Introduction to Accelerator Physics

Part 3

Pedro Castro / Accelerator Physics Group (MPY) Hamburg, 3rd February 2023



Accelerator lectures framework in Summer Student Prog.

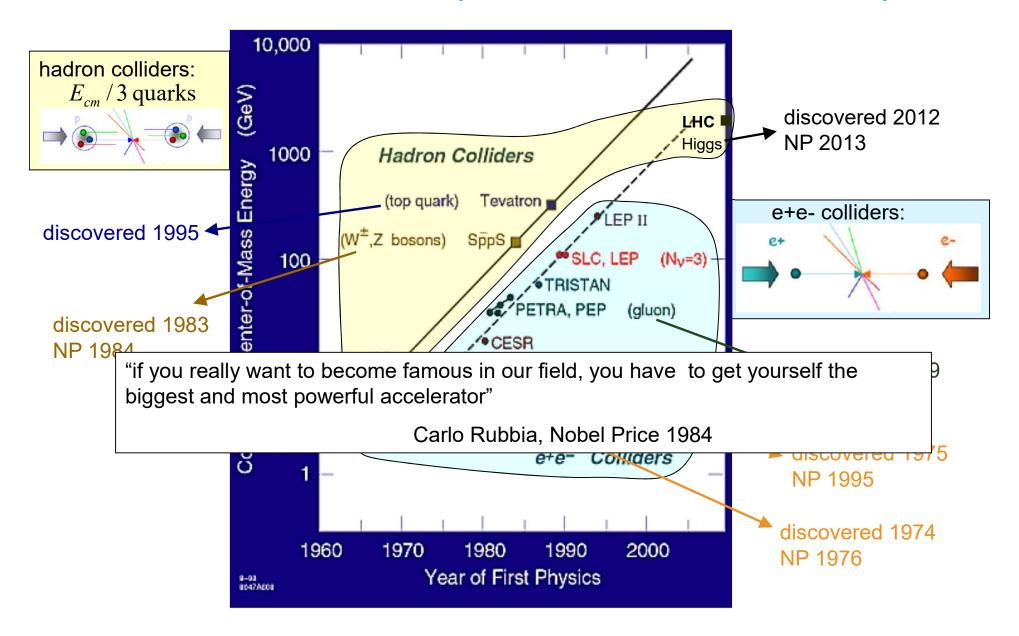
22nd Feb.: Plasma wakefield acceleration, J. Osterhoff

<u>Today:</u> focus on present day <u>and last 50 years</u> accelerator technology

synchrotrons: machines for discoveries

Facility	Particle(s) discovered	Year of discovery	Nobel Price
SPEAR	charm quark	1974	1976
SPEAR	tau lepton	1975	1995
PETRA	gluon	1979	
S $ar{p}$ pS	W^\pm , Z bosons	1983	1984
SLC, LEP	$N_{\rm v}=3$		
Tevatron	top quark	1995	
LHC	Higgs	2012	2013

Main HEP discoveries at synchrotrons in the last 50 years

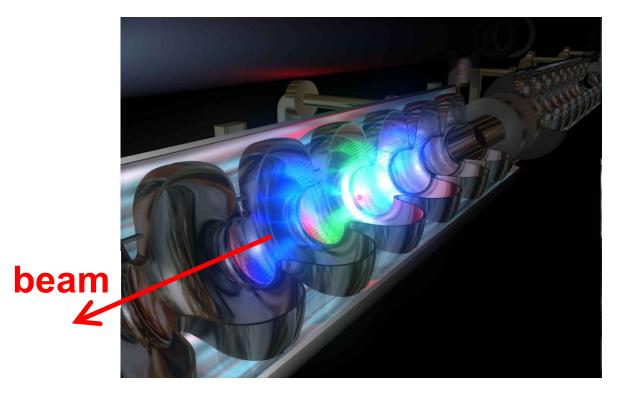


Scope of this lecture:

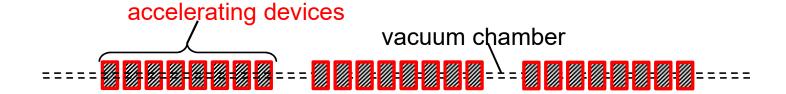
- 1. Synchrotrons: key components and their challenges to reach high energies:
 - Dipole magnetic fields
 - Superconducting dipoles
 - Quadrupole magnets to focus beams

2. Synchrotrons and Linear Accelerators:

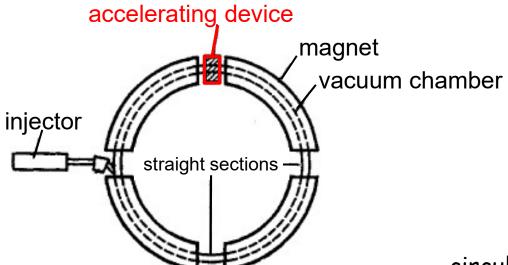
Acceleration using radio-frequency electomagnetic fields



DESY.



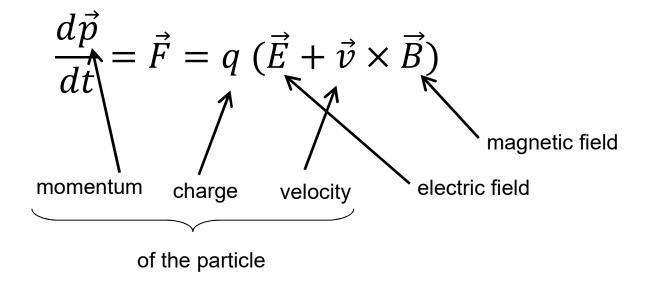
<u>lin</u>ear <u>ac</u>celerator (linac)



circular accelerator: synchrotron

Motion in electric and magnetic fields

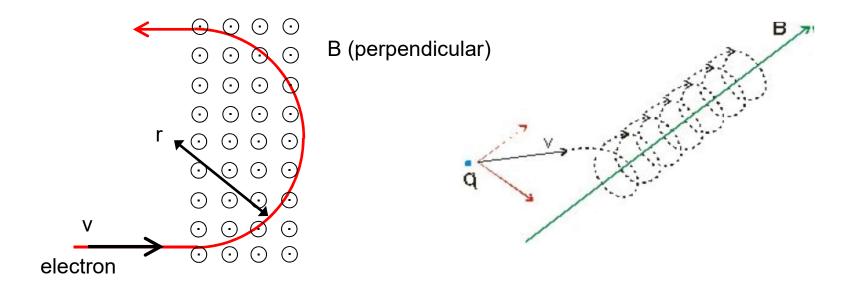
Equation of motion under Lorentz Force



Motion in magnetic fields

if the electric field is zero ($\vec{E} = 0$), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B} \quad \rightarrow \quad \vec{F} \perp \vec{v}$$

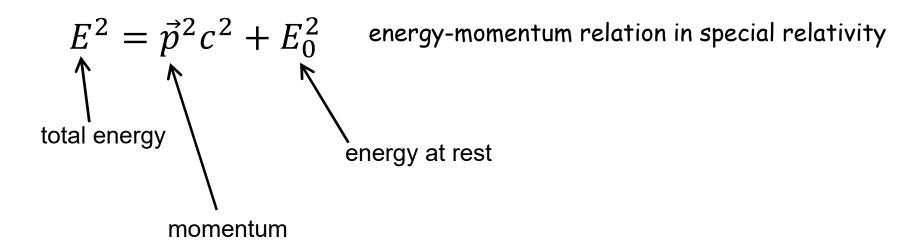


Magnetic fields do not change the particles energy

Motion in magnetic fields

if the electric field is zero (E=0), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B}$$



DESY.

Motion in magnetic fields

if the electric field is zero (E=0), then

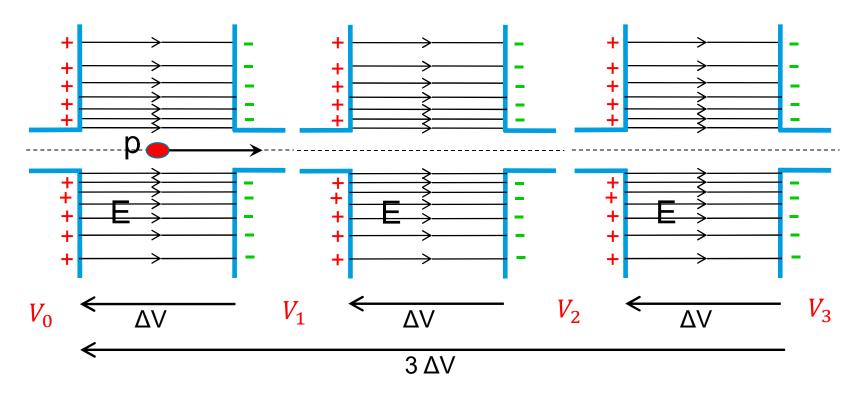
$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B}$$

$$E^2 = \vec{p}^2 c^2 + E_0^2$$

$$E \frac{dE}{dt} = c^2 \vec{p} \frac{d\vec{p}}{dt} = c^2 q \vec{p} (\vec{v} \times \vec{B}) = c^2 q |\vec{p}| |\vec{v} \times \vec{B}| \cos \emptyset = 0$$
since $\vec{v} \times \vec{B} \perp \vec{v} \implies \emptyset = 90^\circ$

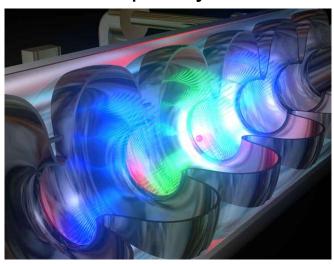
Magnetic fields do not change the particles energy, only electric fields do!

acceleration with DC electric fields

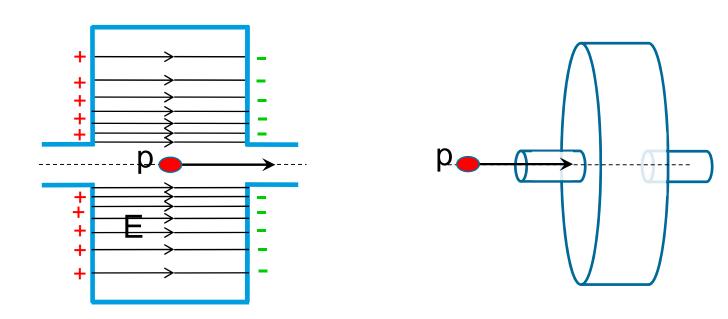


In general:

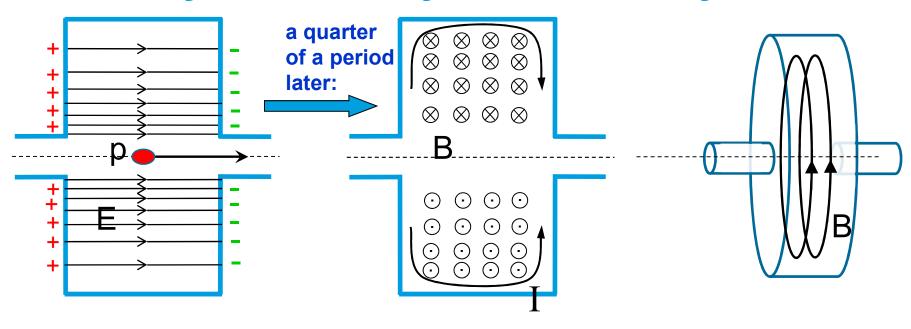
- Static magnetic fields → to guide (bend + focus) particle beams
- Static electric fields → accelerate particle beams (low energy)
- Radio-frequency EM fields → accelerate particle beams (high E)



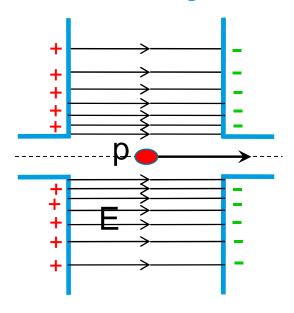
acceleration with RF (radio-frequency) electric fields



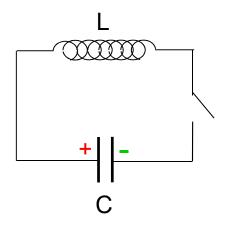
RF cavity basics: a cylindrical cavity



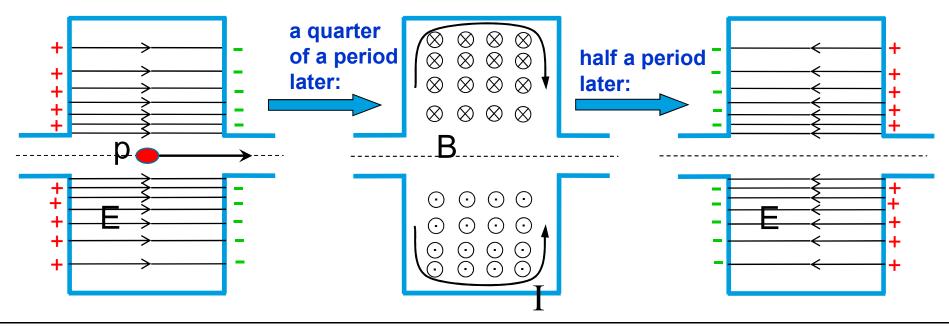
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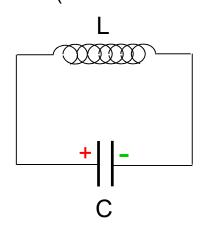
LC circuit (or resonant circuit) analogy:

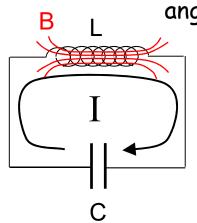


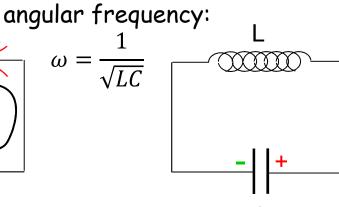
RF cavity basics: a cylindrical cavity



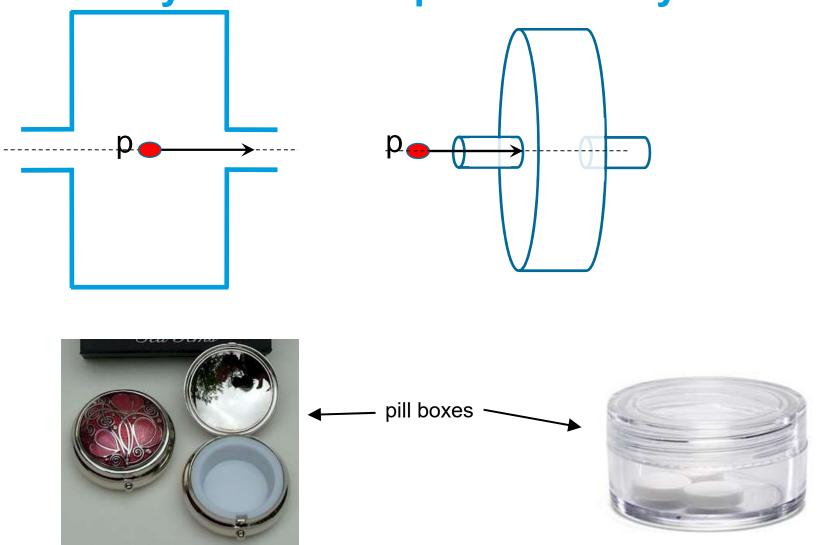
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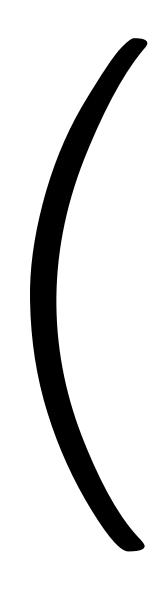




RF cavity basics: the pill box cavity



Equations for the electric and magnetic fields in a pill box cavity



(differential formulation in SI units)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

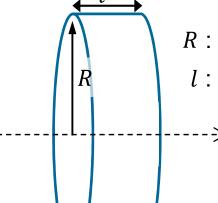
+ boundary conditions

TM modes (transverse magnetic modes)

set of solutions with $\overline{B_z} = 0$ (that is, \overline{B} is transverse)

set of solutions with $E_z = 0$ (that is, \vec{E} is transverse)

TE modes (transverse electric modes)



R: cavity radius

l: cavity length

(differential formulation in SI units)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

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+ boundary conditions

R: cavity radius

l: cavity length

set of solutions with $B_z = 0$ (that is, \vec{B} is transverse)

$$E_{z} = E_{0}J_{m}\left(x_{mn}\frac{r}{R}\right)\cos m\theta \cos\left(\frac{p\pi}{l}z\right)e^{j\omega t}$$

$$E_{r} = -\frac{p\pi}{l}\frac{R}{x_{mn}}E_{0}J'_{m}\left(x_{mn}\frac{r}{R}\right)\cos m\theta \sin\left(\frac{p\pi}{l}z\right)e^{j\omega t}$$

$$E_{\theta} \neq -\frac{p\pi}{l}\frac{mR^{2}}{x_{mn}^{2}r}E_{0}J_{m}\left(x_{mn}\frac{r}{R}\right)\sin m\theta \sin\left(\frac{p\pi}{l}z\right)e^{j\omega t}$$

$$B_{z} = 0$$

$$B_{r} = -j\omega\frac{mR^{2}}{x_{mn}^{2}rc^{2}}E_{0}J_{m}\left(x_{mn}\frac{r}{R}\right)\sin m\theta \cos\left(\frac{p\pi}{l}z\right)e^{j\omega t}$$

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indices:

m=0,1,2,...: number of full period variations in θ of the fields n=1,2,...: number of zeros of the axial field component in \vec{r} p=0,1,2,...: number of half period variations in z of the fields

 J_m : Bessel's functions

 x_{mn} : n-th root of J_m (that is, $J_m(x_{mn}) = 0$)

angular frequency :
$$\omega = c \sqrt{\left(\frac{x_{mn}}{R}\right)^2 + \left(\frac{p\pi}{l}\right)^2}$$

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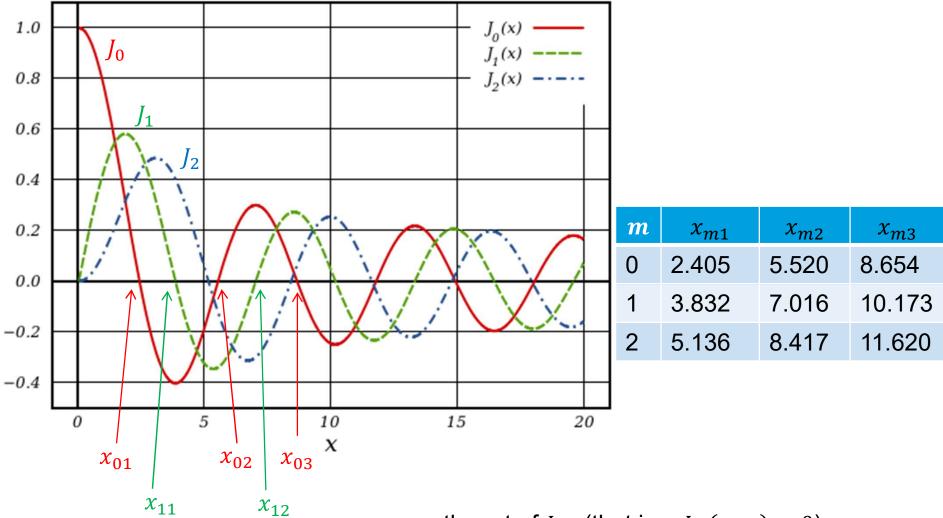
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Maxwell's equations (differential formulation in SI units)

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$$\vec{\nabla} \cdot \vec{B} = 0$$

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indices:

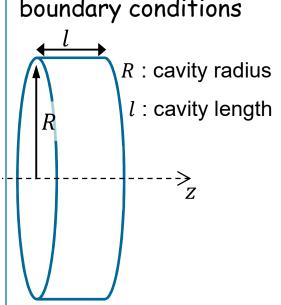
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boundary conditions



fundamental solution with $B_z = 0$ (that is, \vec{B} is transverse)

$$E_z = E_0 J_0 \left(x_{01} \frac{r}{R} \right) e^{j\omega t}$$

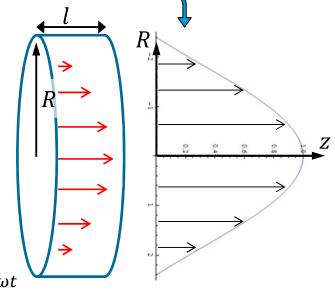
$$E_r = 0$$

$$E_{\theta} = 0$$
$$B_z = 0$$

$$B_z = 0$$

$$B_r = 0$$

$$B_{\theta} = j\omega \frac{R}{x_{01}c^2} E_0 J_1 \left(x_{01} \frac{r}{R} \right) e^{j\omega t}$$



m=0: rotation symmetry of the fields

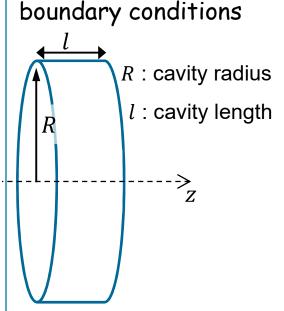
n=1 : no zeros of the axial field component in $ec{r}$

p = 0: no variation in z of the fields

 J_m : Bessel's functions

angular frequency :
$$\omega = c \frac{x_{01}}{R}$$
 $x_{01} = 2.405$

boundary conditions



fundamental solution with $B_z = 0$ (that is, \vec{B} is transverse)

$$E_z = E_0 J_0 \left(x_{01} \frac{r}{R} \right) e^{j\omega t}$$

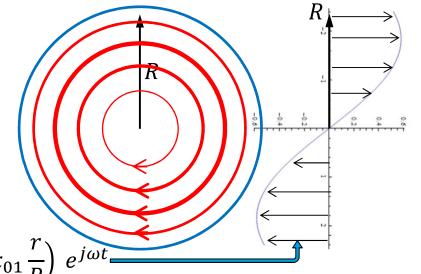
$$E_r = 0$$

$$E_{\theta} = 0$$
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$$B_z = 0$$

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m=0: rotation symmetry of the fields

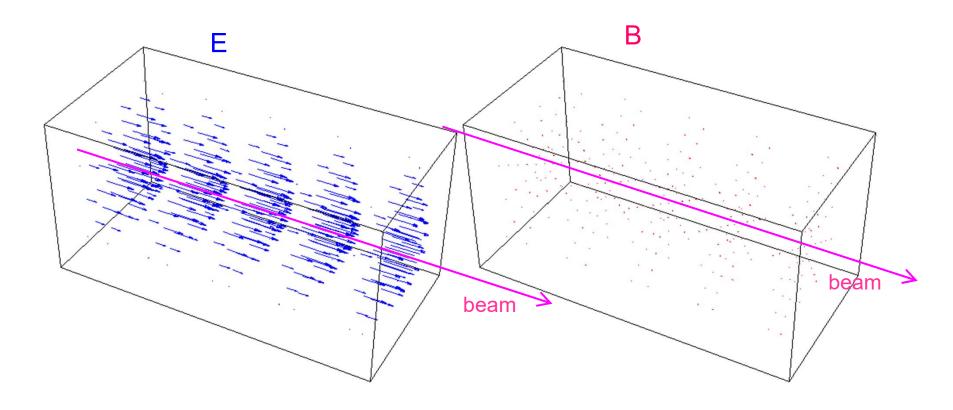
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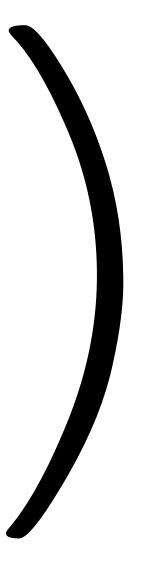
p = 0: no variation in z of the fields

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angular frequency :
$$\omega = c \frac{x_{01}}{R}$$
 $x_{01} = 2.405$

Pill box cavity: 3D visualisation of E and B

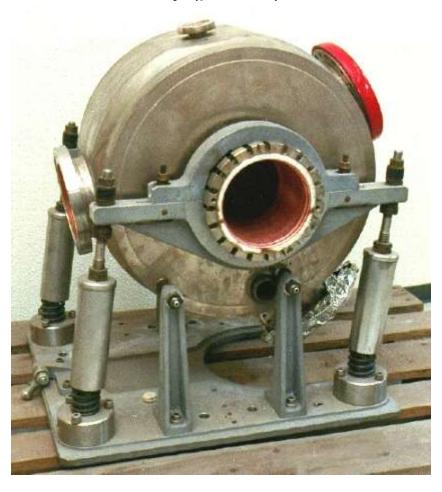




Equations for the electric and magnetic fields in a pill box cavity

Examples of pill box cavities

DESY cavity (pill box)



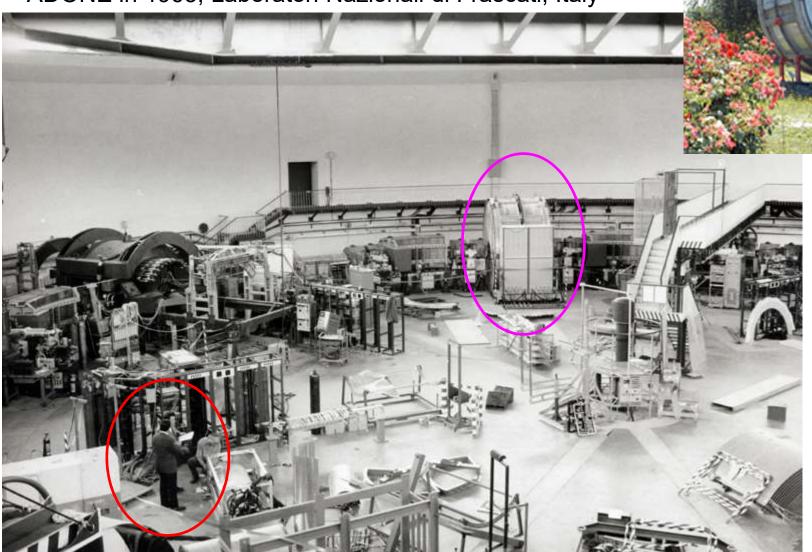
ADONE cavity 51 MHz (pill box) Frascati lab, Italy



Examples of pill box cavities

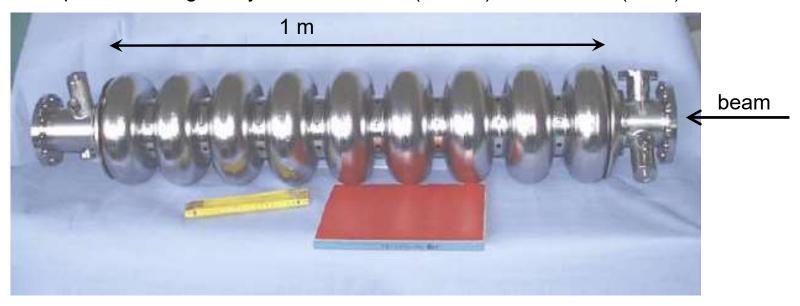
ADONE cavity 51 MHz (pill box) Frascati lab, Italy

ADONE in 1963, Laboratori Nazionali di Frascati, Italy



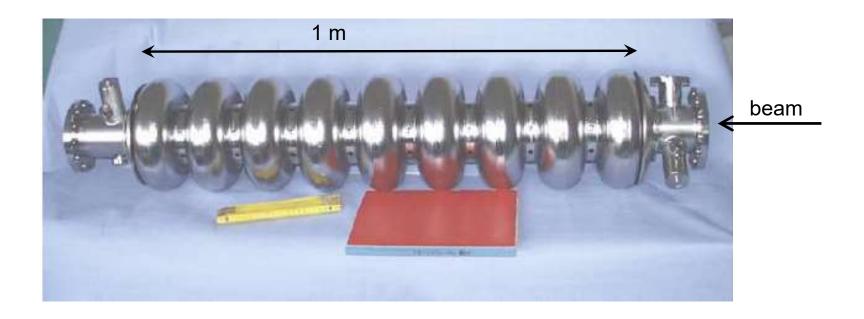
Superconducting cavity used at DESY

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



<u>F</u> ree-electron <u>LAS</u> er in <u>H</u> amburg		0.3 km	DESY	2004-	?	e-	1.2 GeV
European X-ray Free-Electron Laser		3 km	DESY	2016-	?	e-	17.5 GeV
	International Linear Collider	30 km	?	?		e-/e+	2x250 GeV

Superconducting cavity used at DESY

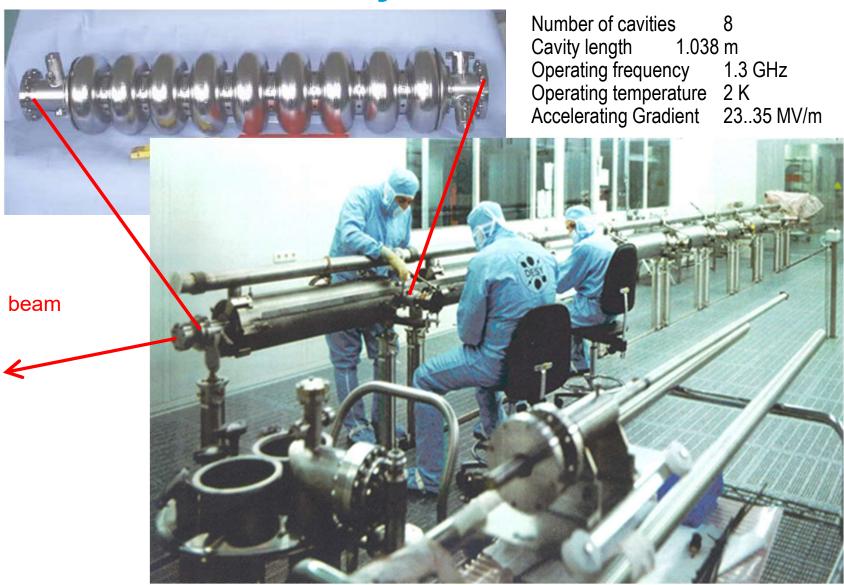


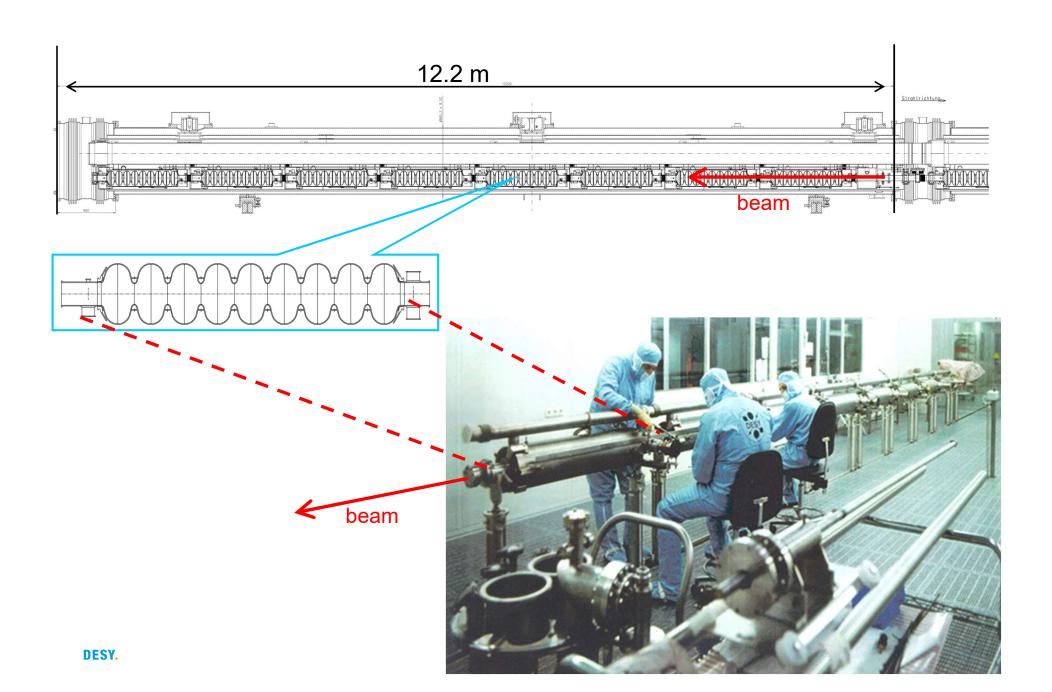
material: pure Niobium

operating temperature: 2 K

accelerating field gradient: up to 35 MV/m

Cavities inside a cryostat

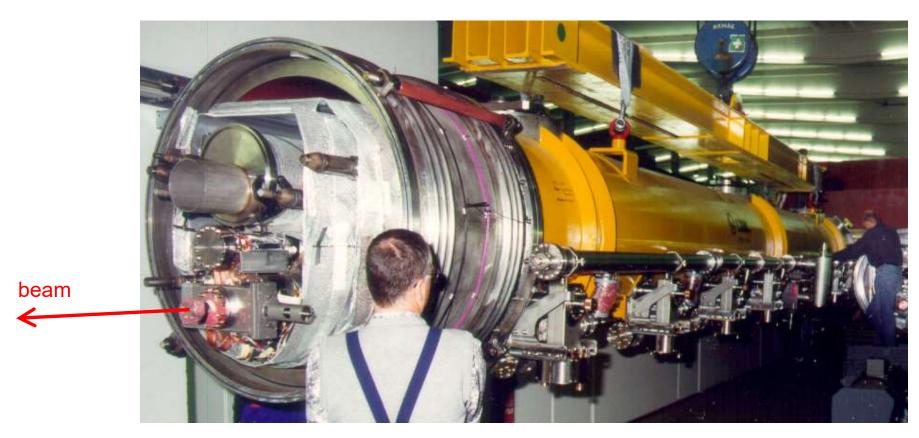




Cavities inside a cryostat



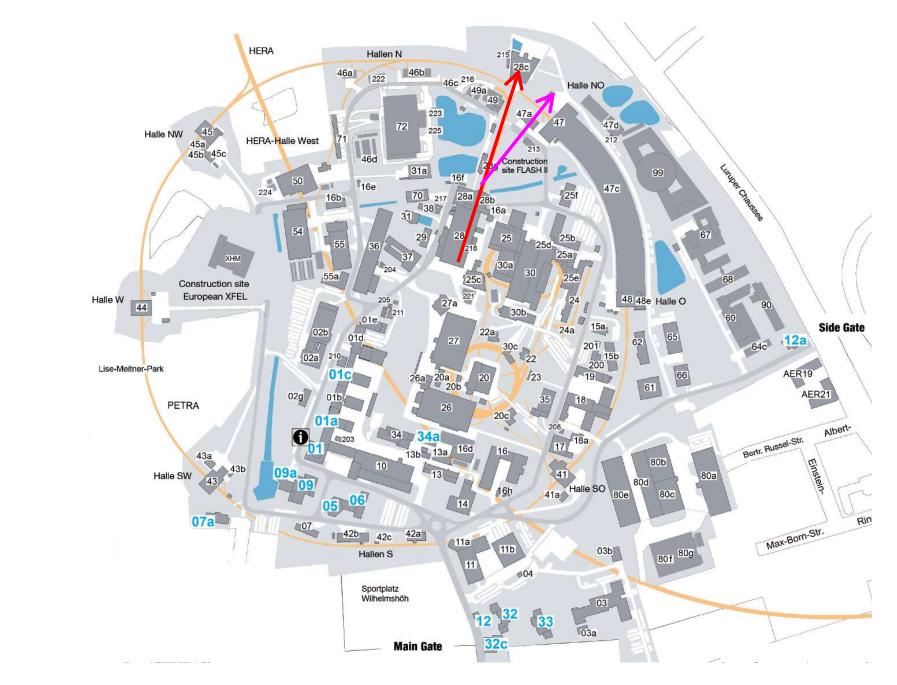
Cavities inside an accelerator module (cryostat)



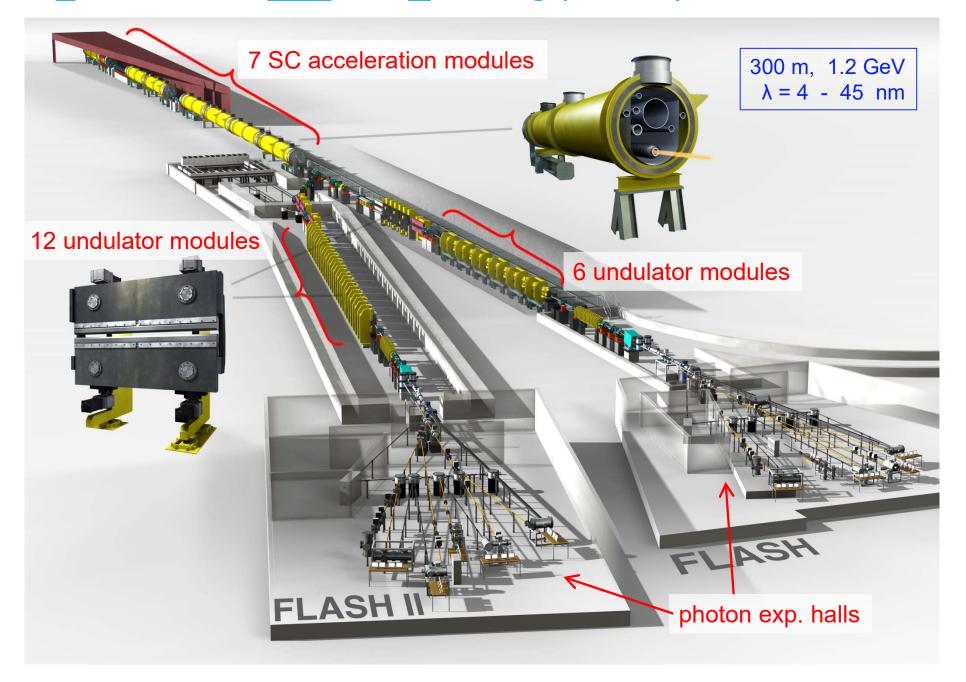
module installation in FLASH (2004)

Free-electron LASer in Hamburg (FLASH)

DESY.



Free-electron LASer in Hamburg (FLASH)



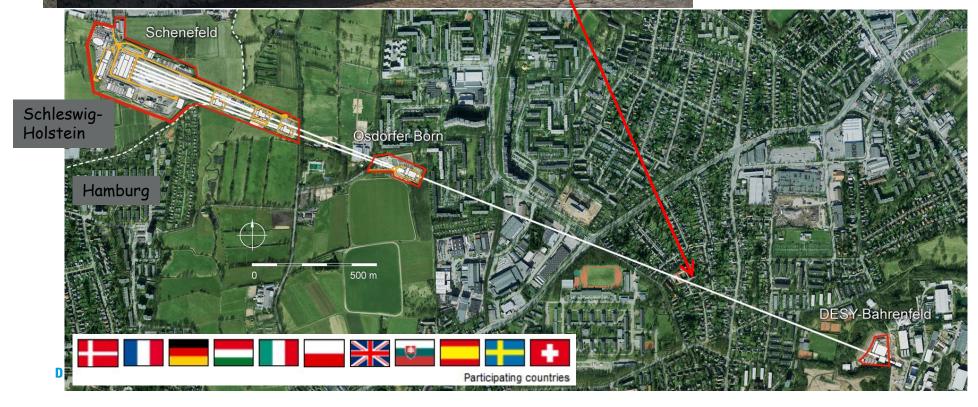
100 accelerator modules (cryostats) in XFEL

European X-ray Free-Electron Laser (XFEL)



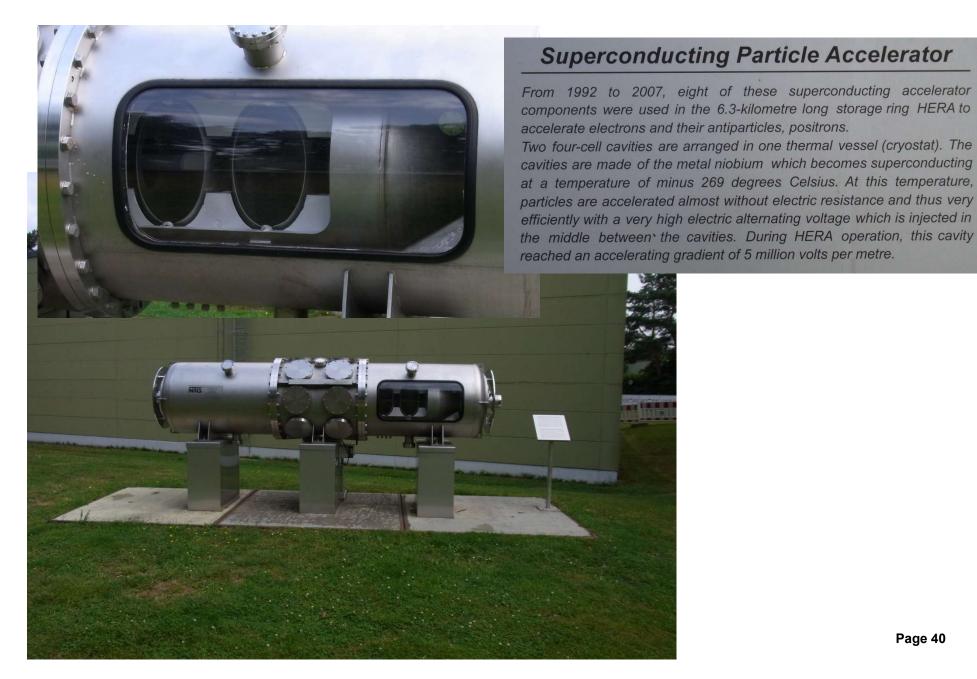
(3 km, 17.5 GeV)

 $\lambda = 0.05 - 6 \text{ nm}$



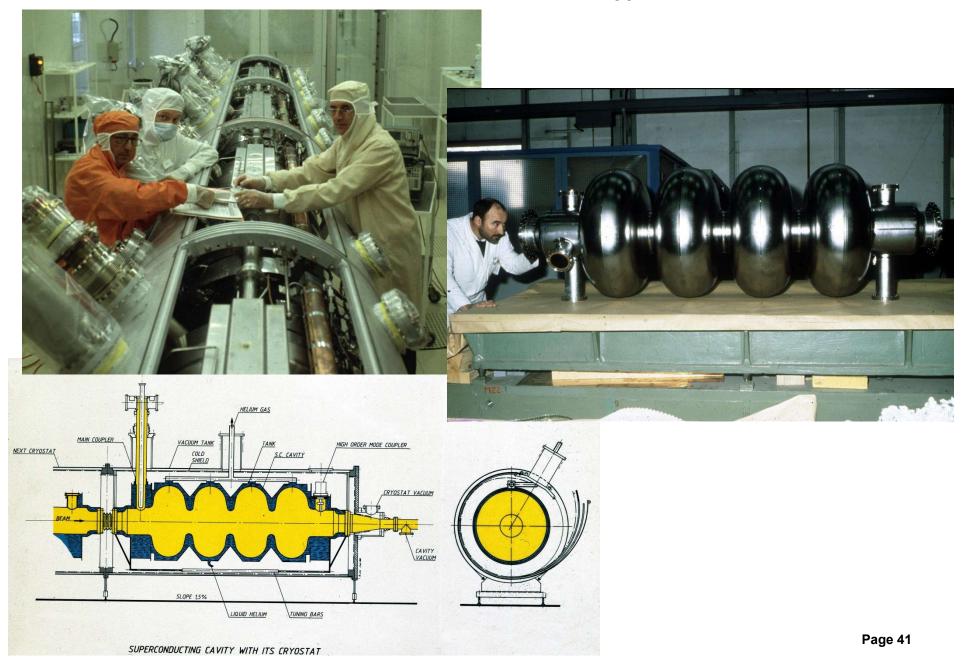
Superconducting cavities at HERA

16 cavities 500 MHz



Superconducting cavities at LEP

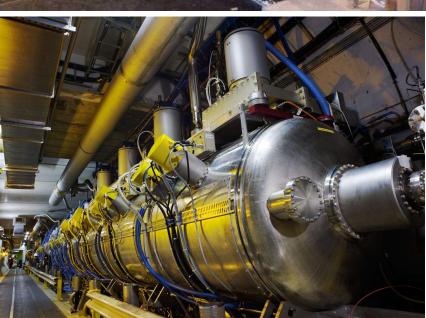
272 cavities 352 MHz



Superconducting cavities at LHC

16 cavities 400 MHz





Other accelerators using superconducting cavities

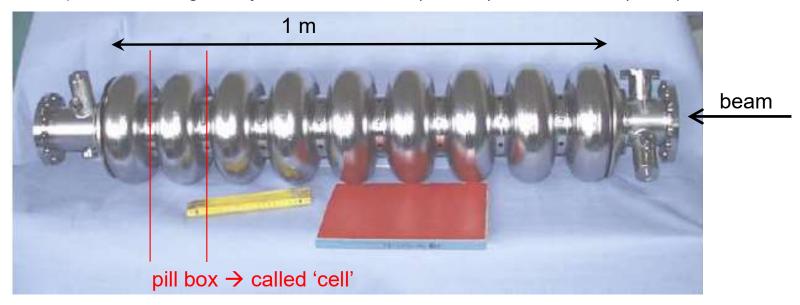
- 5 de-commissioned
- 11 in operation
- 4 in construction
- 10 in design phase

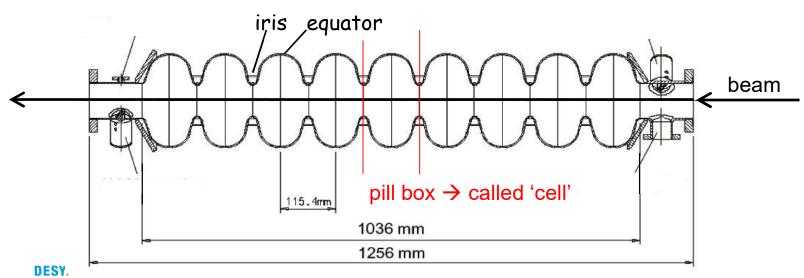
Total = 30

full list: http://tesla-new.desy.de/srf_accelerators

Superconducting cavity used in FLASH and in XFEL

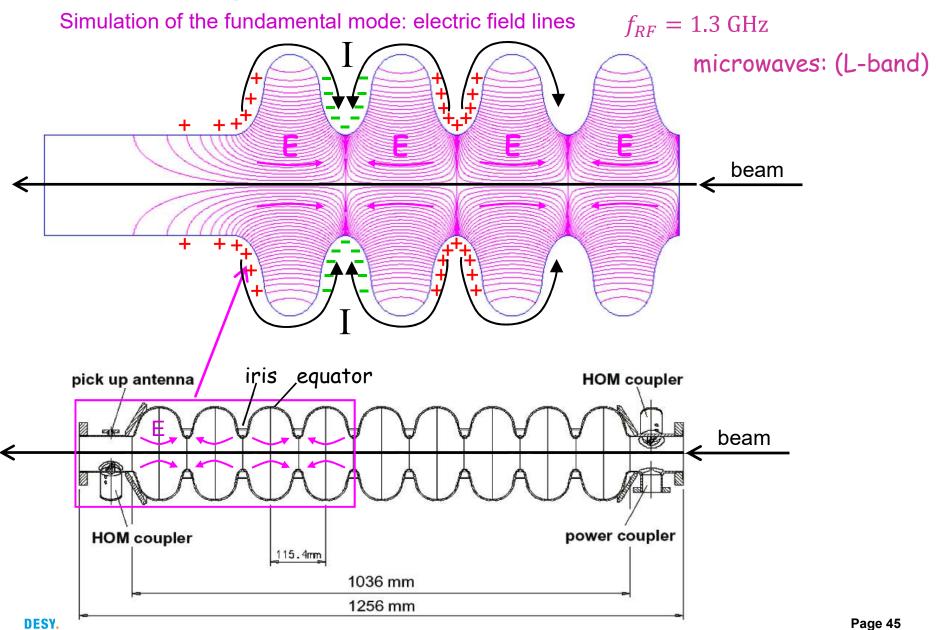
Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)





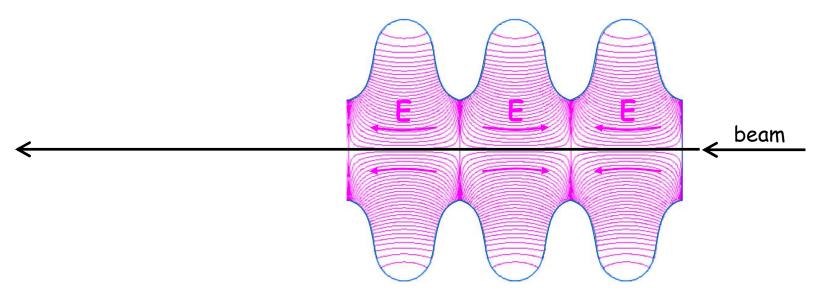
Page 44

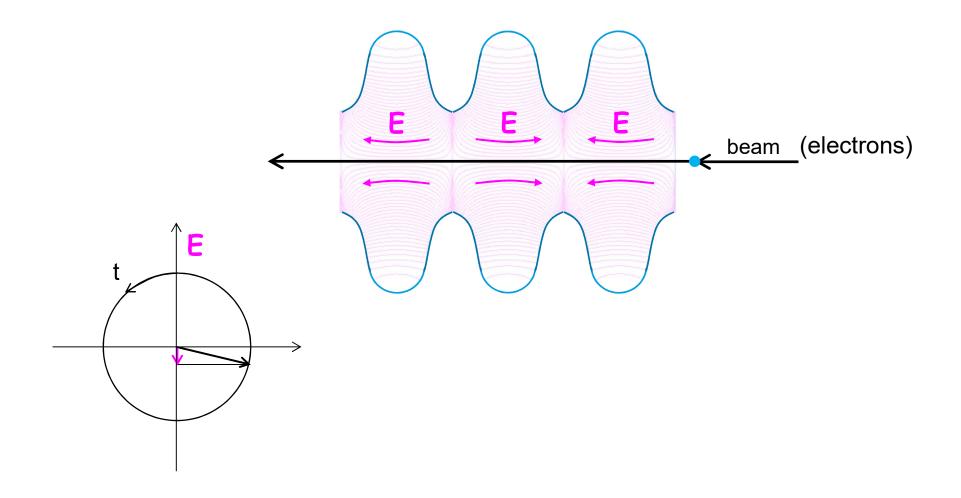
Accelerating field map

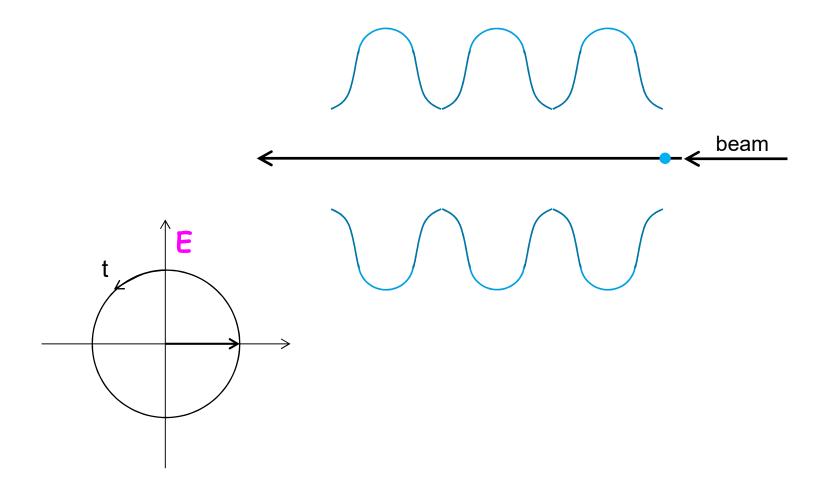


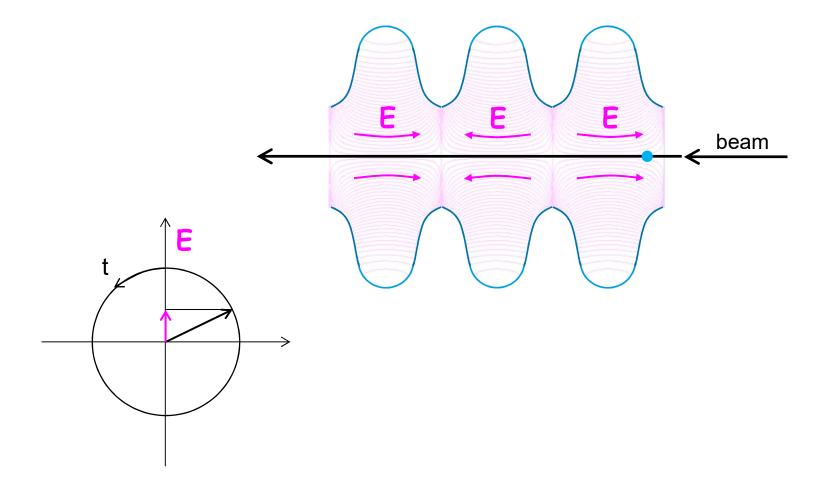
Is there a net acceleration?

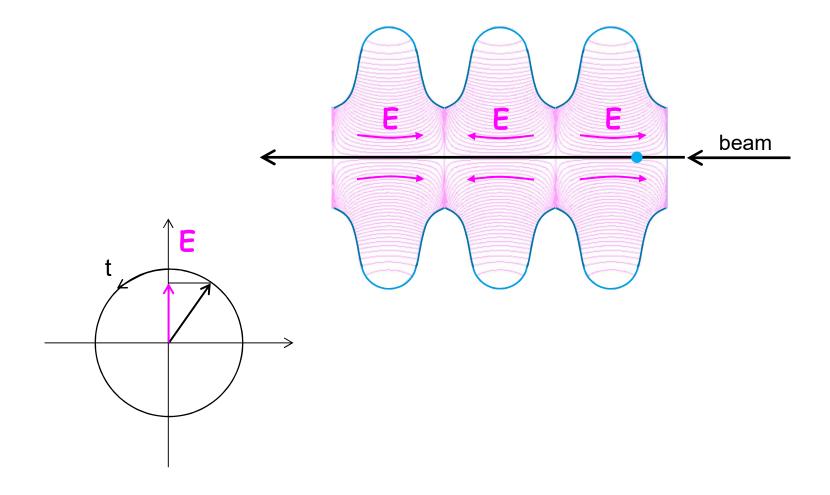
Simulation of the fundamental mode: electric field lines

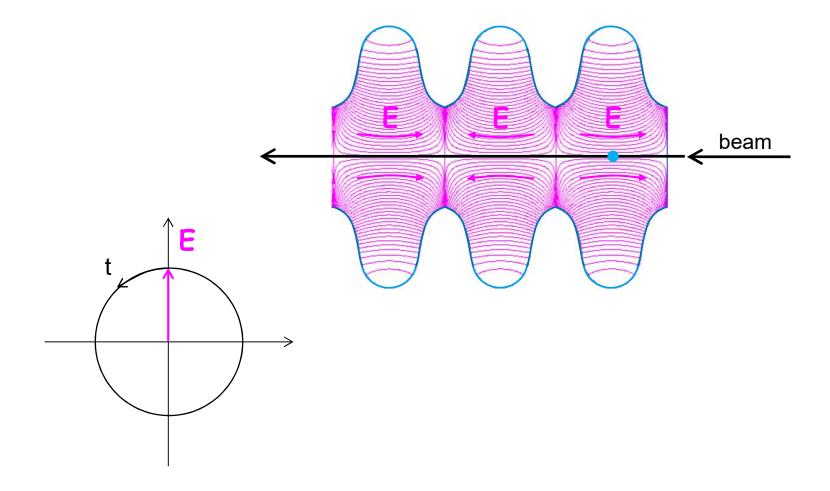


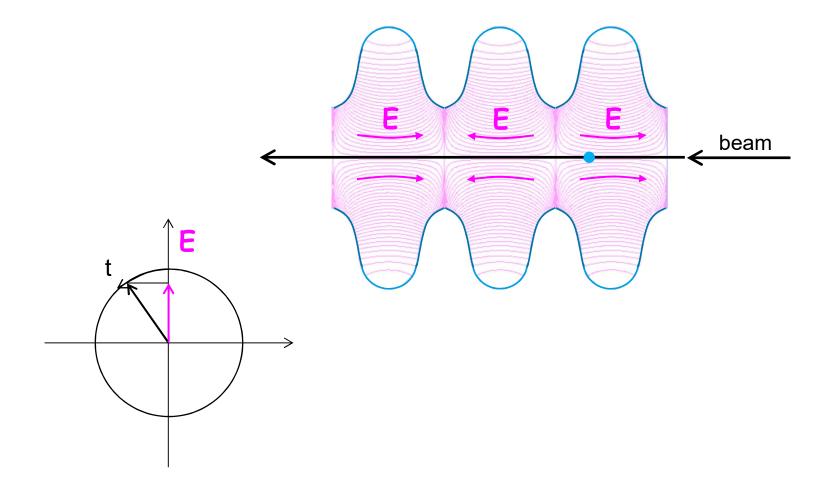


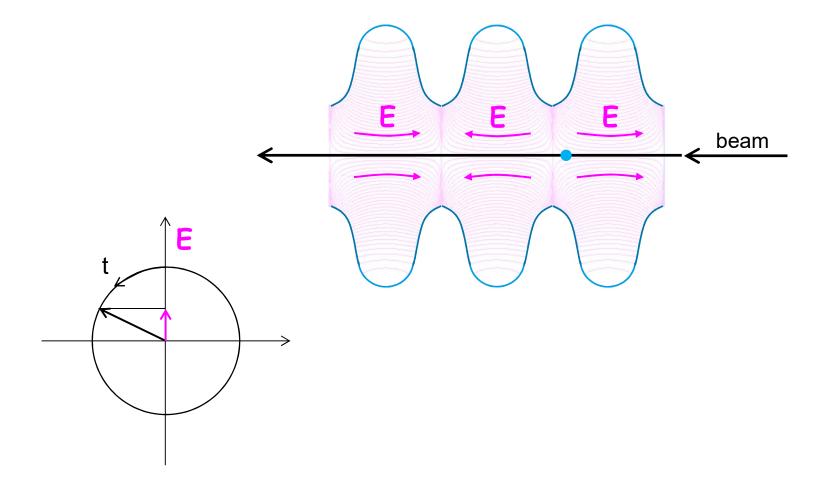


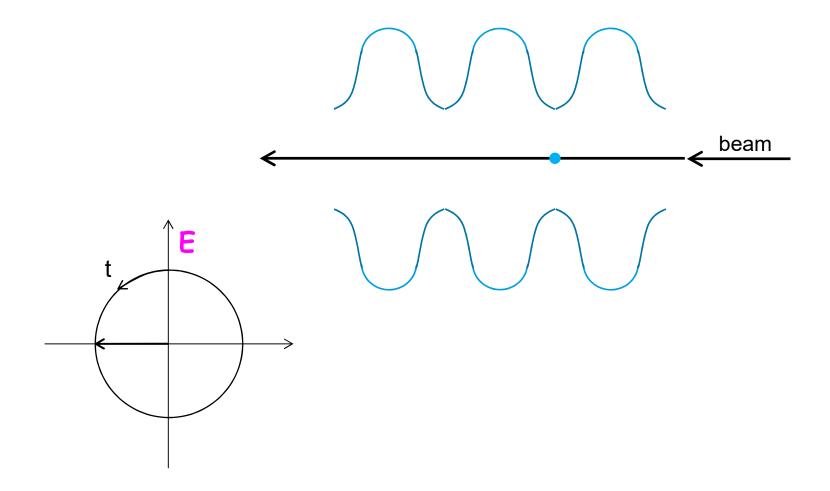


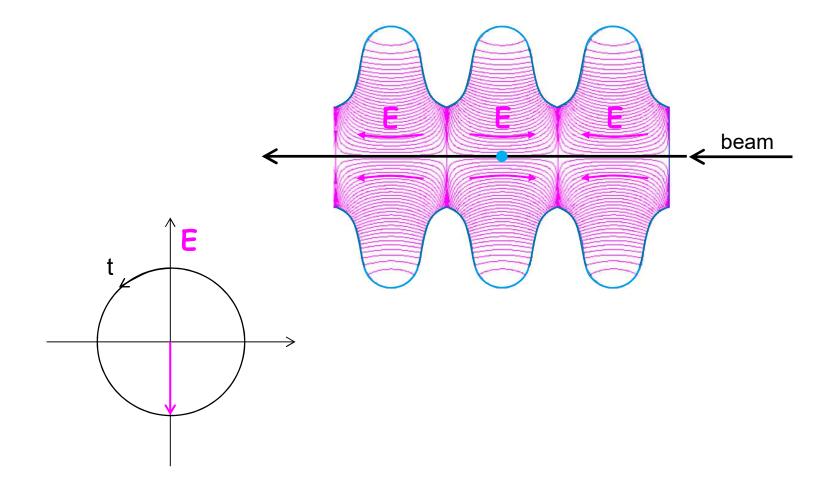


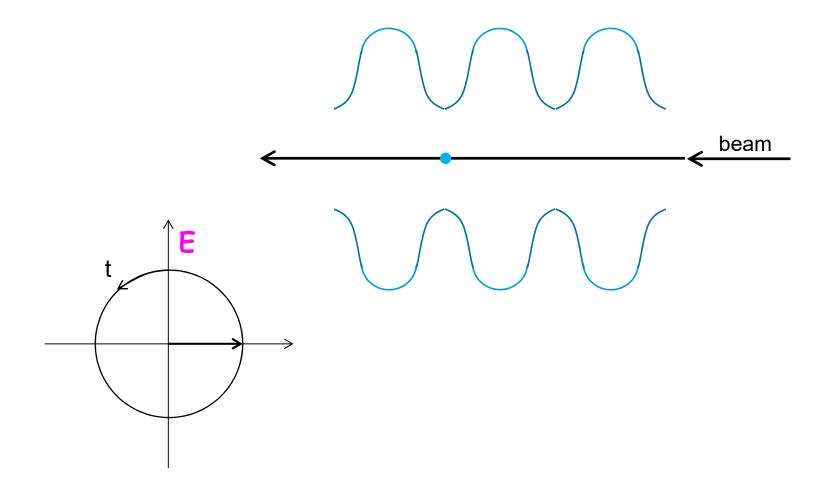


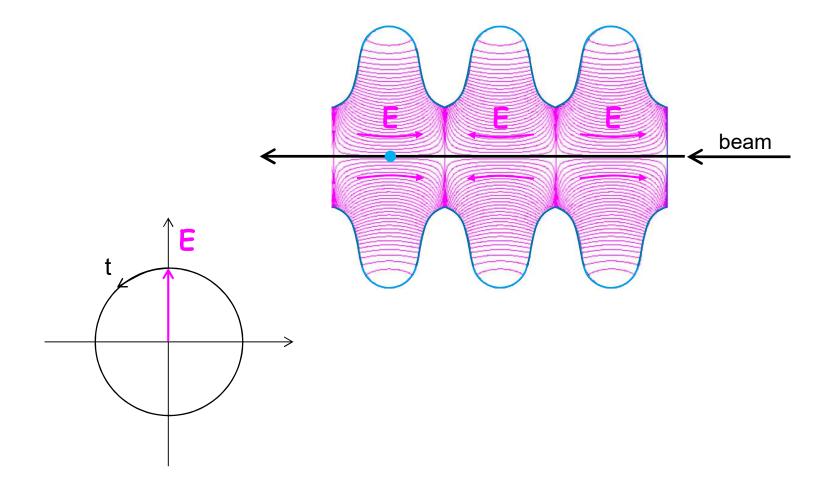


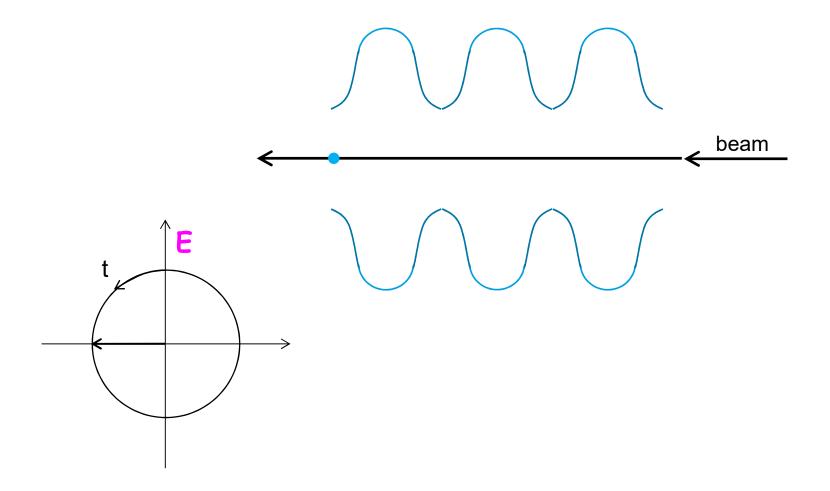






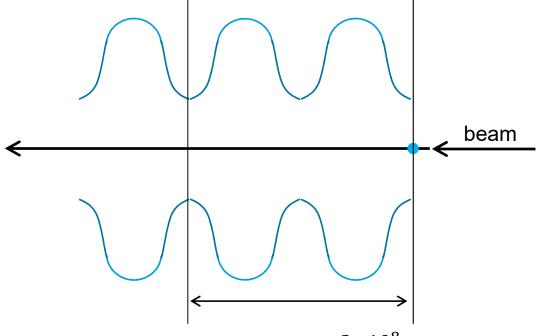


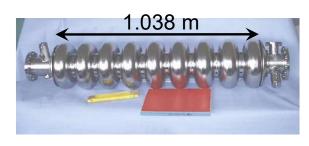




Is there a net acceleration? timing is the key

Is there a net acceleration? timing is the key





for electrons, $\beta \cong 1$

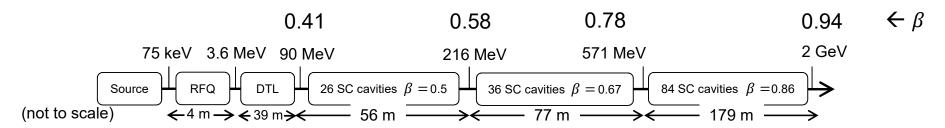
$$l = cT = \frac{c}{f} = \frac{3 \cdot 10^8}{1.3 \cdot 10^9} = 0.23 \, m$$
 (2 cells)

1 cavity (1.038 m) / 9 cells = 0.115 m

Is there a net acceleration? timing is the key

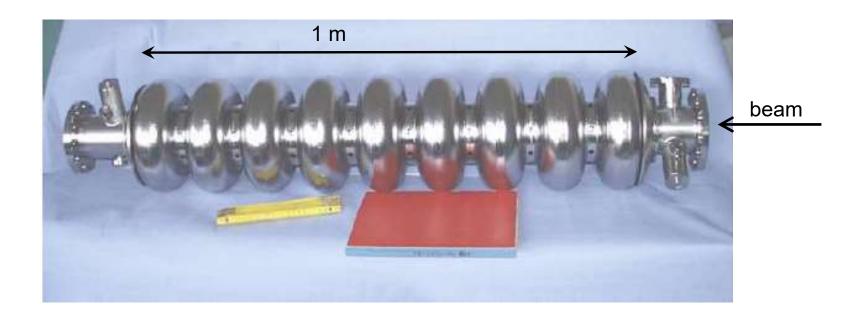
for protrons, $\beta < 1$

example: ESS (European Spallation Source), Lund, Sweden



DESY.

Superconducting cavity used at DESY



Frequently Asked Questions

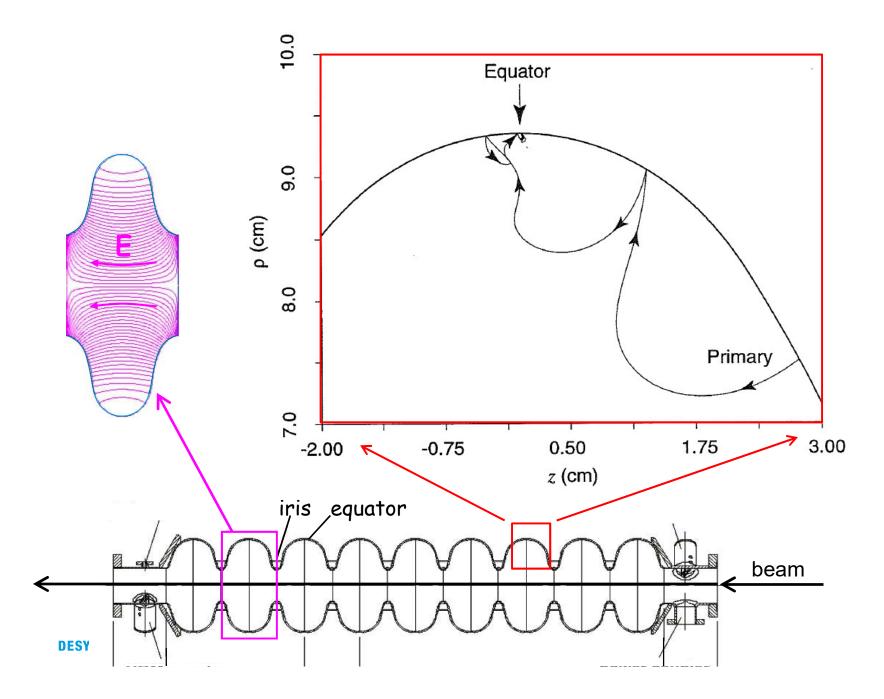
1) Why this shape?

2)

3)

4)

Multipacting mitigation in superconducting cavities

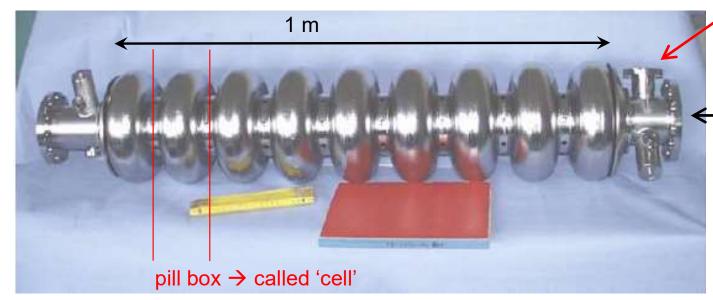


- 1) Why this shape? to reduce/avoid multipacting
- 2) How to feed \vec{E} in?
- 3)
- 4)

DESY.

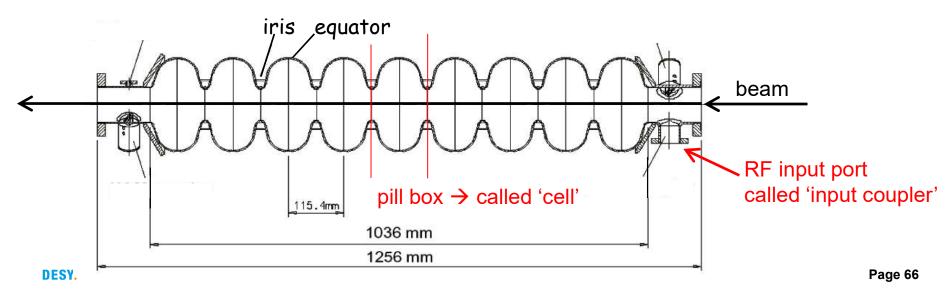
Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)

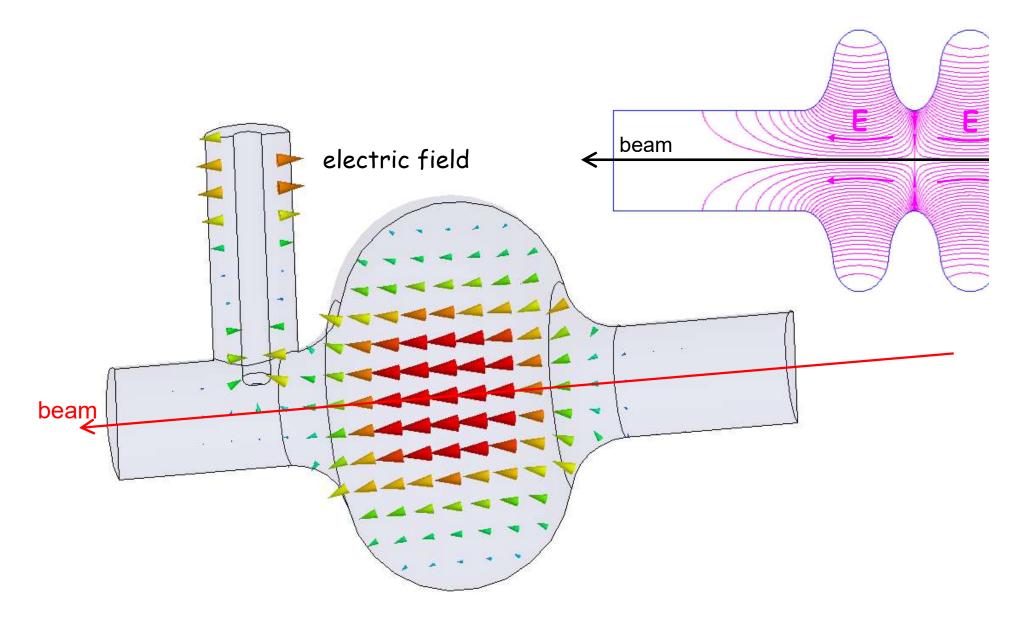


RF input port called 'input coupler' or 'power coupler'

beam



Fundamental mode coupler (input coupler)

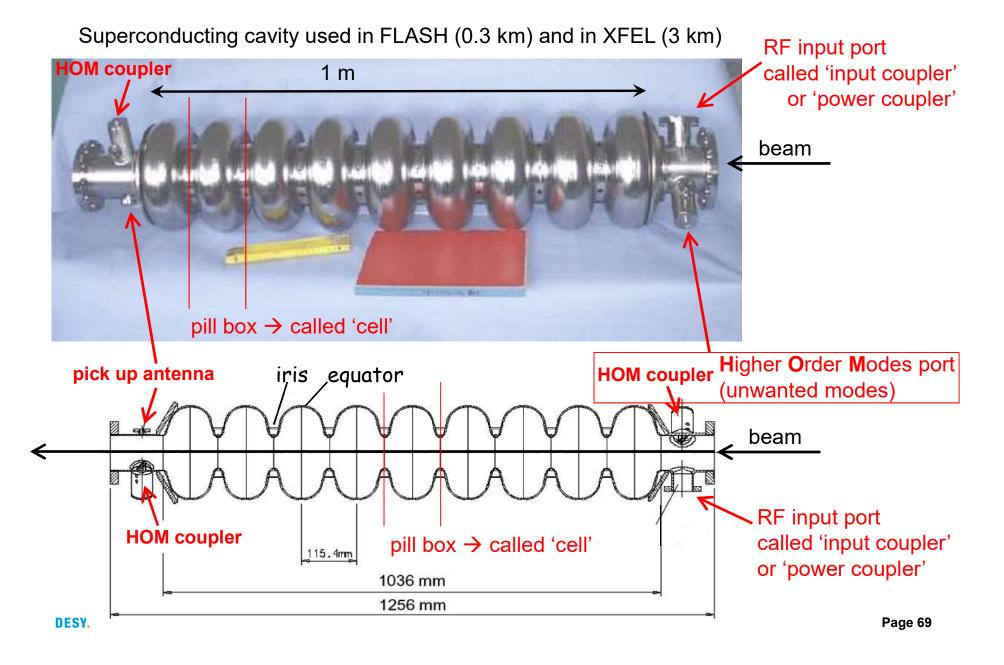


- 1) Why this shape? to reduce/avoid multipacting
- 2) How to feed \vec{E} in? with input couplers
- 3) How to measure \vec{E} ?

4)

DESY.

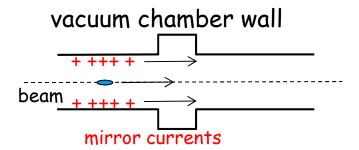
Superconducting cavity used in FLASH and in XFEL

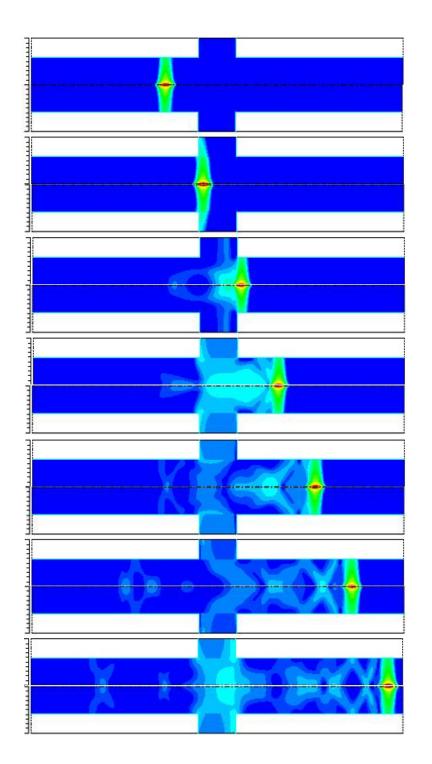


- 1) Why this shape? to reduce/avoid multipacting
- 2) How to feed \vec{E} in? with input couplers
- 3) How to measure \vec{E} ? with pick up antennas
- 4) What are HOM couplers for?

DESY.

Wakefields



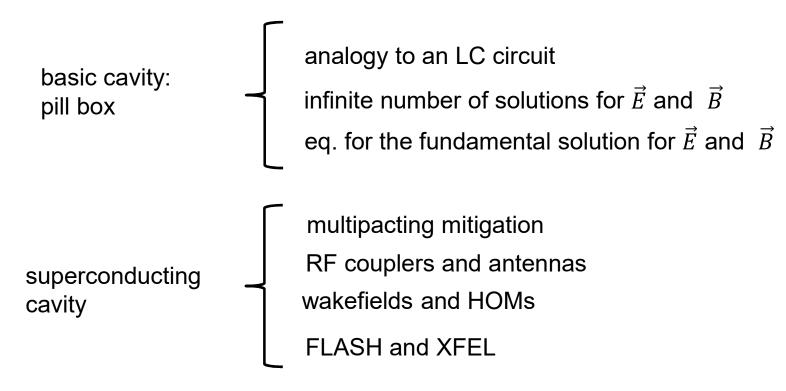


1)	Why this shape?	to reduce/avoid multipacting
2)	How to feed \vec{E} in?	with input couplers
3)	How to measure \vec{E} ?	with pick up antennas
4)	What are HOM couplers for?	. to reduce HOM (wakefields)

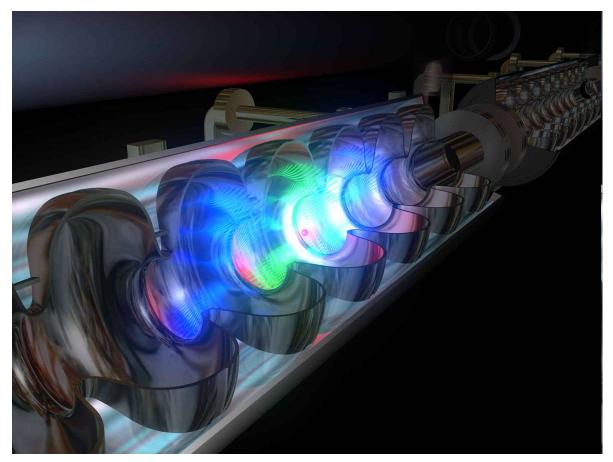
DESY.

Summing-up of this part

Particle acceleration using radio-frequency fields:



MEDIA DATABASE. "Electron acceleration – a virtual simulation"



DESY→Press→Media database→European XFEL (with filter: media type=movies)

YouTube: https://www.youtube.com/watch?v=FJO DmM4q7M

search text: electron acceleration

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