# *Theory Overview / EFT for $\mu \leftrightarrow e$*

Sacha Davidson
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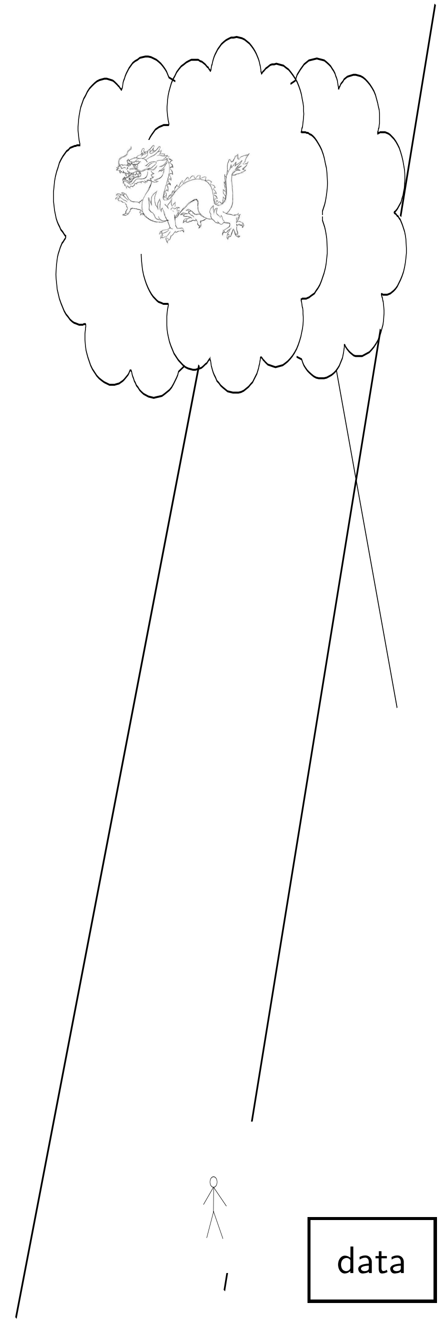



Not an overview (hard to make interesting)

**1:** random comments about LFV +what we know

~~2~~ interesting theory results

**2:** EFT for $\mu \leftrightarrow e$ light LFV NP: **Redigolo**

if $\mu \leftrightarrow e$ is there, will we see it?

if we see it, what can we learn?

Sorry to everyone I forgot to cite!

# Reasons to like LFV

- leptons do not have strong interactions

- leptons can generate the baryon asym. (non-perturbative SM $\not{B}+\not{L}$) without proton decay

- $[m_\nu]$ says there is NP in lepton sector, that must give LFV.

*so LFV exists —yippee!—but we don't see it yet...*

# What we know: categories of LFV constraints

$\boldsymbol{\Delta LF = 1, \Delta QF = 0}$
$\mu A \to eA$, $\tau \to 3l$, $h \to \tau^{\pm} l^{\mp}$... $(l \in \{e, \mu\})$

$\boldsymbol{\Delta LF = 2}$
$\mu \bar{e} \to e \bar{\mu}$, $\tau \to e e \bar{\mu}$...

$\boldsymbol{\Delta LF = \Delta QF = 1}$
$K \to \mu \bar{e}$

**loops $\approx$ not mix categories below $\Lambda_{\mathrm{LFV}}$**

$\boldsymbol{\Delta LF = \Delta QF = 1}$ ... leptoquarks?

**Swallow-NA62, Zuo-$\mu$-ion collider**
**Frau-LHCb, Fulghesu-LHCb**

$\boldsymbol{\Delta LF = 2}$: muonium oscillations

**Uesaka (th), Zhao (expt)**

# what we know about LFV : bounds/upcoming reach

$\Delta LF = 1, \Delta QF = 0 \quad (\Delta LF = \Delta QF = 1)\ ,\ \ (\Delta LF = 2)$

| some processes | current constraints on BR | future sensitivities |
|---|---|---|
| $\mu \to e\gamma$ | $< 4.2 \times 10^{-13}$ | $6 \times 10^{-14}$ (MEG) $\to$ ... |
| $\mu \to e\bar{e}e$ | $< 1.0 \times 10^{-12}$(SINDRUM) | $10^{-16}$ (202x, Mu3e) |
| $\mu Ti \to eTi$ | $< 6 \times 10^{-13}$ , (SINDRUMII) | $10^{-(16\to?)}$ (Mu2e,COMET) |
| $\mu Au \to eAu$ | $< 7 \times 10^{-13}$ , (SINDRUMII) | $10^{-(18\to?)}$ (PRISM/PRIME/ENIGMA) |
| $(\mu \to e\gamma\gamma$ | $< 7.2 \times 10^{-11})$ (CrystalBox) | |
| $\tau \to \{e, \mu\}\gamma$ | $< 3.3, 4.4 \times 10^{-8}$ | few$\times 10^{-9}$ (Belle-II) |
| $\tau \to e\bar{e}e, \mu\bar{\mu}\mu, e\bar{\mu}\mu...$ | $< 1.5 - 2.7 \times 10^{-8}$ | few$\times 10^{-9}$(Belle-II, LHCb?) |
| $\tau \to \begin{Bmatrix} e \\ \mu \end{Bmatrix} \{\pi, \rho, \phi, ...\}$ | $\lesssim$ few $\times 10^{-8}$ | few$\times 10^{-9}$ (Belle-II) |
| $h \to \tau^{\pm}\ell^{\mp}$ | $< 1.5, 2.2 \times 10^{-3}$(ATLAS/CMS) | $< 2 \times 10^{-4}$ (ILC) |
| $h \to \mu^{\pm}e^{\mp}$ | $< 6.1 \times 10^{-5}$ (ATLAS/CMS) | $2 \times 10^{-5}$ (ILC) |
| $Z \to e^{\pm}\mu^{\mp}$ | $< 7.5 \times 10^{-7}$(ATLAS) | |
| $Z \to l^{\pm}\tau^{\mp}$ | $< ... \times 10^{-7}$(ATLAS) | |
| $K^+ \to \pi^+\bar{\mu}e$ | $< 4.7 \times 10^{-12}$ (E865) | $10^{-12}$ (NA62) |
| ... | | |
| muonium | $P_{M\bar{M}} < 8.2 \times 10^{-11}$ (PSI) | $2 \times 10^{-14}$ (MACE) |

# Parametrising LFV data: the many defns of $\Lambda$

1. draw tree diagrams for a process
2. parametrise blob as Lorentz+gauge invariant operator (of dim $n$)
3. write coupling constant $C_I/\Lambda^{n-4}$ for operator $\mathcal{O}_I$
4. add to Lagrangian

data constrains $C_I/\Lambda^{n-4}$; can bound: $\left\{ \begin{array}{ll} \Lambda & \text{for } C_I = 4\pi \\ \Lambda & \text{for } C_I = 1 \\ \Lambda & \text{for } \sum_I C_I^2 = 1 \\ C & \text{for } \Lambda = v \\ C & \text{for } \Lambda = \text{TeV} \end{array} \right.$

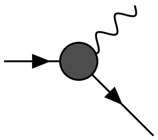
(then there are 2s for $+h.c$, flavour sums, ...)

$$\delta \mathcal{L}_{LFV} = 2\sqrt{2} G_F \sum_I C_I \mathcal{O}_I + \frac{1}{v^3} \sum_J C_J \mathcal{O}_J + ... + h.c. \quad , \quad 2\sqrt{2} G_F \equiv \frac{1}{v^2}$$

**zzz...**

# But what about the dipole?

the dipole operator allows on-shell fermion to emit on-shell $\gamma$: $\mu \to e\gamma$, edms, $g$-2



$$\delta\mathcal{L}_{\mu\to e\gamma} = \frac{M}{\Lambda_{\rm LFV}^2}\left(C_{D,L}\overline{e_R}\sigma^{\alpha\beta}\mu_L + C_{D,R}\overline{e_L}\sigma^{\alpha\beta}\mu_R\right)F_{\alpha\beta}$$

op. is dim5 at low energy, dim6 in SMEFT... what mass upstairs? M : ?$m_f \to v$?

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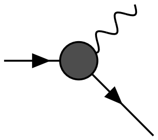

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KunoOkada (me): $M = m_\mu$ for $\mu \to e\gamma$, $M = m_e$ for $d_e$:

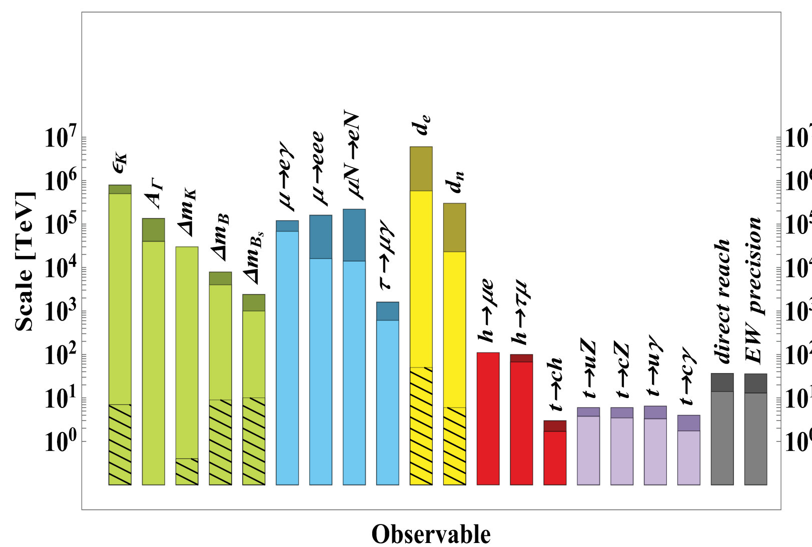
$$BR(\mu\to e\gamma) < 4.2\times 10^{-13} \Rightarrow \Lambda^{e\mu}_{\rm LFV} \gtrsim 10^4 v$$

$$d_e \leq 4.2\times 10^{-30} e{\rm cm} \Rightarrow \Lambda^{ee}_{NP} \gtrsim 3\times 10^4 v$$

EU Strategy : M = v

$$BR(\mu\to e\gamma) \Rightarrow \Lambda^{e\mu}_{\rm LFV} \gtrsim 4\times 10^5 v$$

$$d_e \Rightarrow \Lambda^{ee}_{NP} \gtrsim 4\times 10^6 v$$



## what we know about LFV : bounds/upcoming reach

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| $Z \to l^{\pm}\tau^{\mp}$ | $< ... \times 10^{-7}$(ATLAS) | **Pinsard-CPV in H and Z decays** |
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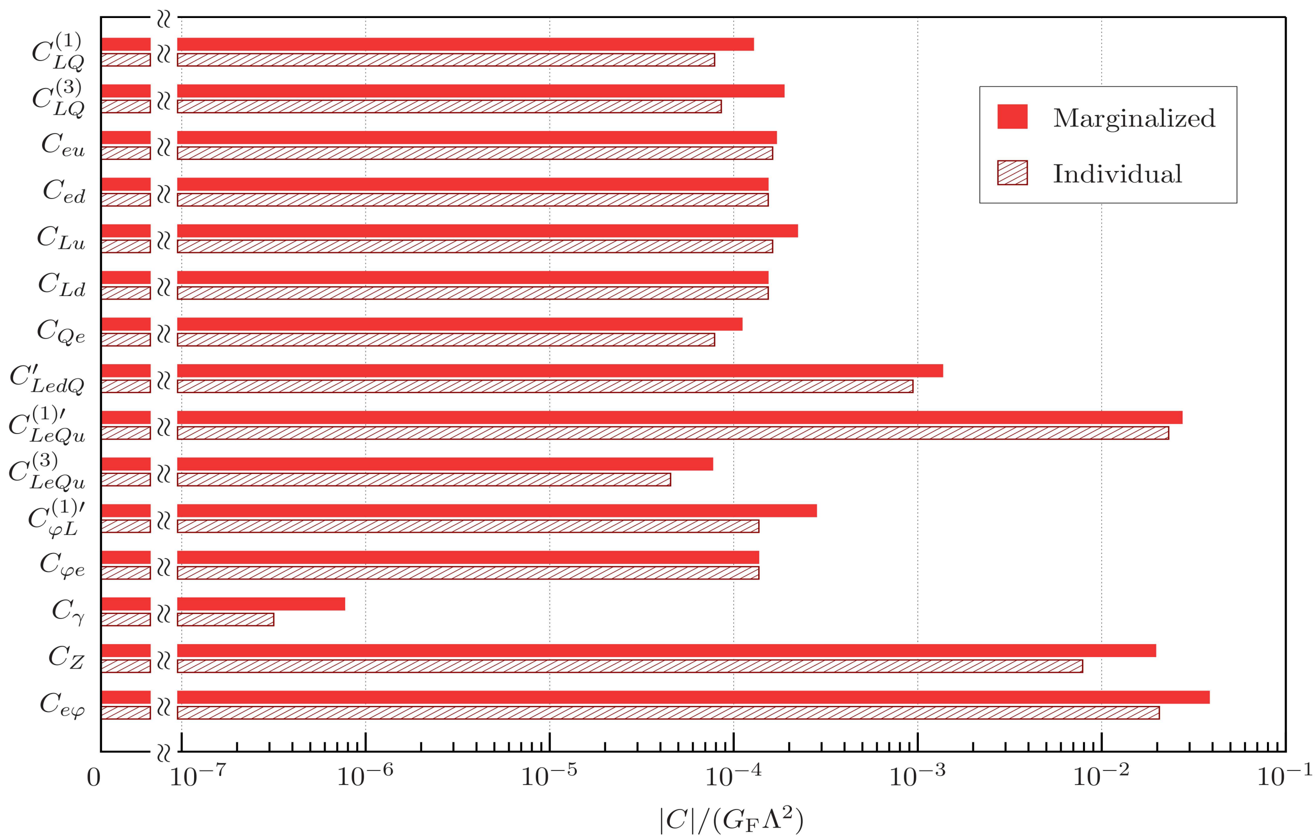
# The $\tau \leftrightarrow l$ sector : *marvellous place to observe LFV*

many processes: current data give indep bounds on magnitude of (almost) all operator coeffs, with $\Lambda_{\rm LFV} \sim 10$ TeV
$\Rightarrow$ promising for distinguishing models (+insensitive to most loops≈theoretically simple)

expected sensitivity of BelleII: BR$\lesssim 10^{-9} \to 10^{-10} \Leftrightarrow \Lambda_{\rm LFV} \sim 30$ TeV.




(taken from BanerjeeEtal, Snowmass WPaper 2203.14919 ) dipole as $C_\gamma v \mathcal{O}_D = C_D m_\tau \mathcal{O}_D$ !

# *EFT for the $\mu \leftrightarrow e$ sector*

M Ardu, B Echenard, S Lavignac

(only) three processes with restrictive bounds
+exceptional upcoming exptal sensitivities

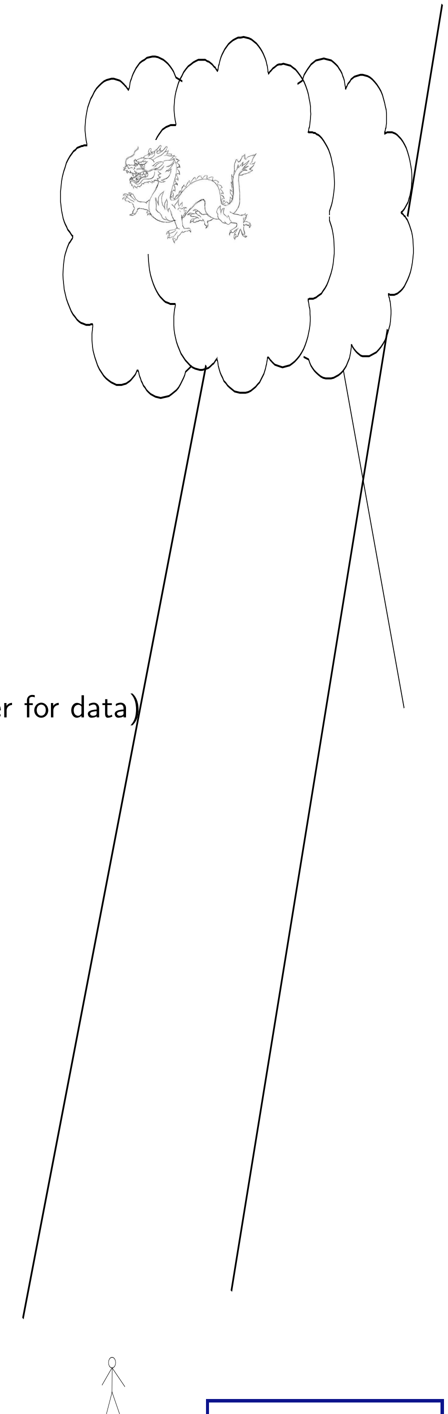
**1.** if $\mu \leftrightarrow e$ LFV is there, will we see it?

I want to know what data tells me about models (not what models prefer for data)
$\Rightarrow$ use EFT...

**3** are there are too many operators in EFT?

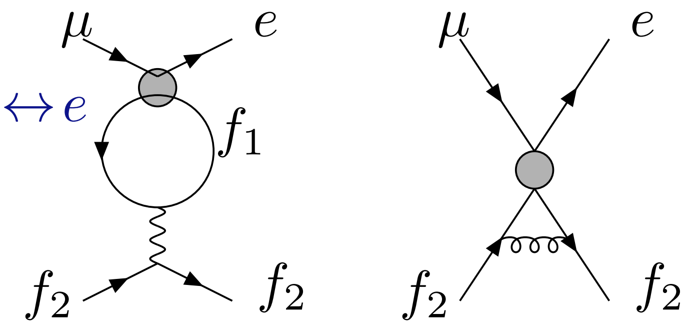
**4.** if we see $\mu \leftrightarrow e$, can we learn something about the model?

**2.** count exptal observables ($\sim$ 12)



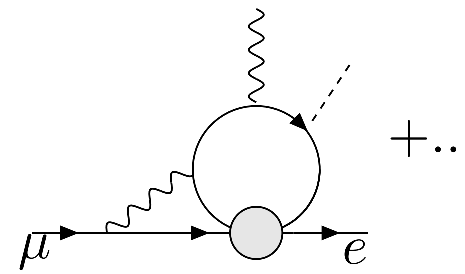
3 processes

# Are $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\mu A \to eA$ sufficient for discovery? 2010.00317




**Problem:** below $m_W$, ($\sim$100) 4-legged $\Delta QF=0$ $\mu \leftrightarrow e$ interactions$\approx$operators, few are measured

**Question:** if $\Delta QF=0$, $\mu \to e$ occurs, will it contribute to $\mu \to e\gamma, \mu \to e\bar{e}e$ or $\mu A \to eA$?




**Can show :** SM loops ensure almost every $\Delta QF = 0$, $\mu \to e$ interaction with $\leq 4$ legs, contributes $\gtrsim \mathcal{O}(10^{-3})$ to amplitudes $\mu \to e\gamma$ , $\mu \to e\bar{e}e$ and/or $\mu A \to eA$ (not $\bar{e}\mu G\widetilde{G}$...)

**Answer:** ?Probably yes? (modulo cancellations)

that is: current bounds sensitive to $\Lambda_{\rm LFV} \lesssim \begin{cases} 100 \to 300 & \text{TeV at tree} \\ 3 \to 10 & \text{TeV at loop} \end{cases}$

with upcoming $\mu \leftrightarrow e$ reach, even probe $\tau$-LFV, via $(\mu \to \tau) \times (\tau \to e)$

Ardu

# What can be measured in $\mu \to e\gamma$ or $\mu \to e\bar{e}e$? (review from KunoOkada)

KunoOkada

$$\delta\mathcal{L}_{\substack{\mu\to e\gamma \\ \mu\to e\bar{e}e}}\Big|_{m_\mu} = \frac{1}{v^2}\Big[C_{DR}(m_\mu\overline{e}\sigma^{\alpha\beta}\mu_R)F_{\alpha\beta} + C_{SRR}(\overline{e}P_R\mu)(\overline{e}P_Re) + C_{VLR}(\overline{e}\gamma^\alpha\mu_L)(\overline{e}\gamma_\alpha e_R)$$

$$+C_{VLL}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_Le)\Big] + \frac{1}{v^2}\Big[R \leftrightarrow L\Big] \quad , \quad \frac{1}{v^2} = 2\sqrt{2}G_F$$

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$\mu \to e\gamma$ with $\mu$-polarisation fraction $P_\mu$, $\theta_e$= angle between $\mu$-spin and $\vec{p}_e$

$$\frac{dBR(\mu\to e\gamma)}{d\cos\theta_e} = 192\pi^2\Big[|C_{DR}|^2(1 - P_\mu\cos\theta_e) + |C_{DL}|^2(1 + P_\mu\cos\theta_e)\Big]$$

KunoOkada

$\mu \to e\bar{e}e$ : ($e$ relativistic $\Rightarrow$ negligeable interference between $e_L$, $e_R$)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2$$

OkadaOkumuraShimizu

$$+ |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

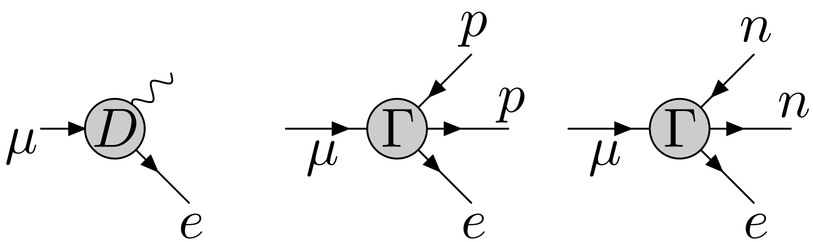
$\mu$ pol. + $e$ angular distributions $\Rightarrow$ measure 4$l$ coefficients + some phases

$\Rightarrow$ **measure magnitude of** $\{C_{DR}, C_{VLL}, C_{VLR}, C_{SRR}, +[L \leftrightarrow R]\}$

# If see $\mu A \to eA$ — what can be measured? (Haxton talk with this title!)

KunoNagamineYamazaki

- $\mu^-$ captured by nucleus, falls to $1s$. (can obtain some $\mu$ polarisation)
- $\mu \leftrightarrow e$ via dipole (with $\vec{E}$) or $\widetilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X \mu)(\bar{N}\Gamma N)$




$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\,,\ A\,,\ T\}$$

# If see $\mu A \to eA$ — what can be measured? (Haxton talk with this title!)

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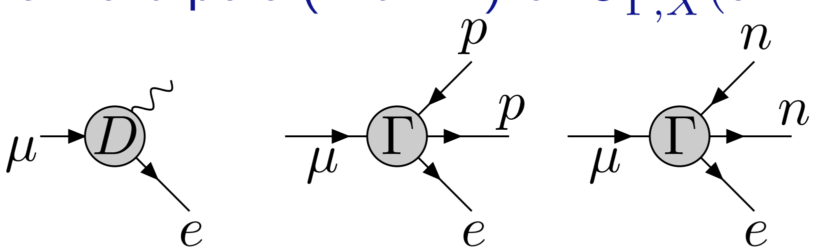



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\ ,\ A\ ,\ T\}$$

- leading "Spin Indep." contribution from $\{D, V, S\}$, coherent across $A$ (BR grows with $A$)

  Spin Indep. conversion ratio on target $A$: KitanoKoikeOkada 2002

$$\frac{32G_F^2 m_\mu^5}{\Gamma_{cap}}\left[|I^p_{V,A}\tilde{C}^p_{V,L} + I^p_{S,A}\tilde{C}^p_{S,R} + I^n_{V,A}\tilde{C}^n_{V,L} + I^b_{S,A}\tilde{C}^n_{S,R} + I_{D,A}C_{D,R}|^2 + |L \leftrightarrow R|^2\right]$$

$I^N_{\Gamma,A} = \int_{\text{nucleus A}}$ lepton wavefns$\times$ S/V density of $N$s — Hitlin, Haxton

CiriglianoDKuno, DKunoSaporta
HoferichterMenendezNoel

- include Spin Dependent (real nuclear phys caln)
- better neutron densities ...
- more targets — HeeckSzafronUesaka
- more operators — DKunoUesakaYamanaka
- NLO $\chi$PT ... — CiriglianoEtal 2203.09547
- with sufficient targets + th. accuracy, measure all $\tilde{C}$s?

(I assume $(\mu A \to eA)_{SI}$ now, constrains $\{\boldsymbol{C_{Al,L}, C_{Al,R}, C_{Au\perp,L}, C_{Au\perp,R}}\}$)

DKunoYamanada

# to define operators for targets:

Spin Indep. conversion ratio on target $A$, KitanoKoikeOkada 2002

$$\frac{32G_F^2 m_\mu^5}{\Gamma_{cap}}\Big[|I^p_{V,A}\tilde{C}^p_{V,L} + I^p_{S,A}\tilde{C}^p_{S,R} + I^n_{V,A}\tilde{C}^n_{V,L} + I^b_{S,A}\tilde{C}^n_{S,R} + I_{D,A}C_{D,R}|^2 + |L \leftrightarrow R|^2\Big]$$

$\Rightarrow$ target $A$ identified by unit vector

$$\vec{u}_A = \frac{1}{\sqrt{\sum I_\Gamma^2}}\Big(I^p_{V,A}, I^p_{S,A}, I^n_{V,A}, I^b_{S,A}, I_{D,A}\Big)$$

and sees coeff. $C_A = \vec{C}\cdot v_A$ of operator $O_A = \vec{O}\cdot v_A$ (check:substitute into BR)
Ex, for Al (all $\{I_\Gamma\}$ comparable)

$$\begin{aligned}\mathcal{O}_{Al} &= \frac{1}{2}\Big(O^p_{V,L} + O^p_{S,R} + O^n_{V,L} + O^n_{S,R} + \frac{1}{2}O_{D,R}\Big)\\ \text{can write}\quad \mathcal{O}_{Au} &= \cos\theta_{Al-Au}O_{Al} + \sin\theta_{Al-Au}O_{Au,\perp}\end{aligned}$$

KKO accuracy $\approx$ 2 indep targets:light + heavy DKunoYamanaka

$\Rightarrow$ $\mu A \to eA$ now constrains $\{\boldsymbol{C_{Al,L}, C_{Al,R}, C_{Au\perp,L}, C_{Au\perp,R}}\}$

## many operators+few constraints=using inconvenient basis

Have 6 (+6) constraints on $e_L$ ($e_R$) operator coefficients. Focus on $e_L$.
**Want to change basis to** *scale -dependent* **basis of constrained 6-d subspace.**

**1.** $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$
Solving RGEs for coefficients (arranged in row vector) gives:

$$\vec{C}(m_\mu) = \vec{C}(\Lambda_{\rm LFV})\boldsymbol{G}(m_\mu, \Lambda_{\rm LFV})$$

so measured $C_{DR} \sim$ weighted sum of many $C$s at $\Lambda_{\rm LFV}$.
Or, a single coeff of a weighted sum of operators...

**2-6.** repeat for other independent constraints.

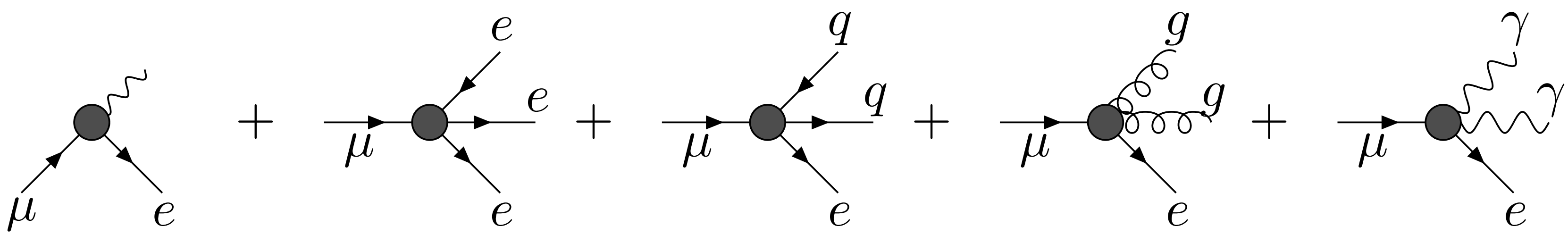
The "excess operators/flat directions" (experimentally inaccessible) are orthogonal, and therefore irrelevant.

Basis should span the finite-eigenvalue subspace of the correlation matrix.

**what to do with this basis?**

# (parenthese: are there too many operators in EFT?

1. operators (more-or-less) correspond to observable interactions




"blob" any Lorentz contraction, coupling of inverse mass dimension.

# Are there too many operators in EFT?

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$$Y_D m_\mu \bar{e}\sigma\cdot F\mu_X + Y_\Gamma^{4l}(\bar{e}\Gamma\mu_X)(\bar{e}\Gamma e_Y) + Y_\Gamma^{2l2q}(\bar{e}\Gamma\mu_X)(\bar{q}\Gamma q_Y) + Y^{GG}(\bar{e}\Gamma\mu_X)GG + Y^{FF}(\bar{e}\Gamma P_X\mu)FF$$

"$\Gamma$" any Lorentz contraction, coupling $Y$ of inverse mass dimension.

2. but few (well-measured) $\mu \leftrightarrow e$ interactions; which exptalists focus on measuring...
3. this is perceived as a fact, not a problem
4. ...? so why is it a problem that there is theory parametrisation for interactions that exptalists don't observe? ??
5. in EFT, do what exptalists do: define an operator basis corresponding to the observables... (no physics in a basis choice. But some bases more convenient than others)

...so with 12 observables, do EFT in 12-d space.

**what to do with this basis?**

# if see $\mu \to e\gamma$, $\mu \to e\bar{e}e$, or $\mu A \to eA$...?can distinguish models?

...model predictions studied for decades...

EFT recipe to study this: (not scan model space—no measure)

- data is a "12-d" ellipse/box in coefficient-space (in an ideal theorist's world)

- with RGEs, can take ellipse to $\Lambda_{\rm LFV}$

- are there parts of ellipse that a model *cannot* fill?

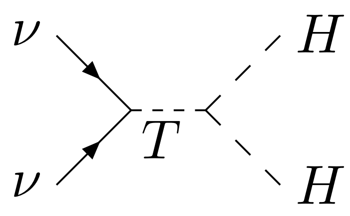
If yes, model can be distinguished/ruled out by $\mu \leftrightarrow e$ data.

Apply recipe:
1) type II seewaw
2) (singlet LQ for $R_D^*$)
3) ...

# Type II seesaw — add SU(2) triplet scalar $\vec{T}$

$$\mathcal{L} \supset \left( [Y]_{\alpha\beta} \overline{\ell^c_\alpha} \varepsilon \vec{\tau} \cdot \vec{T} \ell_\beta + M_T \lambda_H \; H \varepsilon \vec{\tau} \cdot \vec{T}^* H + \text{h.c.} \right) + ...$$

get $[m_\nu]$ at tree (NB: 2 mass scales, so unclear notion of $\Lambda_{\rm LFV}$):




$$[m_\nu]_{\alpha\beta} \sim \frac{[Y]_{\alpha\beta} \lambda_H M_T v^2}{M_T^2} \sim 0.03 \text{ eV} \times [Y]_{\alpha\beta} \frac{\lambda_H}{10^{-12}} \frac{\text{TeV}}{M_T}$$

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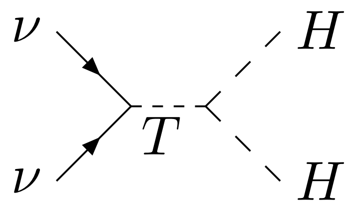


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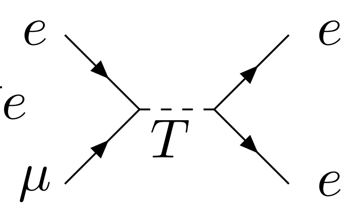
expect $\mu \to e\bar{e}e$ at tree (vanish via Majorana phases $\phi_i$):

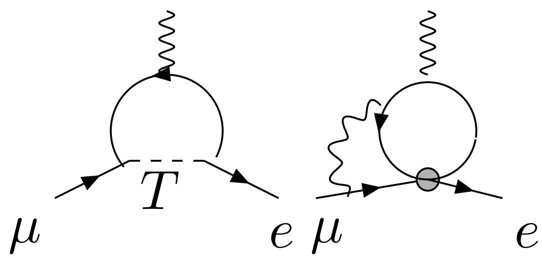
$\mu \to e\bar{e}e$



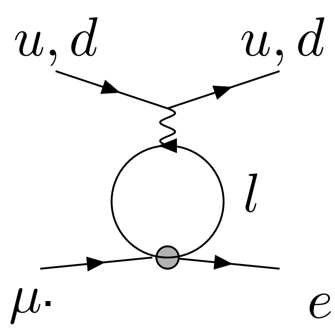

$$C^{e\mu ee}_{V,LL} \sim \frac{[Y]_{\mu e} [Y^*]_{ee} v^2}{M_T^2}$$

and $\mu \to e\gamma$, $\mu A \to eA$ at loop (weaker dependence on unknown model params)

$\mu \to e\gamma$




$\mu A \to eA$



# Type II seesaw: predictions

recall 12 (complex) operator coefficients $\left\{ \begin{array}{l} C_{DR},\ C^{e\mu ee}_{VLL},\ C^{e\mu ee}_{VLR},\ C^{e\mu ee}_{SRR},\ C_{AlightL},\ C_{AheavyR} \\ C_{DL},\ C^{e\mu ee}_{VRL},\ C^{e\mu ee}_{VRR},\ C^{e\mu ee}_{SLL},\ C_{AlightL},\ C_{AheavyR} \end{array} \right.$

- seven coefficients for LFV-involving-singlet-leptons are negligeable

(predicted by all $m_\nu$ models where NP interacts with doublets); test by polarising $\mu$. Kuno Okada

# Type II seesaw: predictions

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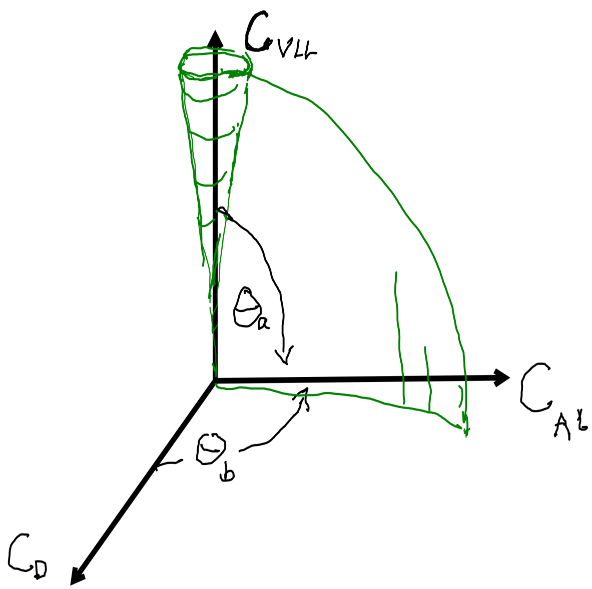
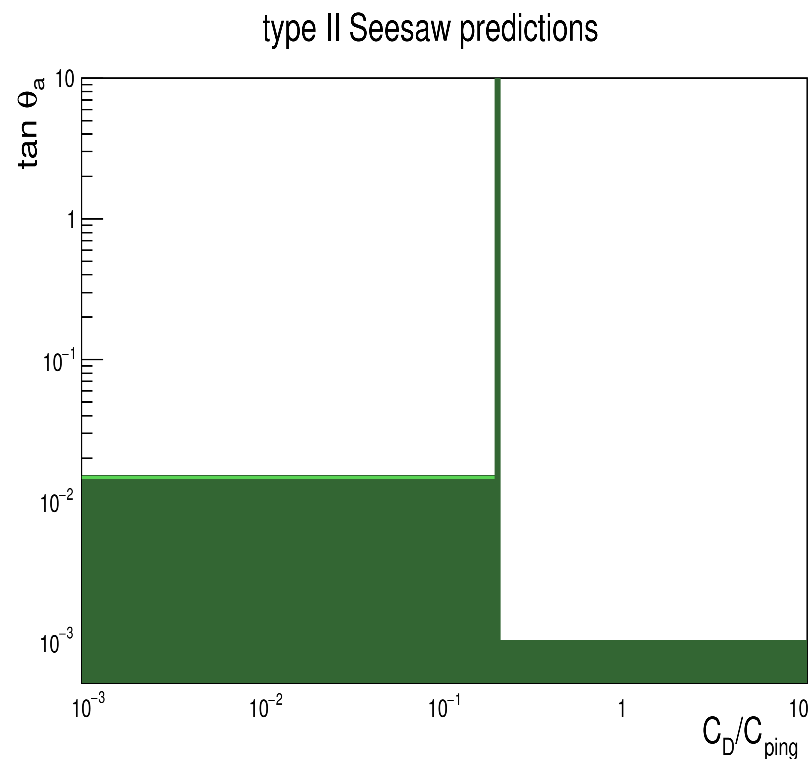
- seven coefficients for LFV-involving-singlet-leptons are negligeable

(predicted by all $m_\nu$ models where NP interacts with doublets); test by polarising $\mu$. Kuno Okada

- $C^{e\mu ee}_{VLL}$ $(\mu \to e\bar{e}e)$ or $C_{Al,L}(\mu A \to eA)$ can vanish (also any of $C_{DR}$ for $m_\nu \gg$)

- $C^{e\mu ee}_{VLL}$ $(\mu \to e\bar{e}e)$ "naturally" large: predict $C_{DR}/C_{Al,L}$ for small $C^{e\mu ee}_{VLL}$.




*prelim!*

model lives in green area expt can probe whole plot: $\tan\theta_{a,b} : 10^{-3} \to 10$

vert. axis ~ loop/tree ; horiz. axis ~ $|C_D|/|C_{Al}|$

# A leptoquark (for $R_{D^*}$)

SU(2) singlet scalar LQ, mass $m_{LQ}$, interactions to all flavours of $l$ and $q$:

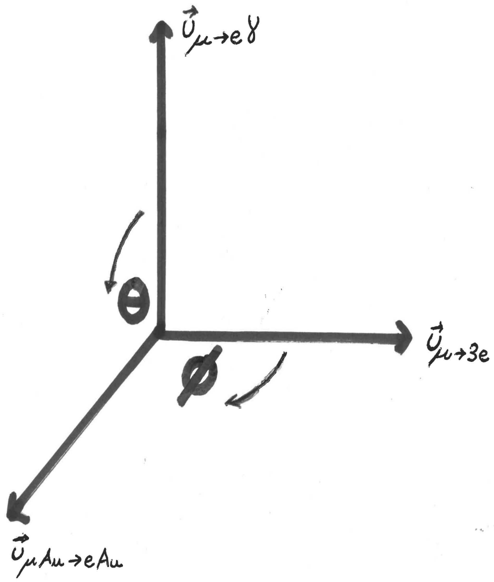
$$(-\lambda_L^{lr}\overline{\ell}_l \varepsilon q_r^c + \lambda_R^{lr}\overline{e}_l u_r^c)S + h.c.$$

⋆ generates scalar (+ vector) $\mu A \to eA$ operators at tree
($\mu A \to eA$ specially sensitive to scalar ops)
⋆ generates LFV operators for singlet leptons as well as doublets

⇒ it can fill all exptally accessible space? Consistent with any $\mu \leftrightarrow e$ observation?
Not quite: not generate $(\bar{e}P_{R,L}\mu)(\bar{e}P_{R,L}e)$ (dim8 in SMEFT), detectable to $\mu \to e\bar{e}e$.

# Plot the exptal bounds and reach

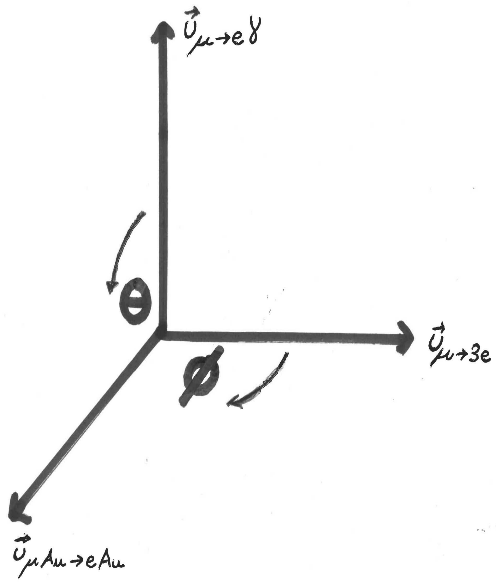
Restrict to 3-d space of coefficients of $\mu \to e_L \gamma, \mu \to 3e_L, \mu Al \to e_L Al (= z, x, y)$.
Model predicts a vector $\vec{C}/\Lambda^2_{\rm LFV}$;

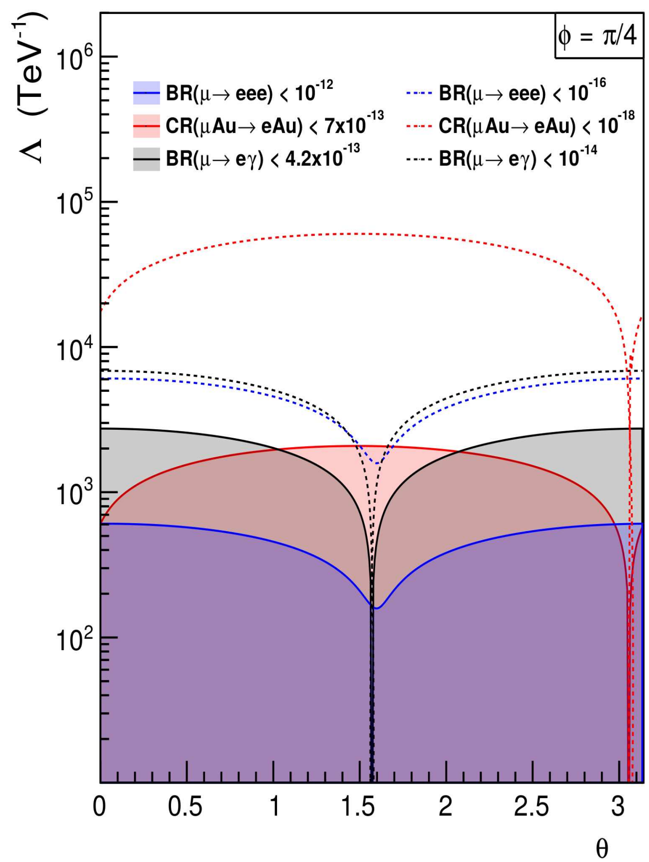


# Plot the allowed parameter space

Restrict to 3-d space of coefficients of $\mu \to e_L\gamma, \mu \to 3e_L, \mu Al \to e_L Al (= z, x, y)$.
Model predicts a vector $\vec{C}/\Lambda^2_{\rm LFV}$; can fix $|\vec{C}| = 1$ and constrain $\Lambda_{\rm LFV}(\theta, \phi)$:

$$\vec{C} \cdot \vec{v}_{\mu \to e_L\gamma} \equiv \frac{v^2 \cos\theta}{\Lambda^2_{\rm LFV}}$$




see 2204.00564

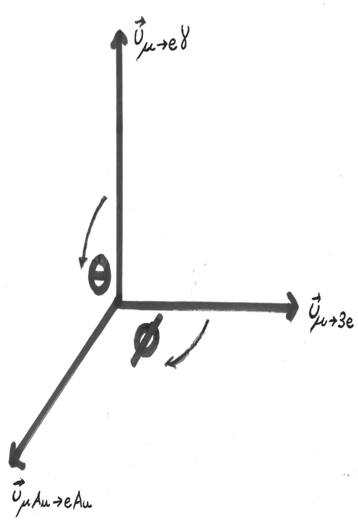
# Plot reach of $\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$

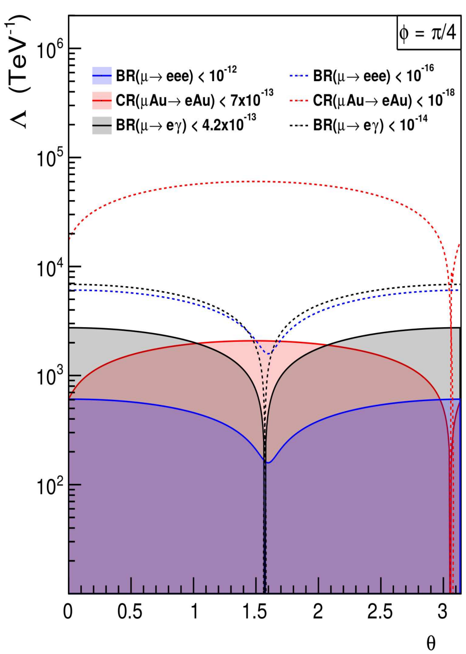
## (in theoretically self-consistent EFT, including LO loops, cancellations...)

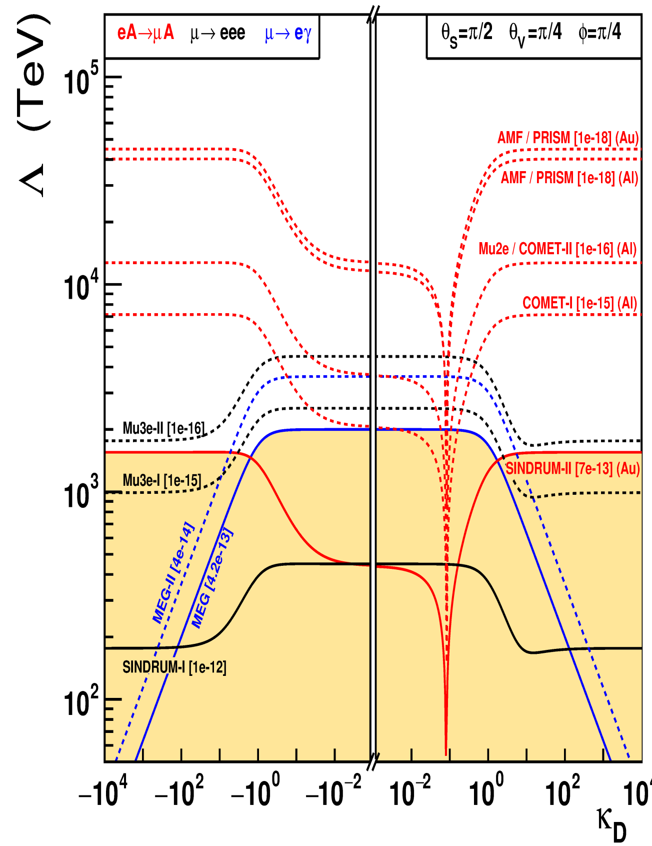
Restrict to 3-d space of coefficients of $\mu \to e_L\gamma, \mu \to 3e_L, \mu Au \to e_L Au (= z, x, y)$.
Impose $|\vec{C}|$=1 and use spher. coord.:

$$\vec{C} \cdot \vec{v}_{\mu \to e_L \gamma} \equiv \frac{v^2 \cos\theta}{\Lambda^2_{\rm LFV}}$$

Define $\kappa_D = {\rm cotg}(\theta_D - \pi/2)$




see 2204.00564

# Summary

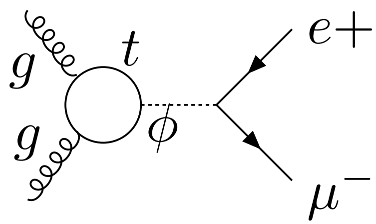
$\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$ have exceptional sensitivity ($\Lambda_{\rm LFV} \lesssim 10^2 \to 10^3$ now, $\Lambda_{\rm LFV} \lesssim 10^3 \to 10^4$ upcoming), to only a few operators at low energy, so:

interesting to include RGEs at leading order, because ensure that almost every $\mu \to e$ operator (in chiral basis) with $\leq 4$ legs contributes at $\gtrsim \mathcal{O}(10^{-3})$ to $\mu \to e\gamma$ and/or $\mu \to e\bar{e}e$ and/or $\mu A \to eA$
Can even have interesting sensitivity to products of some $(\mu \to \tau) \times (\tau \to e)$ interactions!

But many more $\mu \leftrightarrow e$ interactions/operators than observables. In EFT, convenient to restrict to exptally probed subspace of operators/coefficients; this allows to
- plot experimental reach
- explore whether $\mu \leftrightarrow e$ data can test models

*Happy Workshop!*

# *BackUp*



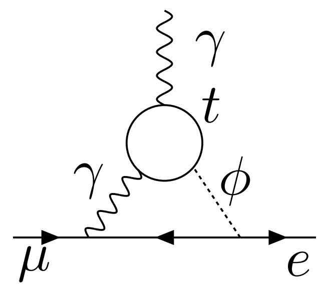

# Heavy decays at colliders $(t, h, Z)$

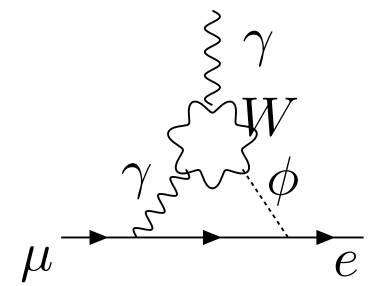
Pezzullo-ATLAS+CMS
Pinsard-CPV in H and Z decays

... its always interesting to measure independent observables!

wrt LFV Higgs decays and $\mu \to e\gamma$:

A boson produced in $gg$ or VBF at colliders,
decaying $\boldsymbol{\phi \to \mu^\pm e^\mp}$, contributes
to $\mu \to e\gamma$ via same diagrams:
but with different weights.
(and many other contributions to $\mu \to e\gamma$...)




So theoretically veery interesting to see $\phi \to \mu^\pm e^\mp$ and $\mu \to e\gamma$:
maybe we could learn something about cancellations?

# ...but: uncertainties in matching to quarks

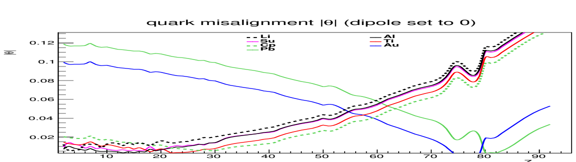
suppose measure coefficients of LFV ops with vector and scalar currents of $n$ or $p$, from $\mu A \to eA$ on different targets
Then match to quarks:

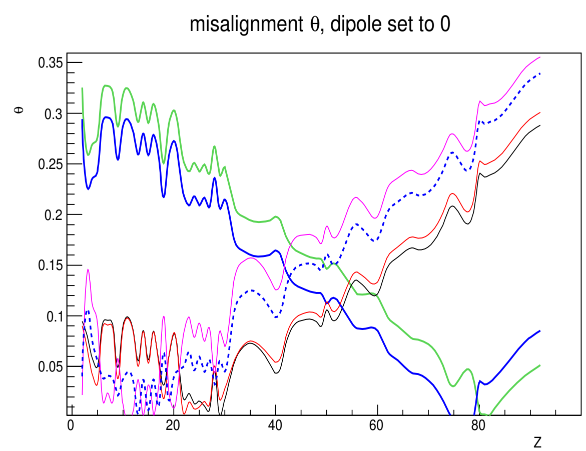
$$\begin{pmatrix} C^{pp}_{V,L} \\ C^{nn}_{V,L} \\ C^{pp}_{S,R} \\ C^{nn}_{S,R} \end{pmatrix} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & G_S^{pu} & G_S^{pd} \\ 0 & 0 & G_S^{nu} & G_S^{nd} \end{bmatrix} \begin{pmatrix} C^{uu}_{V,L} \\ C^{dd}_{V,L} \\ C^{uu}_{S,R} \\ C^{dd}_{S,R} \end{pmatrix}$$

- But for scalar ops, $G_S^{p,u} = G_S^{n,d} \simeq G_S^{p,d} \simeq G_S^{n,u}$

so need great precision to differentiate LFV ops with scalar currents of $u$ or $d$ :(

- and...curent determinations of $G$s from lattice and pions disagree by 50%



$$\mu \to e\gamma\gamma$$

# But to reconstruct $\mu \to e$ bottom-up, need all data?

$eg\ BR(\pi^0 \to e^\pm \mu^\mp) < 3.6 \times 10^{-10}$, or $BR(\Upsilon \to l_1 \bar{l}_2) \lesssim 10^{-6}$?

**Ummm**: $\mu$ decays weakly $\Leftrightarrow \tau_\mu \sim 10^{-6}$ sec.
vs $\tau_{\pi^0} \sim 10^{-16}$ sec (loop-suppressed QED), or $\tau_\Upsilon \sim 10^{-20}$ sec (tree QED/QCD)

Compare *weak* $\mu$ decays to *anomalous QED* $\pi_0$ decay
(write $\delta\mathcal{L} \sim \frac{1}{\Lambda^2_{\rm LFV}}(\bar{e}\mu)(\bar{q}q) + \frac{1}{\Lambda^2_{\rm LFV}}(\bar{e}\gamma\mu)(\bar{e}\gamma e))$ :

$$BR(\mu \to e\bar{e}e) = \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \sim \left|\frac{m_\mu^2/\Lambda^2_{\rm LFV}}{m_\mu^2 G_F}\right|^2 \sim \frac{v^4}{\Lambda^4_{\rm LFV}} \lesssim 10^{-12} \Rightarrow \Lambda_{\rm LFV} \gtrsim 10^5 {\rm GeV}$$

$$BR(\pi_0 \to \bar{e}\mu) = \frac{\Gamma(\pi_0 \to \bar{e}\mu)}{\Gamma(\pi_0 \to \gamma\gamma)} \sim \left|\frac{m_\pi^2/\Lambda^2_{LFV}}{\alpha/4\pi}\right|^2 \sim \left(\sqrt{\frac{4\pi}{\alpha}}\frac{m_\pi}{\Lambda_{LFV}}\right)^4 \Rightarrow \Lambda_{\rm LFV} \gtrsim {\rm TeV}$$

... rare $\mu$ processes have exceptional *sensitivity*, because $\mu$ decay weak.
Other $\mu \to e$ processes constrain "orthogonal" operator coefficients, less well.

# Climbing the mountain for $\mu \to e$: EFT

Renormalisation Group Eqns/matching/scheme-dep./...

(conceptually simple, technically involved)

# Can't we do without RGEs, etc?

in discovery mode for LFV+electroweak loops are small...include later?

counterex: $\mu A \to eA$ in model giving tensor $2\sqrt{2}G_F C_T^{uu}(\overline{e}\sigma P_R \mu)(\overline{u}\sigma u)$ at weak scale

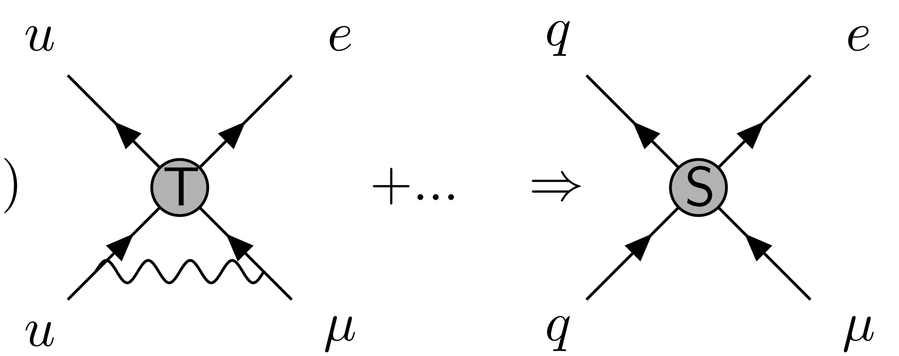
**1: forget loops** quark tensor matches to nucleon spin $\bar{N}\gamma\gamma_5 N$ : $(N \in \{n,p\})$

$$\Rightarrow \boldsymbol{BR(\mu A \to eA) \approx BR_{SD} \approx \frac{1}{2}|C_T^{uu}|^2}$$

(CiriglianoDKuno)
Hoferichter etal

**2: include QED loops $m_W \to 2$ GeV:**

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$




$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\, C_T^{uu}\; (\overline{u}u)(\overline{e}P_Y\mu)$$

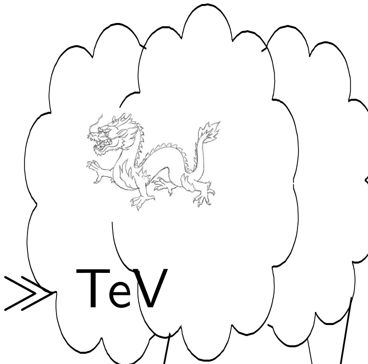
$$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7}C_T^{uu}(m_W)$$

Then, scalar ops have enhanced nuclear matrix elements, and are SpinIndep:

$$\boldsymbol{BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}}$$

**loops can change Lorentz structure/external legs $\Rightarrow$ different operator whose coefficient better constrained. Important for $\mu \to e$.** (?not $\tau \to l$?)

## need operators+bases for 3 EFTs?



$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$

$SU(3) \times SU(2) \times U(1)$

$m_W \sim m_h \sim m_t$

$\{\gamma, g, f\}$

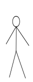
$QCD \times QED$

2 GeV$\sim m_c, m_b, m_\tau$

NB: $\frac{2\text{GeV}}{m_\mu} \sim 20$

$\{n, p, \pi, \gamma, e, \mu\}$ QED $+ \chi PT$

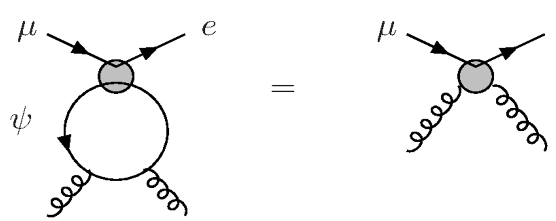
data ($\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA$)



# operators + RGEs: *everything to which data could be sensitive*

**operator basis:** below $m_W$, all gauge invariant operators with $\leq$ 4 legs$\approx$ 100 ops.
add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\bar{e}\gamma\mu)(\bar{e}\gamma e) + ...$
(not dim6: bottom-up perspective/ operator dim. not preserved in matching)
above $m_W$: dim 6 + selected dim 8 (guess by powercounting)
ArduDavidson

**ex:** $(\bar{e}\mu)G_{\alpha\beta}G^{\alpha\beta}$ is dim7 $< m_W$, dim8 in SMEFT. But
- dim6 heavy quark scalar ops $(\bar{e}\mu)(\bar{Q}Q)$ match to $(\bar{e}\mu)GG$ at $m_Q$ (coef.$C_{QQ}/(m_Q\Lambda^2_{\rm LFV})$):




- gluons contribute most of the mass of the nucleon ShifmanVainshteinZahkarov

$$\langle N|m_N\overline{N}N|N\rangle = \sum_{q\in\{u,d,s\}}\langle N|m_q\overline{q}q|N\rangle - \frac{\alpha_s}{8\pi}\beta_0\langle N|GG|N\rangle$$

$\Rightarrow$ dim7 $(\bar{e}\mu)GG$ contributes significantly to $\mu A \rightarrow eA$ via scalar $\mu \rightarrow e$ interactions with nucleons $N$.
CiriglianoKitanoOkadaTuscon

## operators + RGEs: *everything to which data could be sensitive*

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add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\overline{e}\gamma\mu)(\overline{e}\gamma e) + ...$
(not dim6: bottom-up perspective/ operator dim. not preserved in matching)
above $m_W$: dim 6 + selected dim 8 (guess by powercounting)
ArduDavidson

**RGEs+matching:** at "leading order"$\equiv$ largest contribution of each operator to each observable. (2GeV$\to m_W$:resum LL QCD, $\alpha_e \log$, some $\alpha_e^2 \log^2$, $\alpha_e^2 \log$)

**why not just 1-loop RGEs?**

- expand in loops, hierarchical Yukawas, $1/\Lambda^2_{\rm LFV}$,... largest effect maybe not 1-loop (ex: Barr-Zee)
- sometimes 1-loop vanishes...eg: 2-loop $\Delta a_\mu|_{EW} \simeq$ 1-loop $\Delta a_\mu|_{EW}$.
  or 2-loop log-enhanced
  $=$ mixing vector ops to dipole in 2-loop RGEs.

*What can one learn*

*in bottom-up EFT?*

.

# But 3 processes, ∼100 operators ⇒ zoo of flat directions?

DKunoYamanaka

Count constraints: (write $\delta\mathcal{L} = C^{flavour}_{Lorentz,XY}/v^n \; \mathcal{O}^{flav}_{Lorentz,XY} \quad , X, Y \in \{L, R\}$)

$\boldsymbol{\mu \to e\gamma}$ **:** $\quad BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) \quad$ ⇒ **2 constraints**

$\boldsymbol{\mu \to e\bar{e}e}$ **:** ($e$ relativistic ≈ chiral, neglect interference between $e_L, e_R$)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64 \ln \frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2$$
$$+ |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

⇒ **6 more constraints**

$\boldsymbol{\mu A \to eA}$ **:**($S^N_A$,$V^N_A$=integral over nucleus A of $N$ distribution×lepton wavefns, **different** for diff. $A$)

$$BR_{SI} \sim Z^2|V^p_A\tilde{C}^p_{V,L} + S^p_A\tilde{C}^p_{S,R} + V^n_A\tilde{C}^n_{V,L} + S^b_A\tilde{C}^n_{S,R} + D_A C_{D,R}|^2 + |L \leftrightarrow R|^2$$

$$BR_{SD} \sim |\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

SI bds on Au, Ti, (+ SD on ?Ti, Au?) ⇒ **$4 + 2$ more constraints**

future: improved theory, 3SI+2SD targets ⇒ **6+4** constraints

**is 12-20 constraints on** $\sim 100$ **operators a problem?**

## many operators+few constraints=using inconvenient basis

Have 6 (+6) constraints on $e_L$ ($e_R$) operator coefficients. Focus on $e_L$.
**Want to change basis to** *scale -dependent* **basis of constrained 6-d subspace.**

**1.** $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$
Have RGEs for coefficients (arranged in row vector)

$$\mu \frac{\partial}{\partial \mu} \vec{C}(\mu) = \vec{C}(\mu) \mathbf{\Gamma}(\mu, g_s(\mu), \ldots) \quad \Rightarrow \quad \vec{C}(m_\mu) = \vec{C}(m_W) \boldsymbol{G}(m_\mu, m_W)$$

solved as scale-ordered exponential (resummed QCD, $\alpha \log$, some $\alpha^2 \log^2$, $\alpha^2 \log$)
$\Rightarrow$ define scale-dep $\vec{v}_{\mu \to e\gamma}(\Lambda)$, column of **G** such that: $C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu \to e\gamma}(\Lambda)$
$\boldsymbol{\vec{v}_{\mu \to e\gamma}(\Lambda)}$ **is scale-dep basis vector for constrainable subspace**

**2-6.** repeat for other independent constraints. So obtain scale-dep basis vectors for the subspace, defined from the observables.
The "flat directions" (experimentally inaccessible) are orthogonal, and therefore irrelevant.

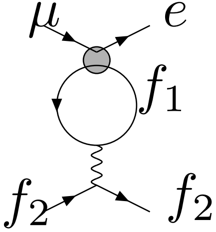
Basis should span the finite-eigenvalue subspace of the correlation matrix.

**what to do with this basis?**

# Wanted to use EFT to take exptal info to models... so:

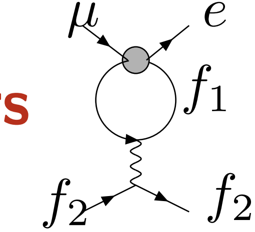
1. *(match to models, and explore what we can learn)*
   (not need to run RGEs at each point in model space)
   are some regions of 6-d space inaccessible to some models?

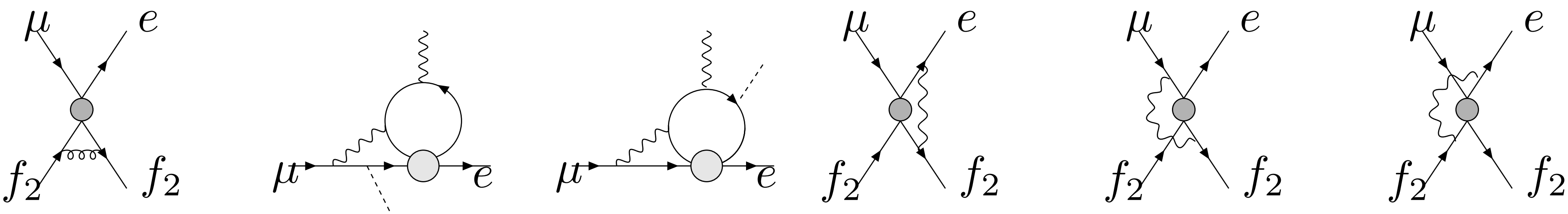
2. make plots of the excluded region in 6-d space ?
   $\Leftrightarrow$ illustrate the reach and complementarity of experiments




# Including SM loop corrections to operators

## ex: 1-loop QED + QCD (+2-loop QED V→D)




$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}$$

solve (analytically/numerically):

$$\vec{C}(m_\mu) = \vec{C}(\Lambda_{\rm LFV})\boldsymbol{G} \quad , \quad \boldsymbol{G} = \text{fn of SM parameters}, \log(\Lambda_{\rm LFV}/\Lambda_{exp})$$

For ex: $BR(\mu\to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 4.1\times 10^{-13} \Rightarrow C_{D,X} \lesssim 10^{-8}$

$$C_{D,X}(m_\mu) = C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) - \frac{\alpha_e}{4\pi e}\left(C^{\mu\mu}_{S,XX} - 8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C_{2loop}\right)\ln\frac{m_W}{m_\mu}$$

$$+16\frac{\alpha_e^2}{2e(4\pi)^2}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right)\ln^2\frac{m_W}{m_\mu} - 8\lambda^{a_T} f_{TD}\frac{\alpha_e}{4\pi e}\left(\frac{2m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_s}{m_\mu}C^{ss}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right)\ln\frac{m}{2}$$

$$+16\frac{\alpha_e^2}{3e(4\pi)^2}\left(\sum_{u,c} 4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)\ln^2\frac{m_W}{2\text{GeV}}$$

$C^\zeta_{Lor}(m_W)$ on right. $\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

operator list:Kuno-Okada, +CiriglianoKitanoOTuzon +BowmanChengLiMatis

# Operator basis $m_\tau \to m_W$ : $\sim 90$ operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of $\mu$ with $e$ and *flavour-diagonal* combination of $\gamma, g, u, d, s, c, b$. $Y \in L, R$.

$$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad \textit{dim } 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad \textit{dim } 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$$

$$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$$

$$(\overline{e}P_Y\mu)(\overline{f}P_Y f) \qquad (\overline{e}P_Y\mu)(\overline{f}P_X f) \qquad f \in \{u, d, s, c, b, \tau\}$$

$$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \qquad \textit{dim } 7$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

**...zzz...but $\sim 90$ coeffs!**

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.

# operators at exptal scale

Kuno Okada

There are dipoles of 2 chiralities

$D \qquad \overline{e}\sigma^{\alpha\beta}P_L\mu F_{\alpha\beta} \qquad \overline{e}\sigma^{\alpha\beta}P_R\mu F_{\alpha\beta}$

which also contribute in $\mu \to e\gamma$, $\mu \to e\bar{e}e$.

Six 4-fermions for $\mu \to e\bar{e}e$, $Y, X \in \{L, R\}, Y \neq X$

$V \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$

$S \qquad (\overline{e}P_Y\mu)(\overline{e}P_Y e)$

For $\mu A \to eA$, interactions with nucleons $N \in \{n, p\}$ parametrised by :

$S,\ V \qquad \overline{e}P_X\mu\overline{N}N \qquad \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha N \qquad X \in \{L, R\}$

$A,\ T \qquad \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha\gamma_5 N \qquad \overline{e}\sigma^{\alpha\beta}P_X\mu\overline{N}\sigma_{\alpha\beta}N$

$P,\ Der \qquad \overline{e}P_X\mu\overline{N}\gamma_5 N \qquad \overline{e}\gamma^\alpha P_X\mu(\overline{N}i\overleftrightarrow{\partial}_\alpha\gamma_5 N)$

Matching in $\chi$PT gives Derivative. But absorb in matching into $G_O^{N,q}$= quark matrix elements in nucleons.

chiral basis for the lepton current (relativistic $e$), but not for the non-rel. nucleons.

# Quantifying which targets give independent information (on nucleons)

1. neglect Dipole (better sensitivity of $\mu \to e\gamma$ (MEGII) and $\mu \to e\bar{e}e$(Mu3e).
   remain to determine: $\vec{C} \equiv (\widetilde{C}^{pp}_{VR}, \widetilde{C}^{pp}_{SL}, \widetilde{C}^{nn}_{VR}, \widetilde{C}^{nn}_{SL})$

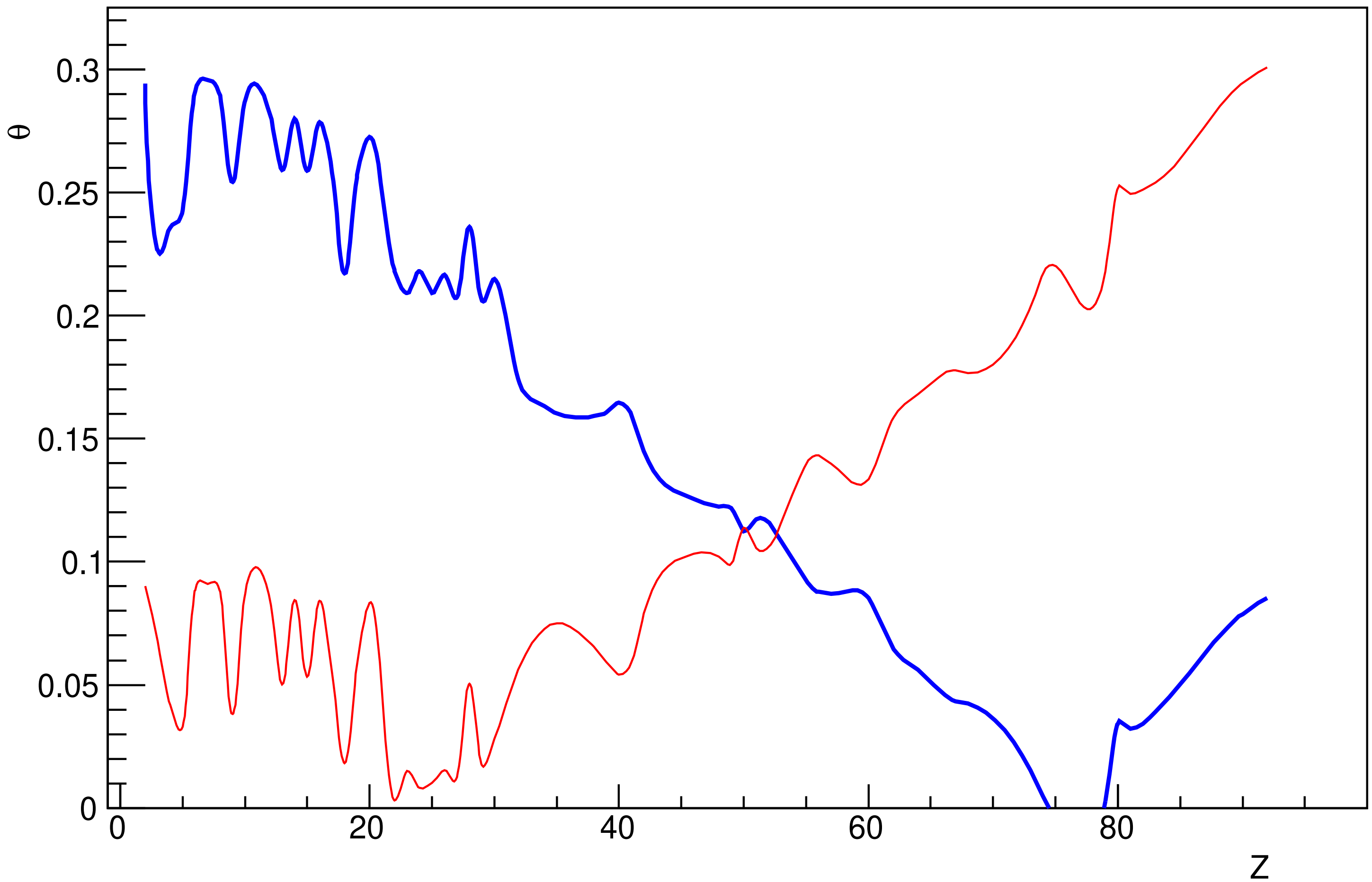
2. recall that
   $$BR_{SI}(A\mu \to Ae) \propto \left|\vec{C} \cdot \vec{v}_A\right|^2$$
   where target vector for nucleus $A$
   $$\vec{v}_A \equiv \left(V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}\right)$$

3. So first experimental search ($eg$ on Aluminium) probes projection of $\vec{C}$ of $\vec{v}_{Al}$
   ... next target needs to have component $\perp$ to Aluminium!
   $\Leftrightarrow$ plot misalignment angle $\theta$ between target vectors

4. how big does $\theta$ need to be?
   overlap integrals have theory uncertainty: $\Delta\theta \left\{ \begin{array}{cc} \text{nuclear} & \sim 5\%(KKO) \\ NLO\ \chi\text{PT} & \sim 10\%(?) \end{array} \right.$
   Both vectors uncertain by $\Delta\theta$; need misaligned by $2\Delta\theta \approx 10 \to 20\%$

**Current data+ theory uncertainty $\sim 10\%$: want $\Delta\theta > 0.2$**

$$\boldsymbol{BR(\mu Au \to e Au) \leq 7 \times 10^{-13}} \qquad (Au : Z = 79)$$
$$BR(\mu Ti \to e Ti) \leq 4.3 \times 10^{-12} \qquad (Ti : Z = 22)$$




$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)})$, and $BR \propto |\vec{v}_A \cdot \vec{C}|^2$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot $\theta$ on vertical axis

## **In the future...with a 5% theory uncertainty:**

First target of Mu2e, COMET: Aluminium (Z=13, A=27)

$$\hat{v}_{Al} \approx \frac{1}{2}(1,1,1,1)$$

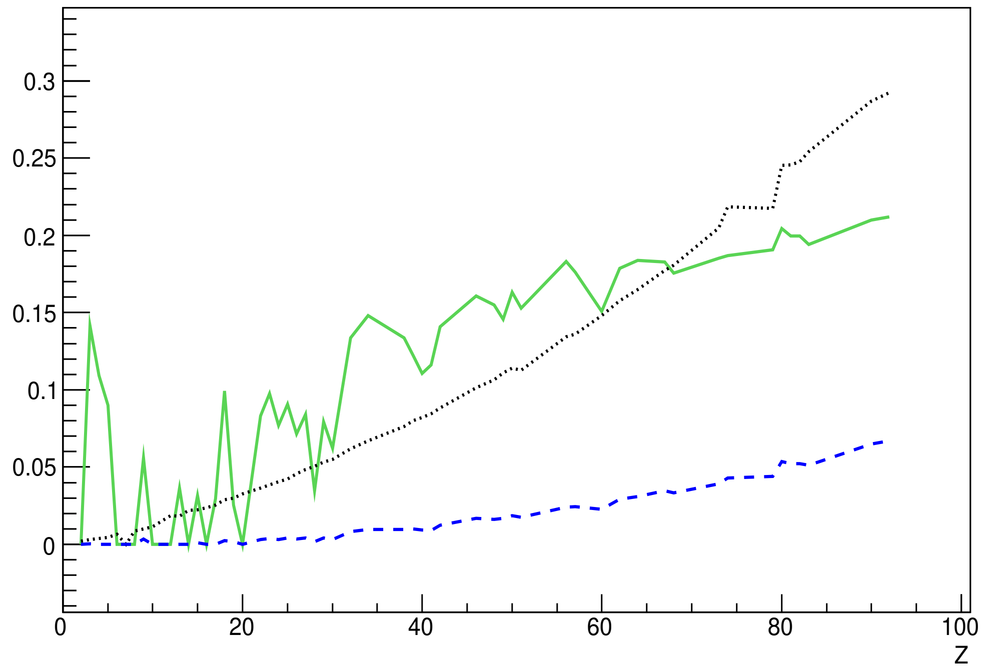
(recall $\tilde{C}_V^{pp}, \tilde{C}_S^{pp}, \tilde{C}_V^{nn}, \tilde{C}_S^{nn}$)

basis of three other "directions":

$$\hat{v}_{np} \equiv \frac{1}{2}(-1,-1,1,1)$$

$$\hat{v}_{VS} \equiv \frac{1}{2}(1,-1,1,-1)$$

$$\hat{v}_{IsoSV} \equiv \frac{1}{2}(-1,1,1,-1)$$

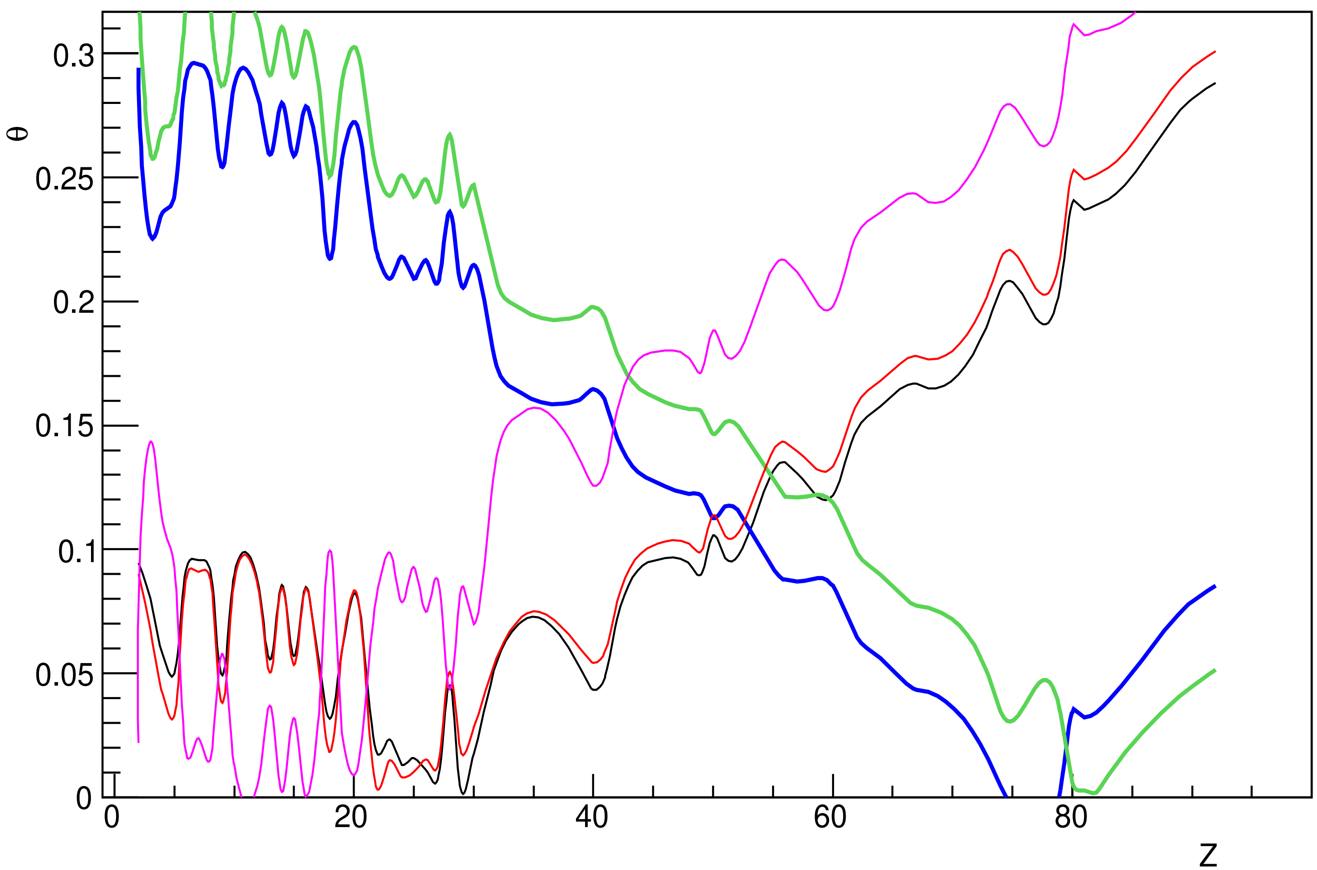



**probe 3 combinations of SI coeffs**

# All current data...

$$BR(\mu Au \to e Au) \leq 7 \times 10^{-13} \qquad (Au : Z = 79)$$

$$BR(\mu Ti \to e Ti) \leq 4.3 \times 10^{-12} \qquad (Ti : Z = 22)$$




$$BR(\mu Pb \to e Pb) \leq 4.6 \times 10^{-11}$$

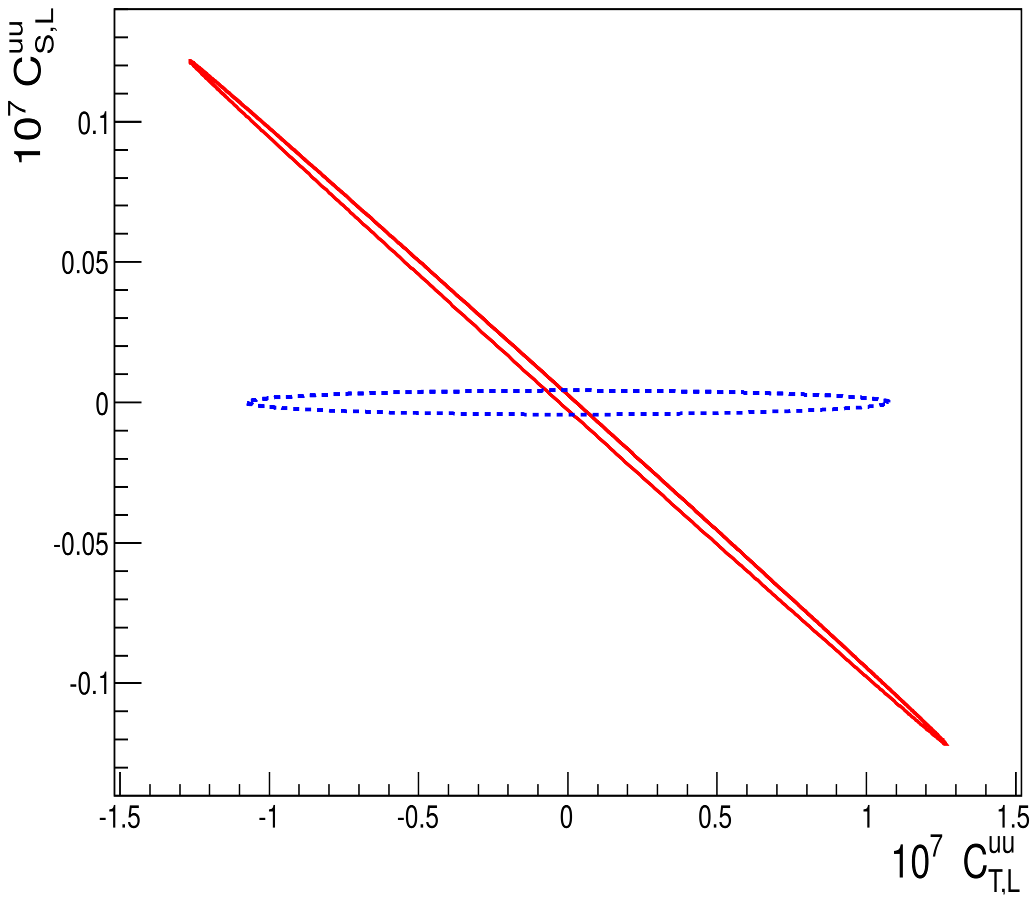
$BR(\mu S \to e S) \leq 7 \times 10^{-11}$ S = Sulpher, Z = 16

$BR(\mu Cu \to e Cu) \leq 1.6 \times 10^{-8}$ Cu = Copper, Z = 29

# sensitivity *vs* constraint

Suppose that $BR(\mu Al \to eAl) \lesssim 10^{-14}$, and :

$$\delta\mathcal{L}(m_W) = C_T^{uu}(\overline{e}\sigma P_Y \mu)(\overline{u}\sigma u) + C_S^{uu}(\overline{e}P_Y\mu)(\overline{u}u)$$




$C_T^{uu}, C_S^{uu}$ constrained to live inside blue (red) ellipse at exptal scale (at $m_W$): sensitivity to $C_S^{uu}$ = cut ellipse @ $C_T^{uu} = 0$; constraint = live in projection of ellipse onto $C_S^{uu}$ axis.