Radiative corrections to dark matter annihilation (and the relic density)

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Outline

Relic density calculation, neutralino dark matter, and motivation for radiative corrections

DM@NLO – Strong corrections

Herrmann, Klasen, Phys. Rev. D76: 117704 (2007)
Herrmann, Klasen, Kovarik, Phys. Rev. D79: 061701 (2009)
Herrmann, Klasen, Kovarik, Phys. Rev. D80: 085025 (2009)
Harz, Herrmann, Klasen, Kovarik, Le Boulc'h (ongoing work)
Freitas, Phys. Lett. B652: 280 (2007)

SloopS – Electroweak corrections

Boudjema, Semenov, Temes, Phys. Rev. D72: 055024 (2005)
Baro, Boudjema, Semenov, Phys. Lett. B660: 550-560 (2008)
Baro, Boudjema, Sememov, Phys. Rev. D78: 115003 (2008)
Baro, Boudjema, Phys. Rev. D80: 076010 (2009)
Baro, Boudjema, Chalons, Hao, Phys. Rev. D81: 015005 (2010)

Conclusion and perspectives

Relic density calculation

Number density of thermal relic (dark matter) governed by Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\rm ann} v \rangle \left(n^2 - n_{\rm eq}^2 \right)$$

$$\Omega_{\rm CDM} h^2 = \frac{m_\chi n_0}{\rho_c} \sim \frac{1}{\langle \sigma_{\rm ann} v \rangle}$$

Particle physics enters via (co-)annihilation of dark matter into SM particles

Thermal average accounts for velocity distribution of non-relativistic dark matter

$$\langle \sigma_{\rm ann} v \rangle = \int {\rm d}s \; f_v(s) \; \sigma_{\rm ann}(s)$$

Cosmological measurements allow to identify (dis)favoured regions of parameter space

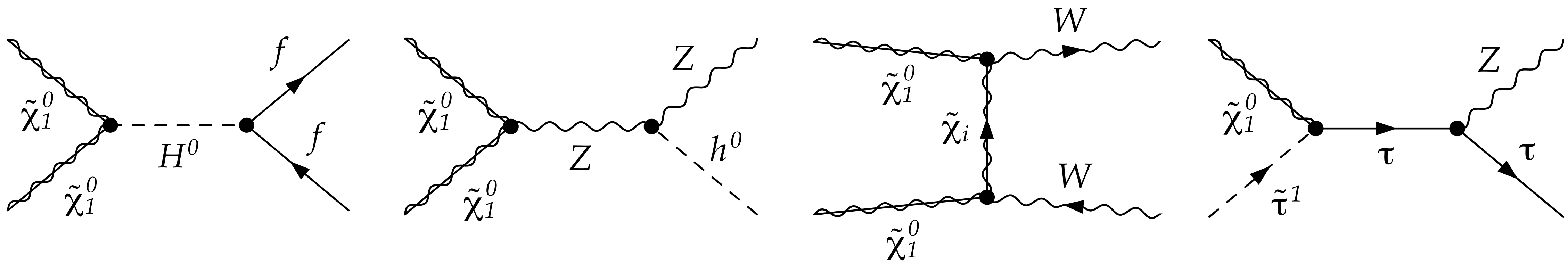
$$0.1018 < \Omega_{\rm CDM} h^2 < 0.1228$$

Public computational tools:

- DarkSUSY [Bergström, Edsjö, Gondolo, Ullio *et al.* 2004-2011]
- micrOMEGAs [Boulanger, Boudjema, Pukhov *et al.* 2003-2011]
- SuperIso Relic [Arbey, Mahmoudi, 2008-2011]

Motivation for radiative corrections

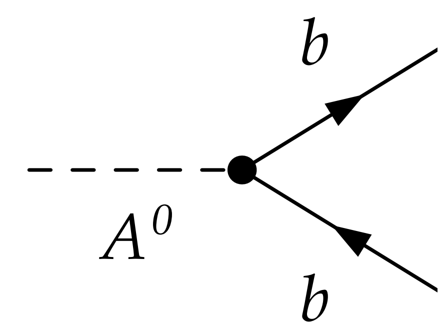
All relevant (co-)annihilation processes implemented − but only at leading order



Higher order effects included only for very sensitive quantities

e.g. effective bottom Yukawa coupling

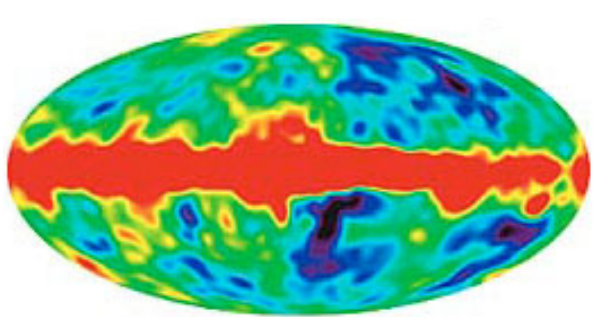
$$h_{Abb} \propto \frac{\overline{m}_b(Q)}{1+\Delta_b}\tan\beta$$



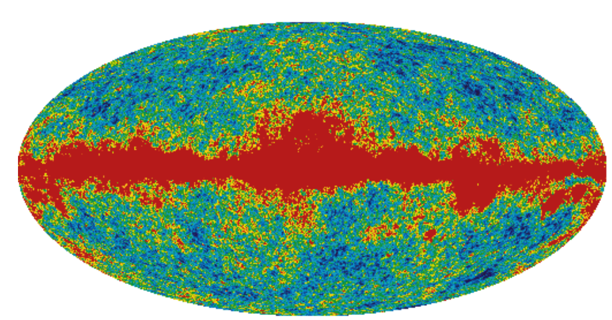
Loop and real emission corrections give important corrections to cross-sections

Sizeable impact on relic density of dark matter from strong and electroweak corrections

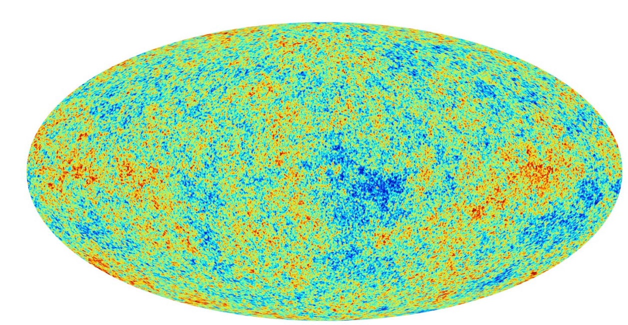
More precise theoretical predictions needed to match the experimental improvements



COBE 1989

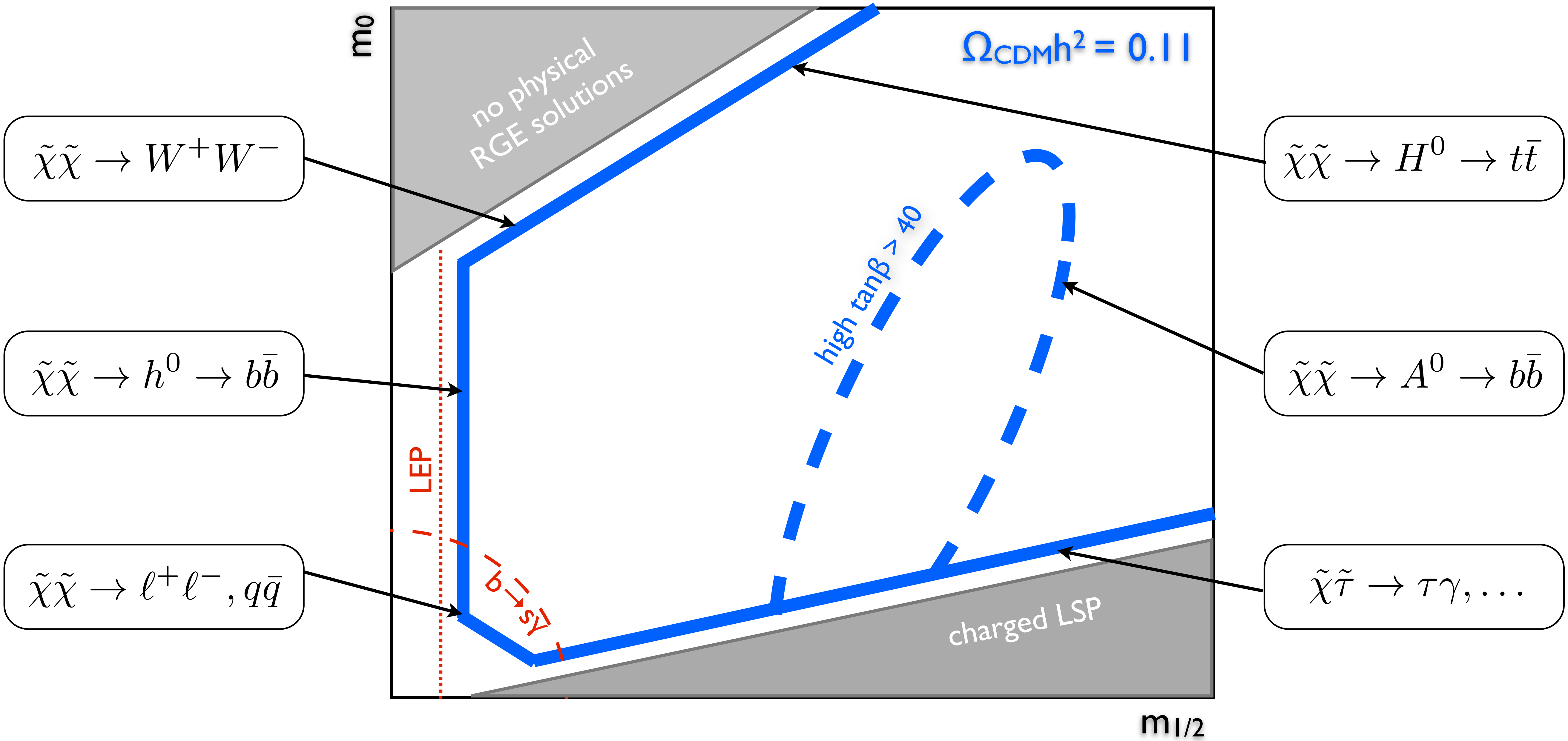


WMAP 2002



Planck 2011

Neutralino annihilation in the CMSSM



Constrained MSSM ("mSUGRA"): Five universal parameters at the GUT scale

$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \text{sgn}(\mu)$$

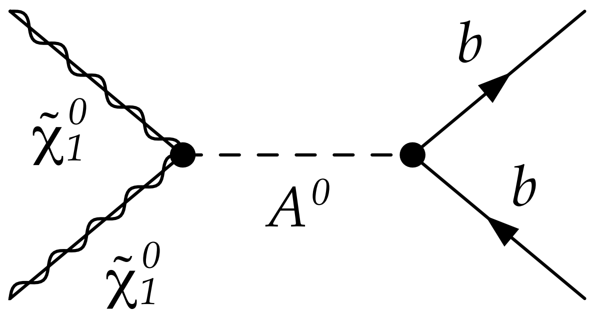
Neutralino annihilation in the MSSM

$\tilde{\chi}^0_1\tilde{\chi}^0_1 \to h^0, H^0, A^0 \to b\bar{b}, t\bar{t}$	Constrained MSSM
$\tilde{\chi}^0_1\tilde{\chi}^0_1 \to Z^0, \tilde{q} \to q\bar{q}$	CMSSM variants with non-universal Higgs/gaugino masses
$\tilde{\chi}^0_1\tilde{\chi}^0_1 \to W^+W^-$	Constrained MSSM
$\tilde{\chi}^0_1\tilde{\chi}^0_1 \to Z^0Z^0, Z^0h^0$	General MSSM
$\tilde{\chi}^0_1\tilde{\tau}_1 \to \tau\gamma, \tau Z^0, \ldots$	Constrained MSSM, MSSM beyond MFV
$\tilde{\chi}^0_1\tilde{t}_1 \to tg, th^0, tZ^0, \ldots$	CMSSM with large trilinear couplings, MSSM beyond MFV
$\tilde{\chi}^0_1\tilde{\chi}^\pm_1 \to u\bar{d}, W^\pm Z^0, \ldots$	MSSM with wino-LSP
$\tilde{\chi}^0_1\tilde{\chi}^0_2 \to u\bar{d}, Z^0Z^0, W^+W^-, \ldots$	MSSM with mixed bino/wino-LSP

Example: Corrections in the "A-Funnel" region

Herrmann, Klasen (2007)

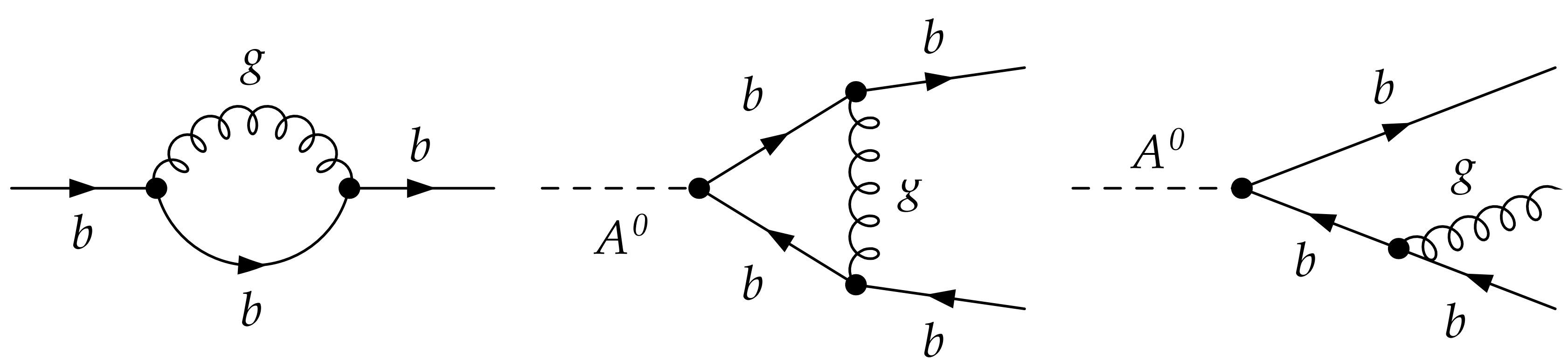
At large tanβ, annihilation into bottom-quarks through pseudoscalar Higgs exchange dominant



$$\sigma_{\rm ann} v \;\sim\; \frac{\beta_b\; h^2_{Abb}\; T^2_{A11}}{\left|s - m_A^2 + i\Gamma_A m_A\right|^2} \;\sim\; m_b^2 \tan^2\beta$$

Standard model QCD corrections include self-energy, vertex correction, and real emission

On-shell renormalization to cancel UV-divergences within the virtual part



Dipole subtraction method to combine virtual and real contributions (and cancel IR-divergences)

$$\sigma_{\rm NLO} \;=\; \left[\sigma_{\rm V} + \int {\rm d}\sigma_{\rm aux}\right]_{\epsilon=0} + \int \left[{\rm d}\sigma_{\rm R} - {\rm d}\sigma_{\rm aux}\right]_{\epsilon=0} \;=\; \sigma_{\rm LO}\left[1 + \Delta_{\rm QCD}\right]$$

[Catani, Seymour *et al.* 2000]

Example: Corrections in the "A-Funnel" region

Herrmann, Klasen (2007)

Resummation of mass singularity using the renormalization group (running quark mass)

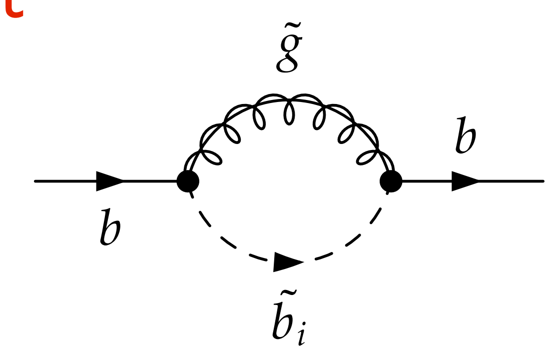
$$\Delta^{(\mathrm{HE})}_{\mathrm{QCD}} \simeq \frac{\alpha_s C_F}{\pi}\left[-\frac{3}{2}\log\frac{s}{m_b^2}+\frac{9}{4}\right] \qquad h_{Abb} = \frac{g m_b \tan\beta}{2m_W} \longrightarrow \frac{g\overline{m}_b(Q)\tan\beta}{2m_W}$$

Higher order QCD corrections known up to $O(\alpha_s^4)$

Top-quark induced two-loop correction negligible for large tanβ

Supersymmetric QCD corrections include self-energy, vertex correction, and real emission

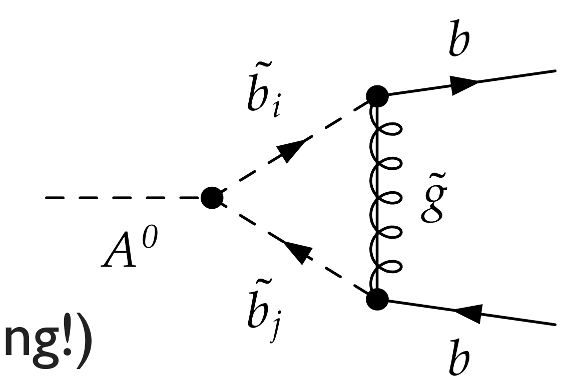
On-shell renormalization to cancel UV-divergences within the virtual part



$$\Delta_{\mathrm{SUSY}} = \frac{\alpha_s(s)}{\pi}C_F\left[1+\frac{1}{\tan^2\beta}\right]m_{\tilde{g}}\mu\tan\beta\; I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})$$

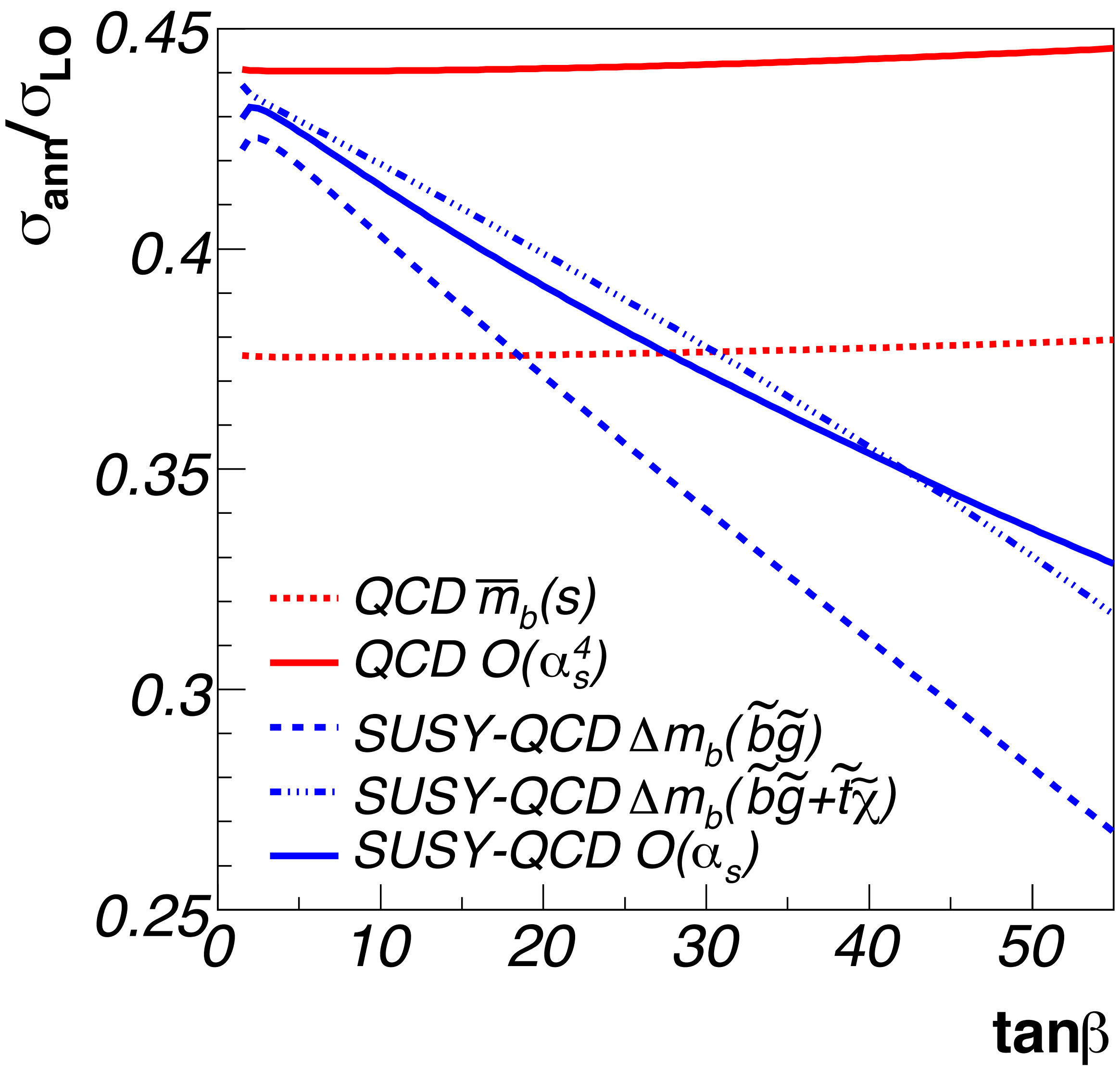
Resummation of mass corrections significant for high tanβ or large A_b

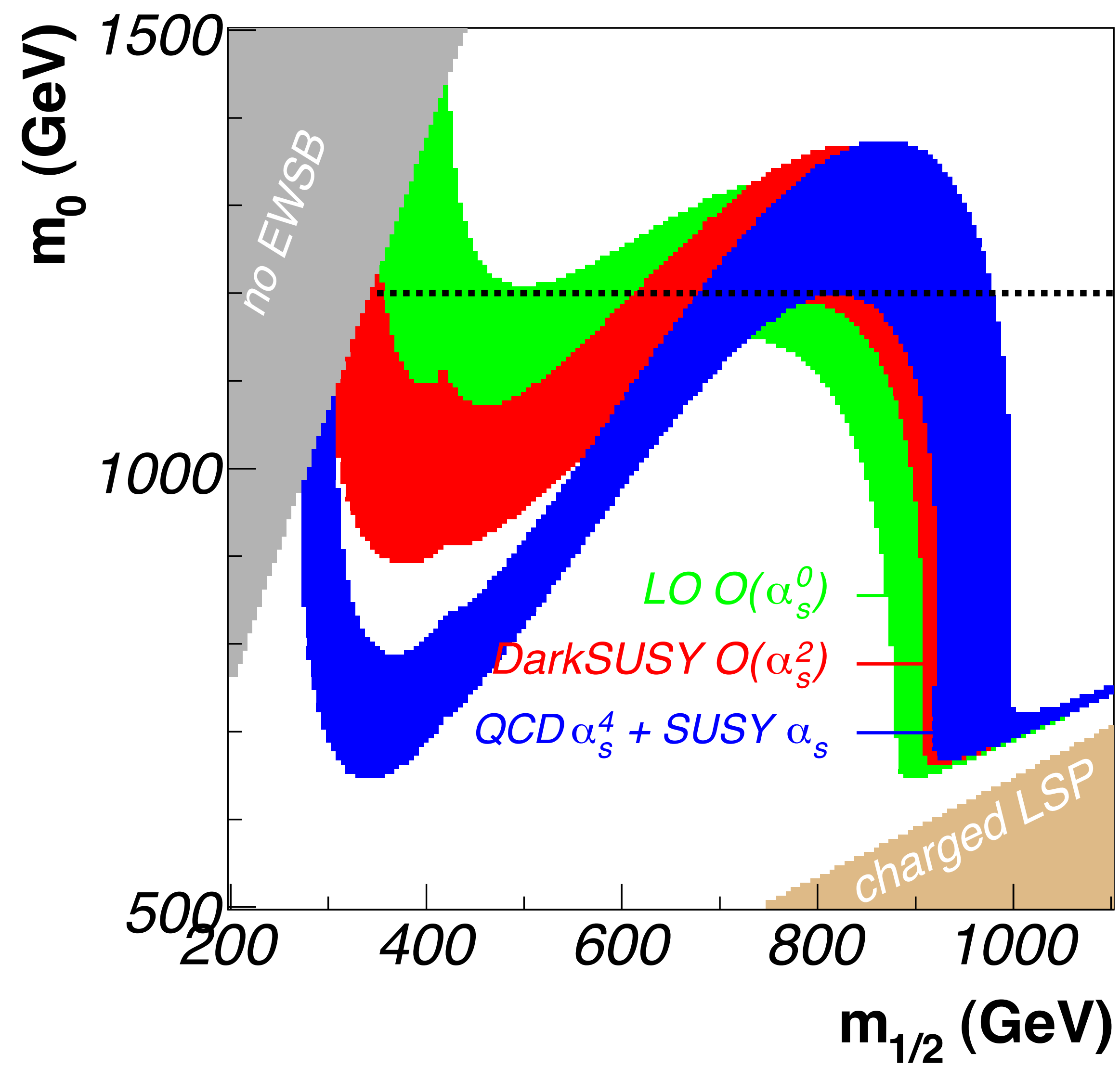
$$\overline{m}_b(s) \longrightarrow \frac{\overline{m}_b(s)}{1+\Delta m_b/m_b}$$



Include also contribution from stop-chargino loop (large top Yukawa coupling!)

Example: Corrections in the "A-Funnel" region





Cosmologically favoured regions shifted to smaller masses to compensate smaller cross section

Effect reversed around the resonance due to simultaneously corrected Higgs width

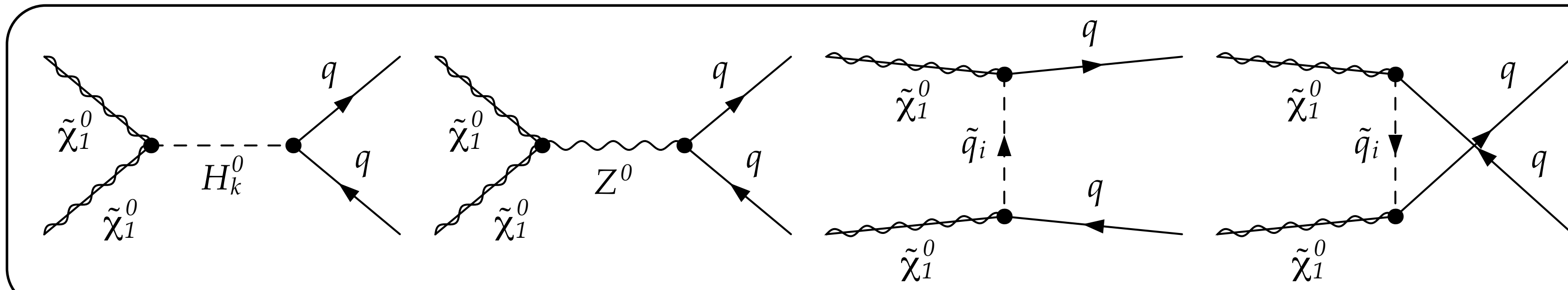
Main impact on cross-section from QCD and SUSY-QCD mass resummation

Impact of corrections larger than current experimental uncertainty

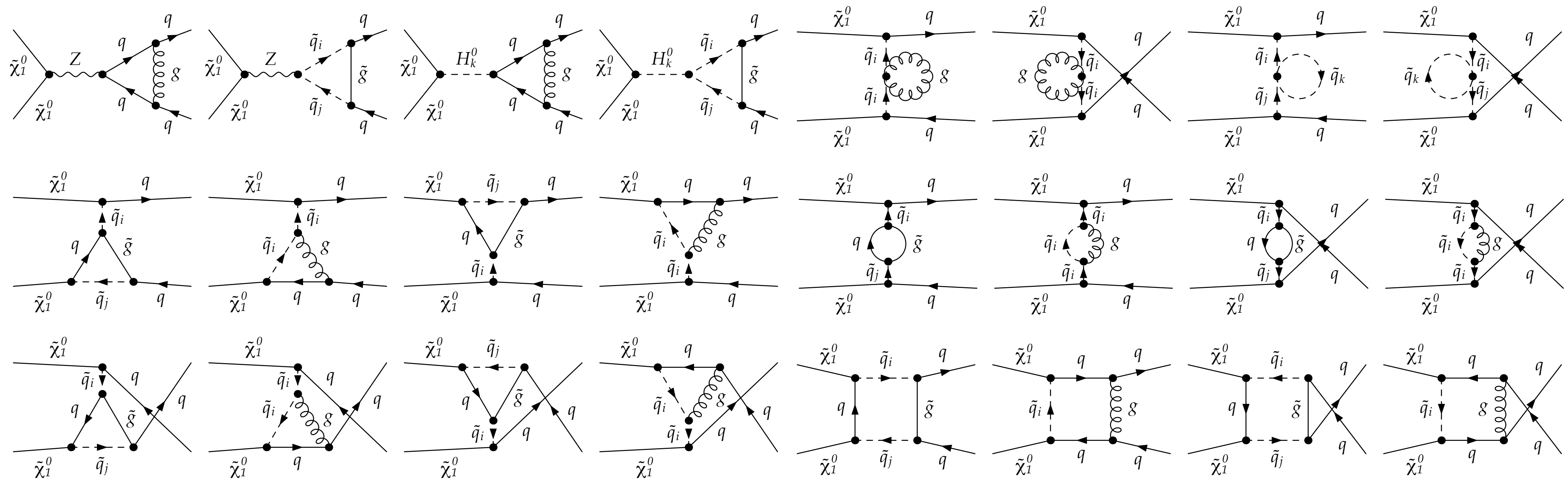
QCD corrections to neutralino pair annihilation

Herrmann, Klasen, Kovarik (2009)

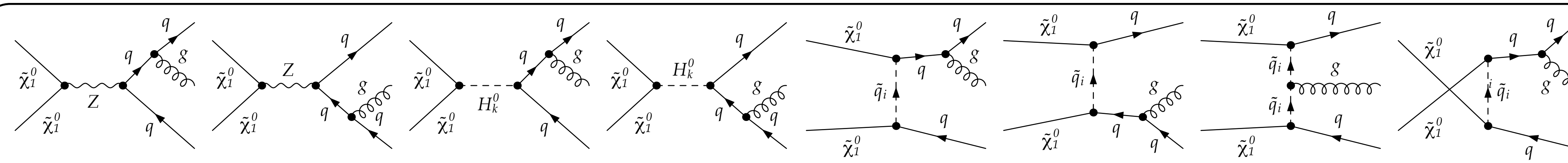
Tree level diagrams

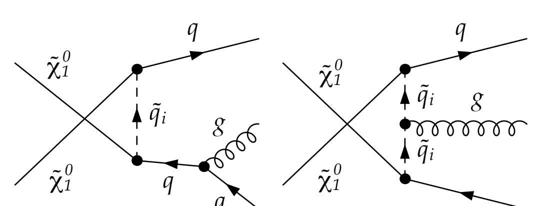


Virtual one-loop contributions



Real gluon emission

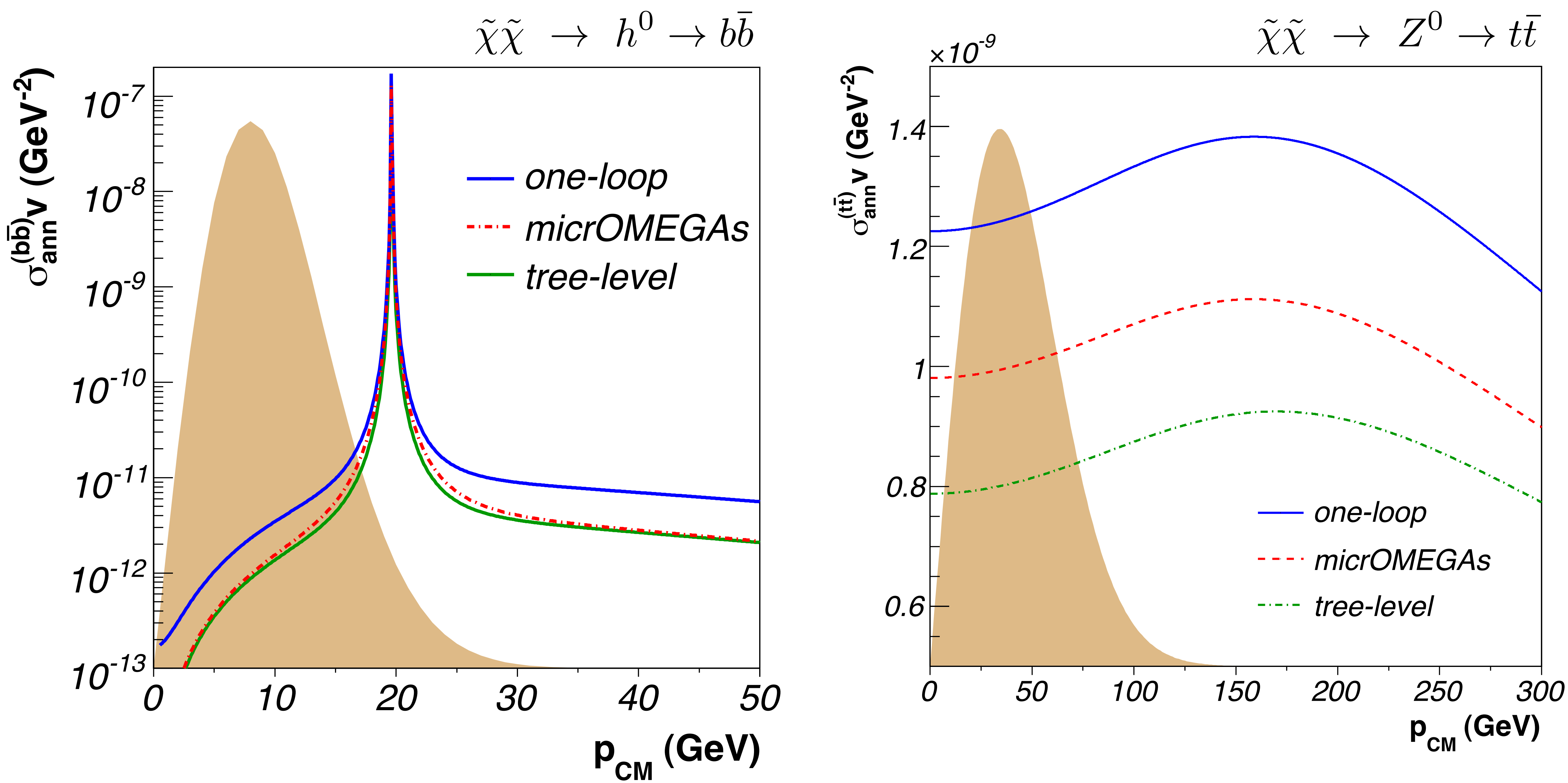




All contributions calculated analytically and implemented in the numerical package "**DM@NLO**"

Extension to the public codes DarkSUSY or micrOMEGAs

Herrmann, Klasen, Kovarik (2009)



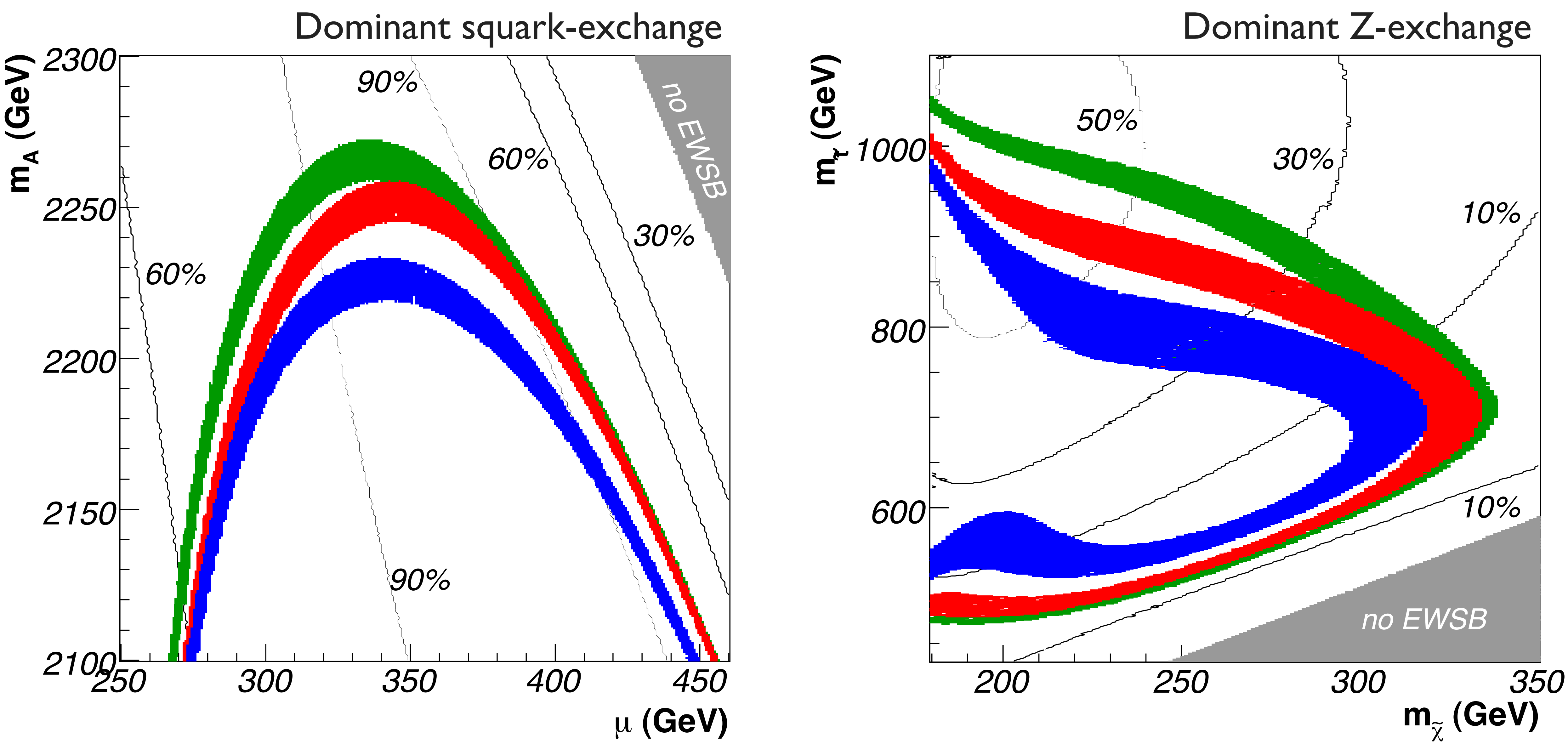
Annihilation cross-section into (heavy) quarks significantly enhanced by full one-loop corrections

Effective Yukawa couplings very good approximation around Higgs resonances

Full one-loop corrections increase cross-section by up to 50% w.r.t. tree-level approximation

Impact on neutralino relic density

Herrmann, Klasen, Kovarik (2009)



Impact of corrections on relic density more important than current experimental uncertainty

Main impact on cross-section from QCD and SUSY-QCD mass resummation

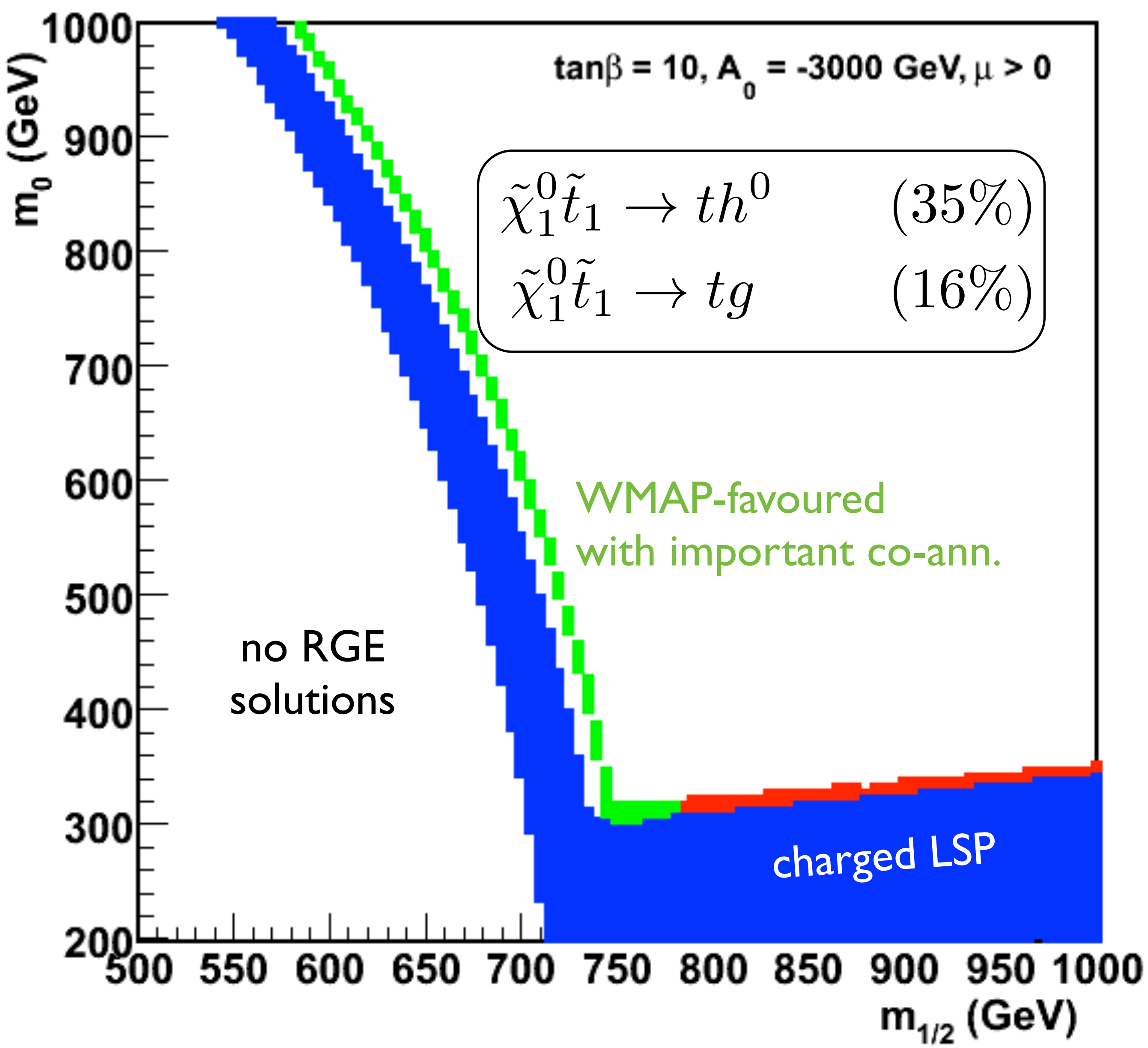
Favoured regions of parameter space shifted by up to 50 GeV in m_A or 200 GeV in m_{stop}

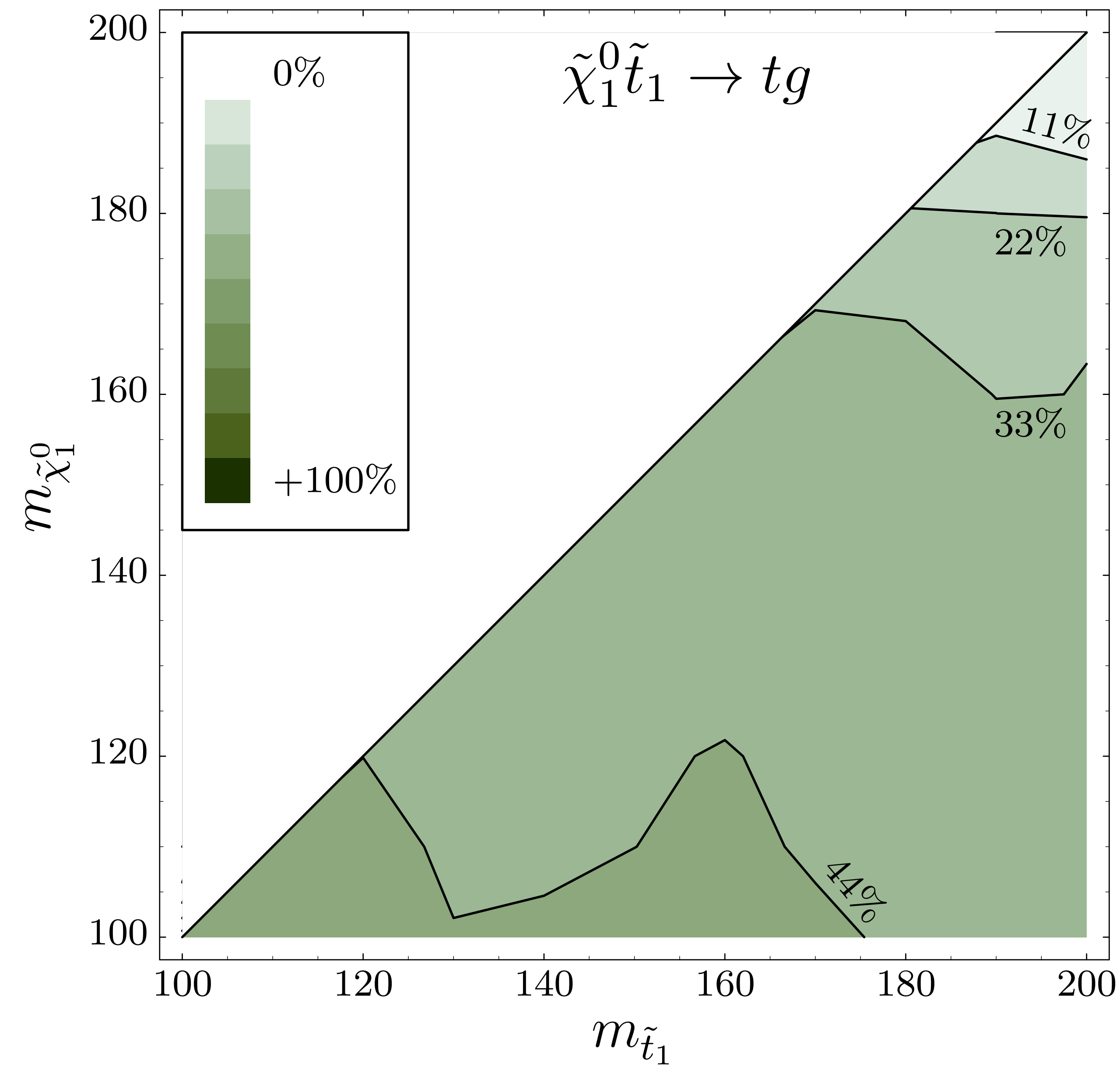
Neutralino-squark co-annihilation also receives corrections from gluon or gluino exchanges

Corrections shown to be sizeable for certain channels and configurations [Freitas 2007]

Calculation and implementation of full corrections to all channels in progress

Previous results generalized to neutralino-neutralino or neutralino-chargino co-annihilation



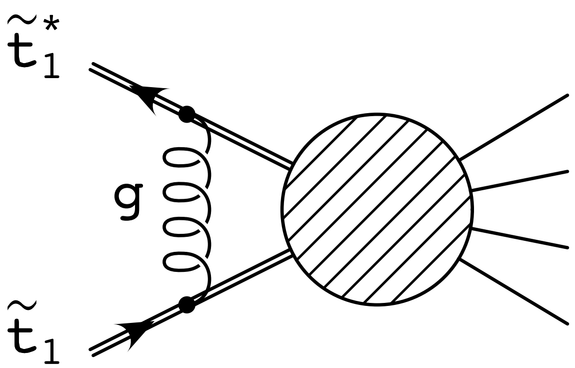


Coulombic corrections to stop pair annihilation

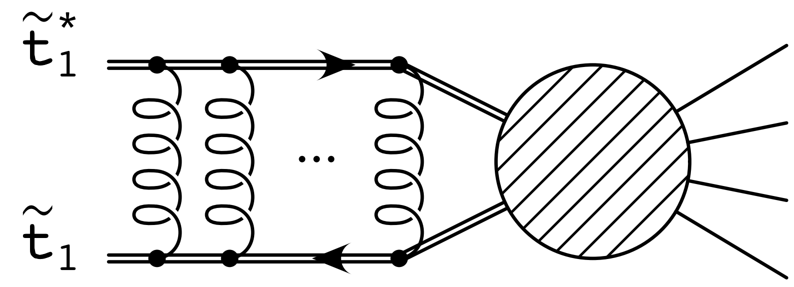
Freitas (2007)

Long range gluon exchange between non-relativistic stops leads to strong enhancement

Resummation of gluonic corrections in NRQCD (or more elaborate framework) necessary



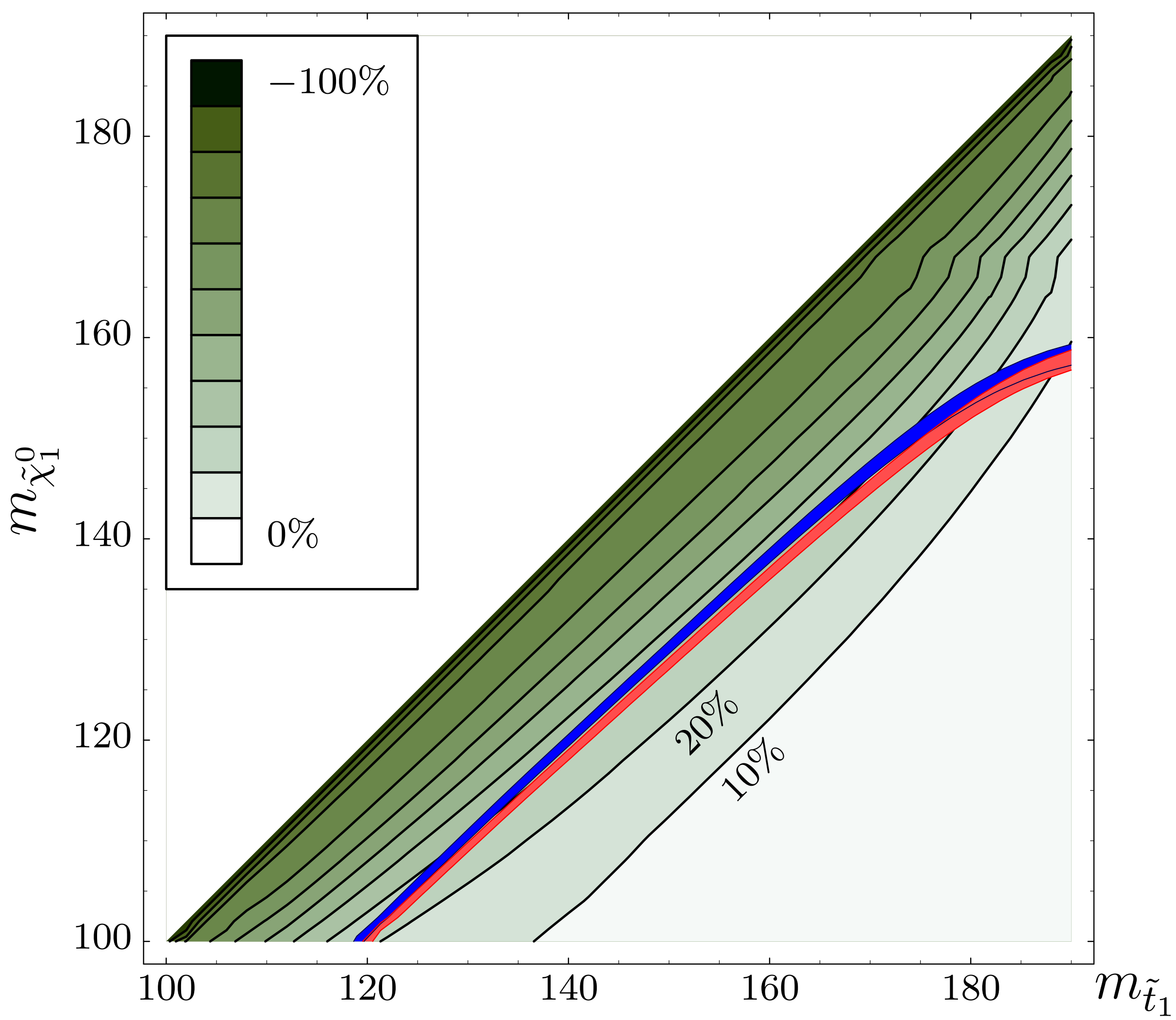
$$\frac{\sigma_{\rm coul}}{\sigma_{\rm LO}} = \frac{2\pi\alpha_s}{3v}$$

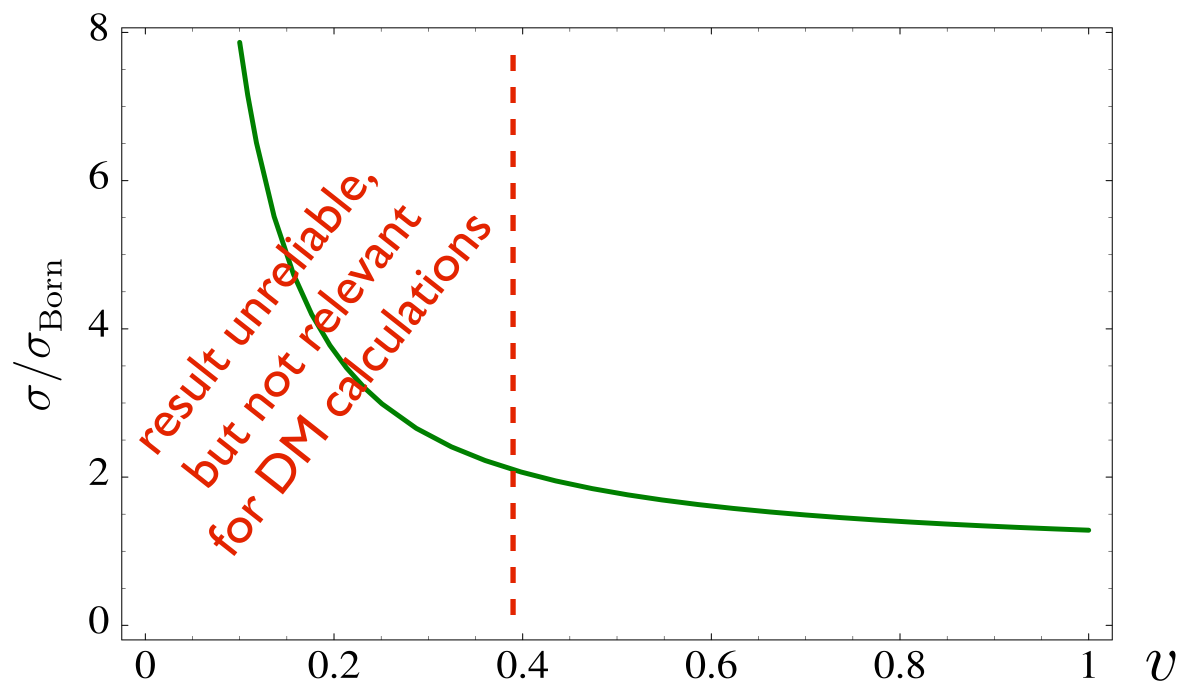


$$\frac{\sigma_{\rm coul}}{\sigma_{\rm LO}} = |\psi(0)|^2$$

$$\left[-\frac{\Delta}{m_{\tilde{t}_1}} + V(\vec{r})\right]\psi(\vec{r}) = (E + i\Gamma)\,\psi(\vec{r})$$

$$V(\vec{r}) = -C_F\frac{\alpha_s}{r} + \mathcal{O}(\alpha_s^2)$$





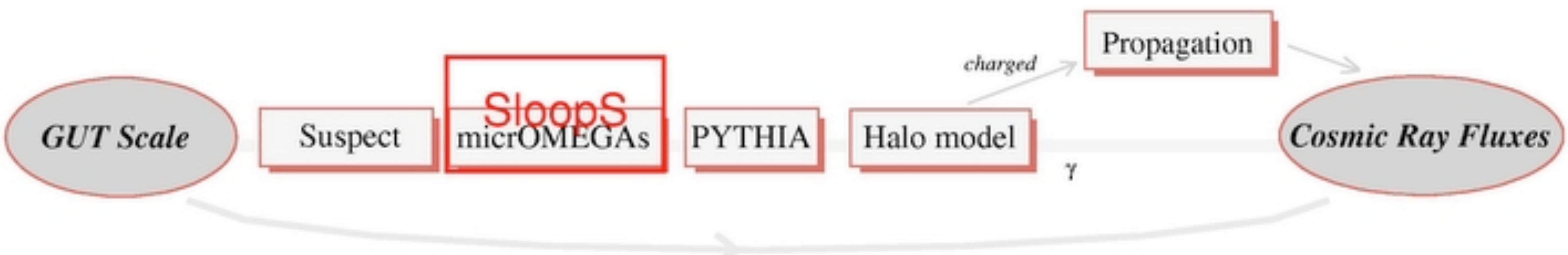
Electroweak corrections – towards an automatic calculation

Baro, Boudjema, Chalons, Semenov (2007-2011)

Electroweak corrections also important − especially for “uncoloured” dark matter

Large number of relevant processes and even larger number of involved diagrams (already at tree-level, but in particular at one-loop!)

Automatization of cross-section calculation unavoidable (~$10^3 - 10^4$ diagrams)



Following results presented in the small velocity expansion

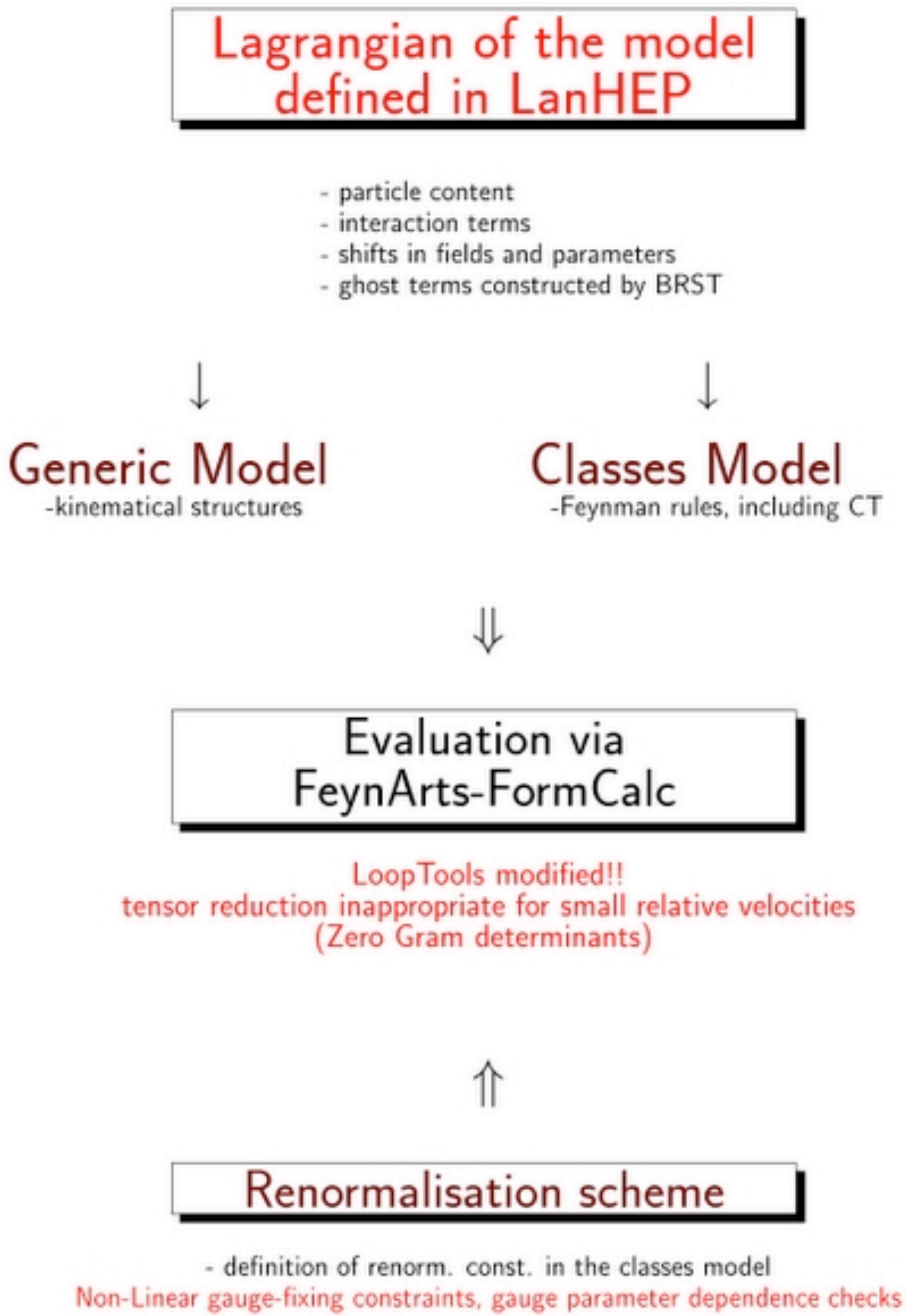
$$\sigma_{ij} v_{ij} = a_{ij} + b_{ij} v^2 + \mathcal{O}(v^4) \qquad v = 2\sqrt{1 - 4m_{\tilde{\chi}}^2/s}$$

$$\langle \sigma_{ij} v_{ij} \rangle = a_{ij} + \frac{6}{x}\left(b_{ij} - \frac{a_{ij}}{4}\right) \qquad x = \frac{m_{\tilde{\chi}}}{T}$$

$$\langle \sigma_{\rm ann} v \rangle = \sum_{i,j} \frac{g_i^2 g_j^2}{g_{\rm tot}^2} \langle \sigma_{ij} v_{ij} \rangle$$

The SloopS project

Baro, Boudjema, Chalons, Semenov (2007-2010)



Cross-sections and other observables in MSSM

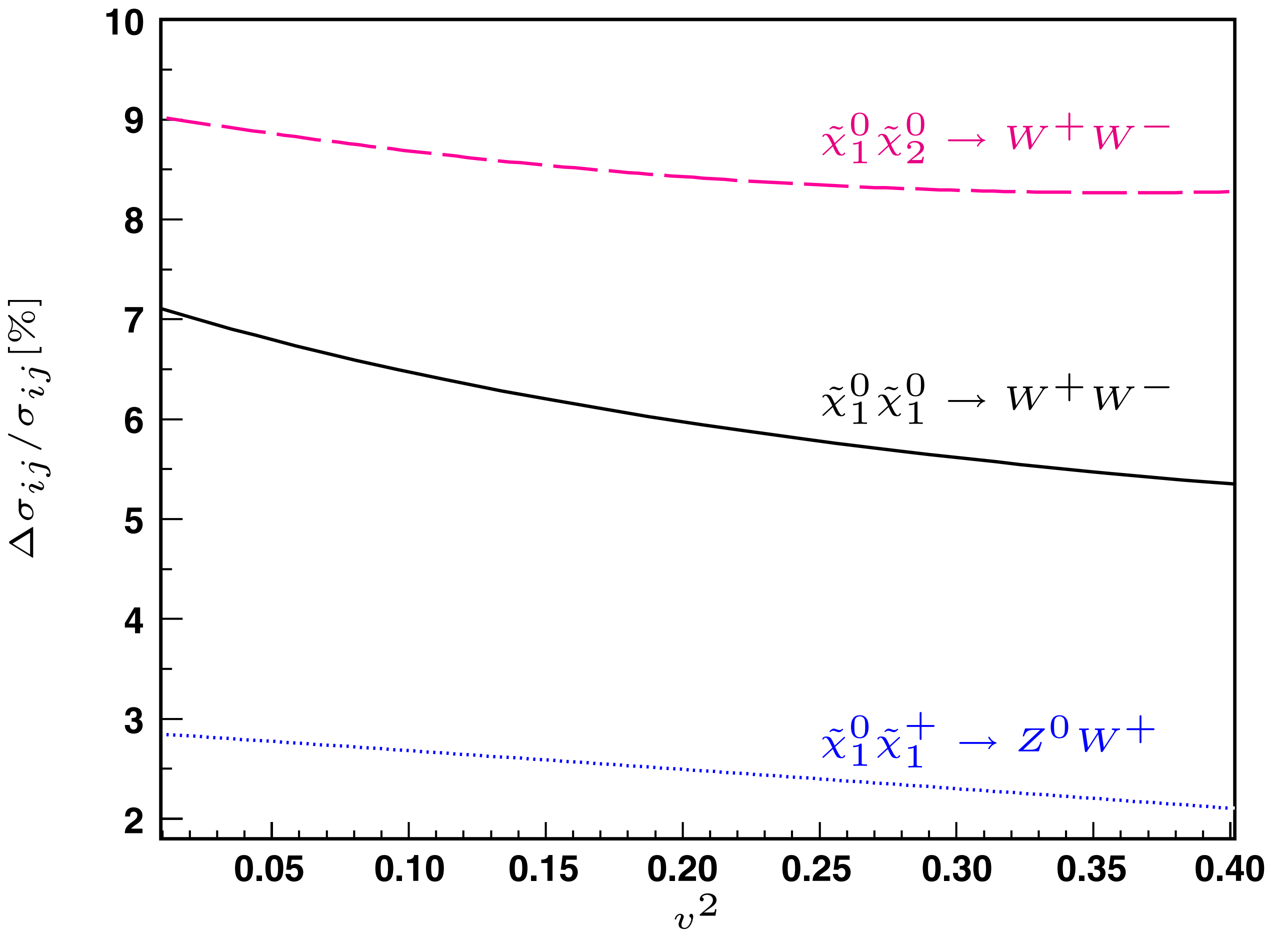
Exploiting and interfacing modules from different codes

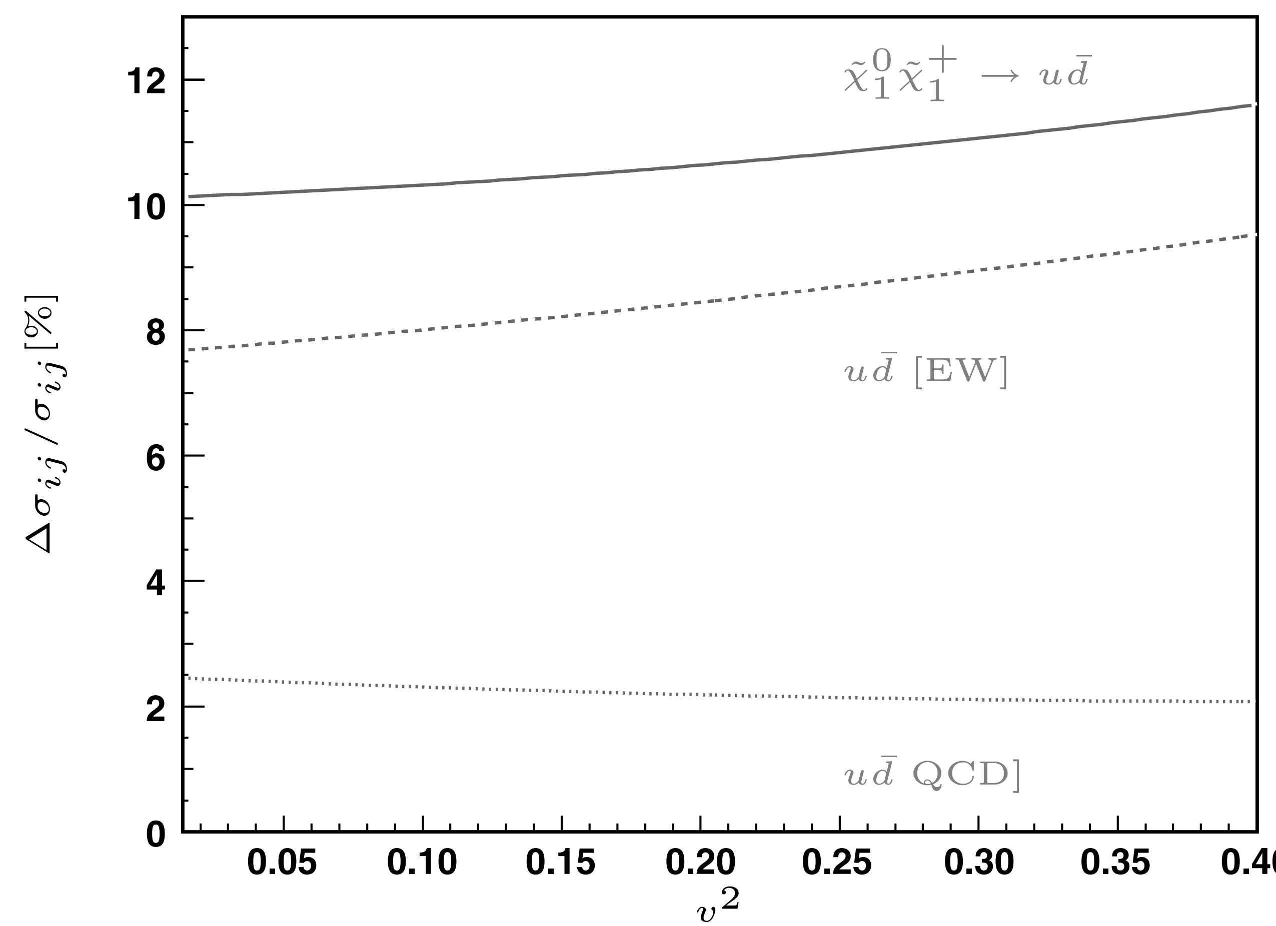
LanHEP [Semenov 1996]
FeynArts/FormCalc [Hahn 2000-2011]

Link to micrOMEGAs allows application to dark matter related calculations...

http://code.sloops.free.fr/

Baro, Boudjema, Chalons, Semenov (2007-2010)





$$
\begin{aligned}
m_{\tilde{\chi}_1^0} &= 106.9 \text{ GeV} \\
m_{\tilde{\chi}_2^0} &= 125.3 \text{ GeV} \\
m_{\tilde{\chi}_1^\pm} &= 124.6 \text{ GeV} \\
\tilde{\chi}_1^0 &= 0.94\tilde{B}^0 + 0.20\tilde{W}^0 - 0.27\tilde{H}_1^0 - 0.10\tilde{H}_2^0
\end{aligned}
$$

44%	$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$
5%	$\tilde{\chi}_1^0\tilde{\chi}_1^\pm \to u\bar{d}, c\bar{s}$
5%	$\tilde{\chi}_1^0\tilde{\chi}_2^0 \to W^+W^-$
5%	$\tilde{\chi}_1^0\tilde{\chi}_1^\pm \to Z^0W^\pm$

Corrections to the dominant channels amount to about 10% (velocity independent)

The issue of tanβ...

Baro, Boudjema, Semenov (2008)

Definition and renormalization of tanβ is conceptually the trickiest part of the calculation

At tree-level ratio of Higgs vacuum expectation values, but no unique expression at one-loop

Usual $\overline{\rm DR}$-scheme − Counterterm pure divergence of the UV factor in dimensional reduction (not gauge-invariant in non-linear gauge and not universal in linear gauge!)

$$\frac{\delta t_\beta^{\overline{\rm DR}}}{t_\beta} = \frac{1}{\cos 2\alpha}\left[\Sigma'_{h^0h^0} - \Sigma'_{H^0H^0}\right]_\infty$$

On-shell scheme with m_H as input − Counterterm defined by measurement of m_H (not well behaved in the decoupling limit $m_H \sim m_A$)

$$\frac{\delta t_\beta^{\rm MH}}{t_\beta} \simeq \frac{1}{m^2_{H^0}/m^2_{A^0} - 1}\left[\frac{\delta m^2_{H^0}}{m^2_{H^0}} - \frac{\delta m^2_{A^0}}{m^2_{A^0}}\right]$$

On-shell scheme with $A_{\tau\tau}$ as input − Counterterm defined by measurement of $A_{\tau\tau}$ (appears most natural, but often considered as too process-dependent...)

$$\frac{\delta t_\beta^{A\tau\tau}}{t_\beta} \simeq -\left[\delta_{\rm V}^{A\tau\tau} + \delta_{\rm CT}^{A\tau\tau} - \delta_v^{\rm QED}\right]$$

Scheme dependence non negligible – especially for two neutralinos in the initial state

Weaker dependence for coannihilation channels with only one neutralino

		Tree	$A_{\tau\tau}$	$\overline{DR}$	MH
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ [44%]	a	+0.81	+7.6%	+12.16%	+29.6%
	b	+1.219	+0.78%	+7.1%	+24.2%
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to u\bar{d}$ [8%]	a	+15.61	+7.2%	+9.8%	+18.8%
	b	−5.81	+5.7%	+8.3%	+17.4%
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to Z^0W^+$ [5%]	a	+8.26	+2.9%	+4.4%	+9.7%
	b	+1.42	−7.3%	−3.3%	+10.7%
$\tilde{\chi}_1^0\tilde{\chi}_2^0 \to W^+W^-$ [5%]	a	+17.81	+9.0%	+11.1%	+18.2%
	b	+11.86	+4.8%	+7.3%	+16.1%
$\Omega_\chi h^2$		0.108	0.105	0.102	0.097
$\frac{\delta\Omega_\chi h^2}{\Omega_\chi h^2}$			−2.8%	−5.6%	−10.2%

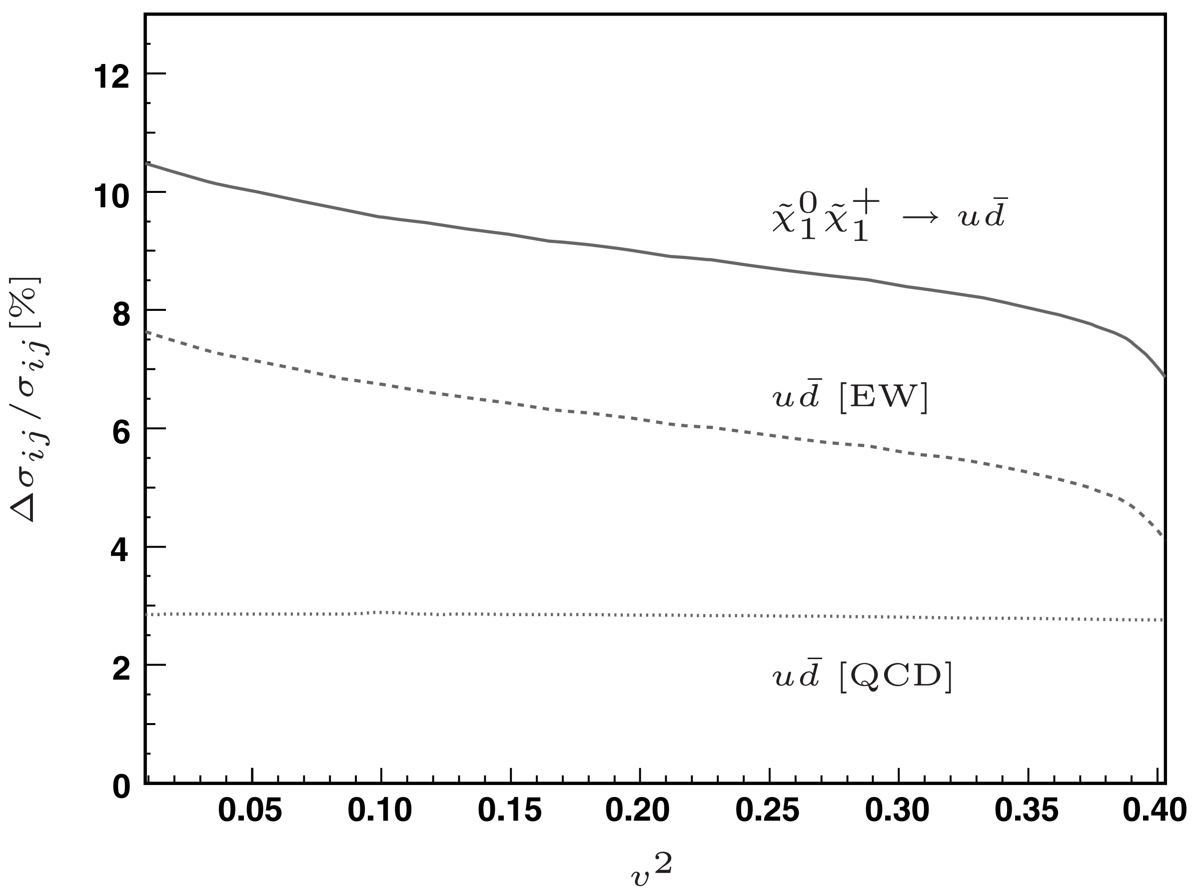
Also masses show scheme dependence at the one-loop level

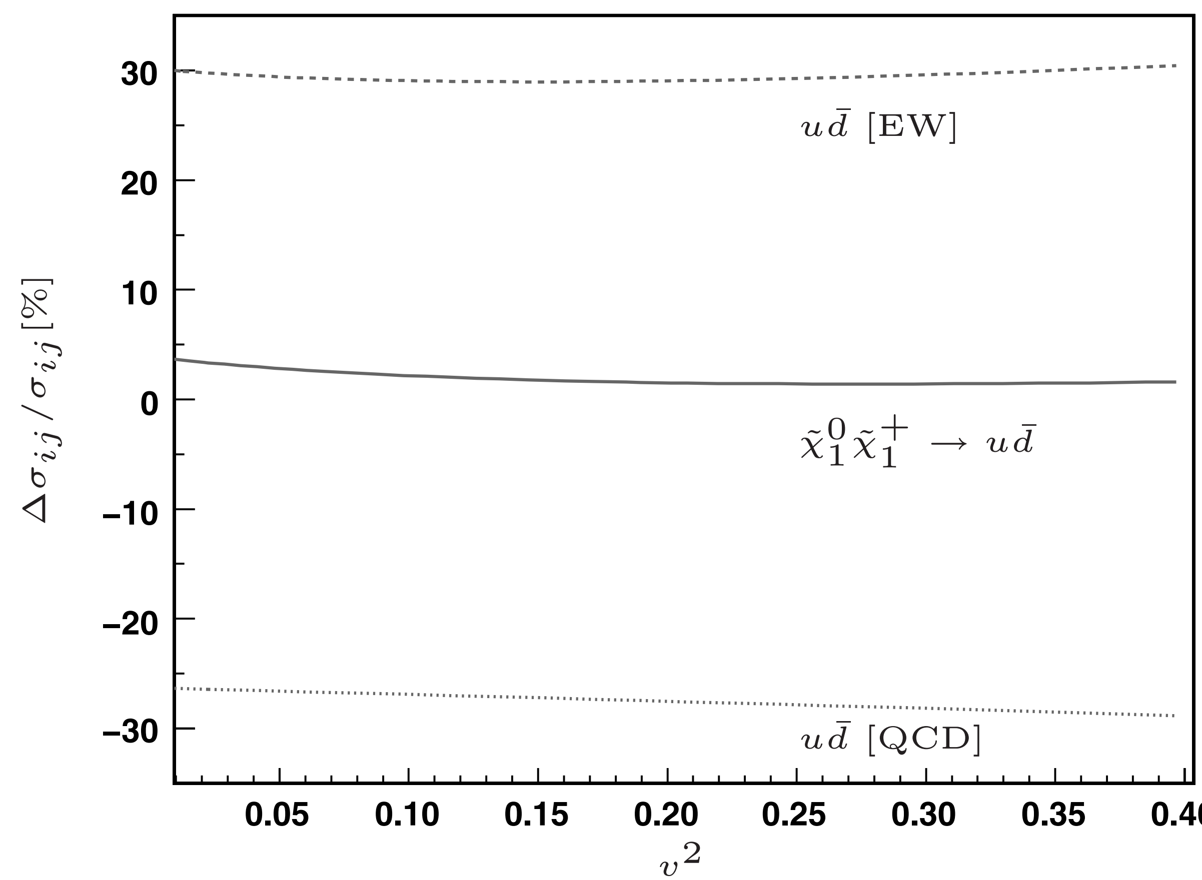
Masses [GeV]		$m_{\tilde{\chi}_2^0}$	$m_{\chi_3^0}$	$m_{\chi_4^0}$
Tree Level		125.3	258.1	270.4
One Loop	- $A_{\tau\tau}$ scheme	125.13	258.58	270.42
	- MH scheme	125.31	258.05	270.65
	- $\overline{DR}$ scheme	125.17	258.46	270.47

Baro, Boudjema, Chalons, Semenov (2010)

Both QCD and EW corrections can be sizeable − sometimes also compensate each other

$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to b\bar{b}$	$A_{\tau\tau}$	$\overline{\mathrm{DR}}$	MH
$\delta a/a$ EW	-1%	$+3\%$	$+31\%$
$\delta a/a$ QCD	-26%	-26%	-26%
$\delta b/b$ EW	-1%	$+3\%$	$+29\%$
$\delta b/b$ QCD	-30%	-30%	-30%





Conclusion

Relic density calculation is a powerful tool to constrain the parameter space of new physics

Public tools for performing such calculations within the MSSM (and other models) are available

Radiative corrections have sizeable impact on (co)annihilation cross-section of dark matter

Resulting impact on neutralino relic density more important than current experimental uncertainty

QCD and EW corrections are equally important

Radiative corrections should be taken into account in the analysis of supersymmetric models

Other sources of uncertainty affect the prediction of the dark matter relic density

(mass spectrum, cosmological model, three-body final states, QCD equation of state, ...)