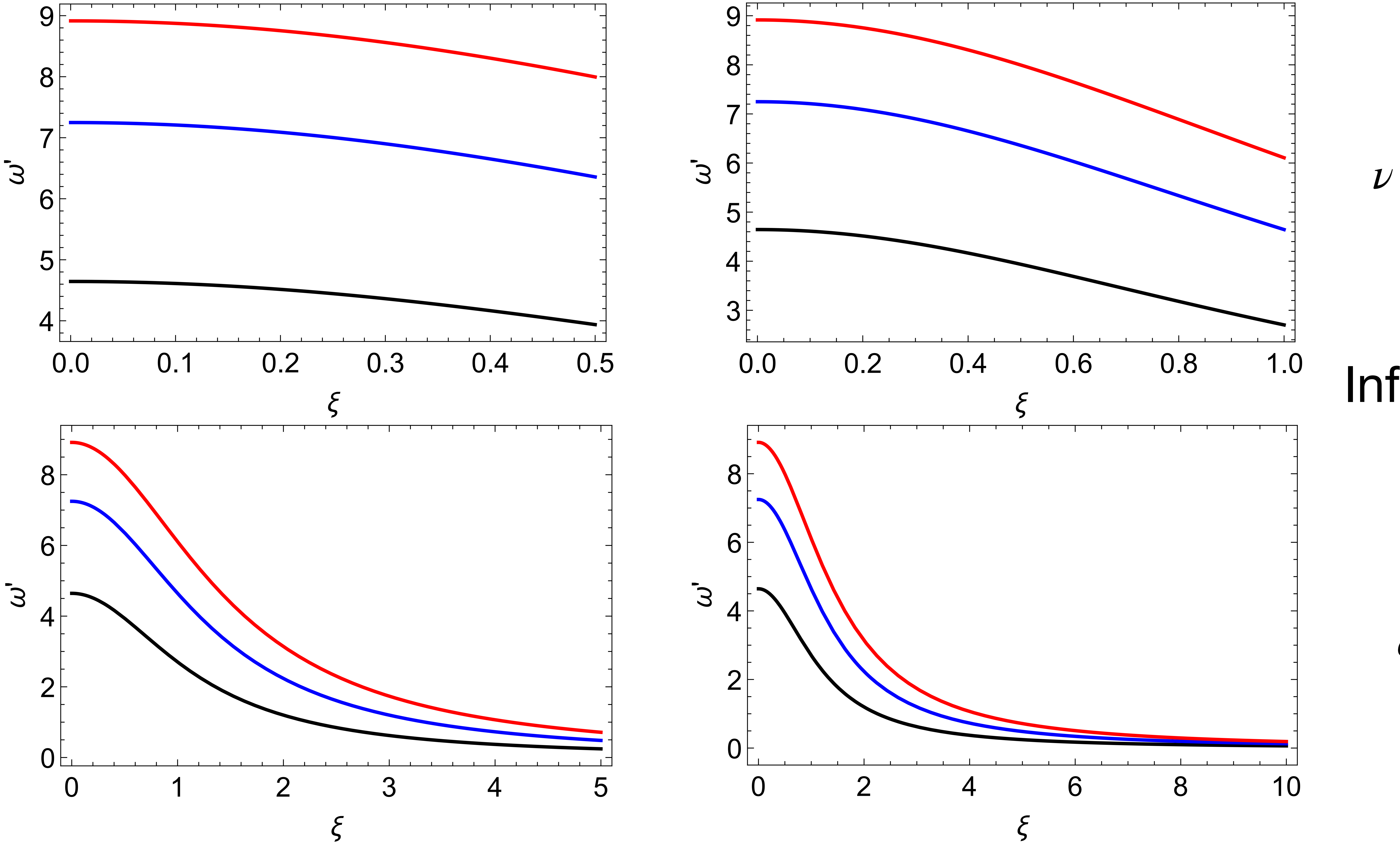
Ptarmigan LMA simulations

Laser intensity at creation with energy



Ptarmigan LMA simulations

Compton harmonics for IPWs



General case

$$\omega' = \frac{\nu k \cdot p}{(p + \nu k) \cdot n'}$$

$$\nu = \frac{k' \cdot p}{k \cdot p'} = \frac{k'^{-} + p'^{-} - p^{-}}{k^{-}}$$

Infinite plane wave case

$$\nu \to \nu_n = n - \frac{\xi^2}{4\eta} \frac{s}{1 - s}$$

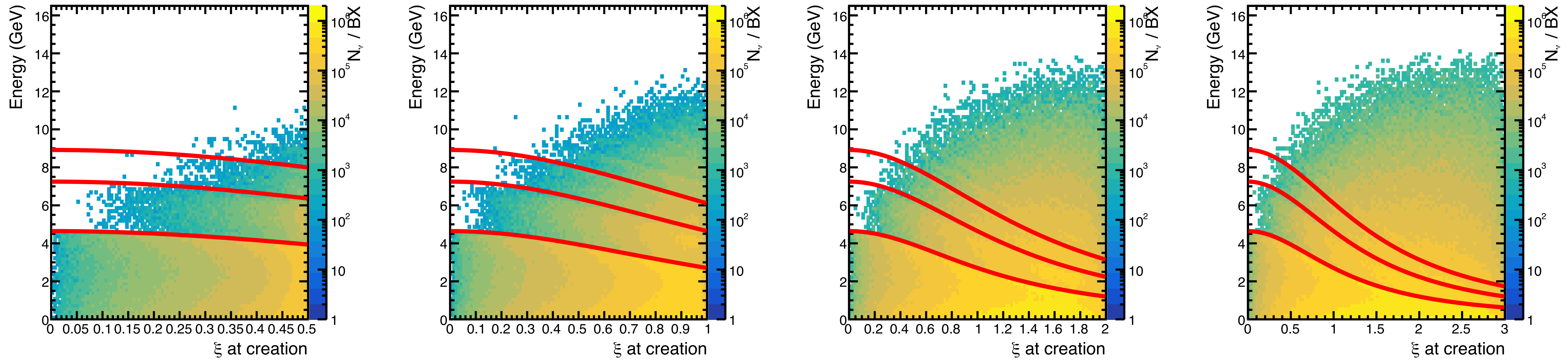
$$\omega'(n) = \frac{n \omega e^{2\zeta}}{1 + 2n \frac{\omega}{m} e^{\zeta} + \xi^2}$$

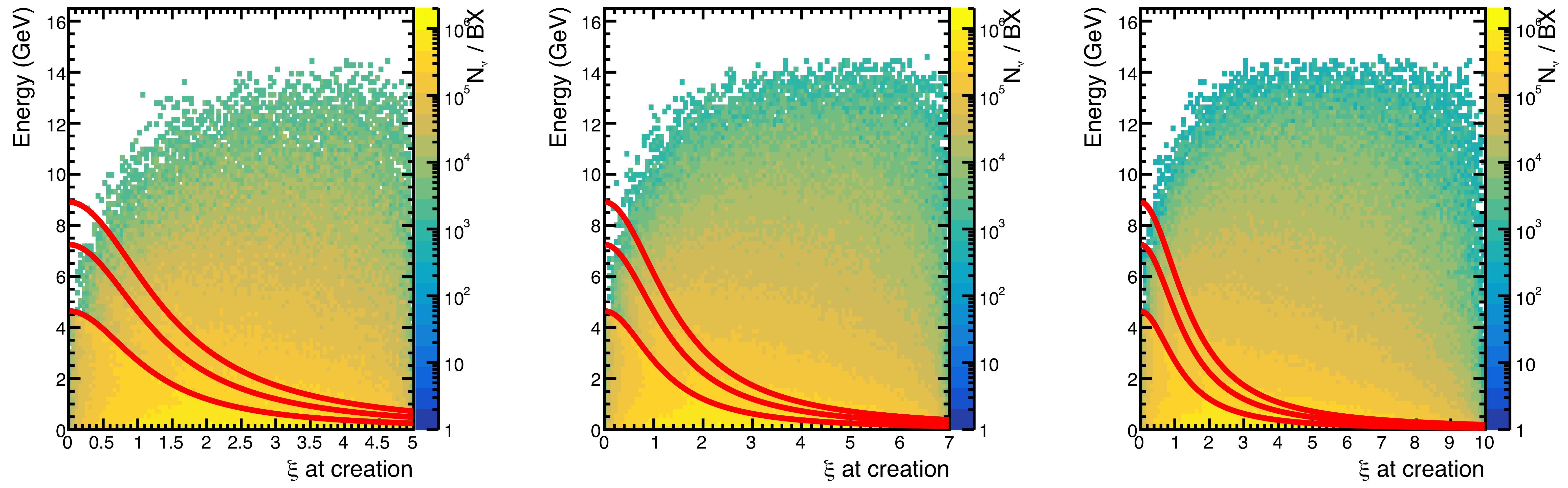
$$\zeta = \operatorname{arccosh} \gamma$$

Ptarmigan LMA simulations

Laser intensity at creation with energy with harmonics

Same plots as slide 6 but with first, second and third IPW harmonics overlayed

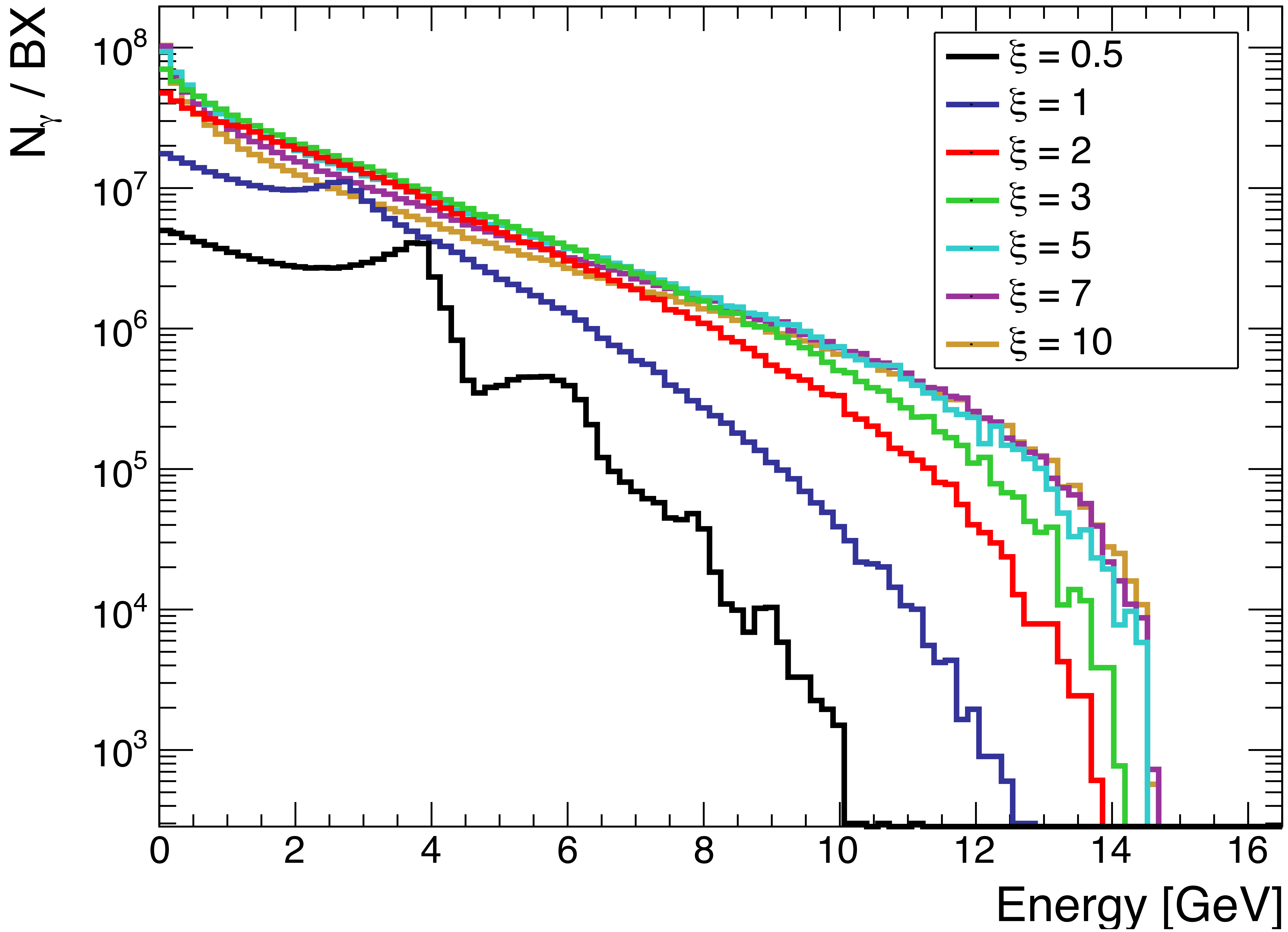




Broadening of harmonics in a focused beam is due to electrons seeing a range of ξ values

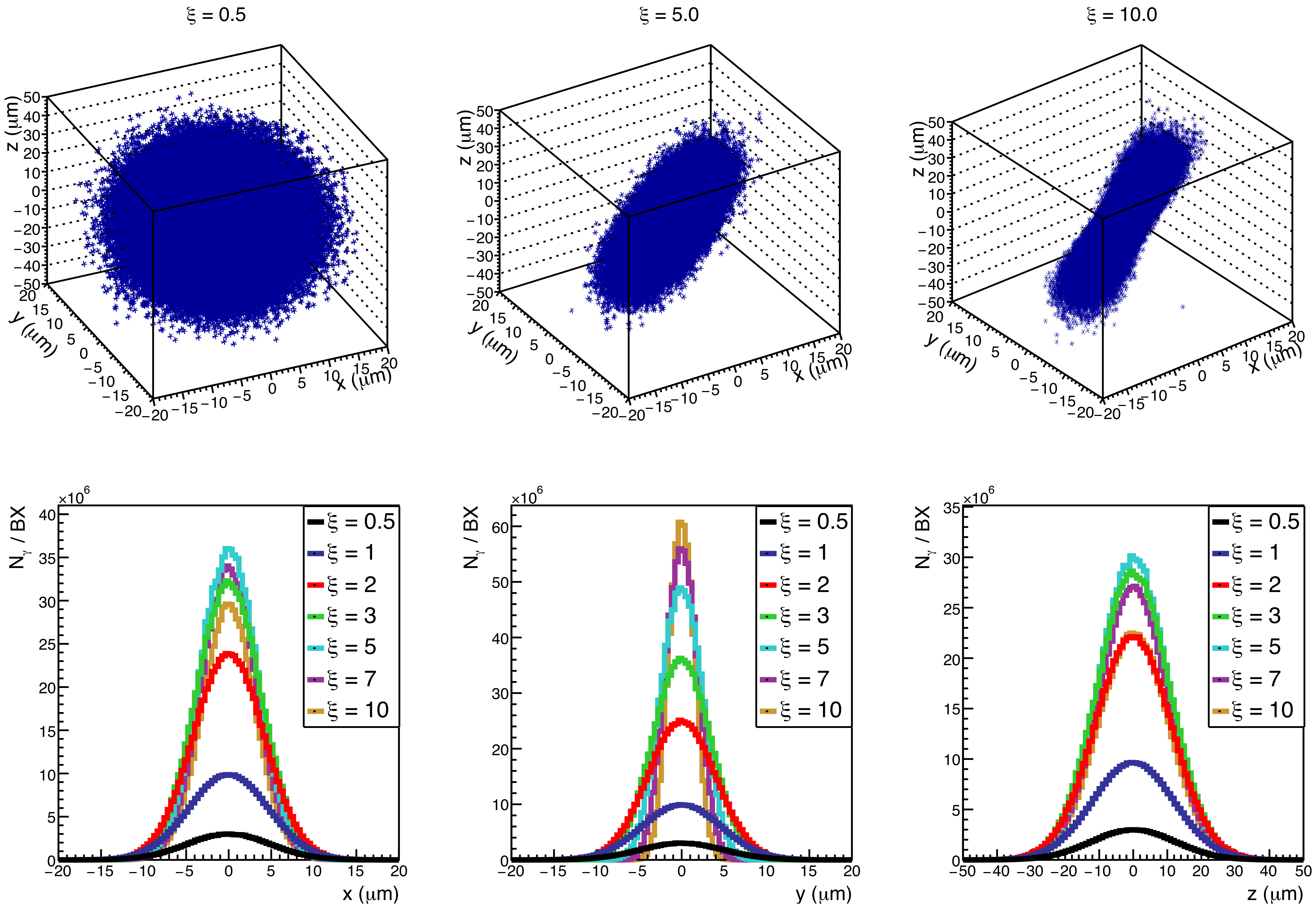
Ptarmigan LMA simulations

Energy spectra



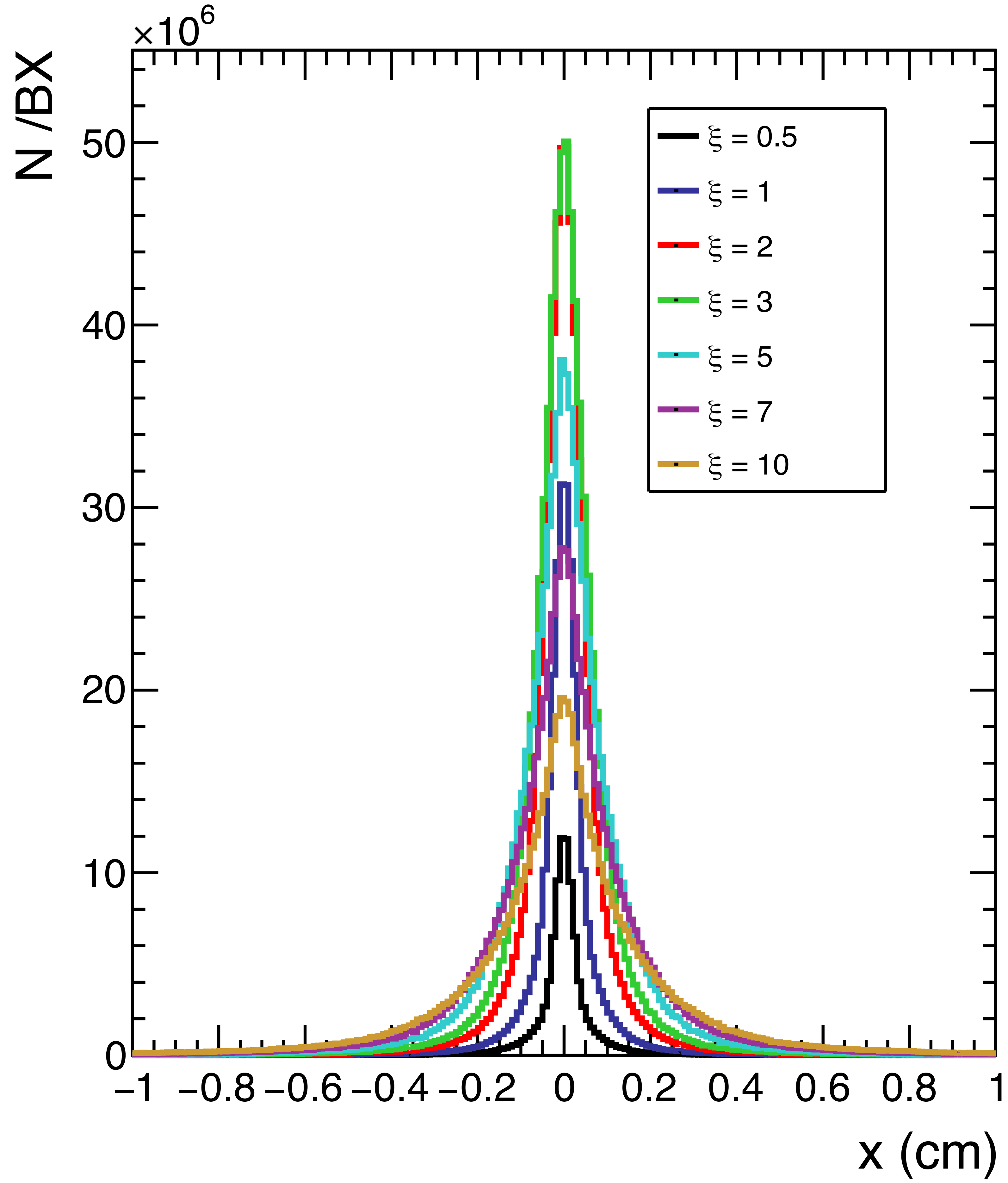
Ptarmigan LMA simulations

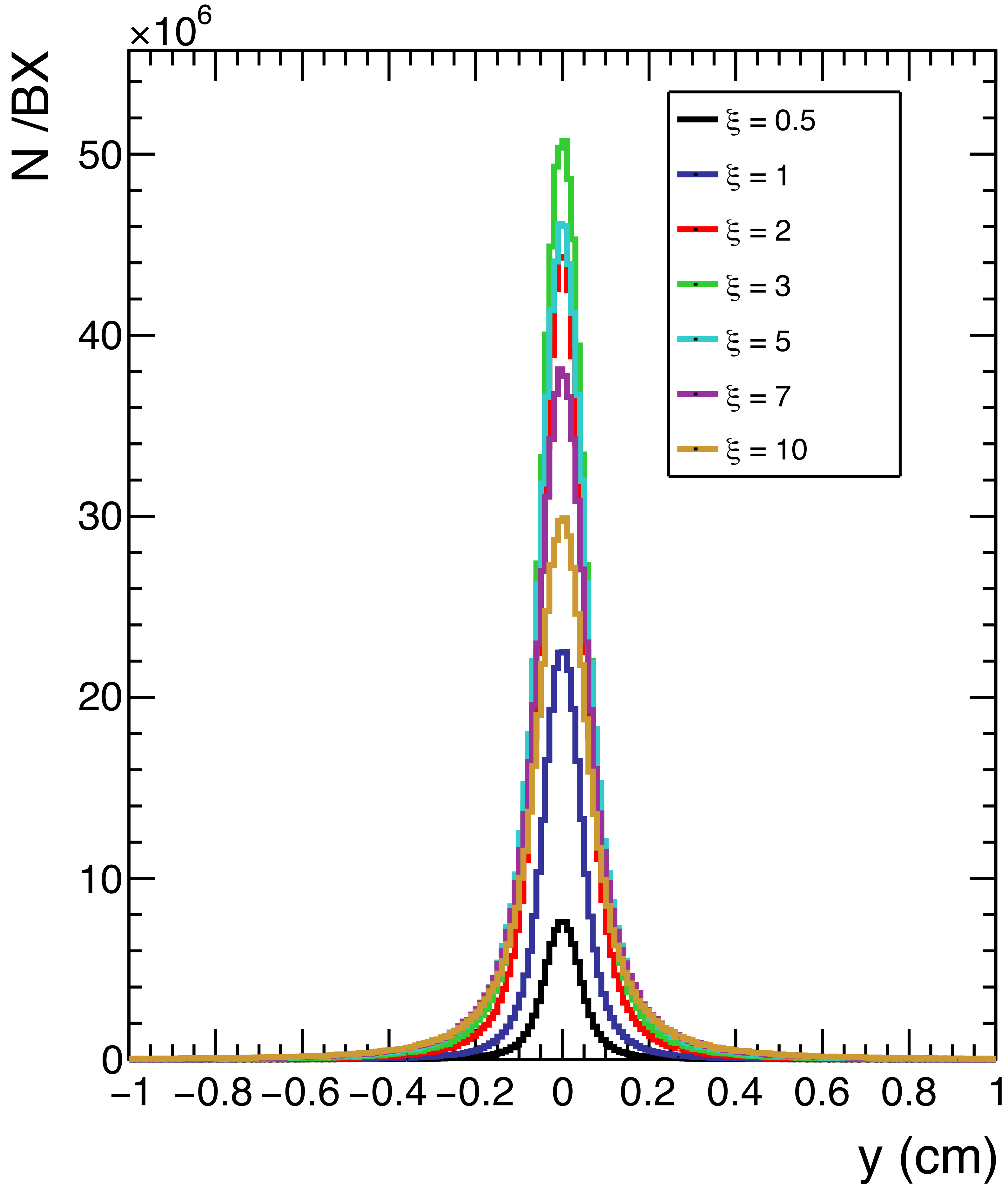
Spatial distribution at creation



Ptarmigan LMA simulations

Spatial distribution at profiler

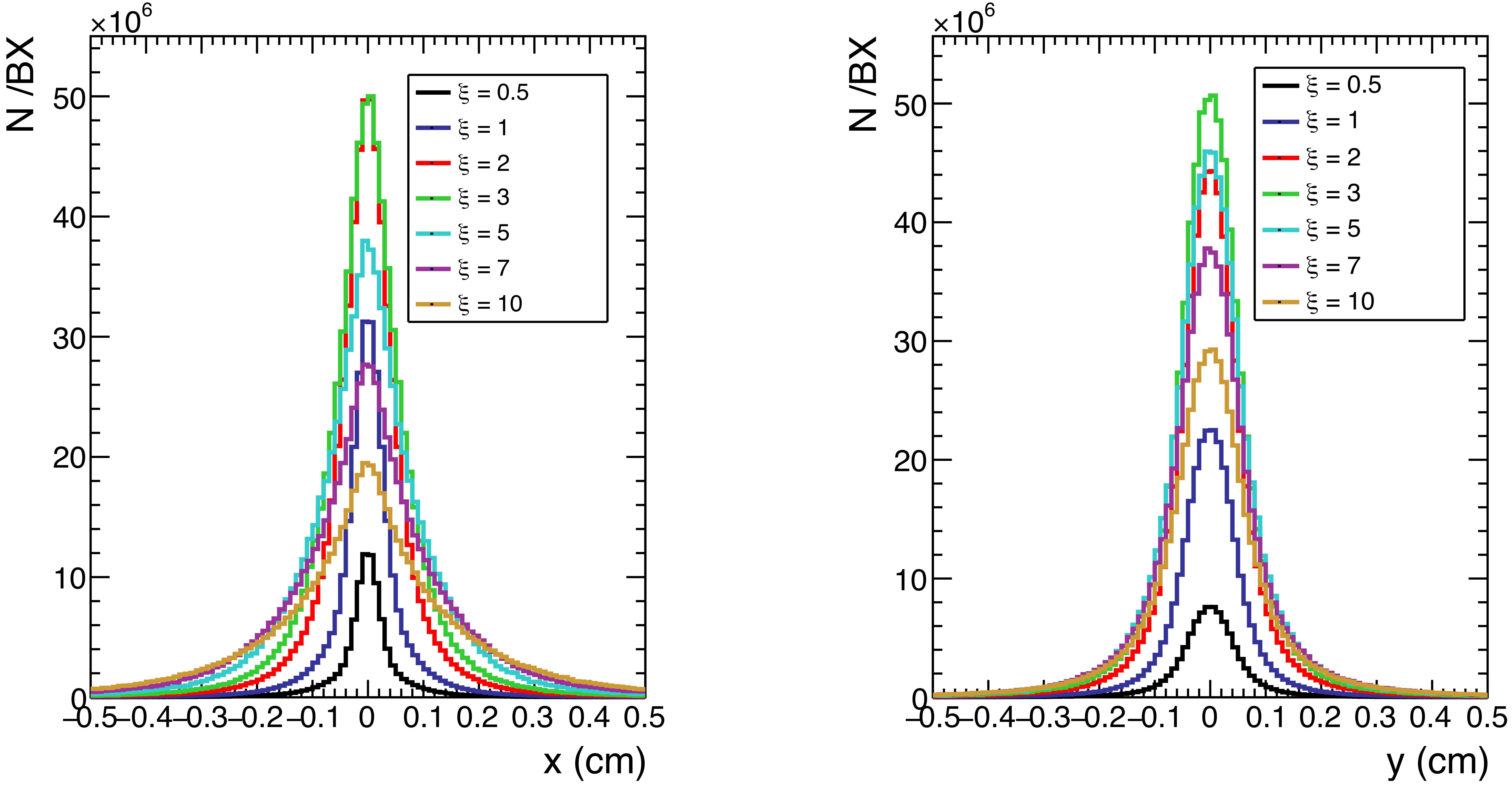




Projection taken at z = 11.5 m downstream of IP

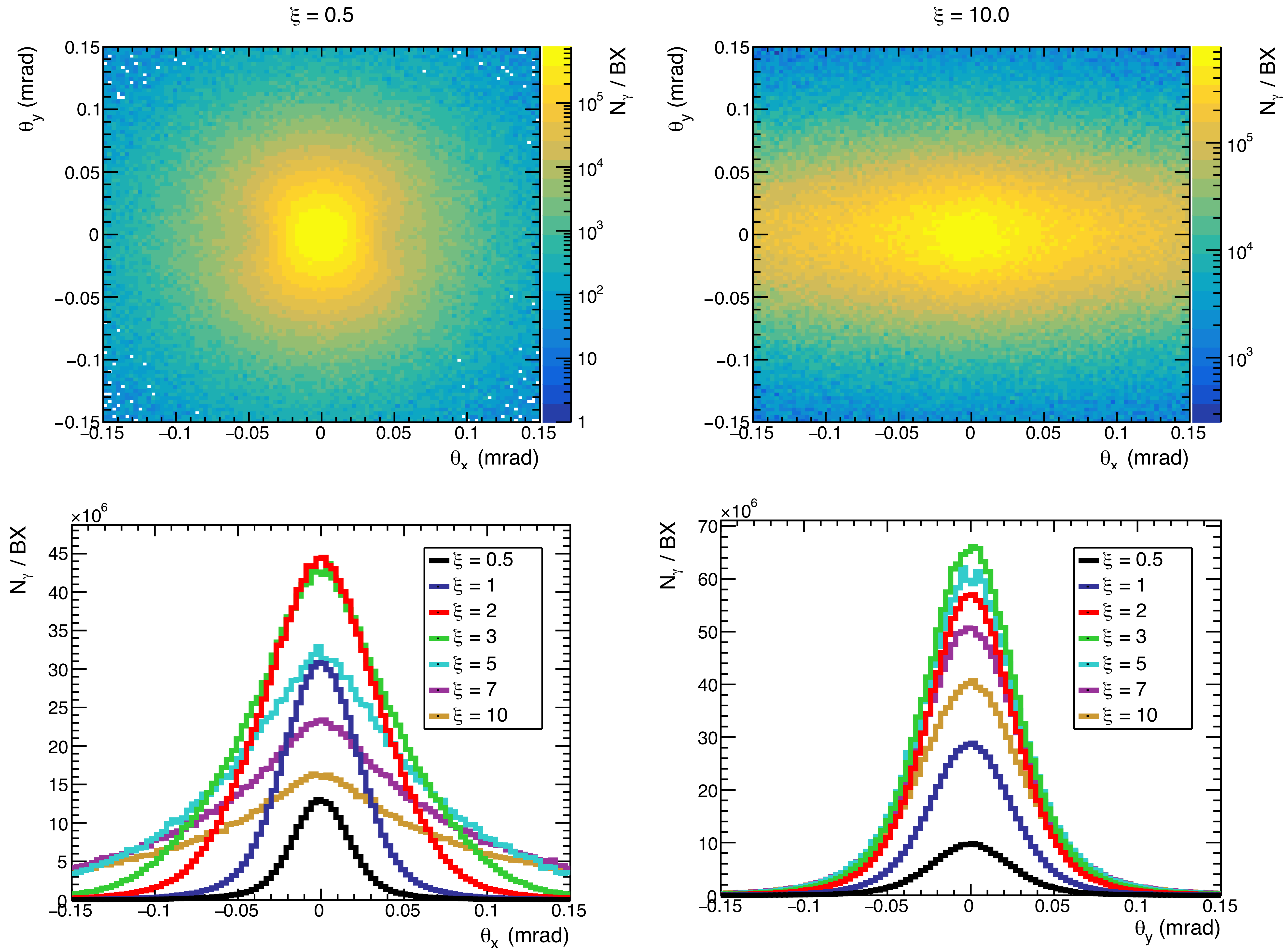
Ptarmigan LMA simulations

Spatial distribution at profiler (zoomed)



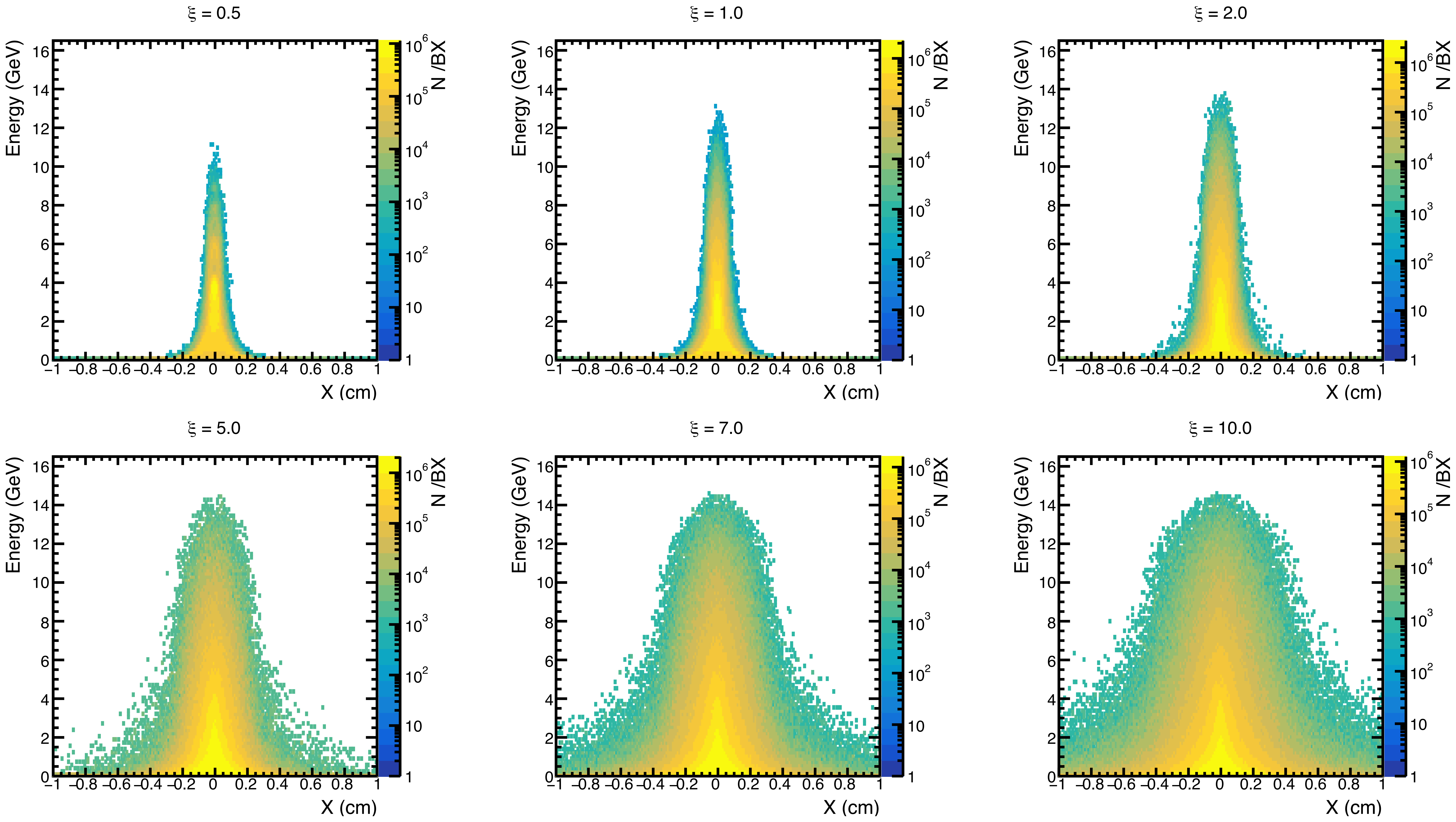
Ptarmigan LMA simulations

Energy weighted radiation profile



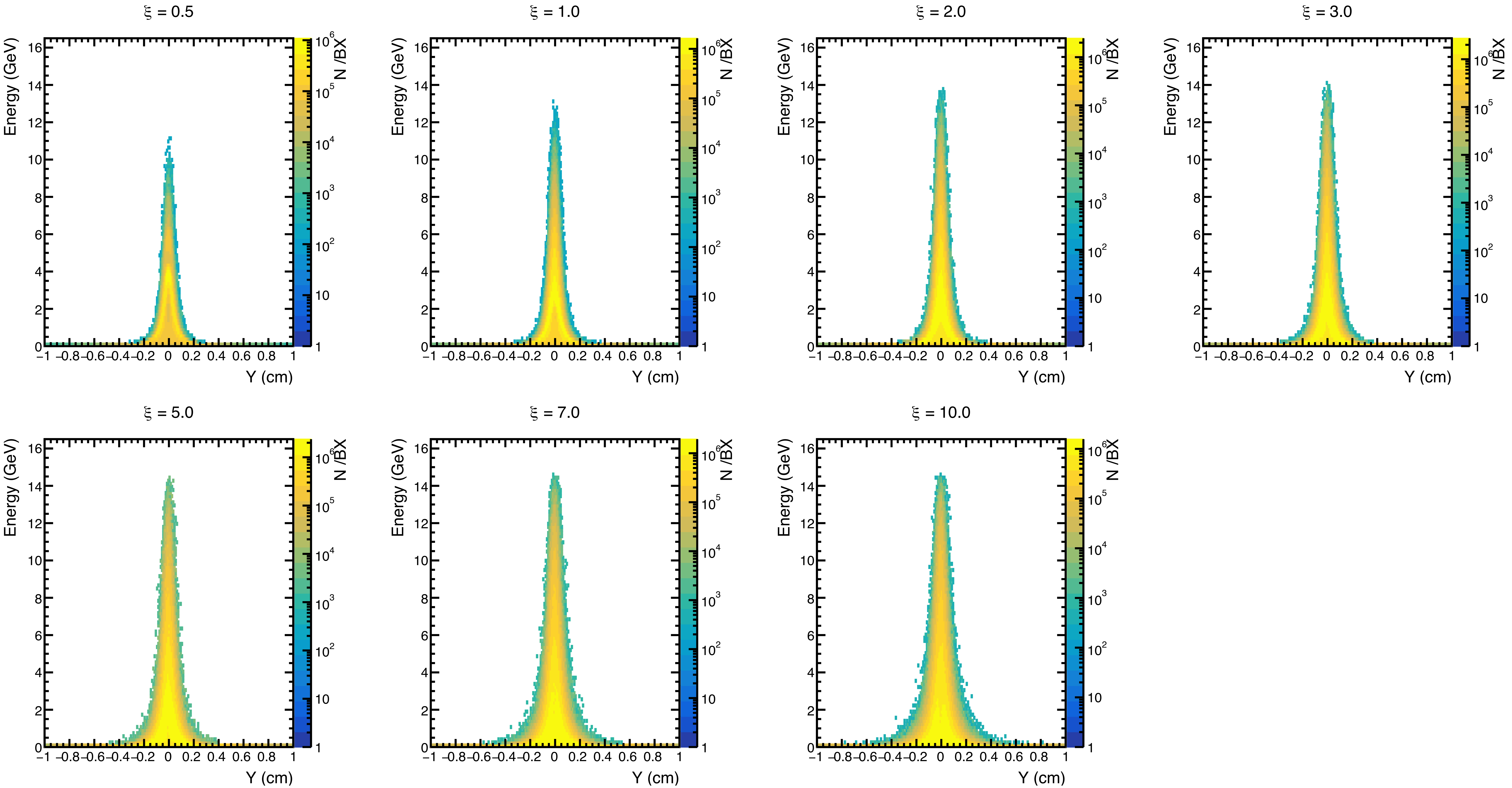
Ptarmigan LMA simulations

Energy - position correlation (parallel to polarisation axis)



Ptarmigan LMA simulations

Energy - position correlation (perpendicular to polarisation axis)



Ptarmigan LMA simulations

Inference of laser intensity

- Inference of laser intensity follows Blackburn et. al. 2020

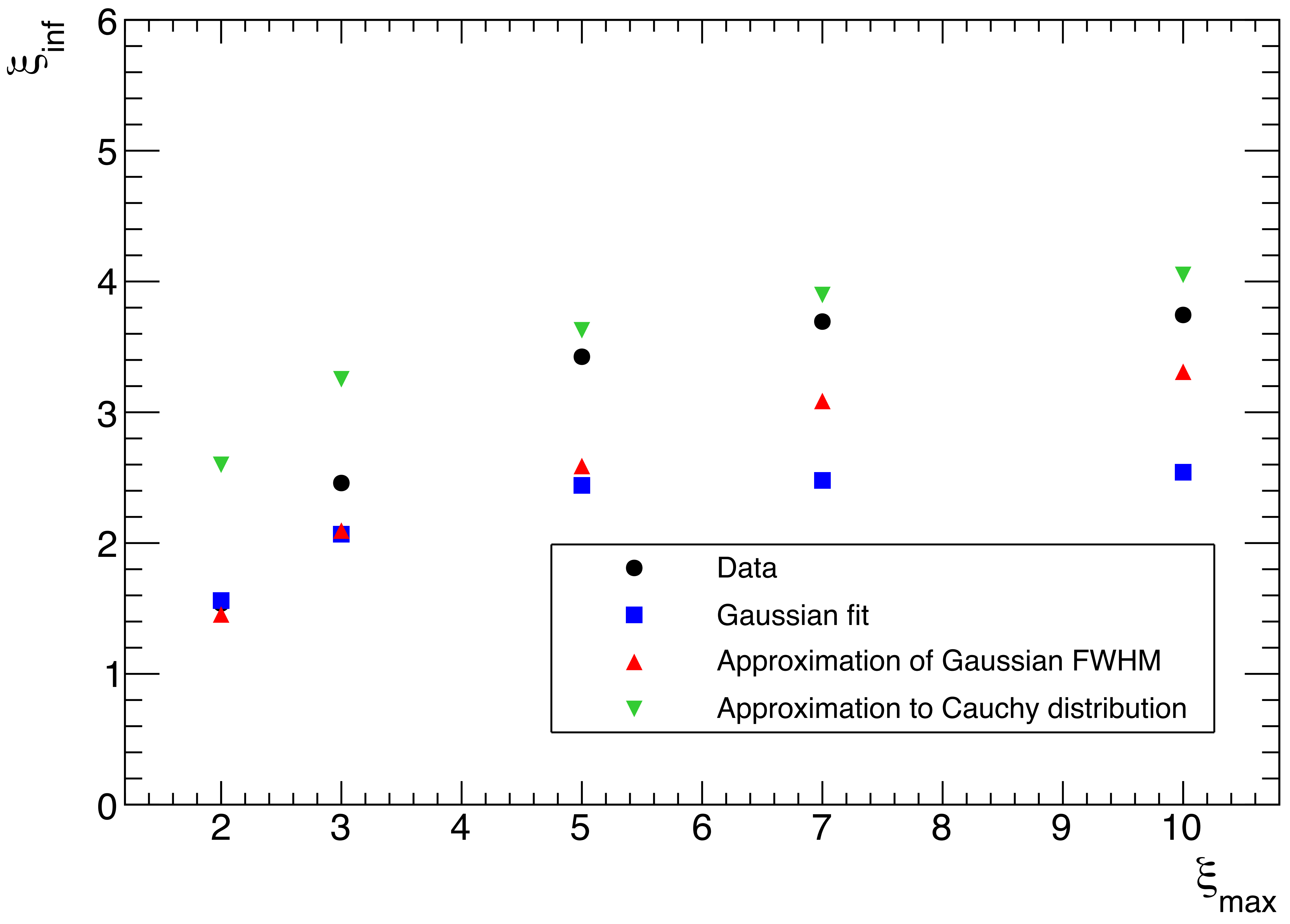
$$\xi = g(\rho)\,\xi_{inf} = \xi_{inf}\sqrt{\frac{1+8\rho^2}{1+4\rho^2}} \quad \rho = \frac{r_b}{w_0}$$

$$\xi_{inf}^2 = 4\sqrt{2}\langle\gamma_i\rangle\langle\gamma_f\rangle(\sigma_{\parallel}^2 - \sigma_{\perp}^2)$$

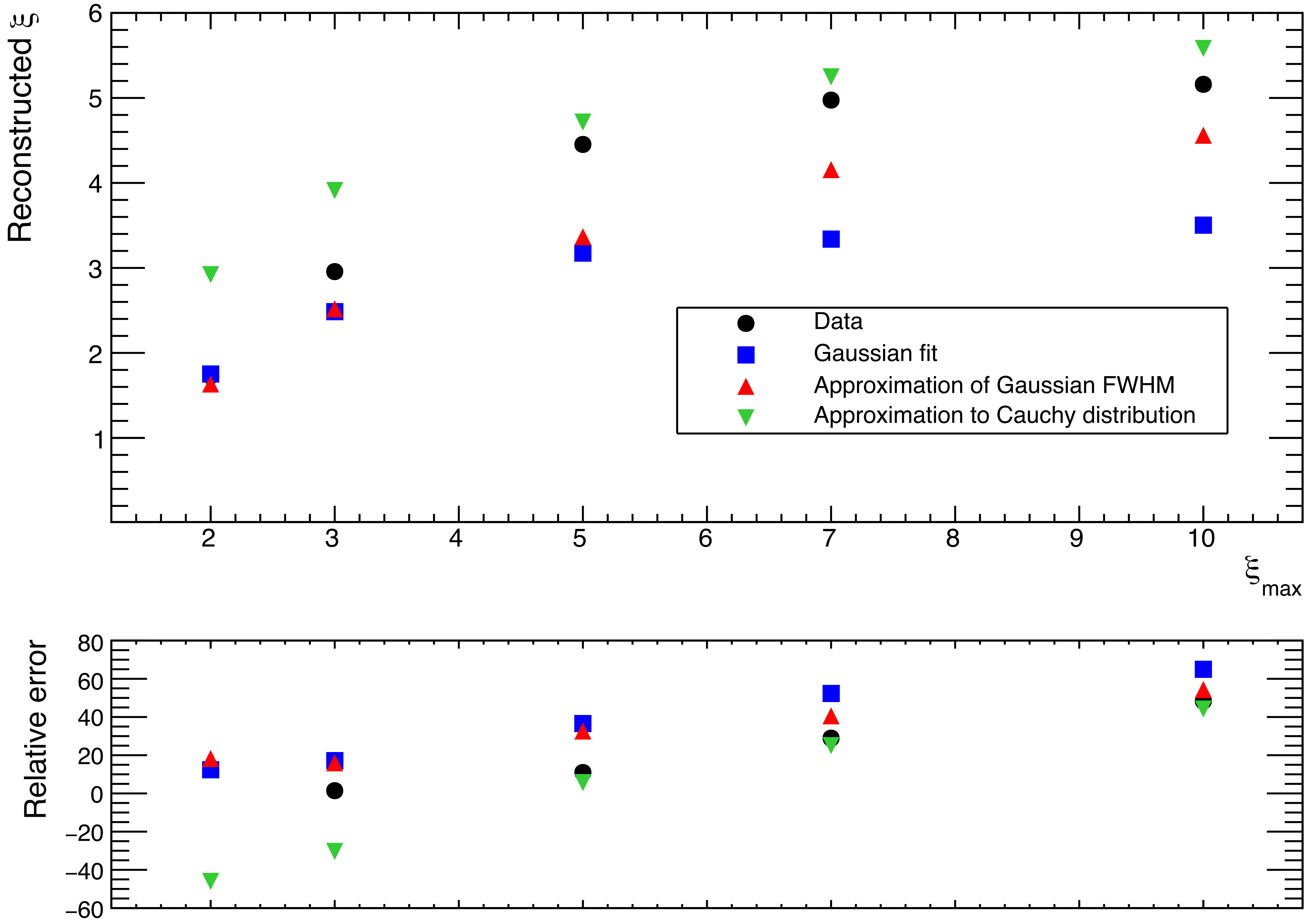
- Variance of angular profiles constructed using different methods:
    - Variance of data
    - Variance of Gaussian fit
    - Variance of approximation to Cauchy fit (see slides 15-17)
    - Approximation of variance assuming a Gaussian FWHM
- Fittings are performed on a central region of width 0.1 mrad

Ptarmigan LMA simulations

Inference of laser intensity



Inferred xi plot seems reasonable based on slide 4



Reconstruction needs more work - alright for low xi but completely off for larger xi

Additional slides

Extra

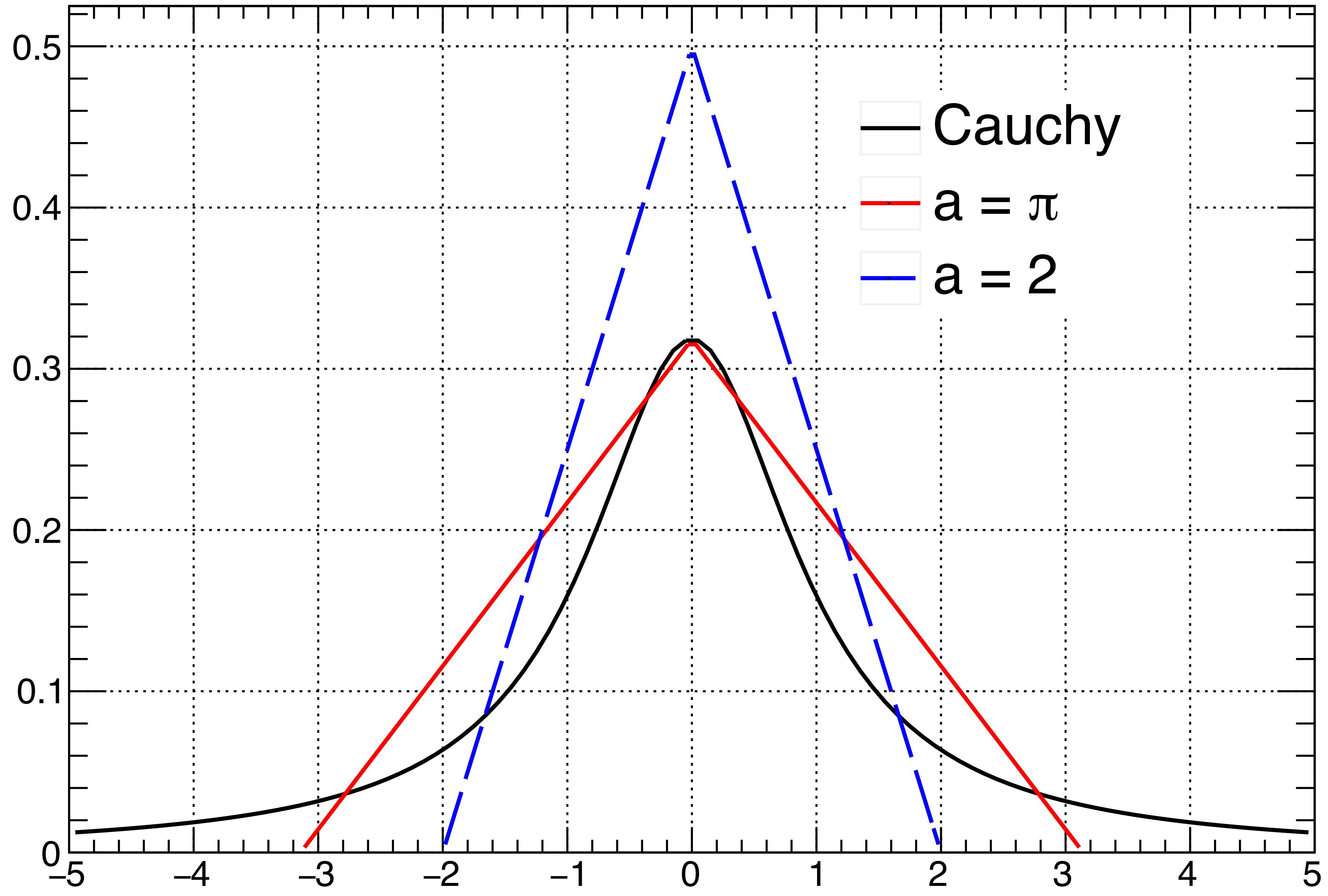
Approximation of Cauchy distribution

- Standard Cauchy distribution is given as

$$X \sim \text{Cauchy}(0,1) \Rightarrow f_X(x) = \frac{1}{\pi}\frac{1}{1+x^2}$$

- A first order approximation to this is a triangle function of the form

$$g(x) = \begin{cases} \frac{1}{a}\left(1 - \frac{|x|}{a}\right) & \text{for } -a \leq x \leq a \\ 0 & \text{elsewhere} \end{cases}$$



- a is determined by the requirements of the approximation; two are considered here:
    - Peak values of distributions coincide $\Rightarrow$ $a = \pi$
    - Function must pass through FWHM points $\Rightarrow$ $a = 2$

Extra

Triangular distribution

- The second moment of a distribution is defined as

$$m_2 = \mathbb{E}_X[X^2] = \int_{\mathbb{R}} dx\, x^2 f_X(x)$$

- Using $f_X(x) = g(x)$ from previous slide

$$m_2 = \frac{a^2}{6} \text{ if } a > 0$$

- Hence, the RMS for each value of a is:
  - $\text{rms} = \frac{\pi}{\sqrt{6}}$ for $a = \pi$
  - $\text{rms} = \sqrt{\frac{2}{3}}$ for $a = 2$

Extra

General triangular distribution

- For a Cauchy distribution with general width parameter γ, the PDF is

$$X \sim \text{Cauchy}(0,\gamma) \Rightarrow f_X(x) = \frac{1}{\pi\gamma}\frac{1}{1+\left(\frac{x}{\gamma}\right)^2}$$

- Triangular approximation can be obtained by the substitution $x \to \dfrac{x}{\gamma}$ or equivalently $a \to \gamma a$:

$$g(x) = \begin{cases} \frac{1}{a\gamma}\left(1 - \frac{|x|}{a\gamma}\right) & \text{for } -a\gamma \le x \le a\gamma \\ 0 & \text{elsewhere} \end{cases}$$

- Hence, the second moments are given by $m_2 = \dfrac{a^2\gamma^2}{6}$ so
  - $\text{rms} = \dfrac{\pi\gamma}{\sqrt{6}} \approx 1.283\,\gamma$ for $a = \pi$
  - $\text{rms} = \gamma\sqrt{\dfrac{2}{3}} \approx 0.816\,\gamma$ for $a = 2$