High-precision muon decay predictions for cLFV experiments

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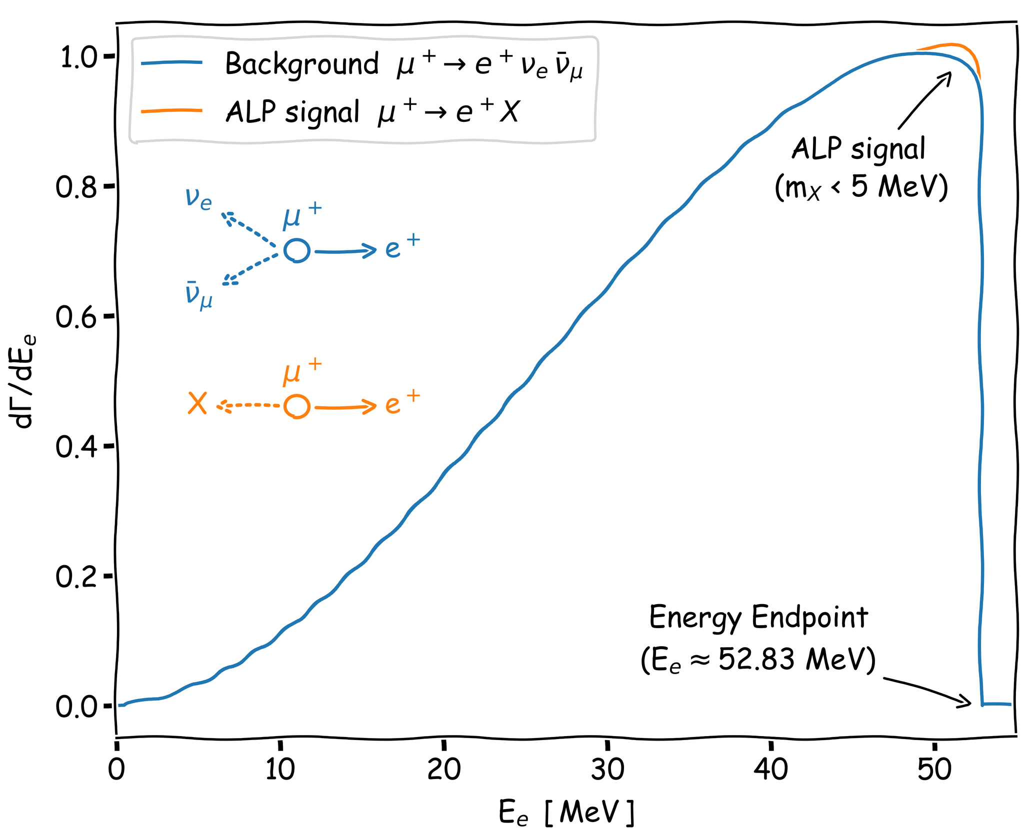
Motivations

- The search for **charged Lepton Flavour Violation** (cLFV) in rare **muon decays** is a key tool to test the Standard Model (SM).
- The **MEG II** ($\mu^+ \to e^+\gamma$) and **Mu3e** ($\mu^+ \to e^+e^-e^+$) experiments at PSI are competitive in searching for decays involving a light neutral boson X, which remains invisible.
- This particle can be an **Axion-Like Particle** (ALP) arising from the spontaneous breaking of a global $U(1)$ symmetry.
- A possible process is the two-body decay $\boldsymbol{\mu^+ \to e^+ X}$.
- Its signature is a monochromatic peak close to the endpoint of the positron spectrum of the $\boldsymbol{\mu^+ \to e^+ \nu_e \bar\nu_\mu}$ background.
- The theoretical uncertainty at the endpoint is enhanced by the emission of **soft photons**, reducing the signal sensitivity.
- The hunt for such an elusive signal requires extremely accurate theoretical predictions for simulations and data analysis.



The McMule framework

- McMule: **M**onte **C**arlo for **MU**ons and other **LE**ptons.
- A numerical framework for the fully differential computation of higher-order **QED corrections** for decay and scattering processes involving leptons, mainly at low energy.
- The precision goal is Next-to-Next-to-Leading Order (**NNLO**).
- All divergences are treated with dimensional regularisation, while renormalisation is performed in OS scheme.
- Soft singularities are subtracted by using the **FKS^2 scheme**.
- Collinear singularities are eliminated by keeping all fermion masses at their physical value ($m \neq 0$).
- Phase space is integrated with the adaptive Vegas algorithm.
- For a process implemented in the code, the user can obtain any differential distribution with any cut, for example to reproduce detector acceptances or analysis selections.
- In addition to **muon and tau decays**, McMule includes leptonic scatterings such as $e\,\mu \to e\,\mu$, $e\,e \to e\,e$, and $e\,e \to \gamma\,\gamma$.

Theoretical predictions

- The positrons produced by **polarised** muon decays are fully characterised by the inclusive distribution

$$\frac{m_\mu}{2}\frac{d^2\Gamma}{dE_e\, d\cos\theta_e} = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}\Big[F(E_e) + P_\mu \cos\theta_e\, G(E_e)\Big]$$

m_μ : Muon mass $\quad$ G_F : Fermi constant

E_e : Positron energy $\quad$ P_μ : Muon polarisation

θ_e : Angle between e^+ momentum and μ^+ spin

F : Isotropic function $\to$ Energy spectrum

G : Anisotropic function $\to$ Polarisation effect

- The positron dynamic is therefore determined by the two dimensionless functions $\boldsymbol{F(E_e)}$ and $\boldsymbol{G(E_e)}$.
- These functions have been computed with McMule for both $\mu^+ \to e^+ X$ and $\mu^+ \to e^+ \nu_e \bar\nu_\mu$ assuming **inclusive photons**.
- A generic muon polarisation is assumed and the positron mass is not neglected. The centre-of-mass frame is always used.

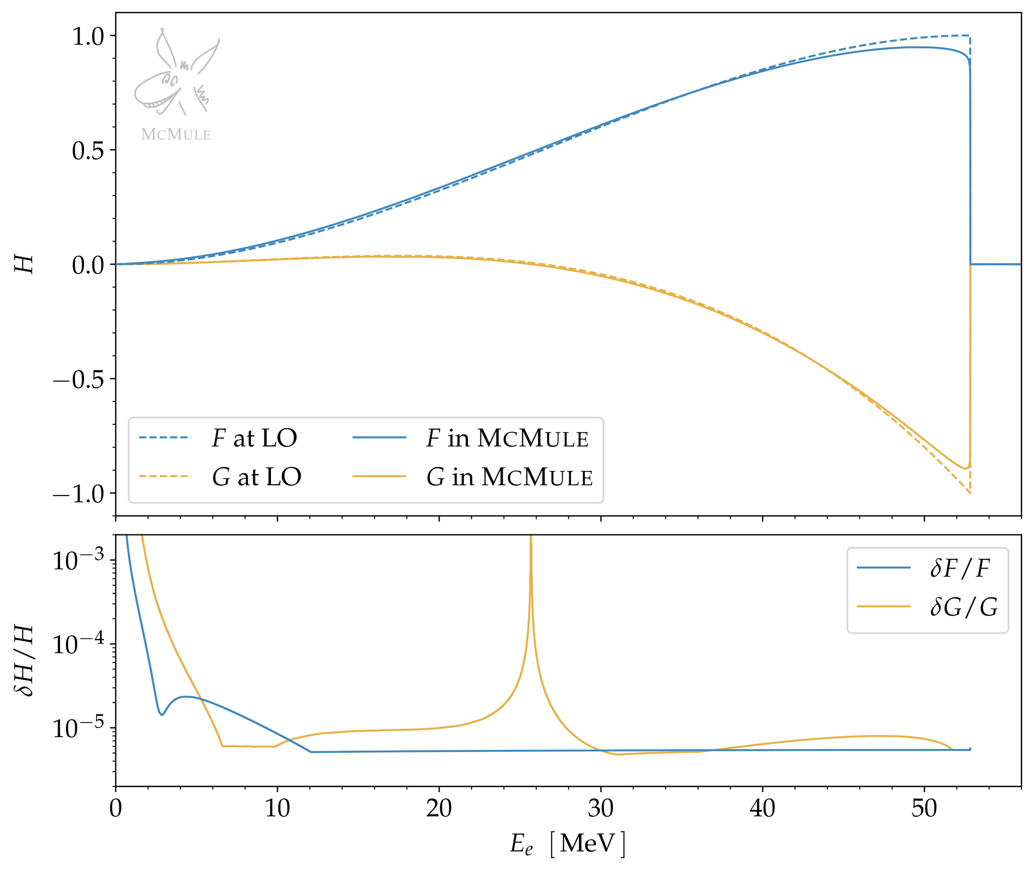
Background $\mu^+ \to e^+ \nu_e \bar\nu_\mu$

- The background $\mu^+ \to e^+ \nu_e \bar\nu_\mu$ is computed in Fermi theory

$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}}\left(\bar\psi_e \gamma^\rho (1-\gamma^5)\,\psi_\mu\right)\left(\bar\psi_{\nu_\mu}\gamma_\rho(1-\gamma^5)\,\psi_{\nu_e}\right) + \mathcal{L}_{\text{QED}}$$

- Full QED corrections are added at **NNLO**, including (hadronic) vacuum polarisation effects and open lepton production.
- The **collinear logarithmic** terms $\propto \log(m_e/m_\mu)$ are included up to N^{4}LO with NLL accuracy, while the **soft logarithmic** terms $\propto \log(1 + m_e^2/m_\mu^2 - 2E_e/m_\mu)$ are analytically resummed to all orders with NNLL accuracy.
- The **theory error** is about $5 \cdot 10^{-6}$, the smallest achieved so far!
- The radiative processes are also implemented: $\boldsymbol{\mu^+ \to e^+ \nu_e \bar\nu_\mu \gamma}$ at NLO and $\boldsymbol{\mu^+ \to e^+ \nu_e \bar\nu_\mu \gamma\gamma}$ at LO.

Theoretical decay functions $H = \{F, G\}$ for $\mu^+ \to e^+ \nu_e \bar\nu_\mu$



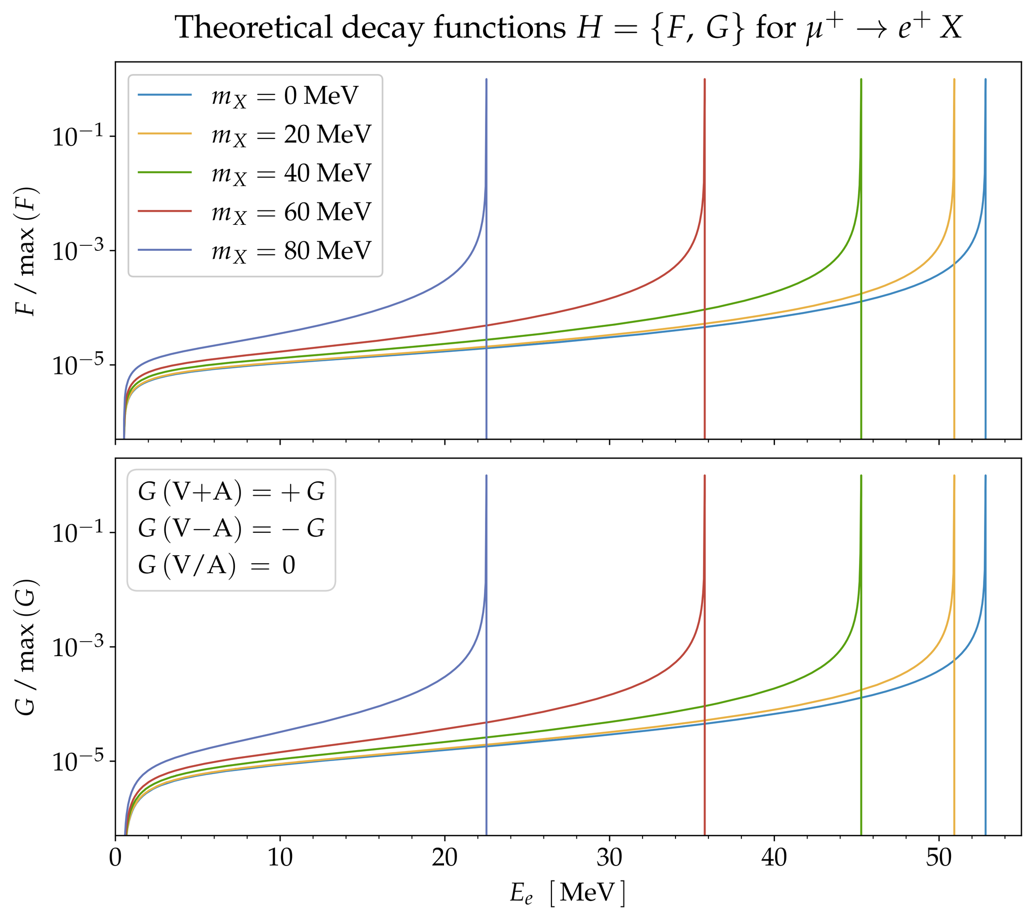
Signal $\mu^+ \to e^+ X$

- The signal $\mu^+ \to e^+ X$ is computed using an **effective model**, which accounts for different ALP masses and couplings

$$\mathcal{L}_X = \frac{1}{\Lambda}(\partial_\rho X)\,\bar\psi_e\,(\gamma^\rho g_V + \gamma^\rho\gamma^5 g_A)\,\psi_\mu + \mathcal{L}_{\text{QED}}$$

- The contribution is suppressed by a **large energy scale** Λ.
- The coupling constants g_V and g_A can be chosen in order to obtain typical chiral structures, such as left-handed (**V–A**), right-handed (**V+A**), vector-like (**V**) or pseudovector-like (**A**).
- The QED corrections at NLO are included with the effect of introducing a **radiative tail** to the positron energy spectrum. The radiative process $\boldsymbol{\mu^+ \to e^+\gamma}$ is implemented at LO.

Theoretical decay functions $H = \{F, G\}$ for $\mu^+ \to e^+ X$



Toy analysis

- The new predictions can be used to estimate the experimental sensitivity and evaluate the impact of the theory error.
- To this end a **simplified model** of the MEG II and Mu3e positron spectrometers has been defined

$$\mathcal{P}_e(E_e) = \int dE_e' \Big[\mathcal{H}_e(E_e') \times \mathcal{A}_e(E_e') \times \mathcal{S}_e(E_e - E_e')\Big]$$

$\mathcal{P}_e$: **Expected** positron energy spectrum

$\mathcal{H}_e$: **Theoretical** positron energy spectrum

$\mathcal{A}_e$: Positron energy **acceptance** function

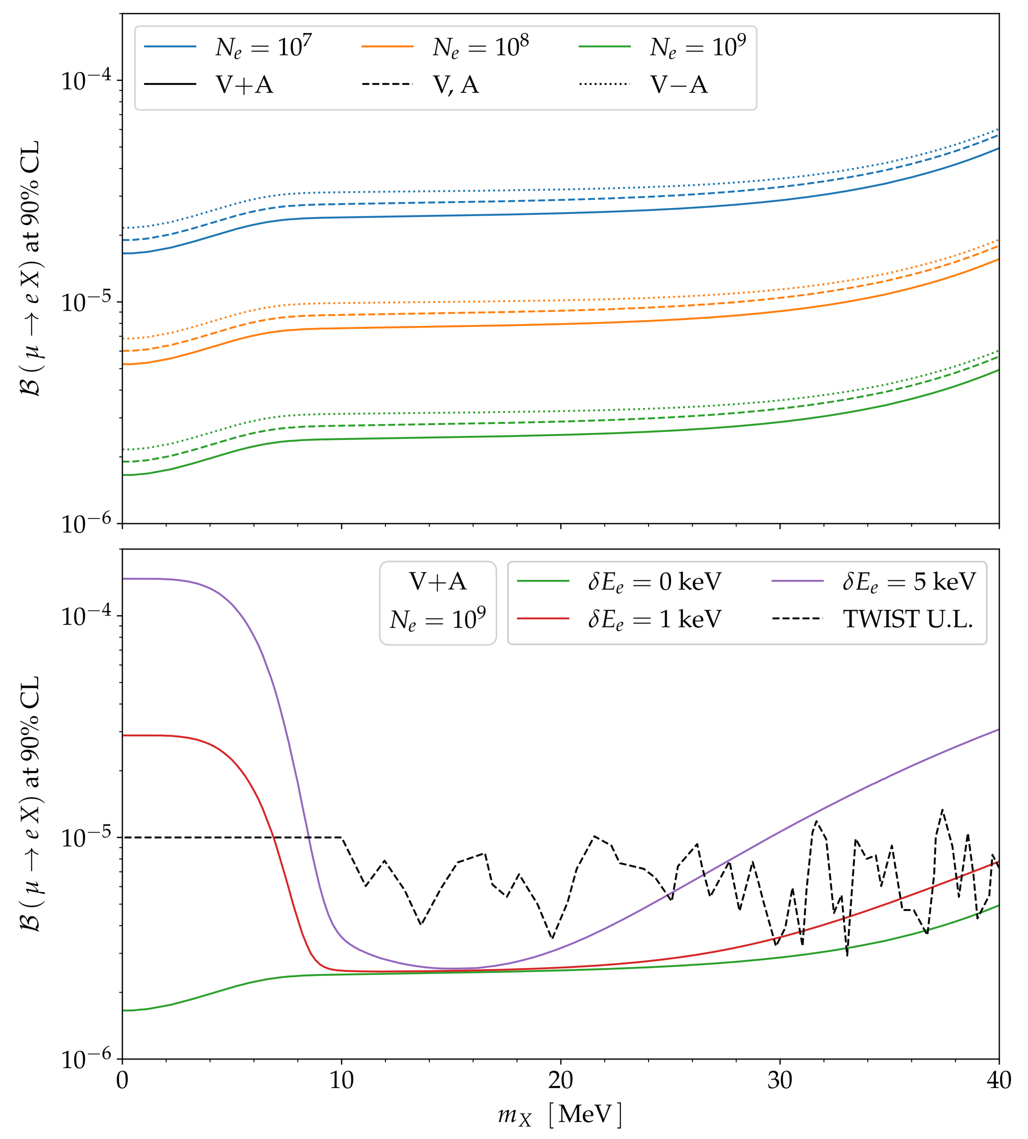
$\mathcal{S}_e$: Positron energy **resolution** function

- The input functions have been parametrised according to the **nominal geometry** and the **expected performance** of the involved detectors.

Expected sensitivity

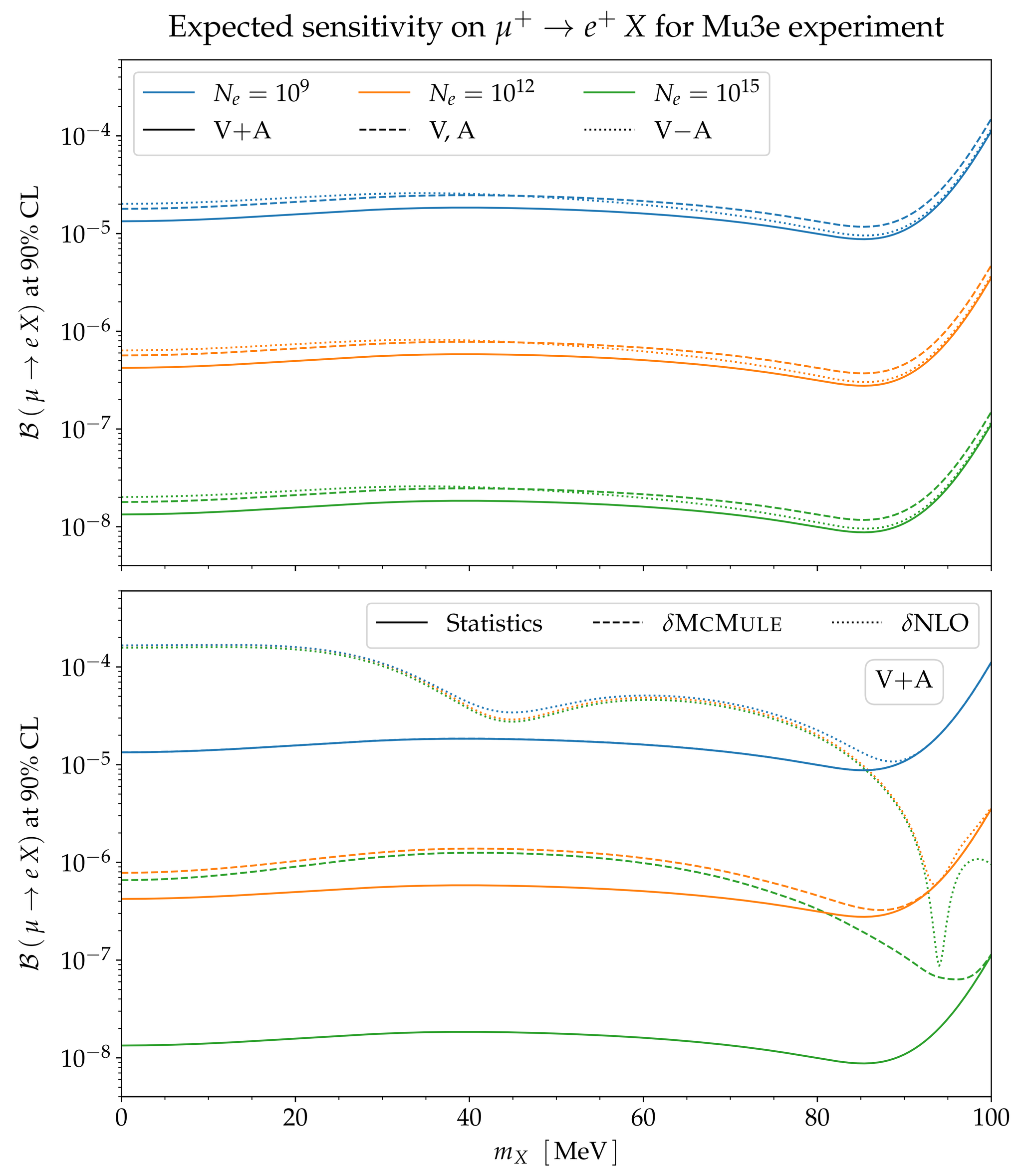
- The sensitivity on $\mathcal{B}(\mu^+ \to e^+ X)$ at 90% CL for MEG II and Mu3e has been estimated by using a **cut-and-count** procedure, accounting for statistical, theoretical and systematic errors.
- Different positron events N_e and ALP masses m_X are assumed.
- In the V+A case, the signal and background positrons tend to be emitted in opposite directions, giving a better sensitivity.
- The impact of a systematic error δE_e in the positron energy reconstruction is reported for MEG II. Since an energy offset in the positron spectrum has the same shape of a signal at the endpoint, the effect is enhanced for small ALP masses.

Expected sensitivity on $\mu^+ \to e^+ X$ for MEG II experiment



- The impact of the theoretical uncertainty is reported for Mu3e. The new McMule predictions make it possible to reach a sensitivity of $\mathcal{B} \sim 10^{-6}$, while a simple NLO computation would have limited it to $\mathcal{B} \sim 10^{-4}$.

Expected sensitivity on $\mu^+ \to e^+ X$ for Mu3e experiment



Conclusions

- The next generation of cLFV experiment based on muon decay requires accurate theoretical predictions to be implemented in a fully differential Monte Carlo framework. This is even more important for **ALP searches**.
- The **McMule** framework features $\mu \to e\,\nu\,\bar\nu$ at NNLO+Logs, $\mu \to e\,\nu\,\bar\nu\,\gamma$ and $\mu \to e\,\nu\,\bar\nu\,e\,e$ at NLO, $\mu \to e\,\nu\,\bar\nu\,\gamma\,\gamma$ at LO, $\mu \to e\,X$ at NLO, and $\mu \to e\,X\,\gamma$ at LO.
- The hunt for flavour-violating ALPs in muon decays is an excellent opportunity for MEG II and Mu3e to extend their physics programme beyond $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^+$.
- The search for small ALP masses is limited by the systematic error on the positron energy. The development of dedicated **calibration tools** is essential to avoid signal biases. In addition, the reduced theoretical error turned out to be indispensable.
- The sensitivity for the V+A case can be improved to $\mathcal{B} \sim 10^{-8}$ with a dedicated **forward detector**, placed opposite the muon polarisation, where the SM background is minimal.

Based on `arXiv:2211.01040`

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