SOMMERFELD ENHANCEMENT, SPIN EFFECTS and RADIATIVE CORRECTIONS
in dark matter interactions

R.Iengo, Hamburg Workshop on Indirect Dark Matter Searches June 2011

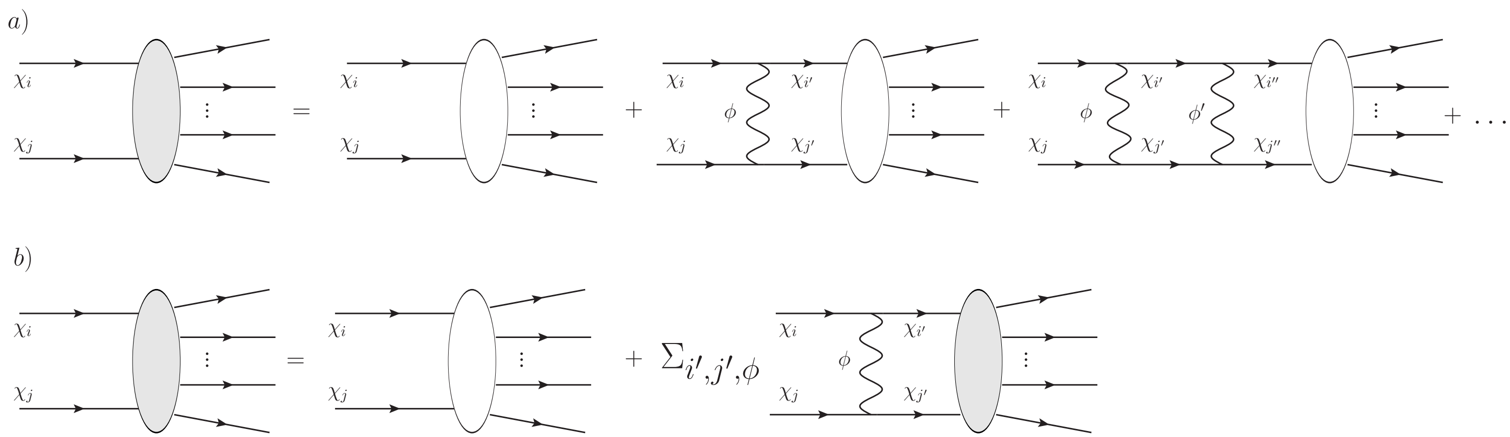
based on R.I. JHEP2009 arXiv:0902.0688
A.Hryczuk,R.I.,P.Ullio JHEP2011 arXiv:1010.2172
and unpublished

SOMMERFELD EFFECT
for coupled channels

Consider a process of the type

$\chi_a\chi_b \to \chi_i\chi_j \to \chi_i'\chi_j' \to \ldots \to$ SM final states

where the intermediate pairs $\chi_i\chi_j$ can be the same or different as the initial pair $\chi_a\chi_b$ (fig.a)



recurrence relation for the annihilation amplitudes

In the non-relativistic limit

(A_{ij} full amplitude, A^0_{ij} tree level value):

$$A_{ij}(p) = A^0_{ij}(p) - \sum_{i'j'\phi} N_{ij,i'j'} \frac{g_{ii'\phi} g_{j'j\phi}}{(2\pi)^3} \int \frac{d^3k}{(\vec{p}-\vec{k})^2 + m_\phi^2} \frac{A_{i'j'}(k)}{\frac{\vec{k}^2}{2m_r^{i'j'}} - \mathcal{E} + 2\delta m_{i'j'}}$$

$\vec{p}$ CM three-momentum, $g_{ii'\phi}$ and $g_{j'j\phi}$ coupling constants,
$N_{ij,i'j'}$ normalization and combinatorial factors.

Our convention :
$\mathcal{E} \; = \; \vec{p}^{\,2}/2m_r^{ab}$ kinetic energy of the reference incoming pair a, b
m_r^{ij} reduced mass, $2\delta m_{ij} = m_i + m_j - (m_a + m_b)$

To rewrite the expression above in the form of an inhomogeneous Schrödinger equations define :

$$
\begin{aligned}
A_{ij}(\vec{p}) &= \left(\frac{\vec{p}^2}{2m_r^{ij}} - \mathcal{E} + 2\delta m_{ij}\right)\tilde{\psi}_{ij}(\vec{p}) && (1) \\
U^0_{ij}(\vec{r}) &= \int d^3\vec{p}\, e^{i\vec{p}\cdot\vec{r}} A^0_{ij}(\vec{p}, P_0) && (2) \\
\psi_{ij}(\vec{r}) &= \int d\vec{p}\, e^{i\vec{p}\cdot\vec{r}} \tilde{\psi}_{ij}(\vec{p}) && (3)
\end{aligned}
$$

which allows us to rewrite the recurrence as a differential equation:

$$-\frac{\partial^2}{2m_r^{ij}}\psi_{ij}(\vec{r}) = U^0_{ij}(\vec{r}) + (\mathcal{E} - 2\delta m_{ij})\,\psi_{ij}(\vec{r}) + \sum_{i'j'\phi} V^{\phi}_{ij,i'j'}\psi_{i'j'}(\vec{r}),$$

ϕ: the exchanged particle (scalar, vector or axial vector boson).

The potential has the form

$$V^{\phi}_{ij,i'j'}(r) = \frac{c_{ij,i'j'}(\phi)}{4\pi}\frac{e^{-m_\phi r}}{r}\,,$$

$c_{ij,i'j'}(\phi)$ depends on the couplings and states involved.

Since the Sommerfeld enhancement factorizes out as a distortion of the incoming wave-function,
we first solve the associate homogeneous equation

with an initial state ab with momentum $\vec{p}$ we get a set of

wave functions $\psi_{ij}^{ab,\vec{p}}(\vec{r})$

Finally one reconstructs the amplitude

$$A_{ab}(\vec{p}) = \lim_{p' \to p} (p'^2 - p^2) \int \frac{d^3k}{(2\pi)^6} \sum_{ij} \frac{[\int d^3r' e^{-i\vec{p}'\vec{r}} \psi_{ij}^{ab,\vec{k}}(\vec{r}')][\int d^3r e^{i\vec{p}\vec{r}} \bar{\psi}_{ij}^{ab,\vec{k}}(\vec{r}) U_{ij}^0(\vec{r})]}{k^2 - p^2 - i\epsilon}$$

$$\to \sum_{ij} \int d^3r \bar{\psi}_{ij}^{ab,\vec{p}}(\vec{r}) \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\vec{r}} A_{ij}^0(\vec{r})$$

The result gets simple in partial waves.

Decomposing in partial waves, consider the reduced radial wave-function $\varphi(x)$:

$$R_{ij,p,l}(r) = Np\frac{\varphi_{ij,l}(x)}{x}, \qquad x = pr$$

getting equation, for the dominant S-wave, $(a, b$ reference incoming pair)

$$\frac{d^2\varphi_{ij}(x)}{dx^2} + \frac{m_r^{ij}}{m_r^{ab}}\left[\left(1 - \frac{2\delta m_{ij}}{\mathcal{E}}\right)\varphi_{ij}(x) + \frac{1}{\mathcal{E}}\sum_{i'j'\phi} V^{\phi}_{ij,i'j'}(x)\varphi_{i'j'}(x)\right] = 0\,.$$

Boundary conditions: $\varphi(0) = 0$

At $x \to \infty$ the solution describes one incoming $\chi_a\chi_b$ state and all the possible $\chi_i\chi_j$ states that can be produced in the ladder.

Two cases for the states $\chi_i\chi_j$ at ∞:

1. $2\delta m_{ij} < \mathcal{E}$ - enough energy to produce states $\chi_i\chi_j$ on-shell
2. $2\delta m_{ij} > \mathcal{E}$ - *not* enough energy; states $\chi_i\chi_j$ are off-shell

The radial wave functions behave at infinity as:

$$R_{ab}(r) \to \frac{C_{ab}}{2i}\frac{e^{ik_{ab}r}}{r} - \frac{1}{2i}\frac{e^{-ik_{ab}r}}{r} \quad \text{incoming pair}$$

$$R_{ij}(r) \to \begin{cases} \frac{C_{ij}}{2i}\frac{e^{ik_{ij}r}}{r} & \text{if on} - \text{shell} \\ \frac{D_{ij}}{2i}\frac{e^{-|k_{ij}|r}}{r} & \text{if off} - \text{shell} \end{cases} \quad \text{every other } \chi_i\chi_j$$

(Normalization consistent with $R_{ab} = \sin(k_{ab}r)/r$ for the non-interacting case)

For the reduced wave functions at $x \to \infty$ (write φ^{ab}_{ij} for the initial state ab)

$$\text{with } q_{ij} = m_r^{ij}/m_r^{ab} \cdot (1 - 2\delta m_{ij}/\mathcal{E})$$

$$i\varphi_{ab} - \partial_x\varphi_{ab} = -e^{-ix}$$

$$\begin{cases} i\sqrt{q_{ij}}\varphi^{ab}_{ij} - \partial_x\varphi^{ab}_{ij} = 0 & \text{if on} - \text{shell} \\ \sqrt{-q_{ij}}\varphi^{ab}_{ij} + \partial_x\varphi^{ab}_{ij} = 0 & \text{if off} - \text{shell} \end{cases}$$

these b.c.'s determine the wave functions $\varphi^{ab}_{ij}(x)$

One then reconstructs the amplitude and find the cross section.

The (co-)annihilation cross section of the pair $\chi_a \chi_b$ is determined, up to the kinematical factor, by

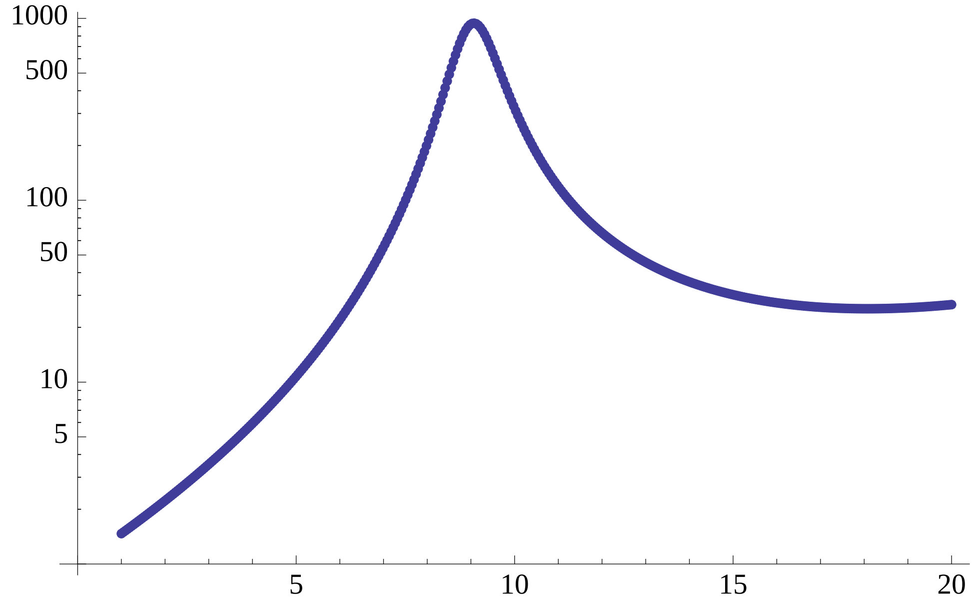
$$\sigma_{(ab)} \propto \sum_{ij} S_{ij}^{ab} \cdot |A_{ij}^{0}|^2$$

where the enhancement factors with our normalization are (for the S-wave)

$$S_{ij}^{ab} = |\partial_x \varphi_{ij}^{ab}|_{x=0}^2 \,.$$

The Sommerfeld effect can be very large when there is a resonance.

An example in the case of just one channel for a Yukawa potential $\frac{\alpha}{r}e^{-\mu r}$

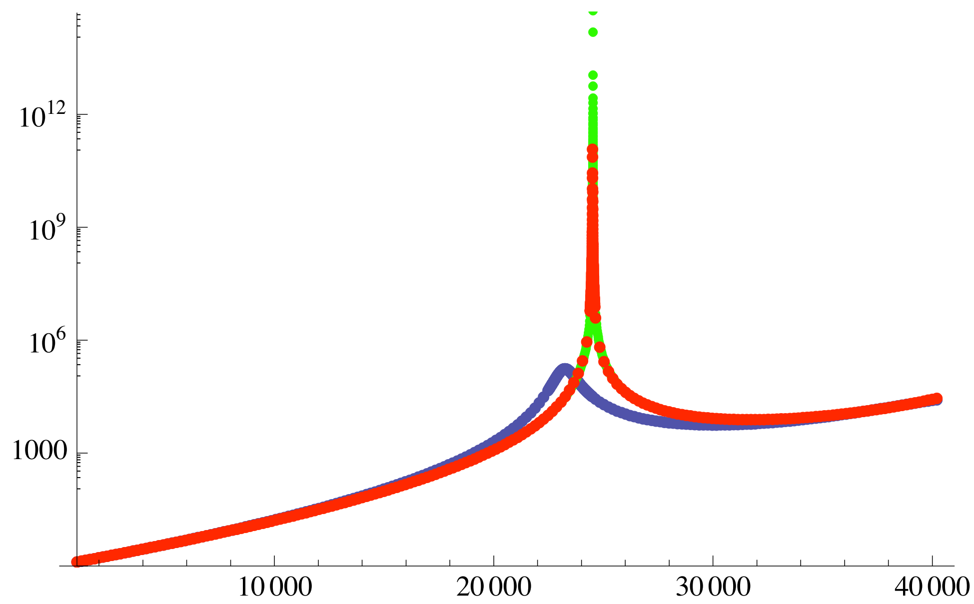


cross-section enhancement as a function of the co-annihilating mass in Tev, for $\mu = 90$ Gev, $\alpha = 1/30$ and an incoming velocity $v = 10^{-3}$

The resonant Sommerfeld enhancement also occurs for higher partial waves.

It is computed by taking higher derivatives of the reduced wave-function at the origin.

An example for the P-wave $l = 1$



enhancement as a function of $m(Gev)$ for $v = 10^{-3}$ $v = 10^{-4}$ $v = 10^{-5}$. Here $\alpha = 1/30$, $\mu = 90Gev$.

In the following applications we consider the dominant $l = 0$ S-wave.

The enhancement depends on the spin of the initial state, taken to be two non-relativistic fermions.

Project the cross section in the singlet and in the triplet spin-state, and multiply each projection by a different enhancement factor.

Describe a fermion-antifermion (Dirac or Majorana) pair by

$$|\Phi_\gamma^{ij}\rangle = N_{ij} \int d\vec{z}\, \bar{\psi}_i(\vec{z}) \mathcal{O}_\gamma \psi_j(-\vec{z}) |0\rangle \Phi_\gamma^{ij}(z).$$

For a (Dirac) fermion-fermion pair take $\psi \to \psi^c$.

The spin singlet $S = 0$ and the spin triplet $S = 1$ are encoded by (extending M.Drees,J.M.Kim,K.I.Nagao PRD2010 arXiv:0911.3795) :

$$S = 0: \quad \mathcal{O}_\gamma \equiv \gamma_5\,, \qquad S = 1: \quad \mathcal{O}_\gamma \equiv \vec{\gamma} \cdot \vec{S}\,,$$

Check, taking the non-relativistic large components :

$$\bar{\psi}_a = \begin{pmatrix} a_\uparrow^\dagger & a_\downarrow^\dagger & 0 & 0 \end{pmatrix} \qquad \psi_b = C \bar{\psi}_b^T$$

$$\bar{\psi}_a \gamma_5 \psi_b = -a_\uparrow^\dagger b_\downarrow^\dagger + a_\downarrow^\dagger b_\uparrow^\dagger \qquad \bar{\psi}_a \gamma_3 \psi_b = -a_\uparrow^\dagger b_\downarrow^\dagger - a_\downarrow^\dagger b_\uparrow^\dagger$$

The normalization is $N_{ij} = 1/\sqrt{2} \quad i \neq j \qquad N_{ij} = 1/2 \quad i = j$

To compute the enhancement, evaluate how the coupling in front of the potential depends on the spin state.

Take the interaction (Γ denoting a generic gamma matrices structure)

$$V_{int} = g_\Gamma^2 \int d\vec{x} d\vec{y}\, \bar{\psi}_k(x)\Gamma\psi_i(x) V^{\phi}_{ki,jl}(\vec{x}-\vec{y})\bar{\psi}_j(y)\Gamma\psi_l(y),$$

where $V^{\phi}_{ki,jl}(r) = \frac{g^\Gamma_{ki} g^\Gamma_{jl}}{4\pi}\frac{e^{-m_\phi r}}{r}$ is the non-relativistic potential due to the propagator of the boson exchanged between the two vertices. In the non-relativistic limit only $\Gamma = 1, \gamma_0, \gamma_j\gamma_5$ can contribute.

Compute $V_{int} \cdot \int d\vec{z}\, N_{ij}\bar{\psi}_i(\vec{z})\mathcal{O}_\gamma\psi_j(-\vec{z})|0\rangle\Phi^{ij}_\gamma(z)$

by doing contractions:

$$\langle\psi_i(x)\bar{\psi}_i(z)\rangle \;\to \delta(\vec{x}-\vec{z})\frac{1+\gamma_0}{2} \qquad \langle\bar{\psi}_l(y)\psi_l(w)\rangle \to -\delta(\vec{y}-\vec{w})\frac{1-\gamma_0}{2},$$

getting

$$V_{int}\cdot\Phi^{ij}_\gamma(z) \to \Phi^{kl}_\gamma(x) = c(klij,\gamma)V^{\phi}_{kl,ij}(|\vec{x}|)\;\Phi^{ij}_\gamma(x)$$

The evaluation of $c(klij,\gamma)$ fixes the strength of the potential in the Schrödinger equations.

A benchmark model

only the fermions of the MSSM and their $SU(2) \times U(1)$ interactions :

- a triplet of Majorana fermions in the adjoint representation
call them Wino's with a gauge invariant mass M

-two left-handed doublets in the fundamental representation
call them Higgsino's with a gauge invariant $\mu-$mass term

One can reorganize those fermions and their Lagrangians into:

$-$ a Wino triplet $\lambda^+, \lambda^- = (\lambda^+)^c, \lambda^0 = (\lambda^0)^c$ $(Y = 0)$

$$L_{Wino} = \bar{\lambda}^+(i\gamma^\mu\partial_\mu - g\gamma^\mu W^3_\mu - M)\lambda^+ + \frac{1}{2}\bar{\lambda}^0(i\gamma^\mu\partial_\mu - M)\lambda^0$$
$$+g\bar{\lambda}^+\gamma^\mu W^+_\mu\lambda^0 + g\bar{\lambda}^0\gamma^\mu W^-_\mu\lambda^+$$

$-$ a Higgsino Dirac-fermions-doublet $\psi = (\psi^0, \psi^-)$ $(Y = -1)$

$$L_h = \bar{\psi}(i\gamma^\mu\partial_\mu - g\gamma^\mu W^a_\mu\tau^a + \frac{g'}{2}B_\mu - M_h)\psi$$

where $M_h = \mu$ of the μ-term.

RELIC DENSITY COMPUTATION

Take a set of N particles χ_1, χ_2, ... χ_N, with mass m_i $m_1 \leq m_2 \leq \ldots \leq m_M$) such that:

i) inelastic scattering on SM thermal bath turn each state into another one
ii) χ_1 is stable.

A system of N coupled Boltzmann equations for each number density n_i

as after freeze out all heavier states decay into the lightest $\Rightarrow$ a single equation for $n = \sum_i n_i$, (e.g. J. Edsjo P.Gondolo PRD1997 he-ph/9704361) :

$$\frac{dn}{dt} + 3\,H\,n = -\langle \sigma_{\mathrm{eff}} v \rangle \left[n^2 - (n^{eq})^2 \right]$$

P.Gondolo J.Edsjo P.Ullio M.Schelke E.A.Baltz JCAP2004 astro-ph/0406204

The effective thermally averaged annihilation cross section

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}}$$

is a weighted sum over the thermally averaged annihilation cross sections $\chi_i + \chi_j \to X$ (in the dilute limit, two-body states dominate):

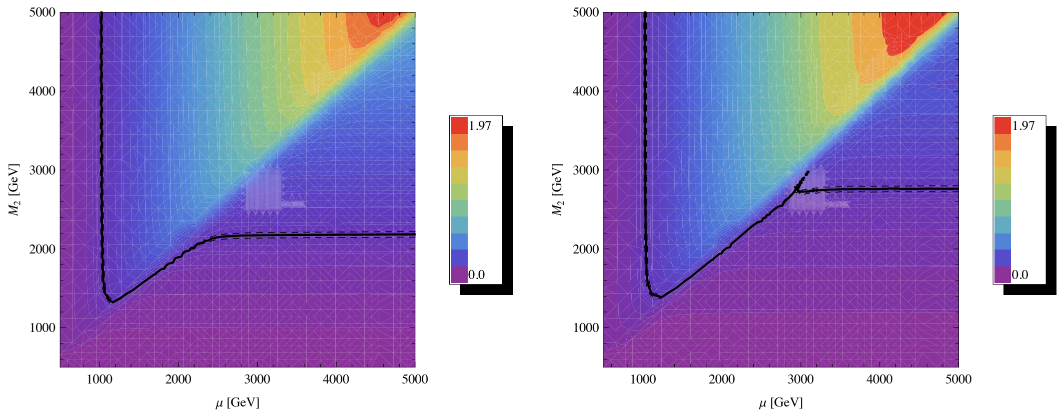
$$\langle \sigma_{ij} v_{ij} \rangle = \frac{1}{n_i^{eq} n_j^{eq}} \sum_X \int \frac{d^3 p_i}{2\,E_i} \frac{d^3 p_j}{2\,E_j} \frac{d^3 p_X}{2\,E_X} \delta^4(p_i + p_j - p_X) f_i^{eq}(p_i) f_j^{eq}(p_j)\, |A_{ij \to X}|^2 \,,$$

$n_i^{eq} = \int d^3 p f_i^{eq}(p_i)$ is the thermal equilibrium number density for the species i, $n^{eq} = \sum_i n_i^{eq}$.

Here we will include the Sommerfeld effect: $|A_{ab}|^2 = \sum_{ij} S_{ij}^{(ab)} \, |A_{ij}^0|^2$,

with $S^{ij}_{(ab)}$ computed as said before.

Results



Relic density Ωh^2 in the $\mu = M_{Higgsino}$-$M_2 = M_{Wino}$ plane,
left): perturbative case, *right)* with Sommerfeld effect included.
The brighter the colour the higher Ωh^2. The solid line corresponds to the value (with 1 σ) for relic density (7-year WMAP) $\Omega h^2 = 0.1123 \pm 0.0035$.

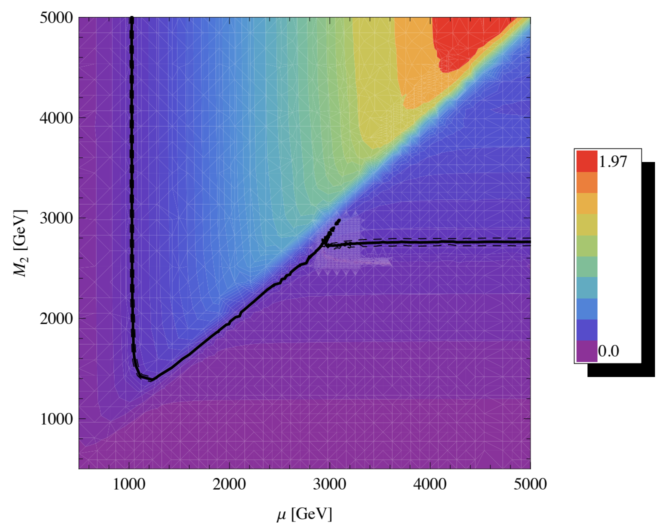
call Wino-region $M_2 << \mu$ and Higgsino-region $\mu << M_2$.

The Sommerfeld enhancement gives a sharp shift in the WMAP-Wino-region to heavier masses; a pure Wino is found at $\Omega h^2 = 0.11$ for a mass of about 2.8 TeV.
Much milder effects of the Sommerfeld enhancement are seen in the Higgsino-region.

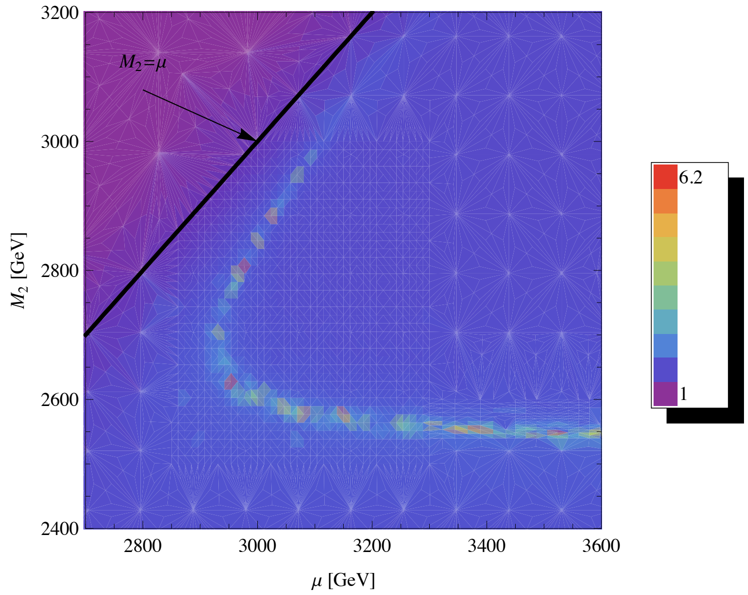
The results found for pure Winos and pure Higgsino are similar to J.Hisano S.Matsumoto M.M. Nojiri O.Saito PRD2006 hep-ph/0212022 and M.Cirelli A.Strumia M.Tamburini NPB 2007 arXiv:0706.4071 in the same limiting cases;

we have more annihilation channels and a careful dependence on the spin state of the annihilating particle pair (e.g. a different coefficient -3 in the axial vector exchange in agreement with Drees et al:2009) and a better control of the numerical solution of the Boltzmann Eq.

Another interesting feature is a spike in the region connecting the pure Higgsino to the pure Wino limit, towards $M_2 \sim \mu$ but still with a predominant Wino component.



In this region, we show in detail the ratio: relic density including Sommerfeld enhancement divided by relic density without it.

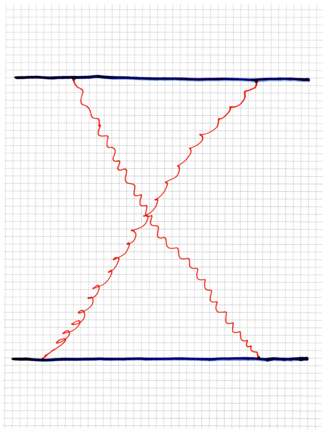


A thin "resonance" slice appears in the plane, starting for pure Winos with mass $m_{\chi^0} \approx 2.5$ TeV and extending to heavier masses into the region with a sizable Higgsino fraction, where the relic density agrees with observations.

The value of the mass we find for a pure Wino $\frac{1}{m_W} \approx \frac{1}{\alpha\ m_{\chi^0}}$, indicating a possible loosely bound state: Bohr radius $\sim$ interaction range.

Radiative corrections to the Sommerfeld enhancement
from corrections to the potential appearing in the Schrödinger equation.

2-body potential $\Leftrightarrow$ sum of the 2-particle-irreducible diagrams. The crossed box is 2-particle-irreducible:



it gives a correction $\sim O(\alpha^2)\frac{e^{-2m_\phi r}}{r}$ compare with the leading term $O(\alpha)\frac{e^{-m_\phi r}}{r}$
It is suppressed by α and it goes faster to zero for large r:
it would give a minor correction to the Sommerfeld enhancement.

Other corrections come from the the radiative corrections to the vertex and to the propagators.

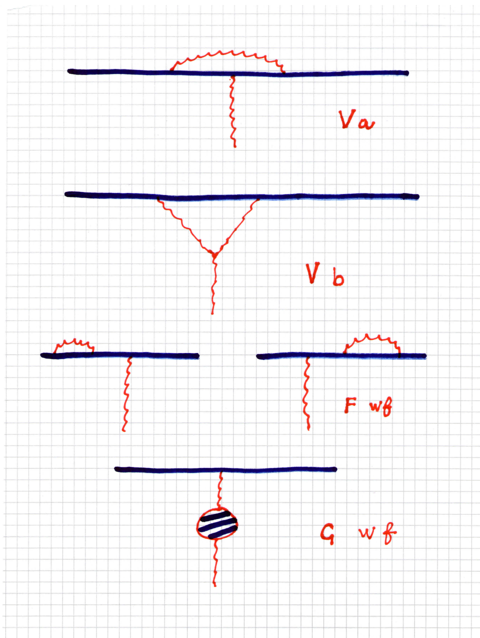
They contain large Log's and could re-sum to the running of the coupling constant.

Question: should we put the coupling at the $m \sim Tev$ scale ?

Here two scales: $m \sim Tev,\ m_w \sim 0.1 Tev$ (and momentum transfer $\leq m_W$)

a situation differing from the standard renormalization-group analysis with one large overall scale;
here more similar to the case of the forward scattering.

Radiative corrections to the vertex and propagators



Take $SU(N)$ ($N=2$), call the vertex (suppressing group indices)

$$-ig\bar{\Psi}\gamma\cdot W\Psi$$

m mass of Ψ, m_w mass of W, μ renormalization scale
$\vec{q}^2$ square momentum transfer $\vec{q}^2 \le m_w^2$.

$$Va: \; -ig\bar{\Psi}\gamma\cdot W\Psi \; \frac{g^2}{(4\pi)^2}(-\frac{N}{2}+C(r))\times(\frac{2}{\epsilon} -$$
$$2\int_0^1 dx\int_0^{1-x} dy \;\; Log[\frac{(x+y)^2m^2+(1-x-y)m_w^2+xy\vec{q}^2}{\mu^2}]$$
$$+2\int_0^1 dx\int_0^{1-x} dy \;\; \frac{(-3+4(x+y)+(1-x-y)^2)m^2-(1-x)(1-y)\vec{q}^2}{(x+y)^2m^2+(1-x-y)m_w^2+xy\vec{q}^2}\;\;)$$
$C(r)=N$ adjoint and $C=\frac{N^2-1}{2N}$ fundamental.

The last term is a UV-finite contribution, however IR divergent for $m_w^2/m^2 \to 0$.

Large Log's corrections, dropping $2/\epsilon$:

$$Va: \; -ig\bar{\Psi}\gamma\cdot W\Psi \; \frac{g^2}{(4\pi)^2}(-\frac{N}{2}+C(r))\times(\,-Log[\frac{m^2}{\mu^2}]-2Log[\frac{m^2}{m_w^2}]\;)$$

$$V b: \; -ig\bar{\Psi}\gamma\cdot W\Psi \; \frac{g^2}{(4\pi)^2}\frac{N}{2} \times (\frac{6}{\epsilon} - 2\int_0^1 dx \;\; Log[\frac{m_w^2+x(1-x)\vec{q}^2}{\mu^2}] -$$

$$2\int_0^1 dx \int_0^{1-x} dy \;\; Log[\frac{(1-x-y)m^2+(x+y)m_w^2+xy\vec{q}^2}{\mu^2}]$$

$$-\int_0^1 dx \int_0^{1-x} \;\; \frac{4(1-x-y)(2-x-y)m^2-2(x+y)\vec{q}^2}{(1-x-y)^2m^2+(x+y)m_w^2+xy\vec{q}^2})$$

The last term is a UV-finite contribution, however IR divergent for $m_w^2/m^2 \rightarrow 0$.

Large Log's corrections, dropping $6/\epsilon$:

$$V b: \; -ig\bar{\Psi}\gamma\cdot W\Psi \; \frac{g^2}{(4\pi)^2}\frac{N}{2} \times (\,-2Log[\frac{m_w^2}{\mu^2}] - Log[\frac{m^2}{\mu^2}] - -2Log[\frac{m^2}{m_w^2}])$$

There are radiative corrections to the vertex induced by the Fermion Ψ wave-function renormalization:

$$F\ wf:\ -ig\bar{\Psi}\gamma\cdot W\Psi\ \frac{g^2}{(4\pi)^2}C(r)\times(-\frac{2}{\epsilon}+2\int_0^1 dxx\ Log[\frac{(1-x)^2m^2+xm_w^2}{\mu^2}]+ 4m^2\int_0^1 dx\ \frac{(2-x)x(1-x))}{(1-x)^2m^2+xm_w^2}\ )$$

We find the large Log's corrections, dropping $2/\epsilon$:

$$F\ wf:\ -ig\bar{\Psi}\gamma\cdot W\Psi\ \frac{g^2}{(4\pi)^2}C(r)\times(Log[\frac{m^2}{\mu^2}]+2Log[\frac{m^2}{m_w^2}]\ )$$

By summing the $Va, Vb, F\ wf$ contributions, the large Log's containing m^2 in the argument disappear:

$$-ig\bar{\Psi}\gamma\cdot W\Psi\ \frac{g^2}{(4\pi)^2}\frac{N}{2}\times(-2Log[\frac{m_w^2}{\mu^2}])$$

Actually, by performing integrations by parts in the integrals over the x, y parameters, it is seen that for $\vec{q}=0$ the sum of the three contributions is exactly equal to

$$-ig\bar{\Psi}\gamma\cdot W\Psi\ \frac{g^2}{(4\pi)^2}\frac{N}{2}\times(\frac{4}{\epsilon}-2\int_0^1 dx\ Log[\frac{xm_w^2}{\mu^2}])$$

This is consistent with the Slavnov-Taylor identities:
$Z_{\bar{\Psi}W\Psi} = 1 + Va + Vb \qquad Z_{\Psi} = 1 - Fp$

$$\frac{Z_{\bar{\Psi}w\Psi}}{Z_{\Psi}}$$

is the same for any Ψ $\Rightarrow$ it cannot depend on the Ψ mass m.

To complete the survey of the radiative corrections, we include the correction induced by (one-half of the) Gauge boson W wave function renormalization:

$$G\ wf: -ig\bar{\Psi}\gamma \cdot W\Psi\ \frac{g^2}{(4\pi)^2} \times (N\frac{5}{3\epsilon} - \frac{5}{6}\ Log[\frac{m_w^2}{\mu^2}]\} + "matt")$$

where $"matt"$ denotes the contribution of the matter, i.e. leptons, quarks and possibly higgs, of the form
$"matt" = -(\#)(\frac{2}{\epsilon} - Log[\frac{m_w^2}{\mu^2}])$ (here we show large Log's only).

This correction does not depend on the fermion mass m

By summing everything we get the total correction to the coupling constant g (dropping c_0/ϵ absorbed by a renormalization at a reference scale):

$$g(1 + \frac{g^2}{(4\pi)^2} \times (\{N\frac{11}{3} - (\#)\}\{-\frac{1}{2}\ Log[\frac{m_w^2}{\mu^2}]\})) \equiv g(m_w^2)$$

At the lowest order we recover indeed, with $g = g(\mu^2)$:

$$g(m_w^2)^2 = \frac{g(\mu^2)^2}{1 + \frac{g(\mu^2)^2}{(4\pi)^2}\{N\frac{11}{3} - (\#)\}\ Log[\frac{m_w^2}{\mu^2}]}$$

that is the standard running of the $SU(N)$ coupling constant at the scale m_w.

In conclusion:

by taking the coupling constant $g(m_w^2)$ at the m_w^2 scale, the radiative corrections to the vertex $\bar{\Psi}\gamma \cdot W\Psi$ (appearing in the evaluation of the Sommerfeld effect) are negligible
for $\vec{q}^2 \leq m_w^2 << m^2$ (actually, no correction at all for $\vec{q}^2 \to 0$).

The same result holds for the $U(1)$ interaction due to the $U(1)$ Ward identity.