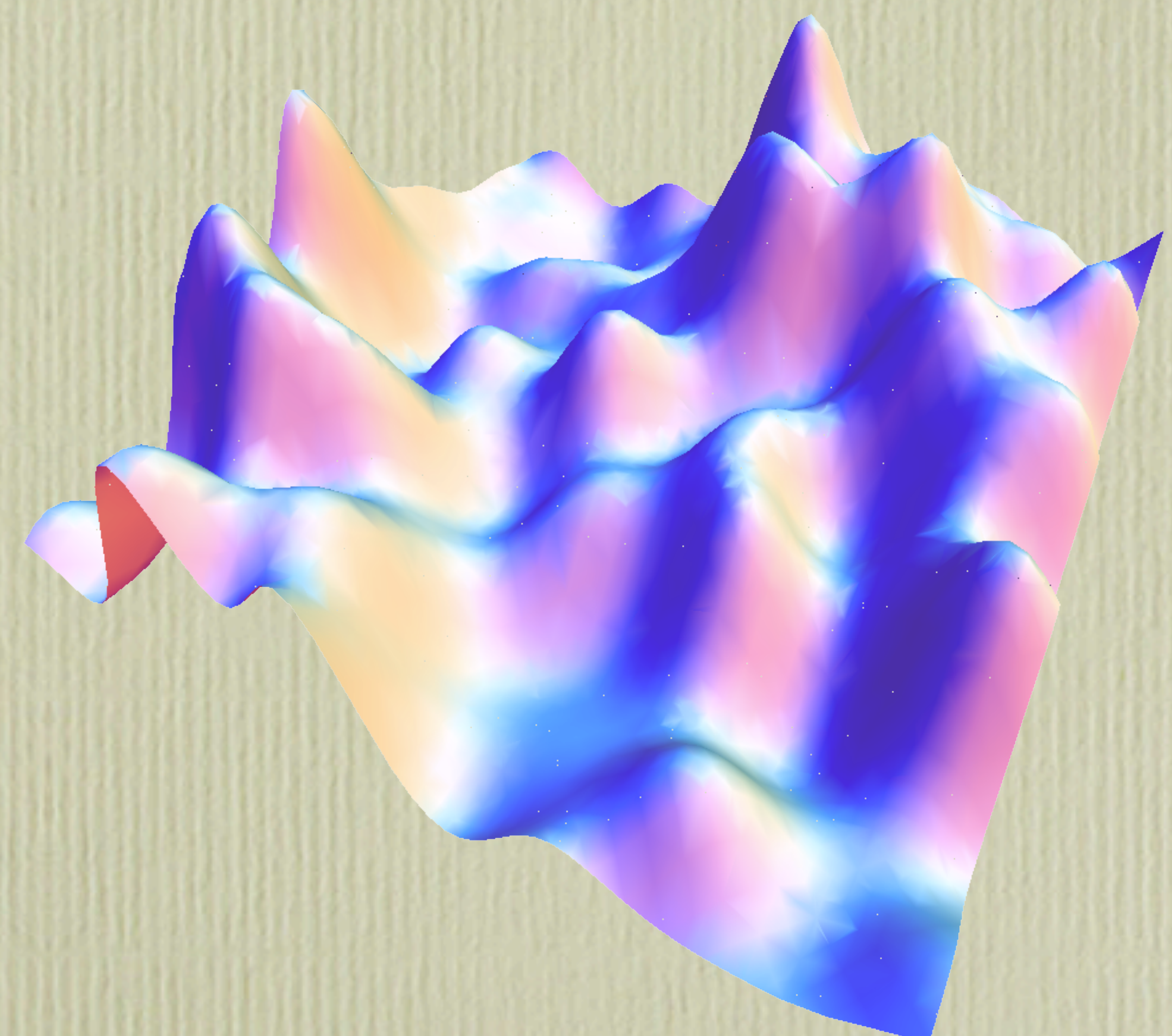
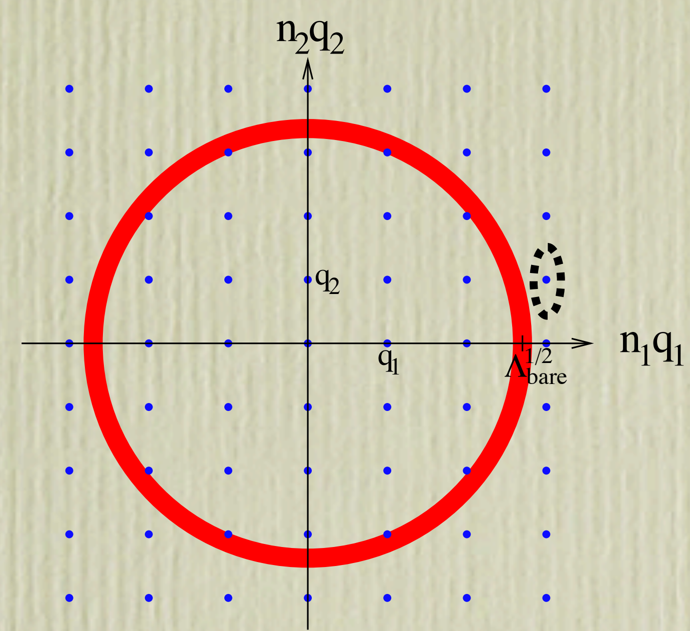
Status and Perspective of Dark Energy



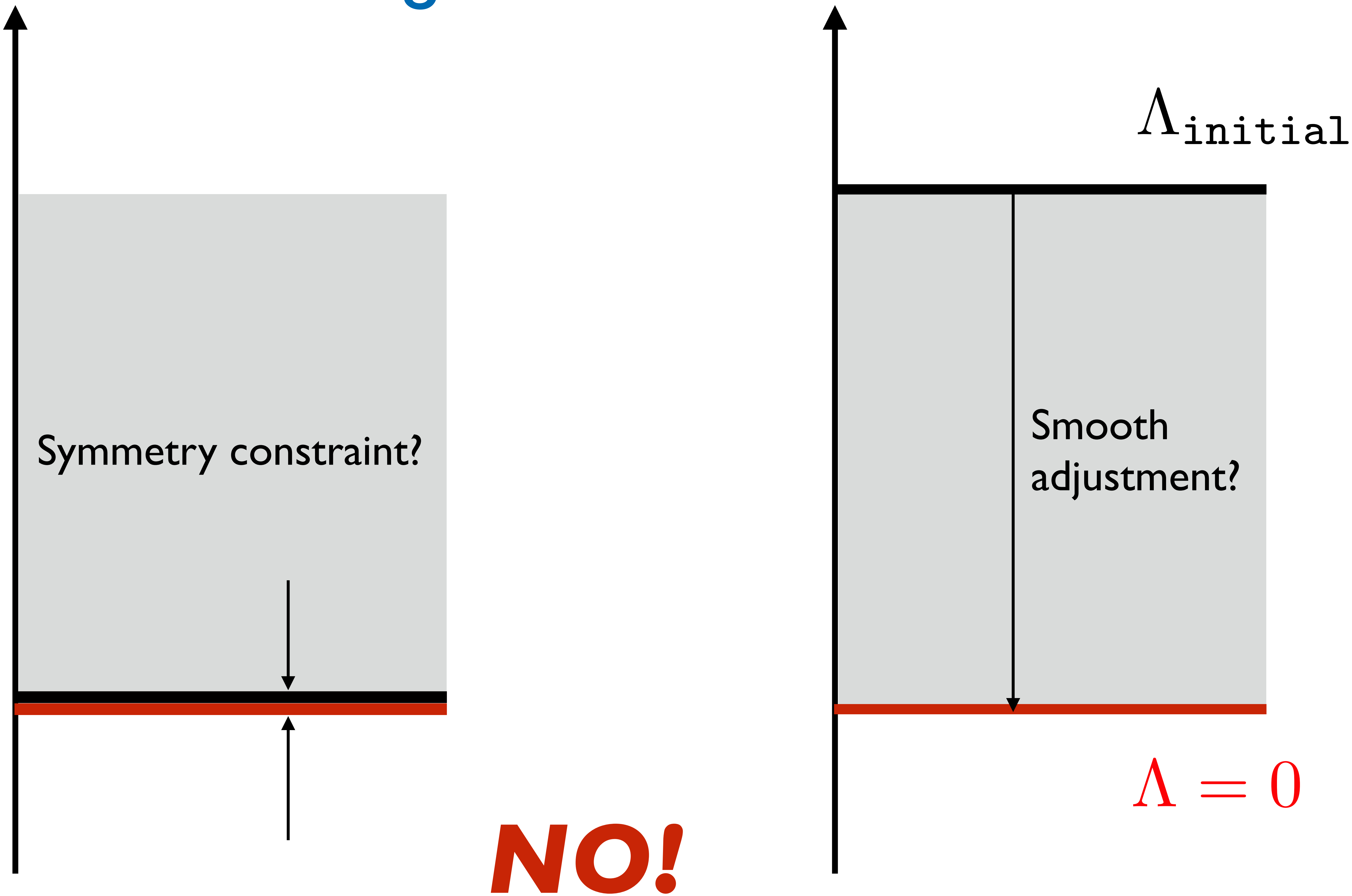


picture from [Bousso & Polchinski '00]

Alexander Westphal

(DESY)

Cosmological Constant Problem



[Weinberg '89]

- further modifying GR - safest form is *f*(*R*)-gravity - not useful:

  - *f*(*R*)-gravity conformally equivalent to scalar field — Weinberg no-go
  - adding higher curvature invariants - ghosts [Stelle ’77] and/or instabilities (massive gravity)

- example of unimodular gravity is suggestive, yet too simple:

  to accommodate a small CC, we need

  - a form of landscape
  - AND a population mechanism

- 1st attempt - couple gravity to a *U*(1) 4-form gauge field strength:

[Brown-Teitelboim '87 & '88] and [Abbott '85]:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}R - \Lambda_0 - |F_{\mu\nu\rho\sigma}|^2\right] \qquad F_4 = dA_3$$

$$+ S_{\rm boundary} - \boxed{\mathcal{T}_A\int d^3\xi\sqrt{\gamma_A} - \mathcal{Q}_A\int A_3}$$

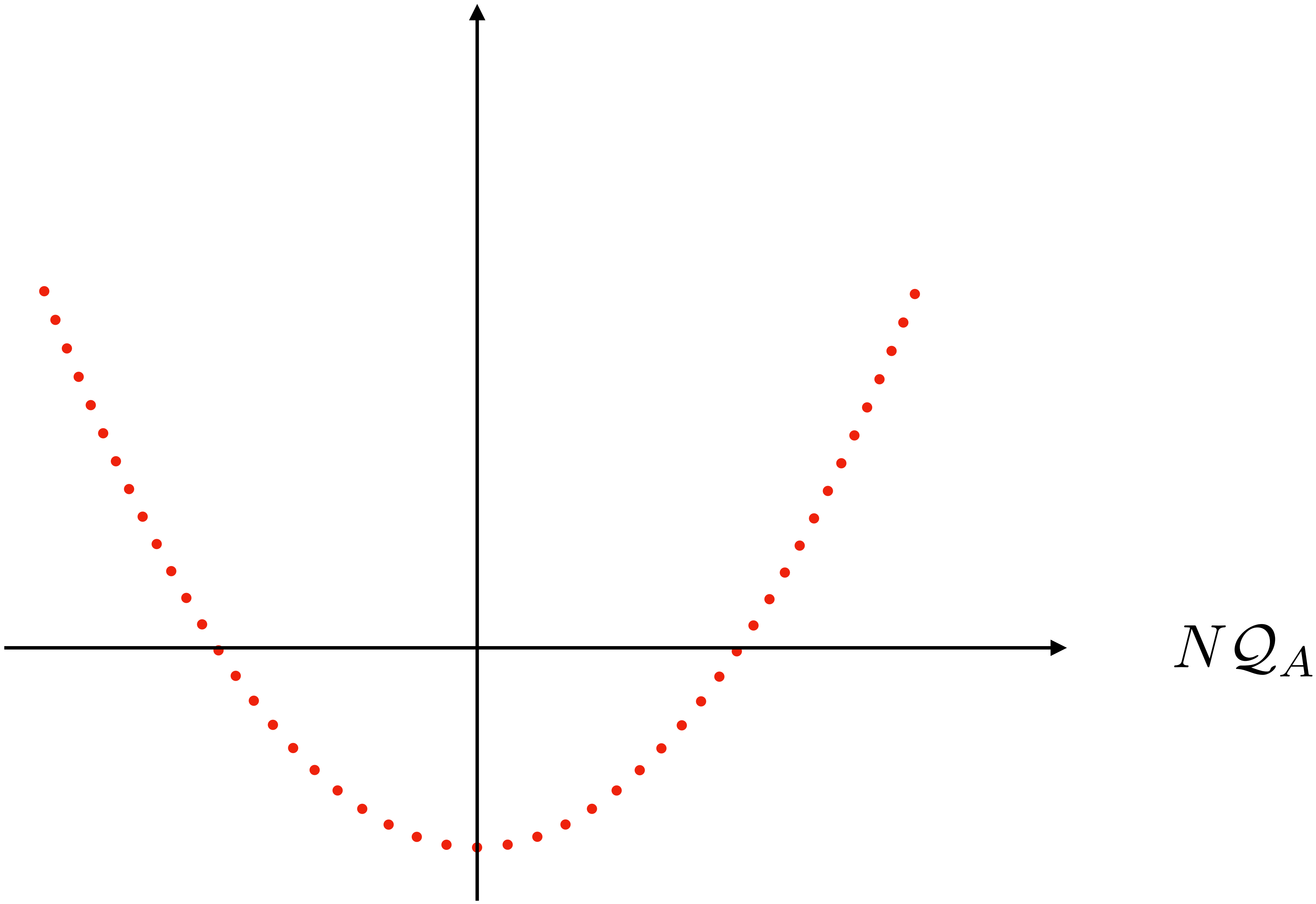
membrane charged under *A*₃

- across a membrane 4-form flux jumps: $\Delta F_4 = \mathcal{Q}_A$

- membranes nucleate via tunneling instanton:
  — analogon of 2D Schwinger process

$$\Rightarrow \quad \Lambda = \Lambda_0 + |F_4|^2 = \Lambda_0 + N^2\mathcal{Q}_A^2$$

$$V = \Lambda_0 + N^2 \mathcal{Q}_A^2 \quad , \quad \Lambda_0 < 0$$



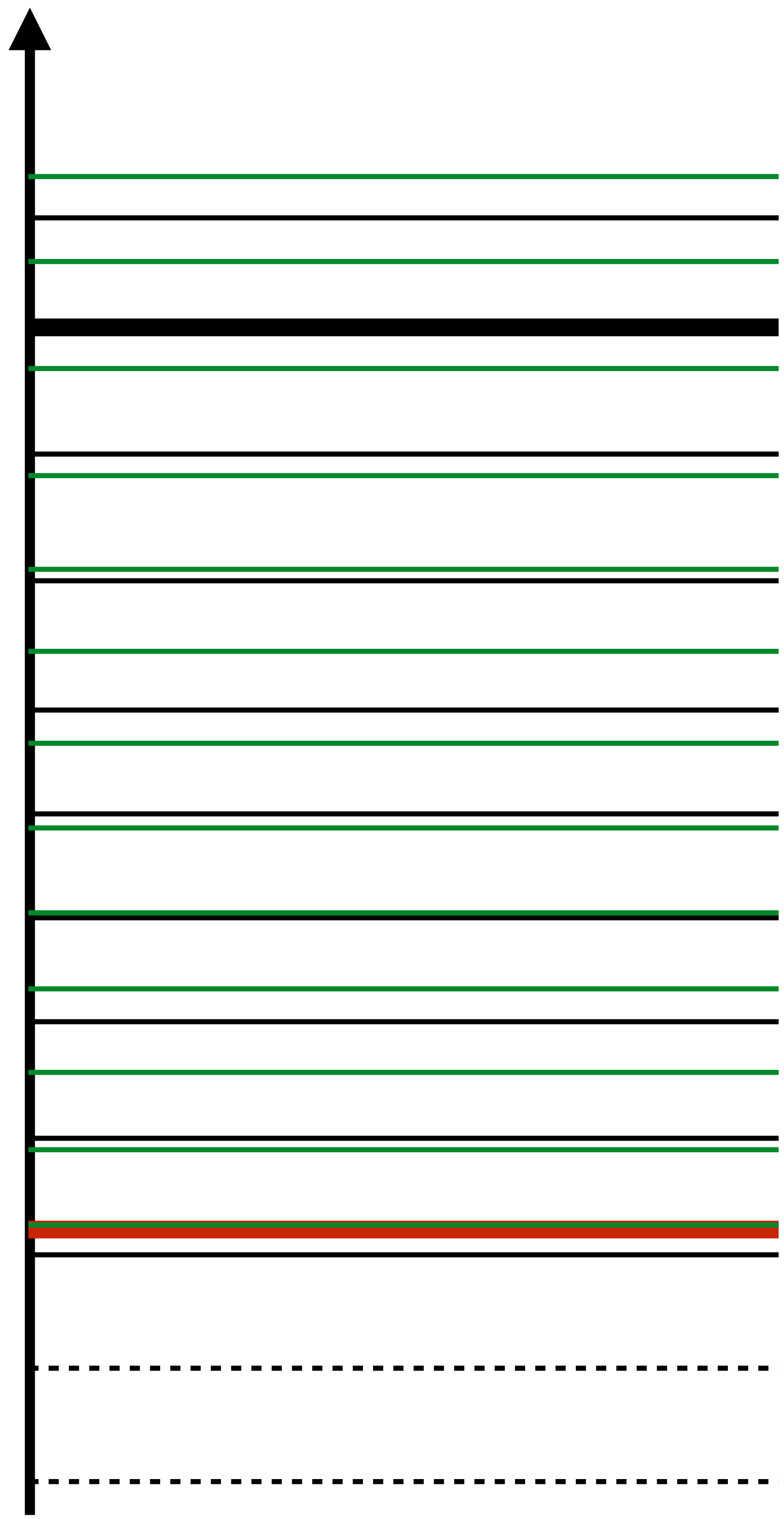
- get small CC and long life-time if $\Delta\Lambda$ very small: $\Rightarrow \mathcal{Q}_A \ll 1$
- but then: universe is empty (no entropy production) **[Abbott '85]**

Stairway in Heaven

CC is unstable, it decays …

As long as the gaps are wide enough, we can fit the "real universe" inside it, all 60ish efolds of inflation, reheating, BBN, etc etc

for a single stairway, steps too tiny [Abbott '85]



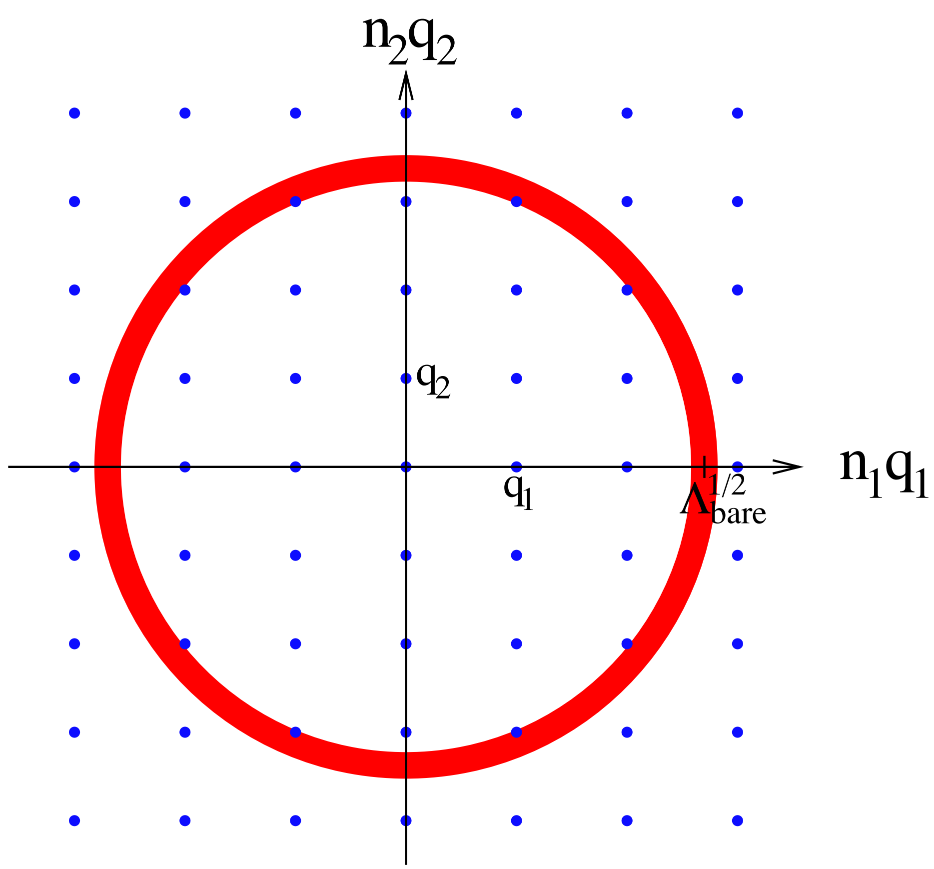
to accommodate small CC,

need ≥ 2 stairways somewhat out of step

… *a landscape*

- 2nd attempt - many 4-forms:

[Bousso-Polchinski '00] **[Feng, March-Russell, Sethi & Wilczek '00]**



picture from [Bousso & Polchinski '00]

$$\Lambda = \underbrace{-\mathcal{O}(M_P^4)}_{\Lambda_0} + \sum_{i=1}^{J} N_i^2 \mathcal{Q}_{A_i}^{\;2}$$

- demand small CC — thinnest $\Delta\Lambda$ - shell containing 1 lattice point:

$$\Delta\Lambda = \frac{|\Lambda_0|}{\#(\text{lattice points in } S^J(r=|\Lambda_0|))} \sim \sqrt{JN^2}^{\,J}$$

$$\text{example}: \mathcal{Q}_{A_i} = 0.1\ ,\ N=1\ ,\ J=100\ \Rightarrow\ \Delta\Lambda \sim 10^{-100}$$

- this toy model has the main features:

  - discrete landscape with small enough CC-spacing

  - vacua with CC > 0 eternally inflate: infinite amount of space-time with high-lying dS space generated

  - tunneling transitions from eternally inflating dS space populate vacua (both down- and up-tunneling present among dS vacua!)
    — discrete, locally null-energy violating quantum effects — no-go evaded!

  - individual membrane 4-form tunneling jumps have large CC-jump:

    > no empty universe problem
    > can have scalar-field slow-roll inflation in there

- central question:

  - produces a multiverse with all possible CC values!

  - why should we observe our tiny one?

- fact - we observe structure formation in our past:

$$\Rightarrow \text{ must have : } -\rho_{DE} \lesssim \Lambda \lesssim \mathcal{O}(10)\rho_{DE}$$

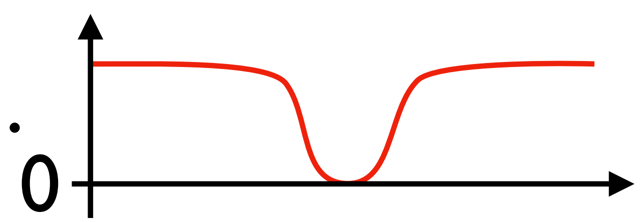
- — a “weak anthropic” argument [Weinberg ’89]

- works only IF there is a CC landscape WITH population mechanism!

- alternative - `quintessence' landscape:

  - we can replace the dS vacua from O(100) 4-form fluxes with a landscape of O(100) scalar field potentials with minimum at CC $\leq$ 0 and slow-roll flat plateaus

$$V = \sum_i \Delta V_i \cdot$$



  - there is no true CC > 0 , but if at least one scalar is on the slow-roll plateau — Coleman-deLuccia tunneling + quasi-dS vacuum fluctuations populate all plateaus:

fine-tuned `quintessence' landscape of quasi-CCs

- but:

  - here as well you need anthropics to explain observed DE magnitude

  - additionally: need to explain slow-roll flatness of quintessence scalar potentials

    there is NO anthropic need for this!

    … so, double fine-tuning !

- can attempt to embed both toy model classes into string theory:

why? - quantum gravity loops are even worsely divergent:

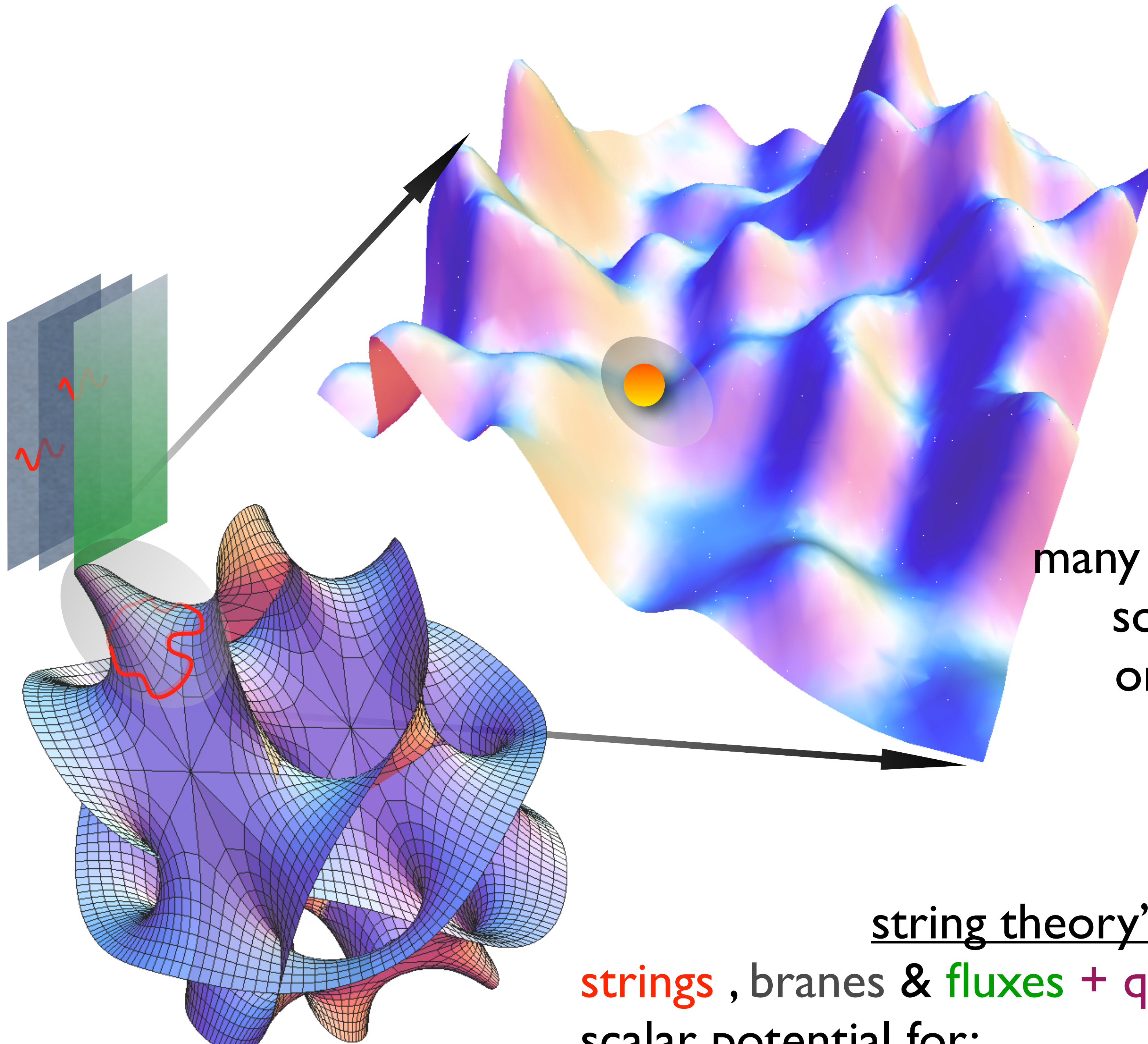
$$\left.\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} \\ h_{\mu\nu} &\to \tfrac{1}{M_{\rm P}} h_{\mu\nu} \end{aligned}\right\} \quad \Rightarrow \quad R \sim (dh)^2 + \frac{1}{M_{\rm P}} h(dh)^2 + \frac{1}{M_{\rm P}^2} h^2 (dh)^2 + \dots$$

- benefit:
  string theory is UV complete,
  all quantum gravity loop contributions to the CC are finite by modular invariance

- problems: string theory has 6 extra dimensions and SUSY

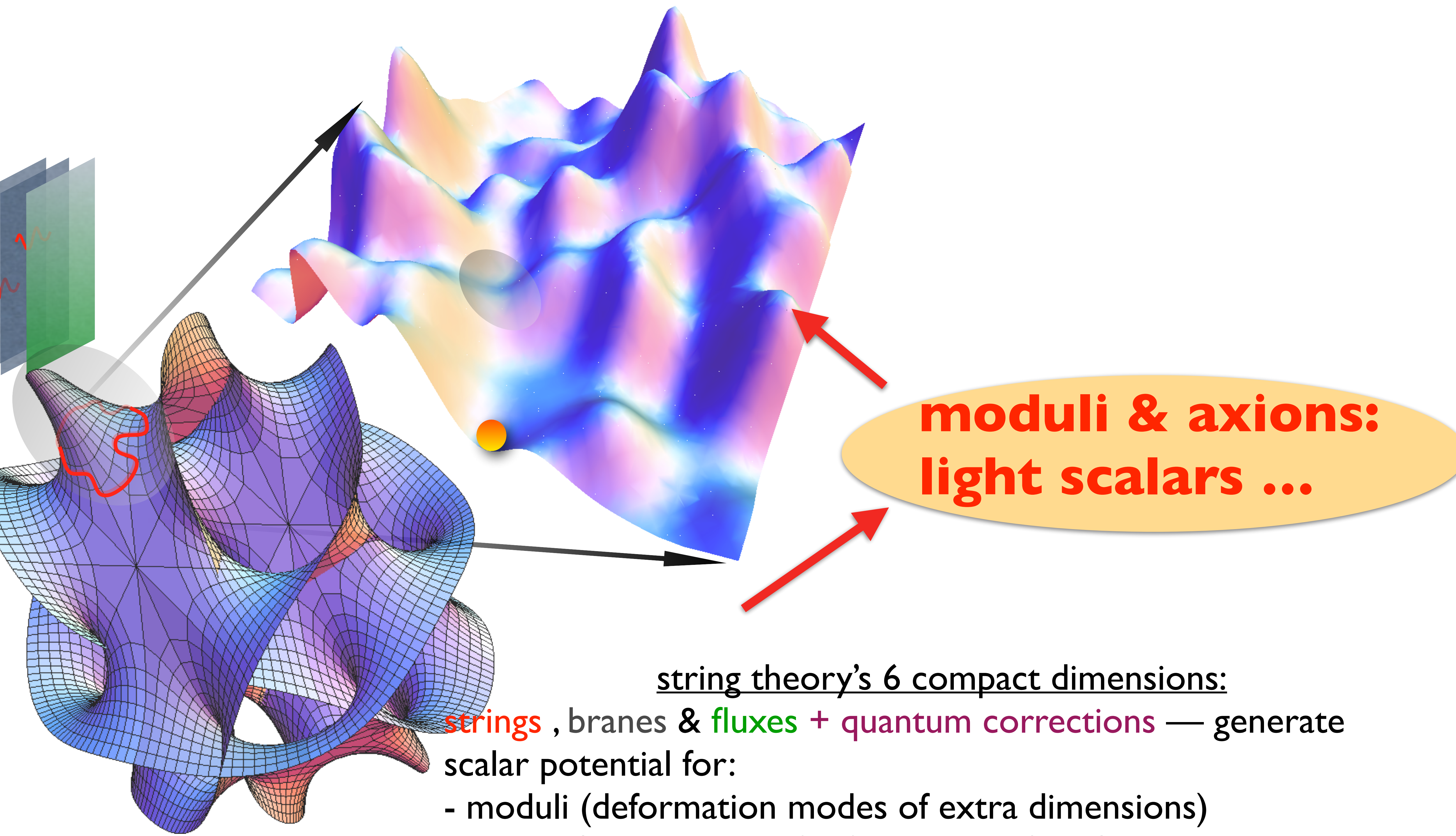
need compactification from 10D to 4D:

6 compact extra-dimensions … and there is more



the string theory *landscape*:
many isolated *vacua*, connected by tunneling
some mountain slopes drive *inflation*
or *slow-roll dark energy (quintessence)*

string theory's 6 compact dimensions:
strings , branes & fluxes + quantum corrections — generate
scalar potential for:
- moduli (deformation modes of extra dimensions)
- axions (zero modes of p-form gauge fields)



<u>string theory's 6 compact dimensions:</u>

strings , branes & fluxes + quantum corrections — generate scalar potential for:

- moduli (deformation modes of extra dimensions)
- axions (zero modes of p-form gauge fields)

- increasingly explicit constructions of dS vacua (minima) in string theory:

  - extra dimensions are a Calabi-Yau manifold (Ricci curvature = 0)

    > low-energy SUSY at tree-level

    > stabilization of all moduli needs quantum effects

    > dS vacua possible IF all string corrections and the hierarchy of EFT mass scales are controlled

for reviews see e.g.:
arXiv:2203.07629
arXiv:2303.04819

  - extra dimensions negatively curved

    > classical contributions from curvature, fluxes, branes and orientifold planes sufficient to generate dS vacua

    > effective action is 10D supergravity — worldsheet string description `ab initio' is unclear

- contrast:

  - network of conjectural constraints on low-energy EFTs from general properties of semi-classical quantum gravity and/or classes of string theory solutions — Swampland Program

- observation in our context:

  in string compactifications with worldsheet description and zero CC at tree level (e.g. CYs) …

  … no dS vacua exist stabilized by classical contributions only — quantum effects seem to be relevant

  conjecture: $V > 0, \phi \to \infty \ : \ |V'| \gtrsim V \text{ or } V'' \lesssim -V$

  there are no dS minima at parametrically extra dimension size and/or parametrically weak string coupling

  [Garg,Krishnan,'18]
  [Ooguri, Palti, Shiu, Vafa, '18]
  [Hebecker, Wrase, '18]

  Study of this asymptotic no-dS conjecture, related swampland conjectures & interplay with string dS constructions provides lasting challenge for coming years!

  for a review see: arXiv:1903.06239

- have we exhausted the possibilities of
  toy model landscapes with population mechanisms?

… one more thing.

compare BP to covariant unimodular GR [Henneaux-Teitelboim '89]:

enters as a Lagrange multiplier scalar field!

$$S = \int d^4x \left[\sqrt{-g}\frac{M_{\rm P}^2}{2} R - \Lambda \left(\sqrt{-g} - \frac{1}{M_{\rm P}^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \right) \right]$$

$$F_4 = dA_3$$

$$\frac{\delta S}{\delta g^{\mu\nu}} \Rightarrow G_{\mu\nu} = -\frac{1}{M_{\rm P}^2} \left(T_{\mu\nu} + \Lambda g_{\mu\nu} \right)$$

$$\frac{\delta S}{\delta \Lambda} \Rightarrow \sqrt{-g}\epsilon_{\mu\nu\rho\sigma} = \frac{1}{M_{\rm P}^2} F_{\mu\nu\rho\sigma}$$

$$\frac{\delta S}{\delta A_3} \Rightarrow d\Lambda = 0 \;\Rightarrow\; \Lambda = const.$$

from now on: $M_{\rm P} = 1$

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$$+ S_{\rm boundary} - \mathcal{T}_A \int d^3\xi \sqrt{\gamma_A} - \mathcal{Q}_A \int A_3$$

[Kaloper; Kaloper & AW '22]

membrane charged under A_3

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Euclidean Field Eqs

- Bulk:

$$ds_E^2 = dr^2 + a^2(r)\, d\Omega_3 \qquad \left(\frac{a'}{a}\right)^2 - \frac{1}{a^2} = -\frac{\Lambda}{3}$$

- Membrane junction conditions:

ambient flux

here: $\Lambda_{out} - \Lambda_{in} = \frac{1}{2}\mathcal{Q}_A$ **BP/BT:** $\Lambda_{out} - \Lambda_{in} = \frac{1}{2}\cdot 2\mathcal{Q}_A\mathcal{Q}_A$

$$\frac{a'_{out}}{a} - \frac{a'_{in}}{a} = -\frac{1}{2}\mathcal{T}_A \qquad a_{out} = a_{in}$$

- 3-form boundary conditions can be neglected since they cancel out

- Bulk solutions are sections of (horo)spheres

$$a(r) = a_0 \sin(\frac{r+\delta}{a_0})\,, \quad \text{for} \quad \Lambda > 0\,; \qquad a(r) = r + \delta\,, \quad \text{for} \quad \Lambda = 0\,;$$

$$a(r) = a_0 \sinh(\frac{r+\delta}{a_0})\,, \quad \text{for} \quad \Lambda < 0$$

[Kaloper; Kaloper & AW '22]

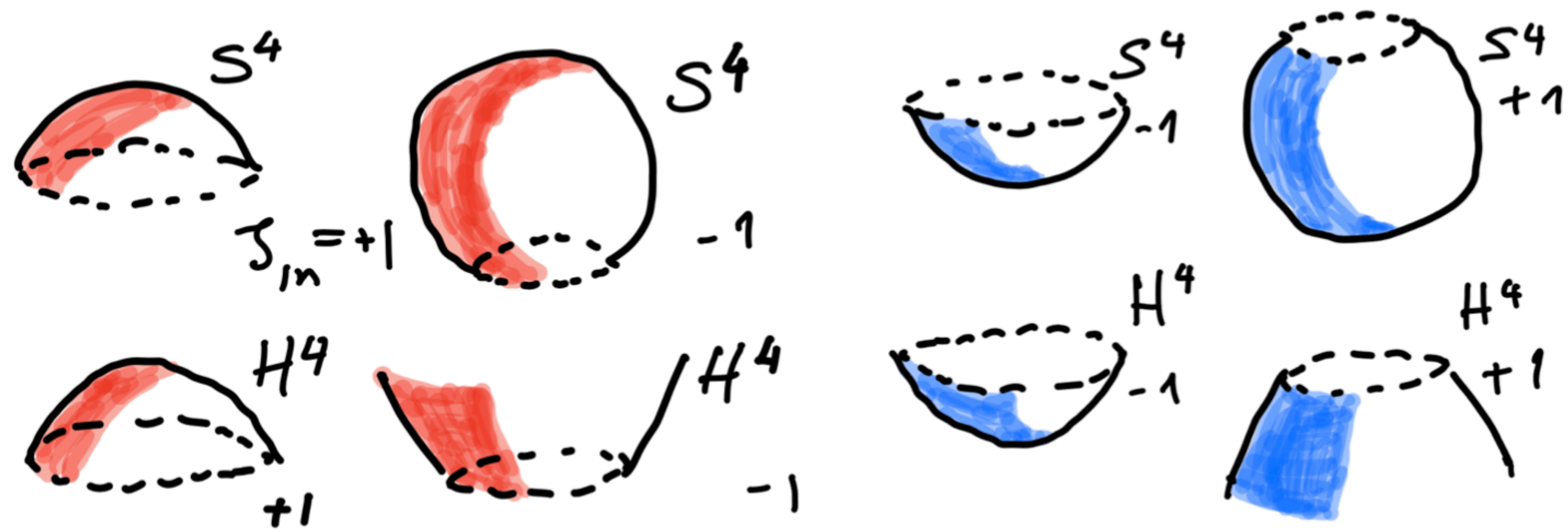
[Kaloper; Kaloper & AW '22]

$\mathcal{T}_A, \mathcal{Q}_A \neq 0$

- Bulk sections:

$\Lambda > 0$

$\Lambda < 0$



$\Lambda > 0$

$\Lambda < 0$

inside **outside**

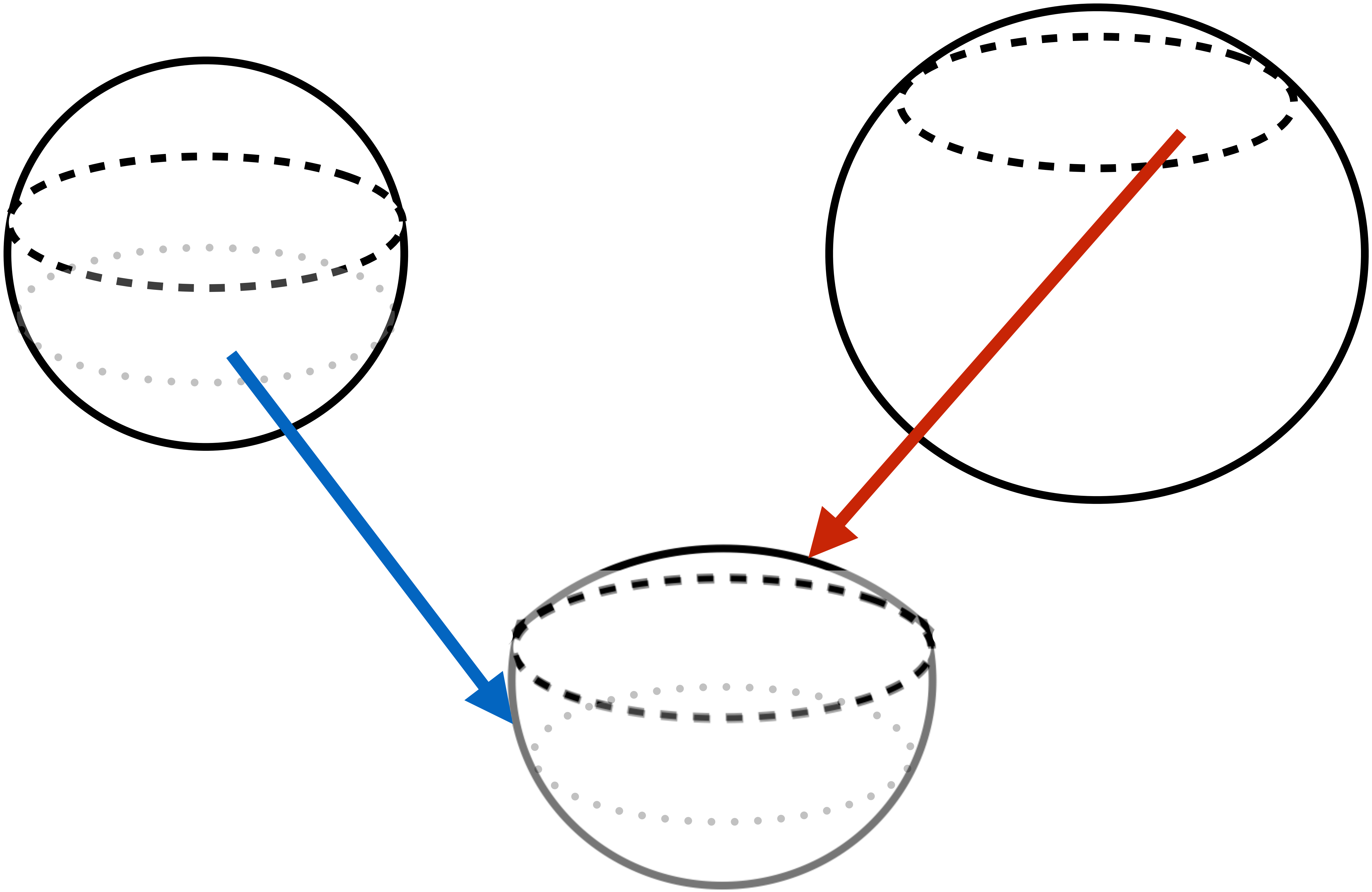
- Junction conditions: massaging the eqs, can rewrite them as

$$\zeta_{out}\sqrt{1-\frac{1}{3}\Lambda_{out}a^2} = -\frac{\mathcal{T}_A}{4}\,(1-q)\;a$$

$$\zeta_{in}\sqrt{1-\frac{1}{3}\Lambda_{in}a^2} = \frac{\mathcal{T}_A}{4}\,(1+q)\;a$$

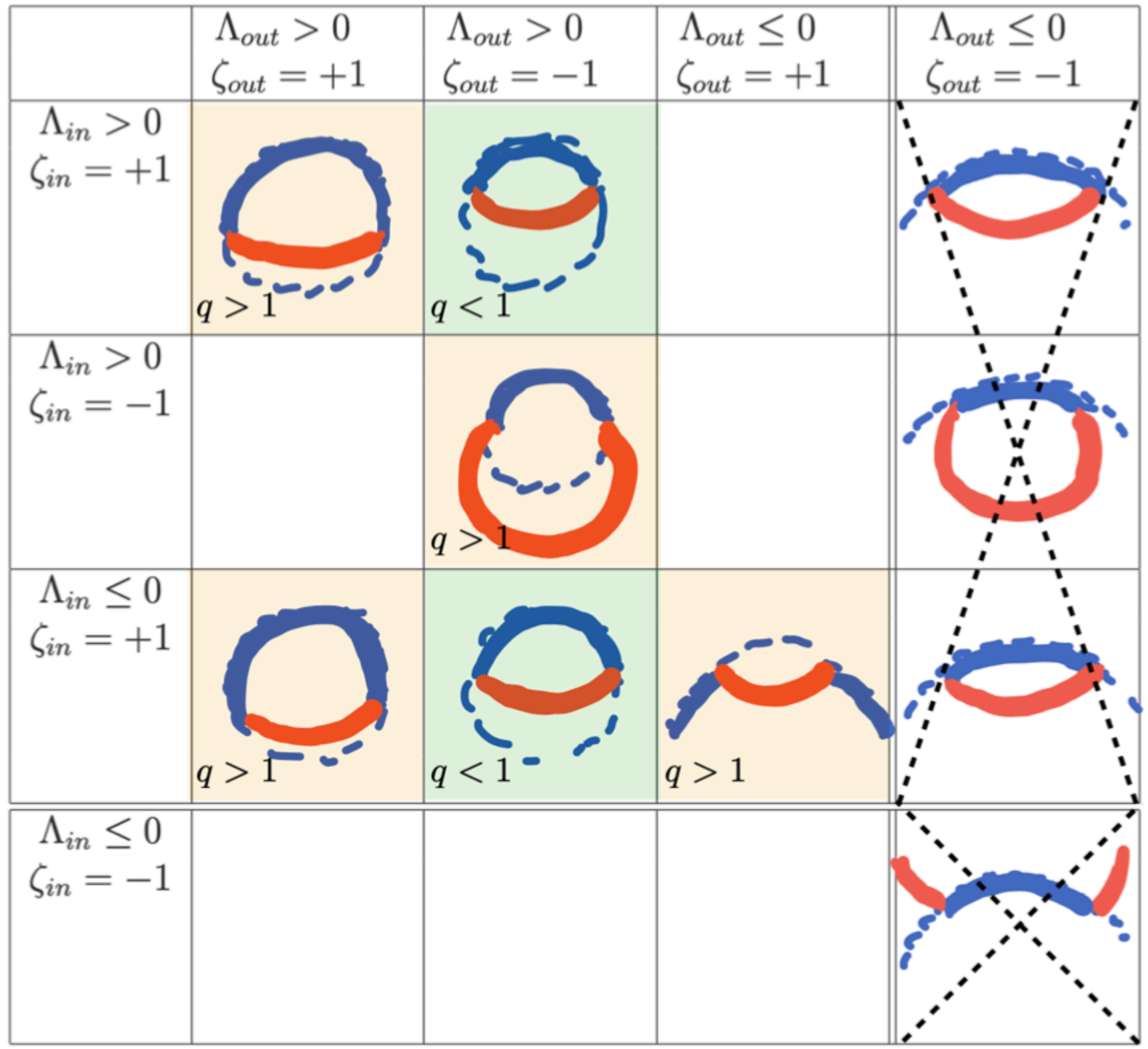
$$q \equiv \frac{2\mathcal{Q}_A}{3\mathcal{T}_A^2}$$

glueing de Sitter Instantons



menu of instantons

[Brown & Teitelboim ’87/’88]



- white: kinematically forbidden (no valid j.c. pairing)
- pale gold: $q > 1$
- pale green: $q < 1$
- crossed-out: divergent bounce action

$$q \equiv \frac{2Q_A}{3\mathcal{T}_A^2}$$

the crucial difference ...

- Junction conditions controlled by

ambient flux

here:
$$\left(1 \mp \underbrace{\frac{2\, M_{\rm P}^4\, \mathcal{Q}_A}{3\mathcal{T}_A^2}}_{q}\right)$$

BP/BT:
$$\left(1 \mp \underbrace{\frac{2M_{\rm P}^2 \cdot 2 Q_A\, \mathcal{Q}_A}{3\mathcal{T}_A^2}}_{q}\right)$$

- in BP/BT ratio *q* changes with decreasing background Q_A ...

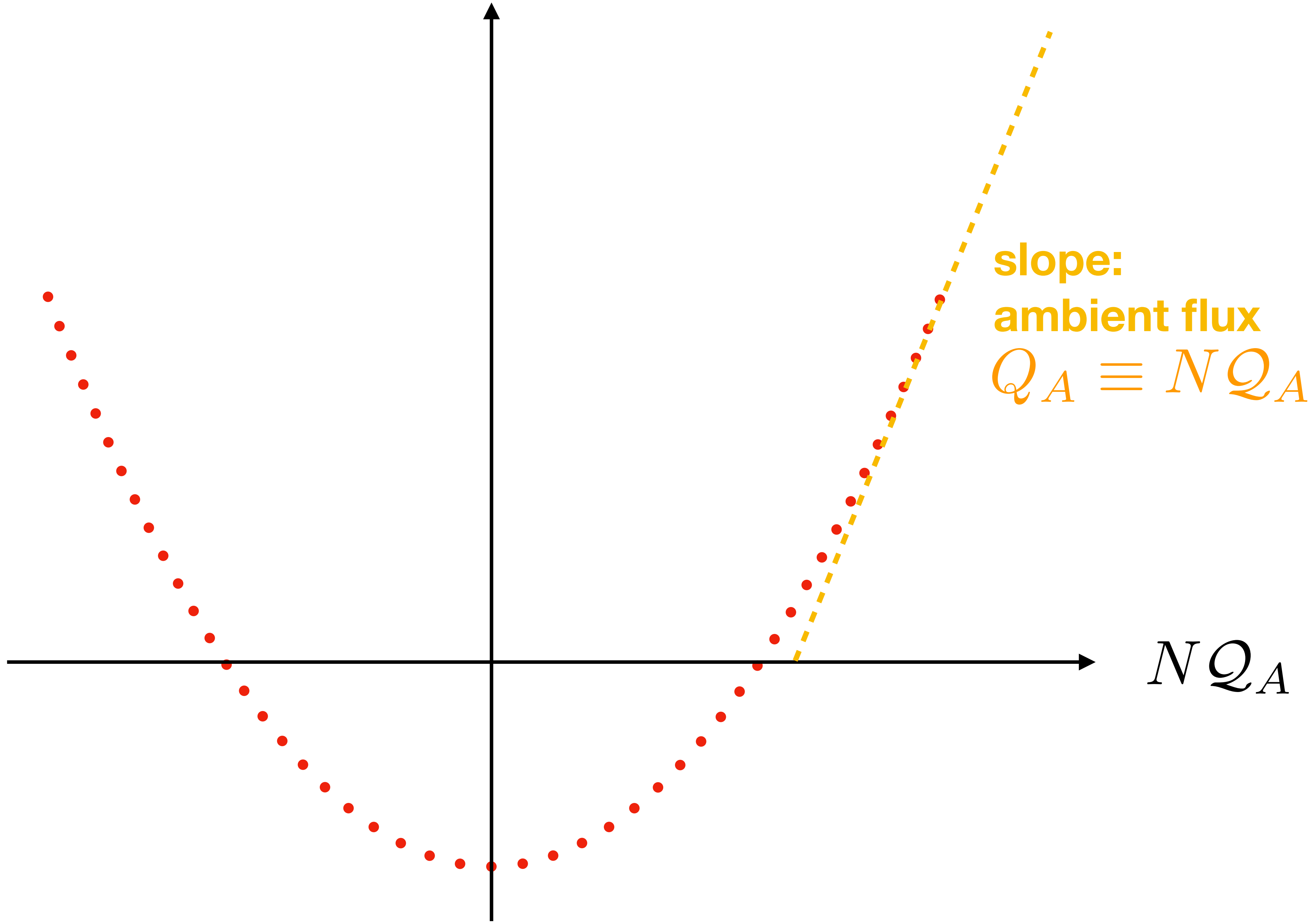
- here, *q* is constant - we can choose!

$$\frac{2\, M_{\rm P}^4\, \mathcal{Q}_A}{3\mathcal{T}_A^2} = q > 1 \quad or \quad < 1$$

[Kaloper; Kaloper & AW '22]

BP/BT:

$$V = \Lambda_0 + N^2 \mathcal{Q}_A^2 \quad , \quad \Lambda_0 < 0$$



BP/BT:

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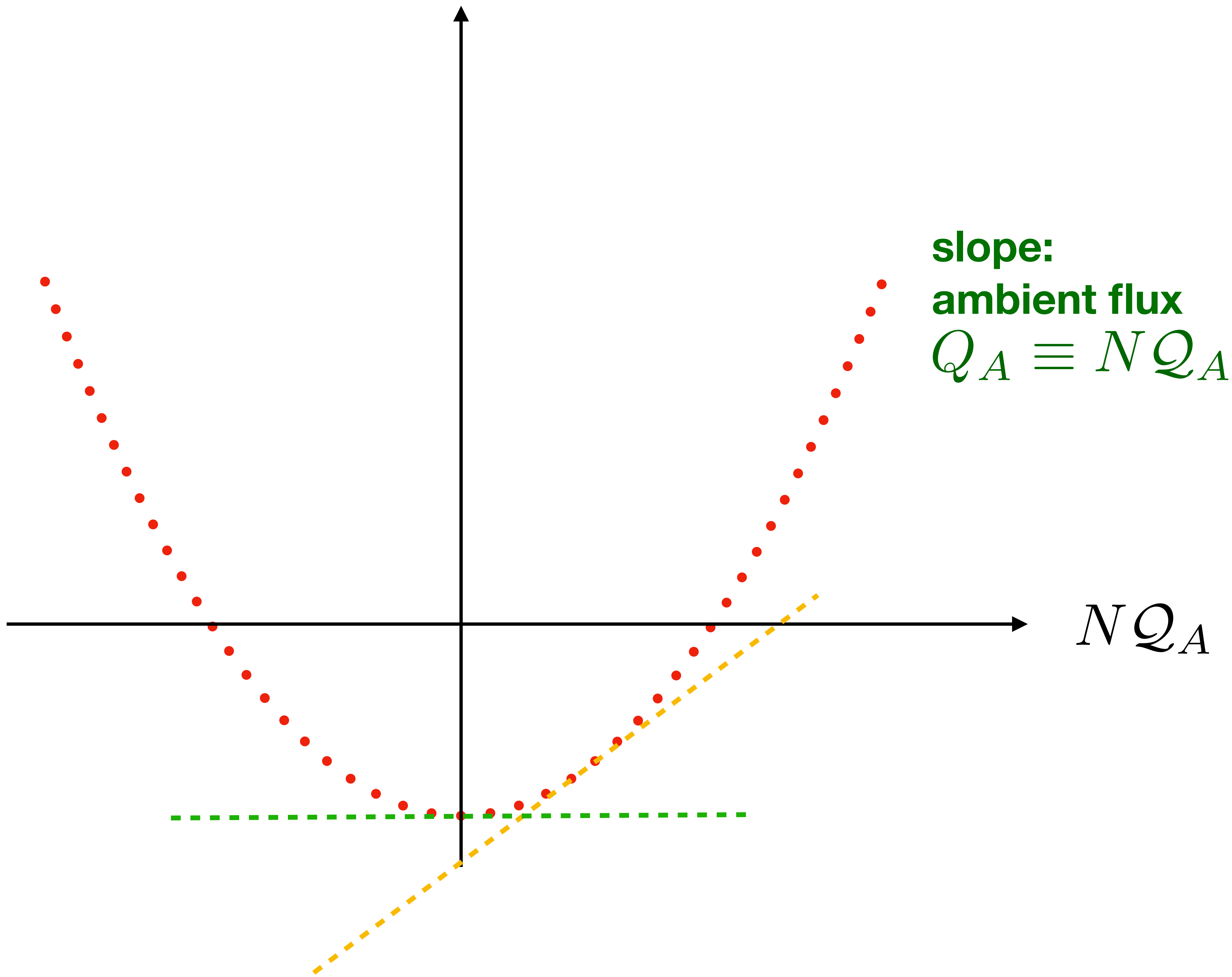


Bounce Action and Decay Rate

- tunneling rate & bounce action:

$$\Gamma \sim e^{-S(\texttt{bounce})} \qquad S(\texttt{bounce}) = S(\texttt{instanton}) - S(\texttt{parent})$$

- on-shell bounce action - evaluated at critical radius:

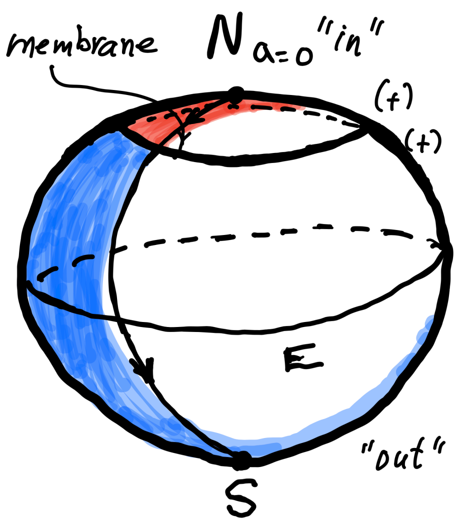
$$S(\texttt{bounce}) = 2\pi^2 \Big\{ \Lambda_{out} \int_{North\ Pole}^{a} da \Big(\frac{a^3}{a'}\Big)_{out} - \Lambda_{in} \int_{North\ Pole}^{a} da \Big(\frac{a^3}{a'}\Big)_{in} \Big\} - \pi^2 a^3 \mathcal{T}_A$$

$$2\pi^2 \Lambda_{in/out} \int_{North\ Pole}^{a} da \Big(\frac{a^3}{a'}\Big) = 18\pi^2 \frac{M_{\rm P}^4}{\Lambda_{in/out}} \Big(\frac{2}{3} - \zeta_{in/out}\big(1 - \frac{\Lambda_{in/out} a^2}{3M_{\rm P}^4}\big)^{1/2} + \frac{\zeta_{in/out}}{3}\big(1 - \frac{\Lambda_{in/out} a^2}{3M_{\rm P}^4}\big)^{3/2}\Big)$$

- rate calculable for instanton menu;
  divergent case are crossed out
- eq.s identical to Brown-Teitelboim;
  final rates depend on junction condition signs

Comparison of Decay Rates

[Brown & Teitelboim '87/'88; Bousso & Polchinski '00]



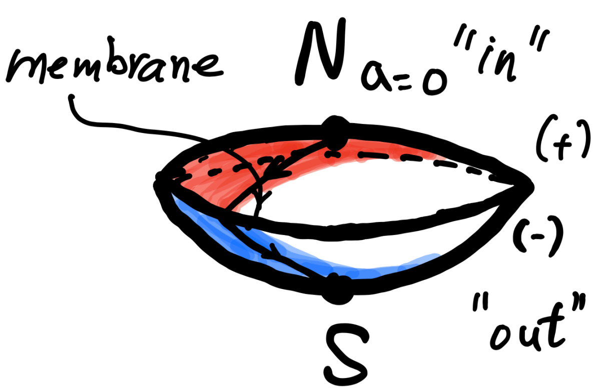
gold

$$S_{\text{bounce}} \simeq \frac{27\pi^2}{2}\frac{\mathcal{T}_A^4}{(\Delta\Lambda)^3} \simeq 108\pi^2\frac{\mathcal{T}_A^4}{M_P^6 Q_A^3} \qquad for \quad q > 1$$

- overshoots $\Lambda = 0$ into AdS

- process absent for $q < 1$ green

[Kaloper; Kaloper & AW '22]



$$S_{\text{bounce}} \simeq \frac{24\pi^2 M_P^4}{\Lambda_{out}}\left(1 - \frac{8}{3}\frac{M_P^2\,\Lambda_{out}}{\mathcal{T}_A^2}\right) \quad for \quad q < 1$$

- dependence on parent Λ persists for dS $\longrightarrow$ AdS transitions

→ this "brakes" the evolution

Cosmological Constant: No Problem!

- Define the problem first

$$\Lambda_{\texttt{total}} = M_{\rm P}^2 \left(\frac{\mathcal{M}_{\texttt{UV}}^4}{\mathcal{M}^2} + \frac{V}{\mathcal{M}^2} + \lambda \right) , \qquad \lambda = \lambda_0 + N \frac{\mathcal{Q}_A}{2} ,$$

- So:

$$\Lambda_{\texttt{total}} = M_{\rm P}^2 \left(\frac{\Lambda_0}{\mathcal{M}^2} + N \frac{\mathcal{Q}_A}{2} \right) ,$$

- Thus the CC is unstable - BUT - to make it arbitrarily small eventually we must either take a tiny membrane charge or fine tune initial value

- This is the problem.

The Fix: add ≥ 1 extra flux & charge

$$S = S[g,A] + \int d^4x \frac{\Lambda}{M_{\mathrm{P}}^2}\epsilon^{\mu\nu\rho\sigma}\hat{F}_{\mu\nu\rho\sigma} - \mathcal{T}_{\hat{A}}\int d^3\xi\sqrt{\gamma_{\hat{A}}} - \mathcal{Q}_{\hat{A}}\int \hat{A}_3$$

$$\frac{\mathcal{Q}_{\hat{A}}}{\mathcal{Q}_A} = \omega \in (\textbf{nearly})\textbf{Irrational Numbers}$$

[Banks, Dine & Seiberg '88]

- As a result: $\Lambda_{\mathtt{total}} = M_{\mathrm{P}}^2\Big(\frac{\Lambda_0}{\mathcal{M}^2} + \frac{\mathcal{Q}_A}{2}(N + \hat{N}\omega)\Big)\,.$
- $N, \hat{N}$ are integers; there exist $N, \hat{N}$, such that CC <<< 1
- long tunneling sequences:
  `green' instantons 'jump' CC down as long as CC > 0
- slow-down near zero CC

$$S_{\mathtt{bounce}} \simeq \frac{24\pi^2 M_{\mathrm{P}}^4}{\Lambda_{out}} \to \infty \quad \Rightarrow \quad \Gamma \to 0$$

[Kaloper & AW '22]

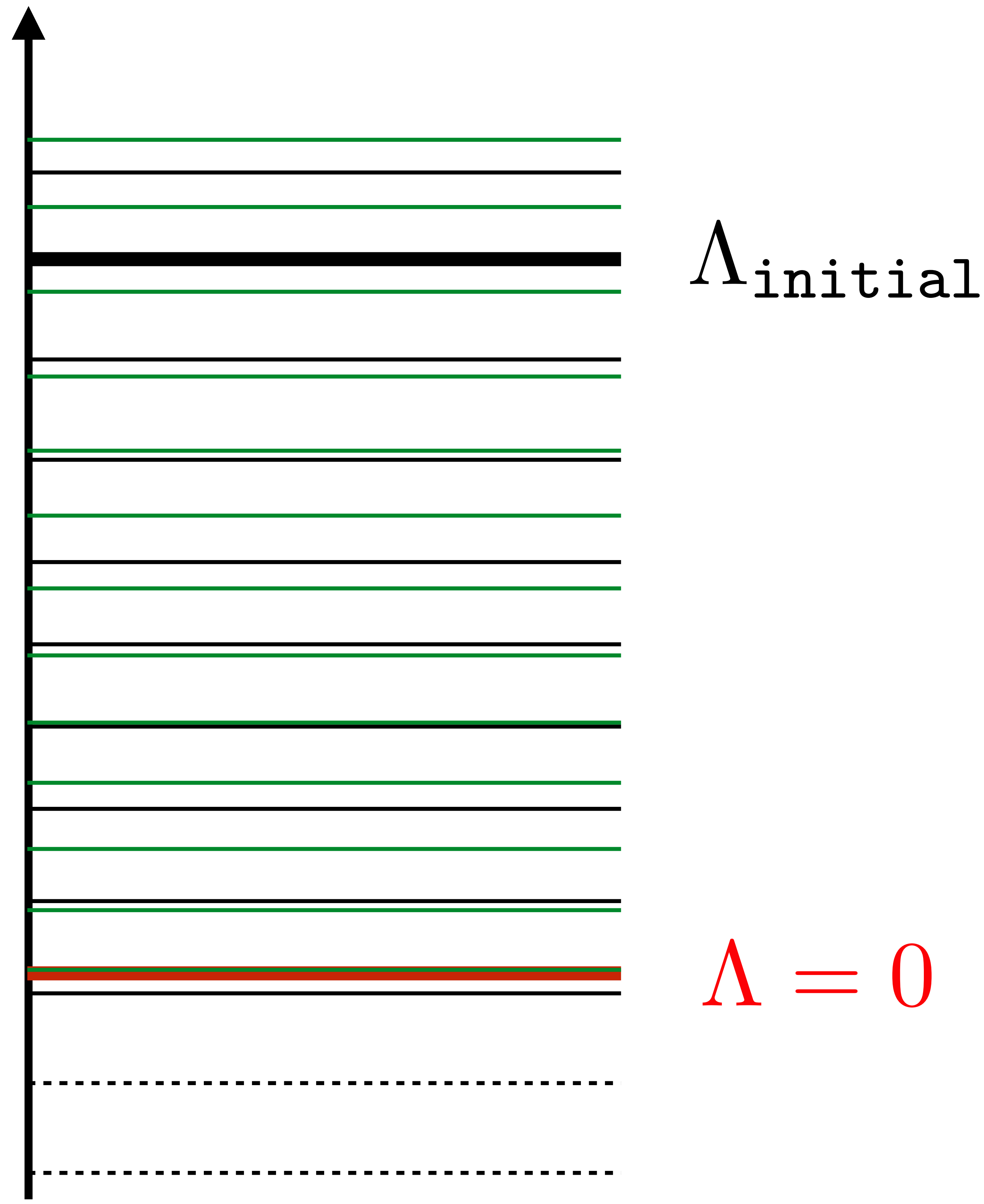
Stairway in Heaven

to dynamically get small CC,

need ≥ 2 stairways somewhat out of step

… *a landscape*

+ jumps stopping at zero CC (green instantons)



[Kaloper & AW ’22]

Approximate Density of States

- discrete evolution ~ Hawking-Baum CC-distro ['84]

$$Z = \int e^{-S_E} \simeq e^{-S_{classical}} = \begin{cases} e^{24\pi^2 \frac{M_P^4}{\Lambda}} = e^{\frac{A_{\rm horizon}}{4G_N}}\,, & \Lambda > 0\,; \\ e^{\Lambda \int d^4x \sqrt{g}} = 1\,, & \Lambda = 0\,; \\ e^{-|\Lambda| \int d^4x \sqrt{g}} \to 0\,, & \Lambda < 0\,, \quad \text{noncompact}\,. \end{cases}$$

- The conclusion is:
  - with irrational charge ratio *or* many fluxes/charges
  - \`green instanton' dominance — $q < 1$

$$\frac{\Lambda}{M_P^4} \to 0 \qquad \text{without anthropics!}$$

[Kaloper & AW '22]

Summary

- Dark energy poses a problem — purest form: CC problem
- The problem is freaking hard! — at the heart of QM vs Gravity
- local, dynamical cancellation mechanisms based on fields in sane QFT don't work
- we are left with accommodation of small dark energy by a landscape with population mechanism
- what landscape: true CC/dS vs fine-tuned quintessence ? stay tuned …
- anthropic arguments for small CC in a landscape necessary ? maybe not — discrete cancellation mechanism(s)? need more research!