Statistical tools for results in IACTs

Maria Kherlakian
Science Club | 23.03.2023

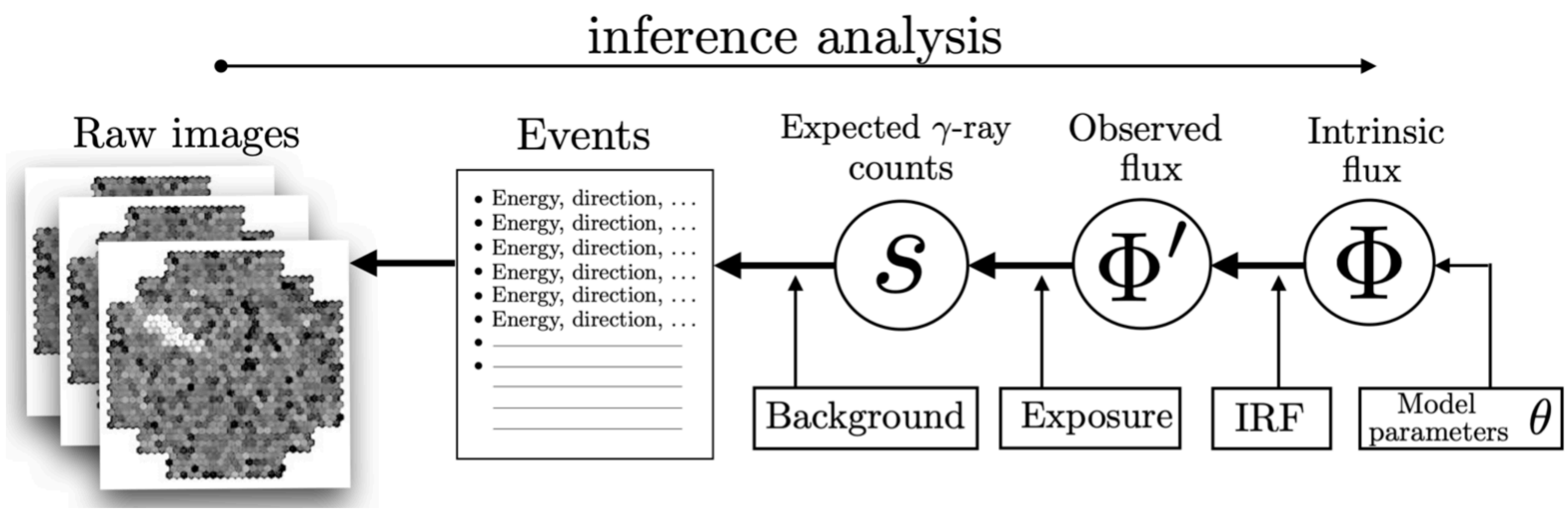
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES
HUMBOLDT-UNIVERSITÄT ZU BERLIN
HELMHOLTZ WEIZMANN RESEARCH SCHOOL MULTIMESSENGER ASTRONOMY
DESY.

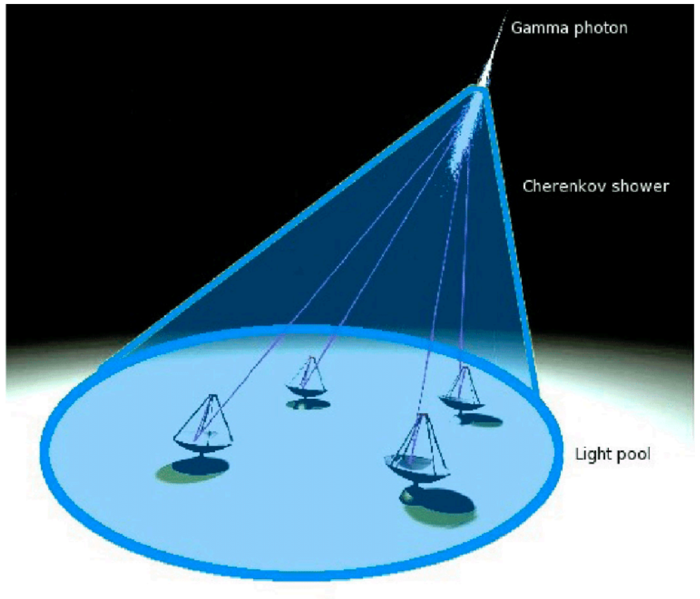
Presentation overview

Reference papers: DOI: 10.48550/arXiv.2202.04590
DOI: 10.1086/161295

- Workflow of an IACTs analysis
- Event reconstruction
- Multivariate Analysis
- Background modelling
- Detection Significance

Workflow of an IACTs analysis

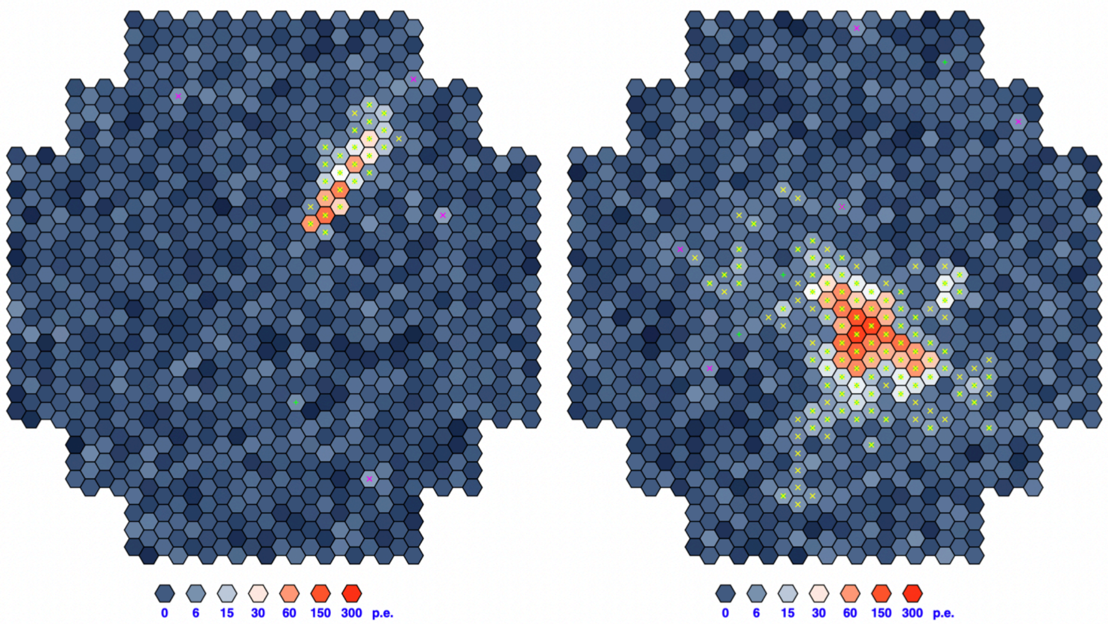




Convolve Φ' with IRFs to obtain intrinsic source flux Φ

$$\Phi(E, t, \hat{n}) = \frac{\mathrm{d}N_\gamma(E, t, \hat{n})}{\mathrm{dE\ dA\ dt}}$$

Event Reconstruction Techniques

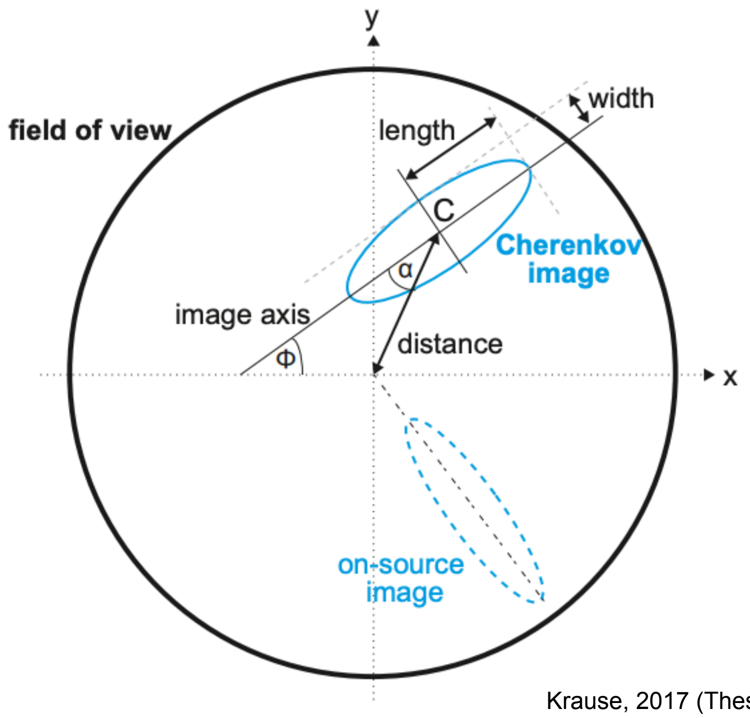


1.0 TeV gamma shower
2.6 TeV proton shower

What we want from the images:

- the energy of the primary gamma-ray
- its arrival direction
- and one or more discriminating variables

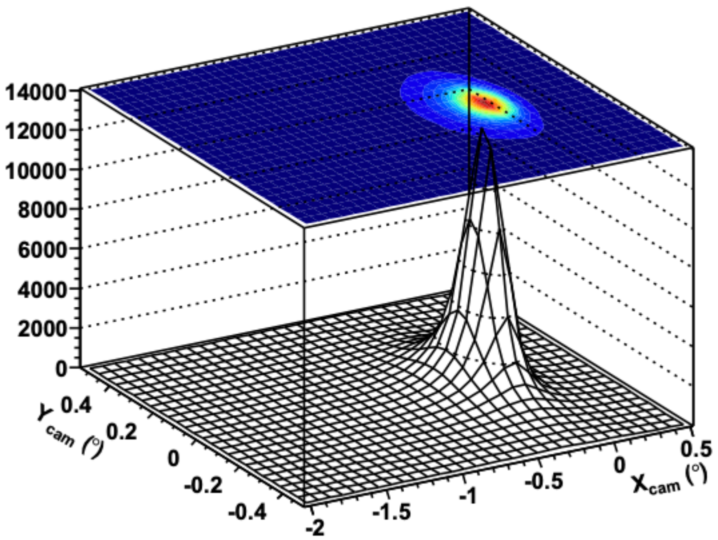
Hillas Method



Krause, 2017 (Thesis)

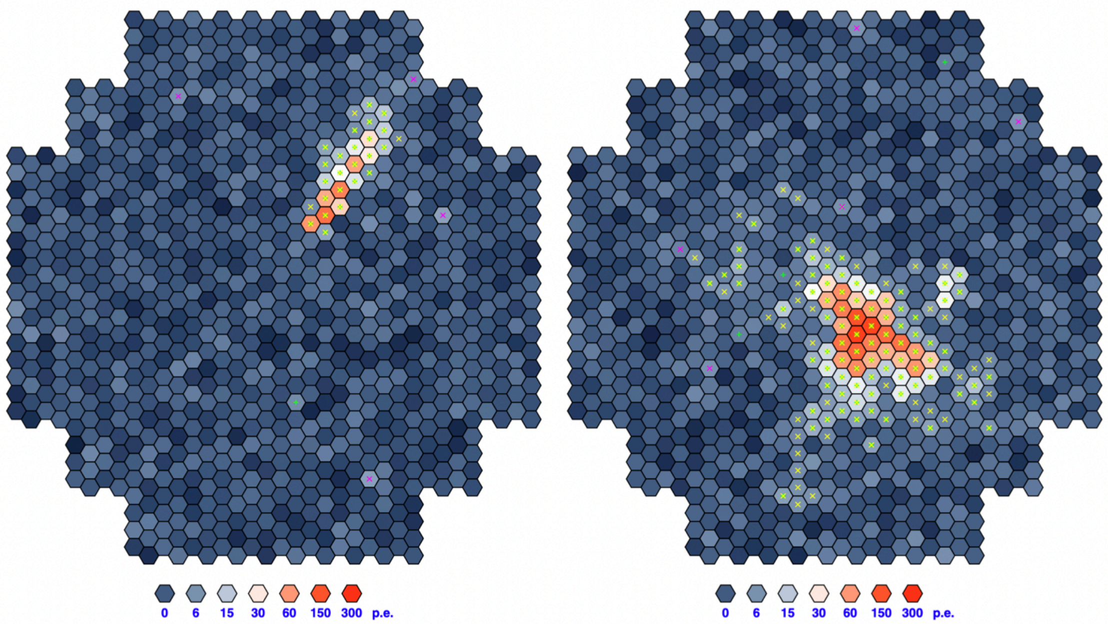
Semi-Analytical Method

MC Template-Based Analysis



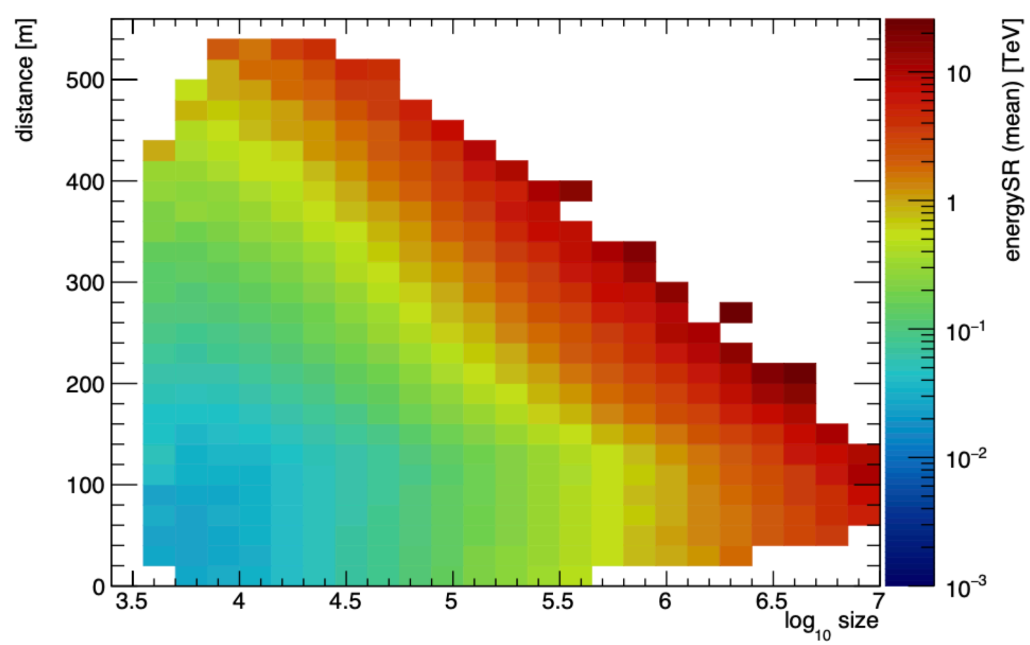
Parsons&Hinton, 2014

Event Reconstruction Techniques

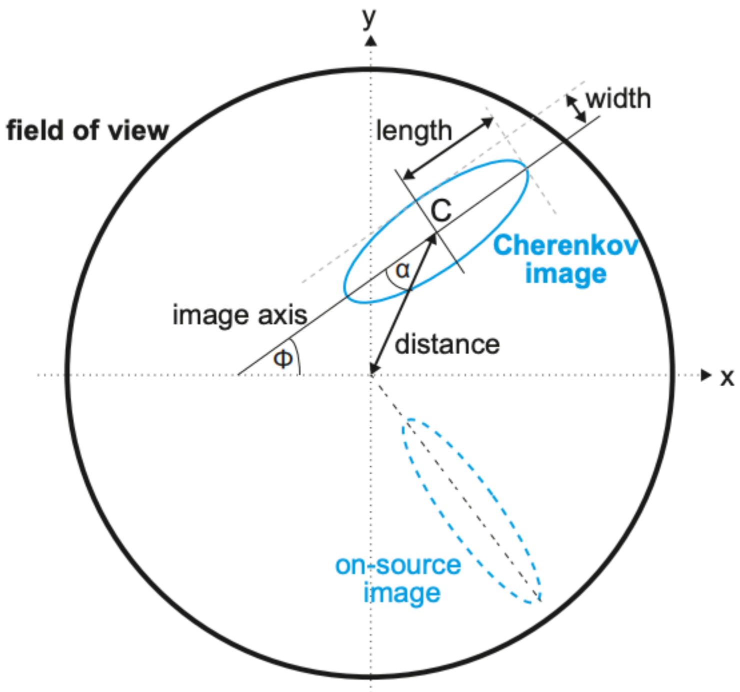


1.0 TeV gamma shower   2.6 TeV proton shower

What we want from the images:



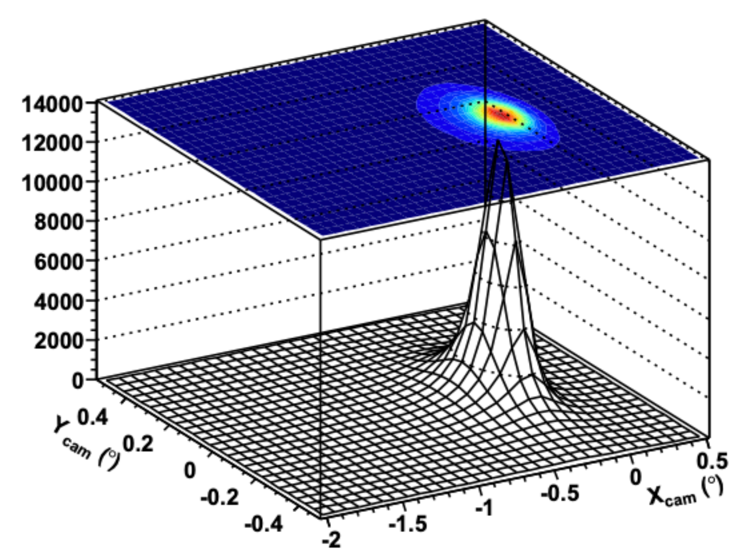
Hillas Method



Krause, 2017 (Thesis)

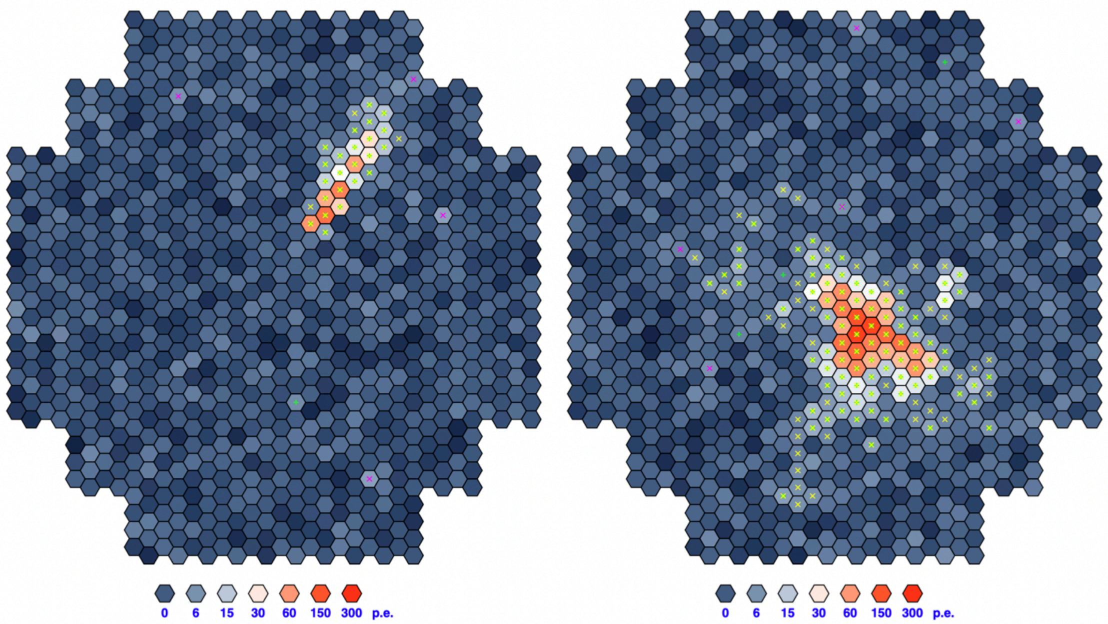
Semi-Analytical Method

MC Template-Based Analysis

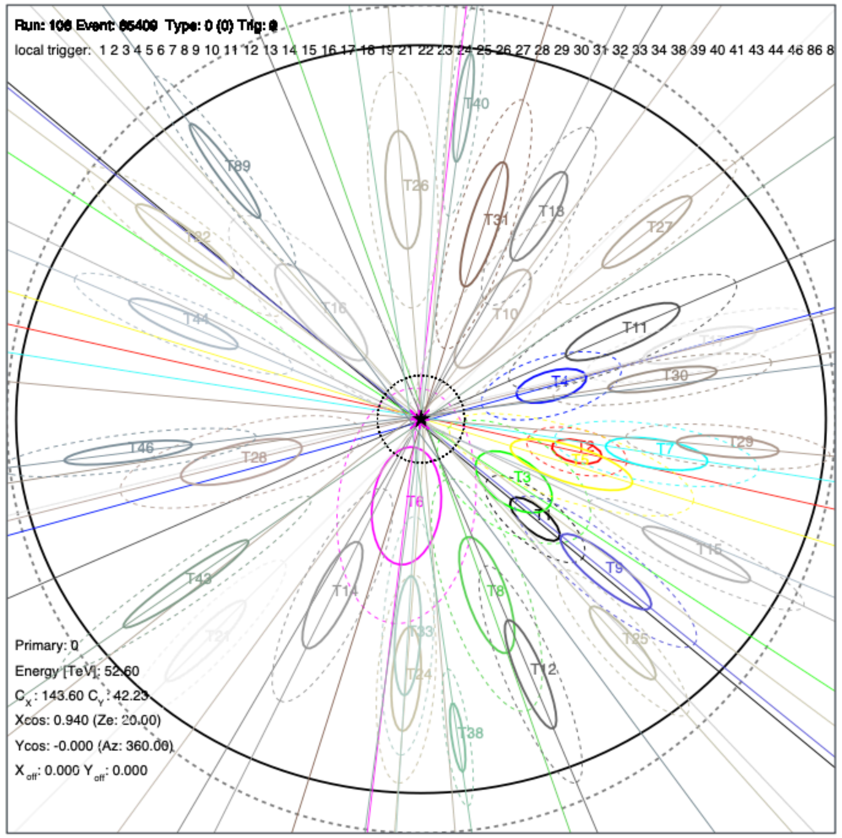


Parsons&Hinton, 2014

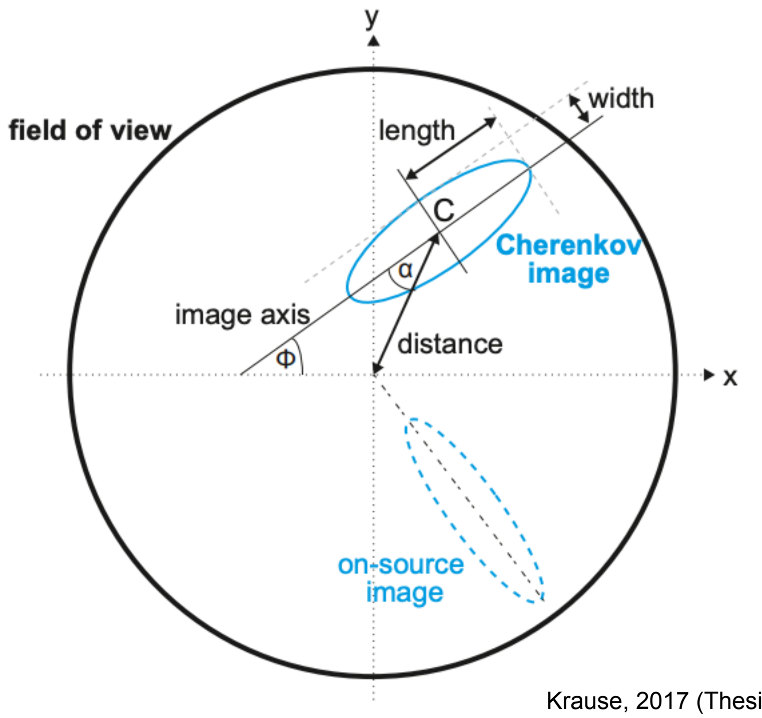
Event Reconstruction Techniques



1.0 TeV gamma shower 2.6 TeV proton shower



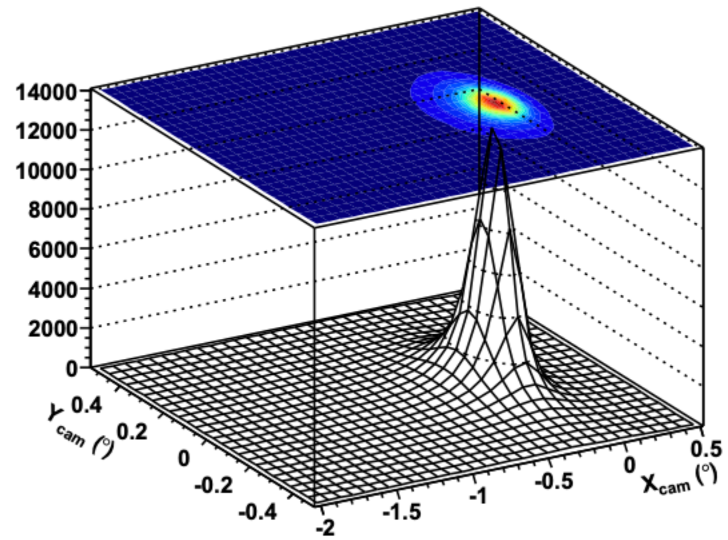
Hillas Method



Krause, 2017 (Thesis)

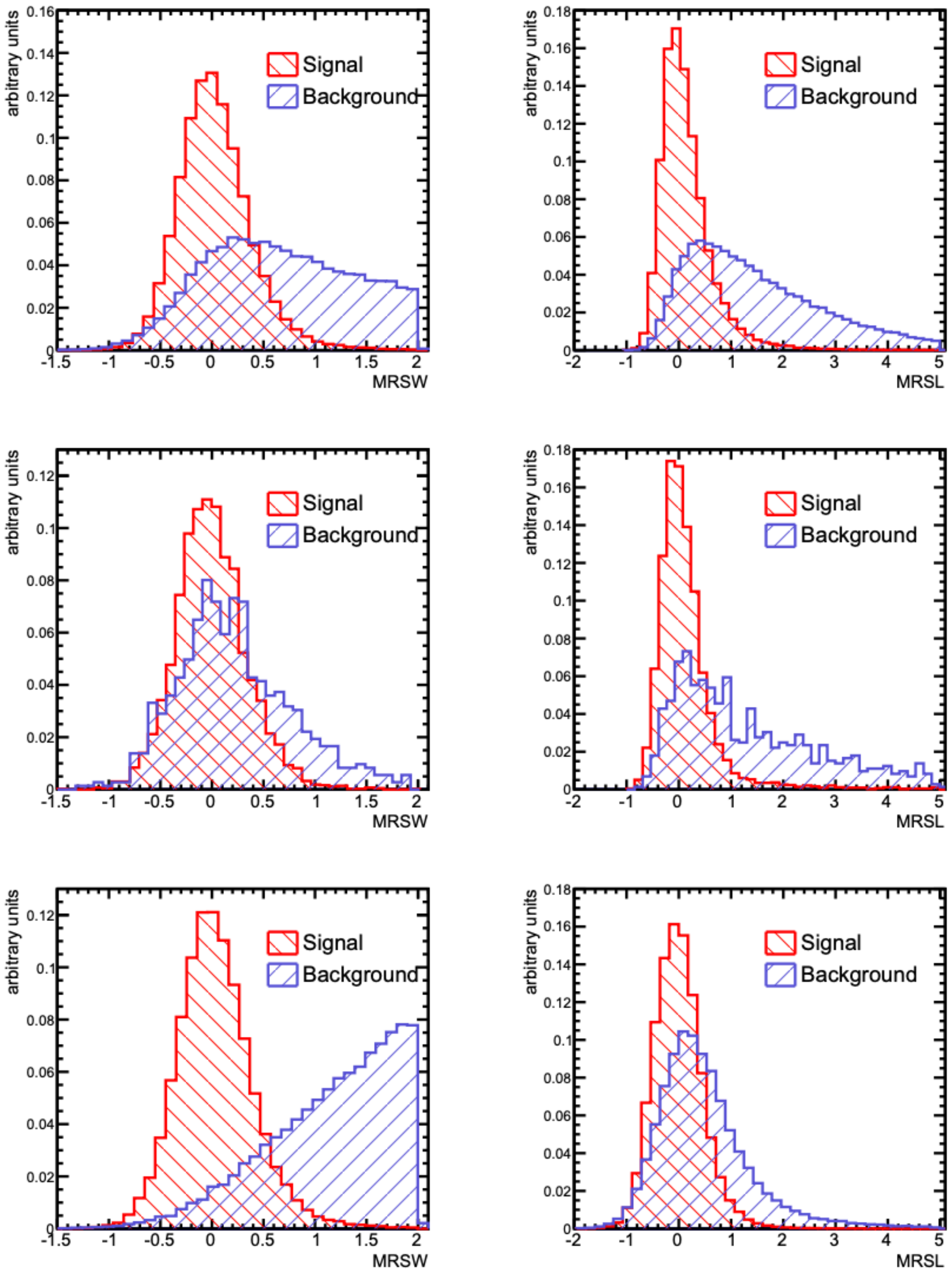
Semi-Analytical Method

MC Template-Based Analysis



Parsons&Hinton, 2014

Multivariate Analysis



Krause, 2015

How likely the event can be associated with a gamma ray

$$Q = \frac{\epsilon_\gamma}{\sqrt{\epsilon_h}} \quad , \quad \epsilon_\gamma = \frac{N_{\gamma,ACC}}{N_\gamma}$$

Increase signal/noise ratio: reducing background

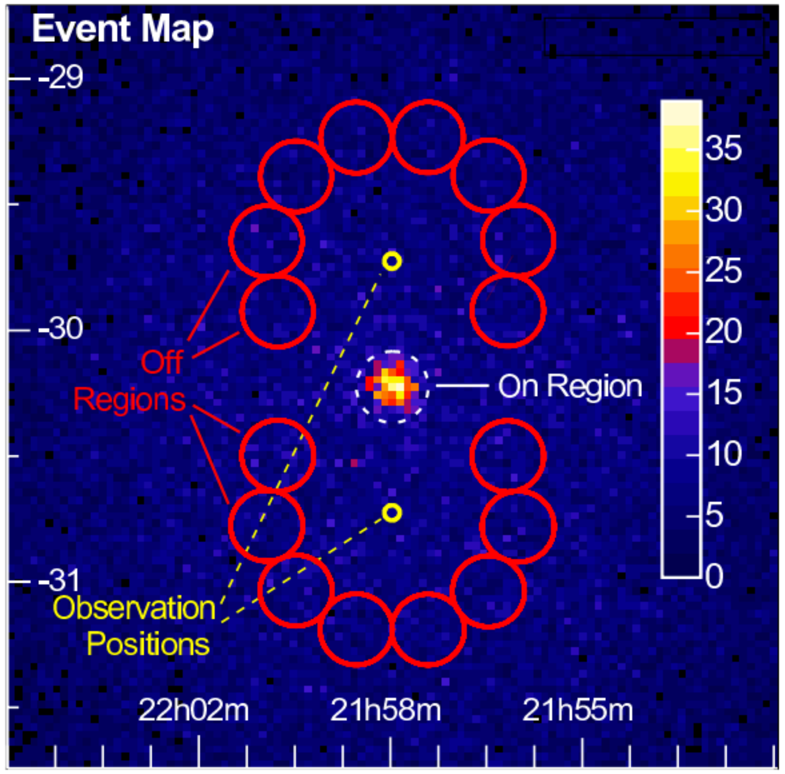
Boosted Decision Trees (BDT): binary tree where events are sorted into small subsets by applying a series of cuts until a given condition is fulfilled. Possible issue: overtraining.

Background Modelling

General case: background unknown

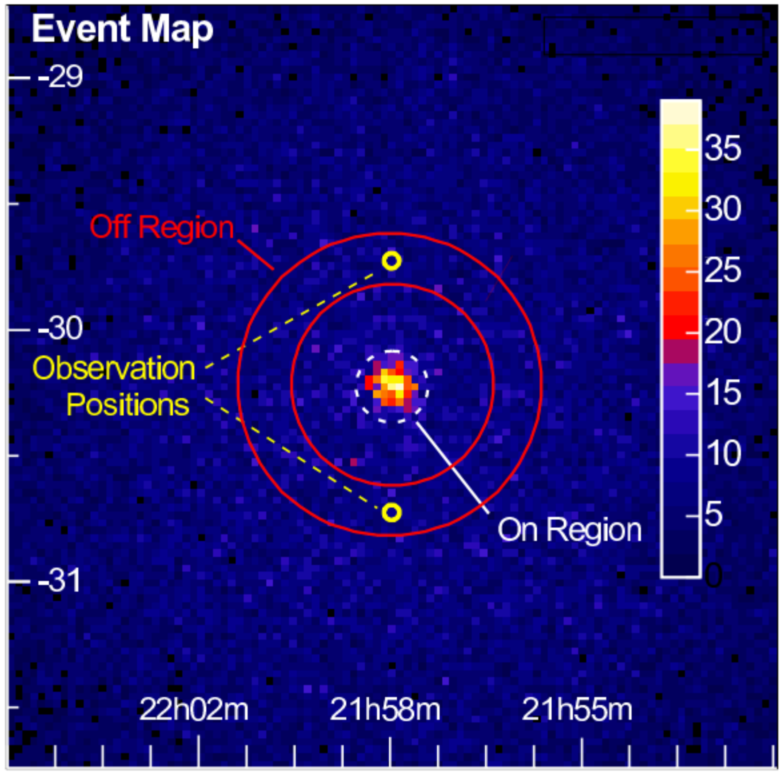
Estimate it by performing OFF measurements, supposedly void of any signal

Wobble or reflected-region background:



Berge, Funk, Hinton, 2006

Ring background



Berge, Funk, Hinton, 2006

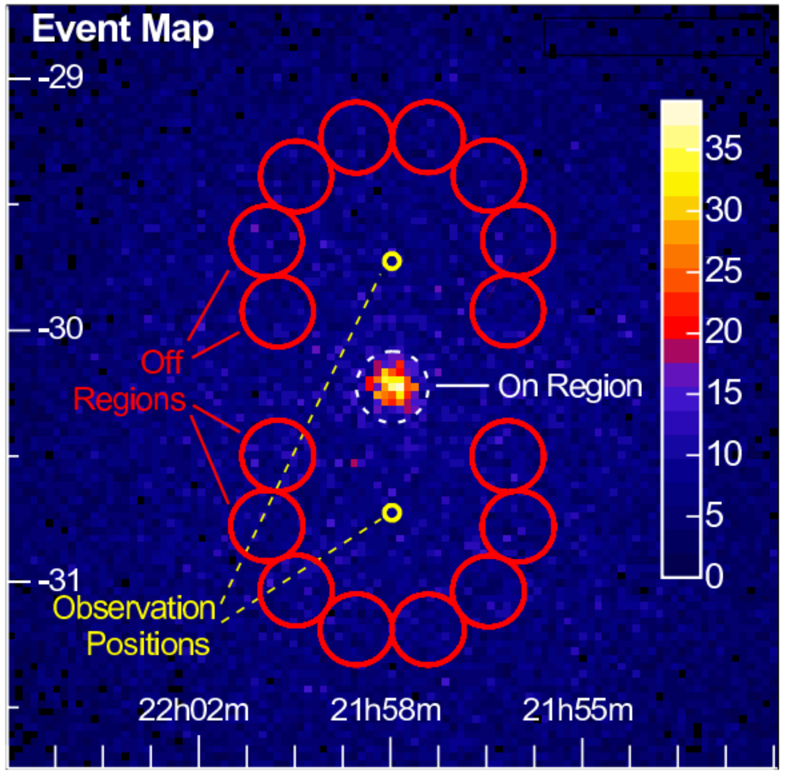
Detection Significance I

Construction of analysis-specific likelihood-ratios and powerful test statistics:

$$-2\ log\ \frac{\mathscr{L}(D_{obs}\,|\,\theta)}{\mathscr{L}(D_{obs}\,|\,\hat{\theta})}$$

Probability of obtaining data D_{obs} assuming parameters θ to be true: $\mathscr{L}(D_{obs}\,|\,\theta) = p(D_{obs}\,|\,\theta)$

What is D_{obs}?



Berge, Funk, Hinton, 2006

ρ number of off regions

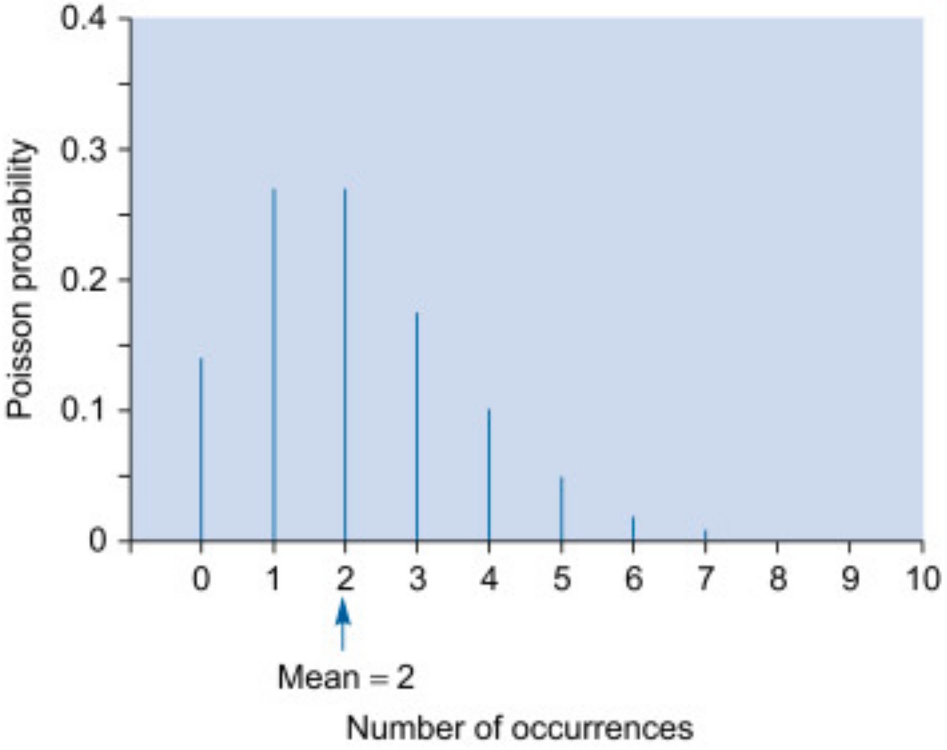
measured counts from all off regions: N_{off}

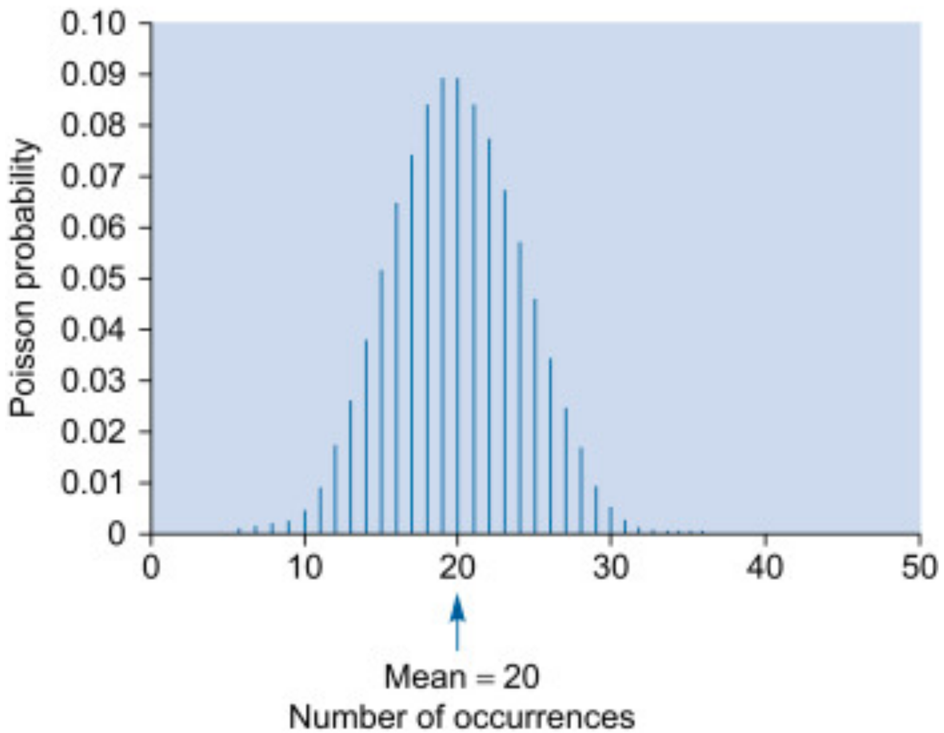
measured counts from on region: N_{on}

We want to estimate number of source events: $< N_s >$
And bkg events: $< N_b >$

Expected bkg events from on region: $\alpha Noff$, $\alpha = 1/\rho$

Photon counts from gamma-ray experiments





For counting experiments:

- integer number of observed events follow a Poisson distribution
- independent events occurring within a fixed time interval
- not always a perfect model for gamma-ray experiments (emission rate is high)

- $P(x = a, \mu) = \dfrac{\mu^a}{a!} e^{-\mu}$
- Standard deviation = square root of the mean: $\sigma = \sqrt{\mu}$
- If a is large, Poisson is approximately normal

Detection Significance III

Let observed data X = (x1, x2, x3, …, xN), unknown parameters $\Theta = (E, T)$ = (e1, ..., en, t1, ..., tn), and statistical hypothesis:

Null hypothesis: $E = E_0$

Alternative hypothesis: $E \neq E_0$

N observed values X given parameters T

$\hat{T}_c$: cond. max. estimate, given $E = E_0$

$$\lambda = \frac{\mathscr{L}(X\,|\,E_0, \hat{T}_c)}{\mathscr{L}(X\,|\,\hat{E}, \hat{T})} = \frac{P(X\,|\,E_0, \hat{T}_c)}{P(X\,|\,\hat{E}, \hat{T})}$$

$\hat{E}, \hat{T}$: maximum likelihood estimates of E and T

Wilk's theorem: if N_{on}, N_{off} are not too few: $-2\ log\ \lambda \sim \chi^2(1)$

If u is a standard normal variable, $u^2 \sim \chi^2(1)$

Detection Significance IV

Let observed data X = (x1, x2, x3, …, xN), unknown parameters $\Theta = (E, T)$ = (e1, ..., en, t1, ..., tn), and statistical hypothesis:

Null hypothesis: $E = E_0$
Alternative hypothesis: $E \neq E_0$

$$\lambda = \frac{\mathscr{L}(X \,|\, E_0, \hat{T}_c)}{\mathscr{L}(X \,|\, \hat{E}, \hat{T})} = \frac{P(X \,|\, E_0, \hat{T}_c)}{P(X \,|\, \hat{E}, \hat{T})}$$

IACT case:

Data:	X = (N_{on}, N_{off})
Unknown par:	T = ($< N_s >$, $< N_b >$)
Null hypothesis:	$< N_s >$ = 0
Alternative hypothesis:	$< N_s > \neq 0$

Detection Significance V

$P(X\,|\,\hat{E},\hat{T}) \longrightarrow$ N_{on}, N_{off} independent. Poisson maximised at:

$$< N_s > = N_{on} - \alpha N_{off}\ ,\ \ < N_b > = \alpha N_{off}$$

$$\boxed{\mathscr{L}(X\,|\,\hat{E},\hat{T}) = P\left[N_{on}, N_{off}\,|\, < N_s > = N_{on} = \alpha N_{off}, < N_b > = \alpha N_{off}\right]}$$

$P(X\,|\,E_0,\hat{T}_c) \longrightarrow$ $< N_s > = 0\ ,\ \ < N_b > = \dfrac{N_{on} + N_{off}}{\rho + 1} = \dfrac{\alpha}{1+\alpha}(N_{on} + N_{off})$

$$\boxed{\mathscr{L}(X\,|\,E_0,\hat{T}_c) = P\left[N_{on}, N_{off}\,|\, < N_s > = 0, < N_b > = \frac{\alpha}{1+\alpha}(N_{on} + N_{off})\right]}$$

$$\mathcal{L}(s,b) = \mathcal{P}(N_{on}\mid s+\alpha b)\cdot\mathcal{P}(N_{off}\mid b) = \\ = \frac{(s+\alpha b)^{N_{on}}}{N_{on}!}e^{-(s+\alpha b)}\cdot\frac{b^{N_{off}}}{N_{off}!}e^{-b}.$$

Detection Significance VI

$$\lambda = \frac{\mathscr{L}(X\,|\,E_0, \hat{T}_c)}{\mathscr{L}(X\,|\,\hat{E}, \hat{T})} = \left[\frac{\alpha}{1+\alpha}\left(\frac{N_{on}+N_{off}}{N_{on}}\right)\right]^{N_{on}} \left[\frac{1}{1+\alpha}\left(\frac{N_{on}+N_{off}}{N_{on}}\right)\right]^{N_{off}}$$

Wilks theorem: if N_{on}, N_{off} are not too few: $-2\ log\ \lambda \sim \chi^2(1)$

If u is a standard normal variable, $u^2 \sim \chi^2(1)$ -> $\boxed{S = \sqrt{-2\ log\ \lambda}}$

Li&Ma

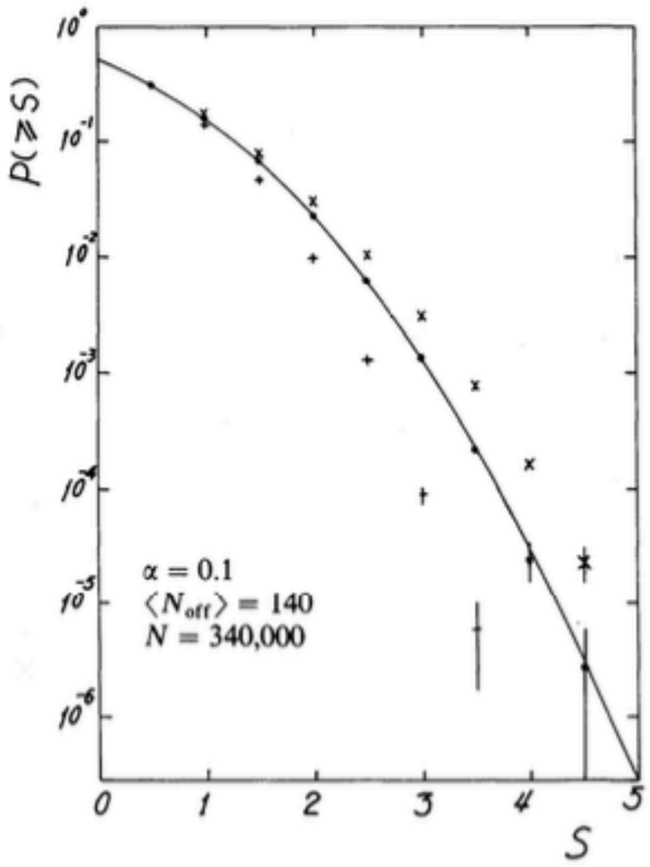


FIG. 2a

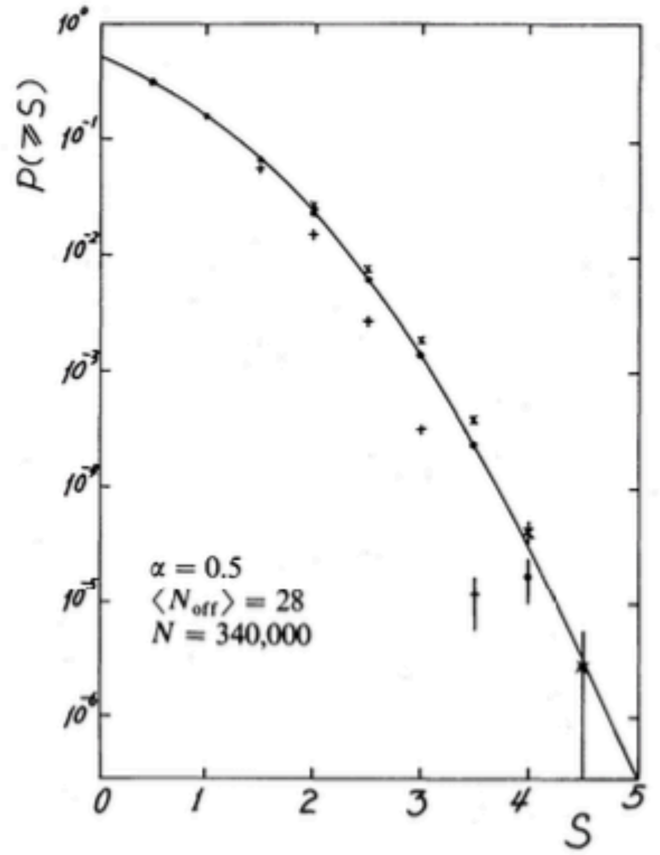


FIG. 2b

+: $S = \frac{N_s}{\sigma(Ns)} = \frac{N_{on} - \alpha N_{off}}{\sqrt{N_{on} + \alpha^2 N_{off}}}$

x: $S = \frac{N_s}{\sigma(Ns)} = \frac{N_{on} - \alpha N_{off}}{\sqrt{\alpha(N_{on} + N_{off})}}$