DARK MATTER CLUMPS (subhalos)

and

ANNIHILATION SIGNAL

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standard scenario:

ADIABATIC and GAUSSIAN FLUCTUATIONS

GENERALITIES

- **Clumps** are the earliest structures produced in the universe.
- They are developed from primordial density fluctuations $\delta(\vec{x}) \equiv \delta\rho/\rho$.
- They are assumed to be adiabatic and Gaussian, with $\nu = \delta/\sigma$ peak-height.

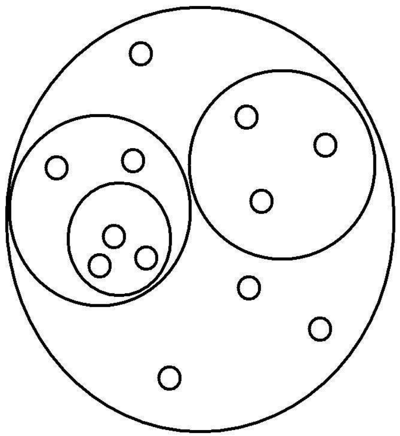
$$P(\delta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\delta^2}{2\sigma^2}\right),$$

- One operates with the Fourier component δ_k.
- The primordial fluctuations are produced at inflation as quantum fluctuations with spectrum $P(k) \equiv \delta_k^2 \sim k^{n_p}$, where $n_p = 1$ for the H-Z spectrum, $n_p = 0.963 \pm 0.014$ WMAP7.
- At radiation-dominated epoch amplitudes grow very slowly $\delta_k \propto \ln t/t_i$, at matter-dominated epoch they grow fast as $\delta_k \sim (t/t_{\rm eq})^{2/3}$, and enter non-linear stage $\delta\rho/\rho \sim 1$ at $t = t_{\rm nl}$.

Press-Schechter (1974) theory: hierarchical structure

- When fluctuation enters non-linear regime, it decouples from expansion of the universe.
- At this moment, t_n, the object reaches maximum size, r_n, and starts collapsing.
- At radius $r_c \sim r_n/2$ the clump is virializing and its contraction stops. The density fluctuation at this moment is $\delta_c = 3(12\pi)^{2/3}/20 \approx 1.7$, and density of the clump is $18\pi^2$ times larger than density of the universe.
- The epoch of formation for clump with mass M and fixed ν (e.g. $\nu = 1$) is determined by $\sigma(M, t_f) = \delta_c$. For clumps with $M \sim 10^{-6} M_\odot$, $z_f \approx 50$.

Hierarchical structures emerge in PS theory due to merging. A small clump belongs to host clump, this host clump is submerged to bigger one etc. The clumps are destroyed in a hierarchical structure by tidal interaction, and number of clumps is small (VB, Dokuchaev, Eroshenko 2003).



NON-LINEAR EVOLUTION OF DENSITY PERTURBATIONS

I. ANALYTIC SOLUTION FOR SINGLE FLUCTUATION

Gurevich and Zybin 1988

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \vec{r}}(\rho \vec{u}) = 0, \qquad \frac{\partial \vec{u}}{\partial t} + (\vec{u}\frac{\partial}{\partial \vec{r}})\vec{u} + \frac{\partial \psi}{\partial \vec{r}} = 0, \qquad \Delta \psi = \rho.$$

. **continuity** **Euler** **Poisson**

Initial conditions at $\delta\rho/\rho \sim 1 : \rho(\vec{r}) = \rho_{\max}(1-\xi^2), \quad \xi^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}.$
Solution for spherically symmetric initial conditions: $\rho(r) \propto r^{-12/7}$.
General case (multistream flow with caustics): $\rho(\mathbf{r}) \sim \mathbf{r}^{-\beta}, \quad \beta = \mathbf{1.7} - \mathbf{1.9}$

II. NUMERICAL SIMULATIONS FOR SMALL CLUMPS

Diemand, Moore, Stadel (2005): $\beta = 1.5 - 2.0,$
Ishiyama, Makino, Ebisuzaki (2010): $\beta = 1.5$.

CORE OF A CLUMP

Its existence follows from convergence of annihilation rate $\int dr\, r^2\, n_X^2(r)$.

What physics provides the core ? (e.g. $n_X = const$ at $r \leq r_c$) ?

1. Instability triggered by slightly non-radial trajectories of DM particles caused by
(i) decaying mode in fluctuation evolution (Gurevich, Zybin 1993) or by
(ii) tidal interaction (VB, Dokuchaev, Eroshenko 2003).
In case **(i)** the core is extremely small $x_c \equiv r_c/R_{\rm cl} \sim \delta_{\rm eq}^3$.
In case **(ii)** the calculated quantity was deflection distance $r_{\rm def} \sim 0.1\ R_{\rm cl}$.
It gives the upper limit $x_c < 0.1$ or $x_c \ll 0.1$.

2. Inverse flux due to annihilation (VB, Gurevich, Zybin 1992), VB, Bottino, Mignola 1995).
3. Fermi degeneration of DM particles (VB, Dokuchaev, Eroshenko, Kachelrieß2010).
4. Phase-space volume conservation (the Liouville theorem).
Numerical simulations are badly needed to determine the size of the core!
Small core $x_c \ll 1$ provides the strong annihilation signal.

TIDAL INTERACTION OF CLUMPS

In spherically symmetric case, including initial conditions, $d\vec{v}/dt = -\vec{\nabla}\phi(r)$ results in singularity $\rho(r) \propto r^{-12/7}$.
Tidal forces produce non-radial velocity $dv_i^{\rm tid}/dt = T_{ij}(t)v_j$, where a force T_{ij} is given by non-spherical gravitational field :

$$T_{ij} = \Phi_{ij} - \frac{1}{3}\Phi_{ll}\delta_{ij}, \quad \Phi_{ij} = \frac{\partial^2\phi}{\partial r_i \partial r_j}$$

Perturbation of radial velocities produces instability and change density distribution.

In case of singularity or small core the density profile is stable. A proof is based on adiabatic invariant

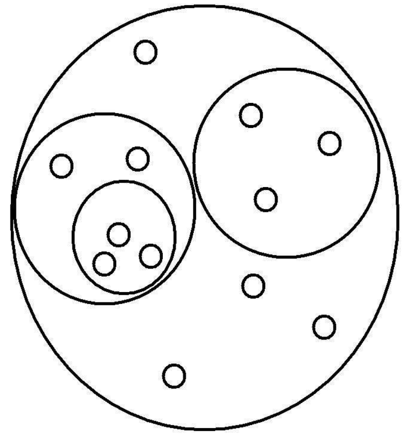
$$I = \int_{-x_m}^{x_m} dx[2(E-\Psi)]^{1/2},$$

where $E = \frac{1}{2}mv^2 + \Psi$, x_m and $-x_m$ are reflection points at $E = \Psi(x_m)$ on caustics.
Physics : At small r DM particles oscillate on orbits with high frequencies, while tidal forces are usually changing slowly, e.g. in hierarchical structures. For large cores and low frequencies, this argument does not work: no adiabatic protection.

TIDAL DESTRUCTION OF CLUMPS IN HIERARCHICAL STRUCTURES

VB, Dokuchaev, Eroshenko 2003 (BDE03)

Hierarchical structure at non-linear stage: A small clump belongs to host clump, this host clump is submerged to bigger one etc. The clumps are destroyed due to asymmetric grav field of smallest host clump. We calculate the density of surviving clumps relative to the total DM density $\xi dM/M$.



Survival probabilities for clumps with given ν and n, index of the power spectrum :

$$\xi(n,\nu) \approx (2\pi)^{-1/2} \exp(-\nu^2/2)(n+3)y(\nu).$$

The PS mass-distribution function (relative mass-density of the clumps) is given by $\xi(n,\nu)d\nu dM/M$ and after integration over ν :

$$\xi_{\rm in} dM/M \approx 0.02(n-3)dM/M \approx 4\times 10^{-3} dM/M$$

Numerical simulations by Moore et al (Nature 2005), recalculated to relative mass density gives : $\xi dM/M \approx 8.3\times 10^{-3} dM/M.$

DESTRUCTION OF CLUMPS IN MILKY WAY

$\rho_{\rm cl} \propto r^{-\beta}$ with $\beta \sim 1.5 - 1.8$: $M \propto \int_{r_c}^{R} dr r^2 \rho_{\rm cl}(r)$, $\dot{N}_{\rm ann} \propto \int_{r_c}^{R} dr r^2 \rho_{\rm cl}^2(r)$.

Mass loss does not influence annihilation rate

Mass loss : (i) grav field of single stars, (ii) bulge, (iii) collective grav field of disc.

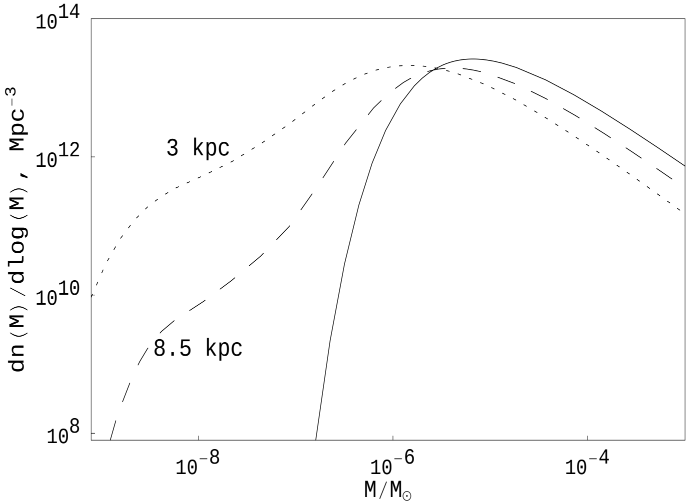
Zhao et al (Silk) 2005: subhalos are fully destroyed producing streams and caustics.

Moore et al 2005: central cores survive and could be detected in γ-rays.

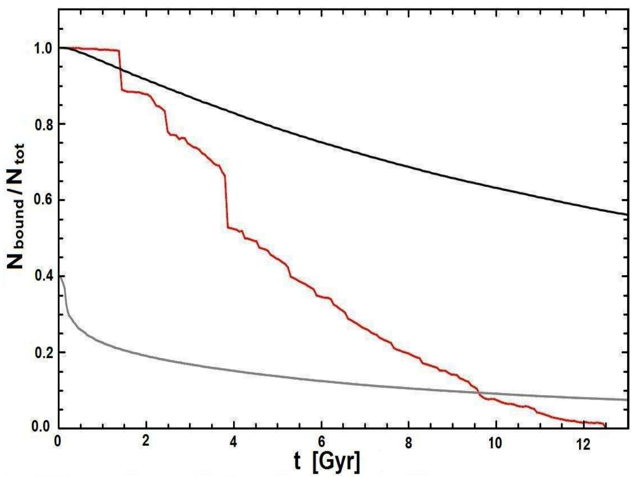
BDE 2008 : adiabatic protection of the core. $\delta E \to \delta E A(a)$. Main effect – from disc-crossing.
Adiabatic parameter $a = \omega\tau_d$. $A(a) = (1 + a^2)^{-3/2}$ is Weinberg adiabatic factor.
At $a \gg 1$, $A \to 0$: full adiabatic protection. For small $x_c \lesssim 10^{-3}$ cores are fully protected.

Schneider, Krauss, Moore 2010 : numerical simulations of mass loss and production of streams, caustics, surviving cores. They do not result in the detectable direct and indirect signal. Disc crossing results in full disruption of the remnant core.

Effect of mass loss in analytic and numerical calculations



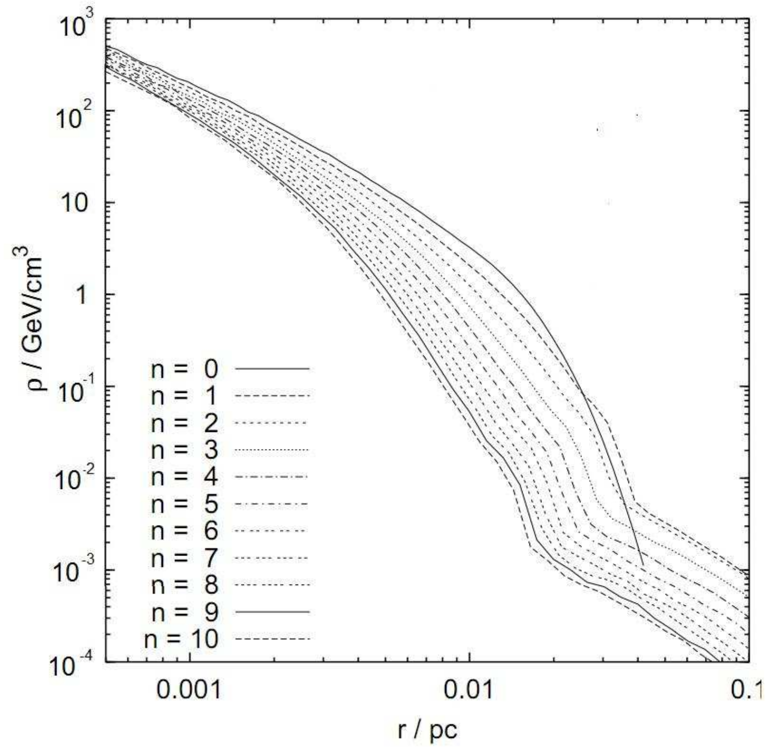
Mass distribution of clumps in Galaxy at 3 and 8.5 kps. The solid curve shows initial mass distribution. BDE 2008.



Ratio of remnant mass to initial mass. Red line corresponds to disc crossings. Schneider, Krauss, Moore 2010.

Clump density profile after successive collisions with stars

B. Moore et al (2006): Numerical simulations.

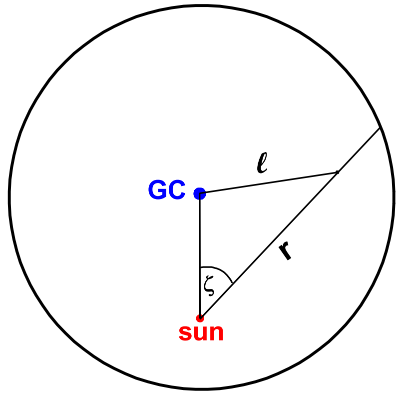


Three effects:

(i) mass loss, (ii) survival of inner part with $\rho(r) \propto r^{-1.6}$, (iii) production of tail.

BOOST OF ANNIHILATION SIGNAL (BDE 2003, 2008).

Because of large density, annihilation in clumps boosts the signal ($\dot{N}_{\rm ann} \propto \rho^2$). Boost relative to single-particle (continuous) component does not depend on properties of DM particles. Gamma-ray flux at Sun position is calculated as:



$$I_{\rm cont}(\zeta) = \int_0^{r_{\rm max}(\zeta)} dr\ \rho_\chi^2(\ell),$$

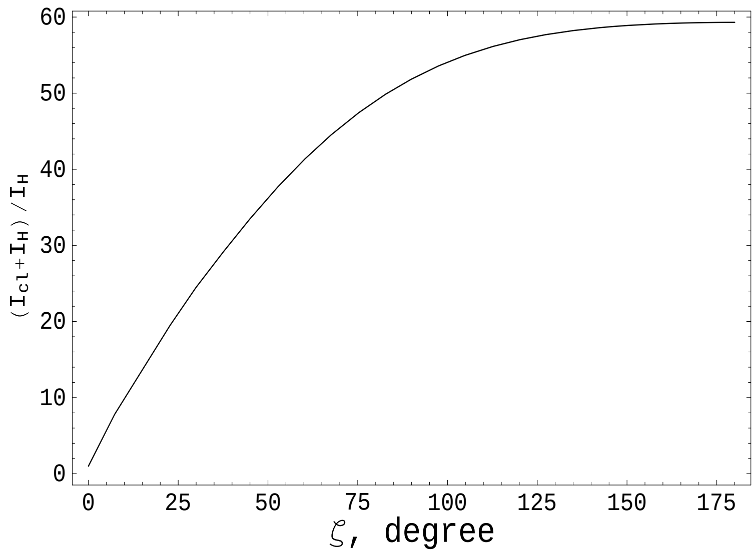
$$I_{\rm cl}(\zeta) = \int_0^{r_{\rm max}(\zeta)} dr \int dM_c \int dr_c n_{\rm cl}^{\rm surv}(\zeta, \ell, M_c, r_c)\dot{N}_{\rm ann}(M_c, r_c).$$

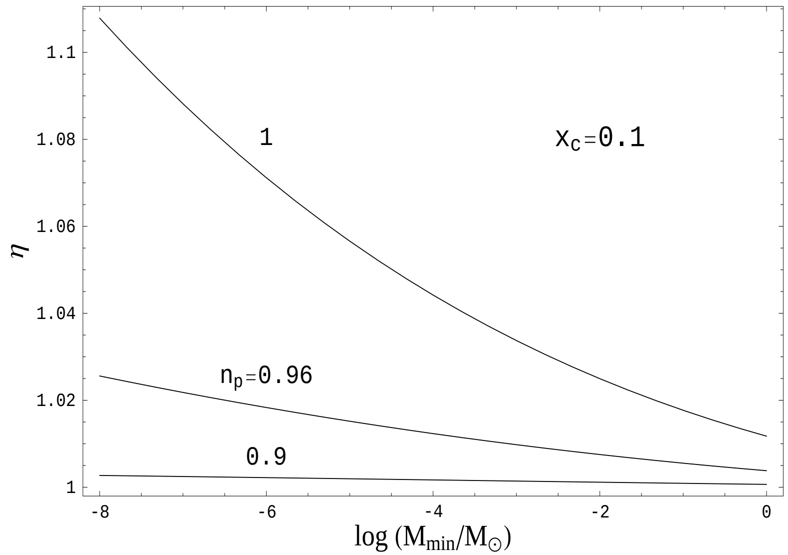
Boost in the direction ζ :

$$\eta(\zeta) = [I_{\rm cont}(\zeta) + I_{\rm cl}(\zeta)]/I_{\rm cont}(\zeta).$$

In directions close to GC the clumps are destructed stronger, and boost is small. At $\zeta \sim \pi$ the boost is strong, but absolute value of signal is small.

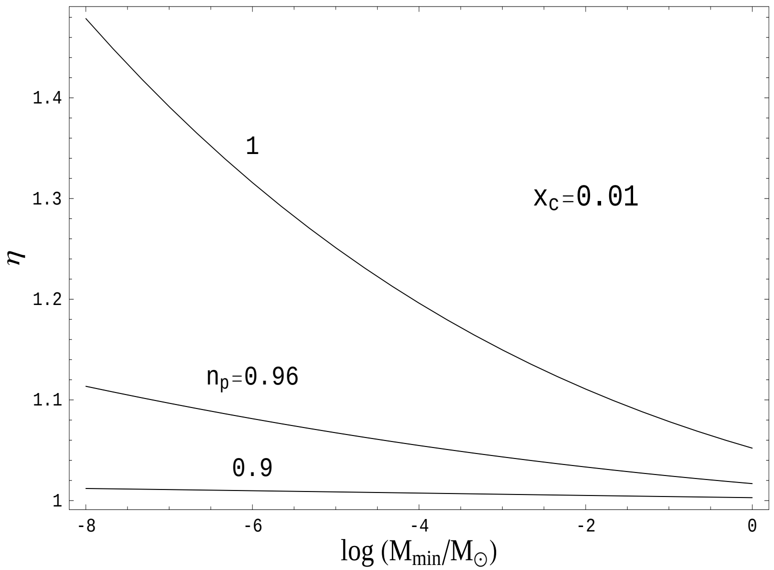
BOOST OF ANNIHILATION SIGNAL (BDE 2003, 2008).

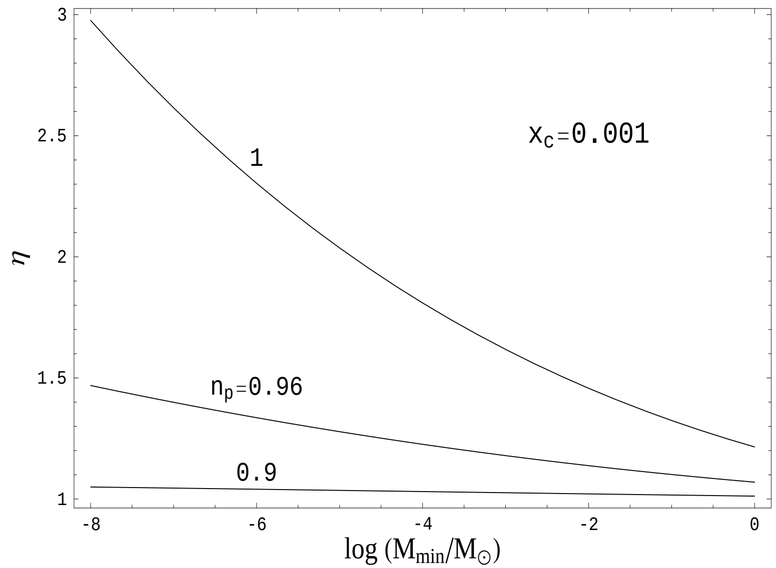




with galactic destruction.

without galactic destruction.





MINIMAL CLUMP - MASS.

provided by wiping off the fluctuations during and after kinetic decoupling.

It occurs in three processes:

(i) Diffusion of DM particles χ from a fluctuation.

(ii) Free streaming (soon after decoupling).

(iii) Damping the oscillation modes crossing the horizon (streaming with friction).

Physics:

In the decoupling process scattering length $\chi\ell \to \chi\ell$ increases and χ can escape from a fluctuation first by diffusion (from small-size inhomogeneity) and after decoupling accomplished – by free streaming. However, the largest scale fluctuations are washed out by streaming with friction.

Interaction of DM particles with plasma particles is crucial.

$M_{\rm min}$ estimate for neutralino with $m_\chi \sim 100$ GeV and sfermions with $\tilde{m} \sim 200$ GeV.

Decoupling temperature: $T_d \sim 10$ MeV $\quad (t_d \sim 7 \times 10^{-3}$ s) .

diffusive cutoff: $M_{\rm diff} \sim 10^{-13}\ M_\odot$, free streaming: $M_{\rm fs} \sim 10^{-7} - 10^{-6}\ M_\odot$,
streaming with friction: $M_{\rm fs} \sim 10^{-6} - 10^{-5}\ M_\odot$.

$$M_{\rm min} = sup(M_i) \sim 10^{-6} - 10^{-5}\ M_\odot,$$

All scales below $M_{\rm min}$ are washed out.

COMPARISON OF EXISTING CALCULATIONS FOR NEUTRALINO.

recalculated to $m_\chi = 100$ GeV and $\tilde{m} = 200$ GeV

authors	SHS 01	BDE 03	GHS 05	LZ 05	Bert 06
T_d, MeV	28	26	25	20	22.6
$M_{\rm min}/M_\odot$	2.5×10^{-7}	1.7×10^{-7}	1.5×10^{-8}	1.3×10^{-5}	8.4×10^{-6}

Supersymmetric models

SUGRA models with soft-breaking parameters $m_0,\ m_{1/2},\ A_0,\ \beta$ provide radiative EWSB. Benchmark scenarios include cosmological and elementary-particle restrictions. E.g. benchmark scenarios by Ellis et al 04 has parameters similar to ones above.

scenario	χ	$\tilde{e}_L$	$\tilde{e}_R$	$\tilde{\nu}$
B' GeV	95	188	117	167
E' CeV	112	1543	1534	1535

Profumo, Sigurdson, Kamionkowski 2006:
a few MeV $\lesssim T_d \lesssim$ a few GeV, $3 \times 10^{-12} \lesssim M_{\rm min}/M_\odot \lesssim 3 \times 10^{-4}$
Inclusion of KK particles further expands the range of allowed parameters.

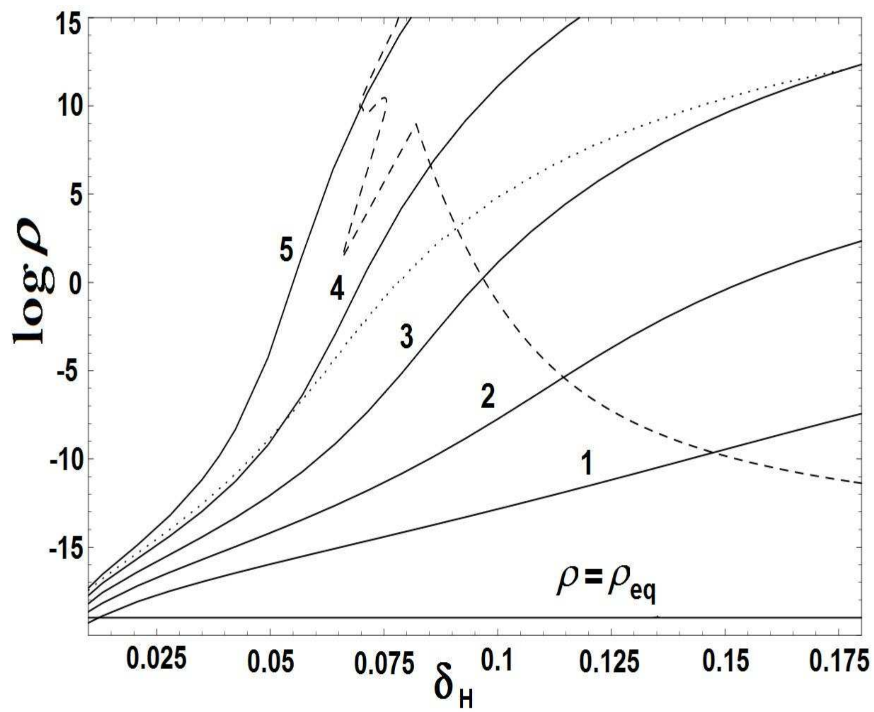
NON-STANDARD LORE: SUPERDENSE CLUMPS.

VB, Dokuchaev, Eroshenko, Kachelrieß, Solberg 2010.

Two possible scenarios for superdense clumps:
(i) Production of large-amplitude fluctuations with continuous spectrum in RD dominated epoch, (ii) production of fluctuation as a sharp peak (spike). Example of (i): seeded fluctuations (e.g. seeded by cosmic strings – Kolb, Tkachev 1994). Example of (ii): peculiarity in inflationary potential, e.g. a flat segment (Starobinsky 1992), $\delta_H \sim V^{3/2}/V' \to \infty$. In spiky scenarios clumps have almost fixed mass.

In scenario (i) fluctuations have continuous spectrum and results in production of the wide range of clump masses. The basic assumption is the large amplitude at production in RD epoch $(\delta\rho/\rho)_{prod} \equiv \Phi \gg 1$. Clump evolves non-linearly in RD epoch and it provides the large density of a clump.

ρ is mean density of a clump in g/cm^3, δ_H is perturbation of radiation density, curves 1 – 5 are for clump masses $M = 10^{30},\ 10^{20},\ 10^{10},\ 1,\ 10^{-10}$ g .



Spiky scenario.

Spike in perturbation spectrum $P(k)$ can be produced at inflation or during RD epoch. Spike is expected to be accompanied by usual power-law spectrum. Spike results in clumps with fixed mass. Fraction of such clumps cannot exceed 1/2. Direct detection by gravitational arrays (e.g. LISA) is possible.

Common properties of superdense clumps.

- Clump density profile $\rho(r) \propto r^{-\beta}$ is produced earlier than hierarchical structure: no tidal destruction.
- Destruction in the Galaxy is less or absent.
- **Ordinary neutralino is strongly disfavoured due to excess of γ-ray flux.**
- Superheavy neutralino is a plausible DM candidate.

Gravithermal instability for superdense clumps from superheavy neutralinos

Gravithermal instability is known for globular clusters. It sets by two-body gravitational relaxation and results in isothermal density profile $\rho(r) \propto r^{-2}$. Case of superheavy particles in superdense clump is identical.

$$t_{\rm rel}^{\rm grav} = \left(\frac{1}{E}\frac{dE}{dt}\right)^{-1} = \frac{1}{4\pi}\frac{v^3}{G^2 m_\chi^2 n_{\rm cl}\ln(0.4N)}$$

CONCLUSIONS

- Small-mass clumps (subhalos) are reliably predicted objects, which number density is calculated analytically (e.g. in the Press-Schechter theory) and in numerical simulations.
- Because of the high clump's density, the annihilation signal is strongly amplified. The boost-factor = $(J_{\rm clump} + J_{\rm single-part})/J_{\rm single-part}$ depends only on clump's phenomenological parameters ($n_p = 0.96$, β, $r_{\rm core}$, $M_{\rm min}$) and does not depend directly on elementary-particle parameters, e.g. $\sigma_{\rm ann}$ (BDE2003).
- The clump's density-profile is $\rho(r) \propto r^{-\beta}$ with $\beta \approx 1.5 - 2.0$. Core, in the form $\rho(r) \to const$ or $\rho(r)$ flattening, must be there to provide convergence of annihilation rate.
- Once produced, cusp and core are protected by adiabatic invariant.
- Clumps are efficiently destroyed by tidal interaction in the hierarchical structures at production, when cusp/core are not yet formed. Surviving probability strongly depends on n_p and $M_{\rm min}$. For $n_p = 0.96$, $M_{\rm min}/M_\odot \gtrsim 10^{-7}$ and $x_c = r_{\rm core}/R \gtrsim 0.01$, the boost-factor is small ($\lesssim 1.4$).

- The survived clumps can be further destroyed in the Galaxy.
  Silk et al 2005: all clumps are fully destroyed,
  Moore et al 2010: disc crossings can fully disrupt a clump.
  BDE 2008: Dense cores survive due to adiabatic protection. Annihilation signal becomes weaker, but little. The crucial parameter is $r_{\rm core}$.

- $M_{\rm min}$ for neutralino clumps (recalculated to $m_\chi \sim 100$ GeV and $\tilde{m} \sim 200$ GeV).
  Diffusion: $M_{\rm min}/M_\odot \sim 10^{-13}$
  Free streaming: $M_{\rm min}/M_\odot \sim 10^{-7} - 10^{-6}$
  Streaming with friction: $M_{\rm min}/M_\odot \sim 10^{-6} - 10^{-6}$
  All scales are too large for the big enough boost-factor !

- **Uncertainties and other possibilities.**
  Kamionkowski et al 2006: Uncertainties in SUSY parameters.
  $3 \times 10^{-12} \leq M_{\rm min}/M_\odot \leq 3 \times 10^{-4}$ – boost-factor is large.
  The other types of DM particles, e.g. KK-particles.
  Superdense clumps: the boost-factor strongly increases.

The large boost factor is possible !

EPILOGUE

Physics is an observational science. However, sometimes, e.g. in Lattice QCD, the direct detailed experiment is impossible. Specialists refer to numerical simulations in this case as to experiment. I think that for study of clumps numerical simulations play the similar role as compared with analytic approach.

“ We rarely missed an opportunity to be led astray, and it was only new, laboriously obtained experimental data that put as back on the right track.”

M. Schwarzschild   from “Structure and evolution of the stars”