Beam-driven plasma acceleration I

Sébastien Corde
Ecole Polytechnique, LOA

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Credits to my PhD students for material: P. San Miguel Claveria and V. Zakharova

Outline

Beam-driven plasma acceleration I

- Exciting plasma wakefields with particle beams
  - ➢ Linear wakes
  - ➢ Nonlinear wakes
  - ➢ How to accelerate: the two-bunch configuration
- Basic concepts for particle beam evolution in plasma
  - ➢ Envelope equation and matching
  - ➢ ~~Evolution of longitudinal phase space~~

Beam-driven plasma acceleration II

- Beam loading and energy-transfer efficiency
  - ➢ In 1D and 3D linear wakes
  - ➢ In nonlinear wakes
- Advanced concepts for particle beam evolution in plasma
  - ➢ Head erosion of drive beam
  - ➢ Instabilities, ion motion and emittance of trailing beam

Two different time scales: plasma and beam

Beam-driven plasma acceleration I

- Exciting plasma wakefields with particle beams
  - ➢ Linear wakes
  - ➢ Nonlinear wakes
  - ➢ How to accelerate: the two-bunch configuration
- Basic concepts for particle beam evolution in plasma
  - ➢ Envelope equation and matching
  - ➢ ~~Evolution of longitudinal phase space~~

Time scale 1: how fast the plasma responds to the beam

Time scale 2: how fast the particle beam evolves

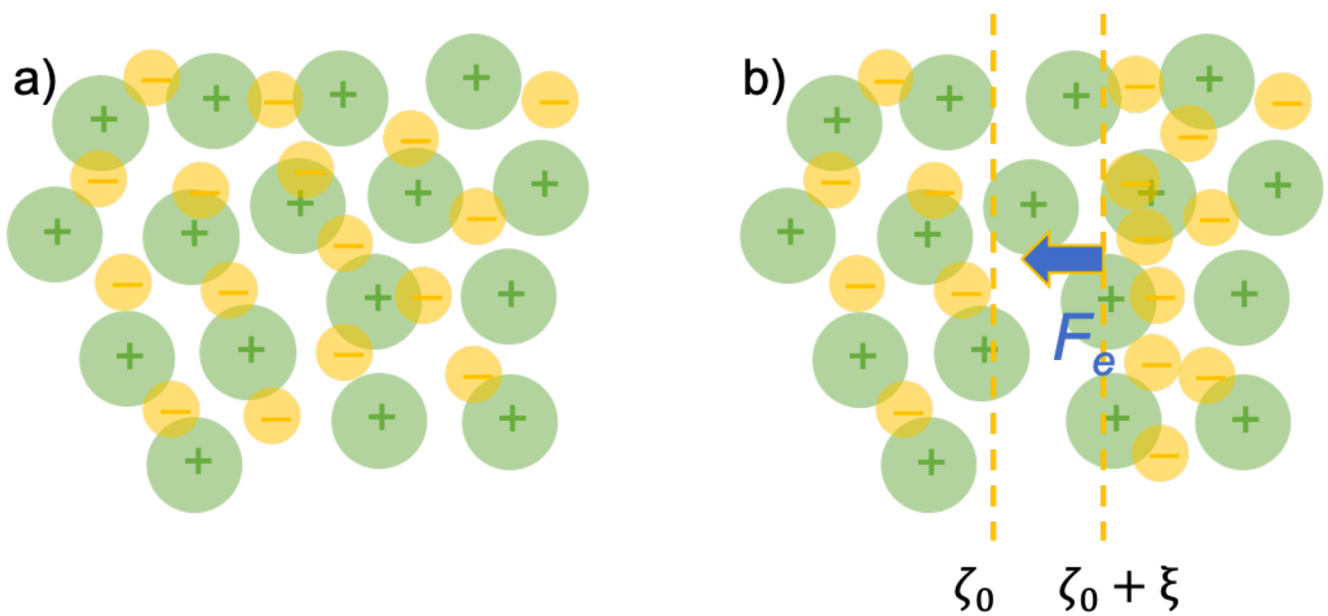
Quasi-static approximation (QSA):

- plasma time scale << beam time scale
- beam considered as « frozen » when plasma response is calculated
- used in most analytical models, and in some particle-in-cell simulation codes
- particularly powerful for beam-driven plasma acceleration

Exciting plasma wakefields with particle beams

Linear plasma wakefield

➢ At the heart of plasma accelerators, the plasma oscillation:



a displaced plasma slice is an harmonic oscillator:

$$d_t^2 \xi = -\omega_p^2 \xi$$

with a characteristic frequency ω_p called the plasma frequency:

$$\omega_p = \frac{n_0 e^2}{m_e \epsilon_0}$$

➢ Using a cold fluid description for plasma electrons and immobile ions:

continuity $$\frac{\partial n_p}{\partial t} + \nabla \cdot (n_p \vec{v_p}) = 0$$

eq. of motion $$\left(\frac{\partial}{\partial t} + \vec{v}_p \cdot \nabla\right) \vec{p}_p = -e(\vec{E} + \vec{v}_p \times \vec{B})$$

Maxwell $$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad \text{etc.}$$

linearize

$$\xrightarrow[n_p = n_0 + n_1, \quad \text{etc.}]{}$$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot (\vec{v_1}) = 0$$

$$m_e \frac{\partial \vec{v}_1}{\partial t} = -e \vec{E}_1$$

$$\nabla \cdot \vec{E}_1 = -e \frac{n_1}{\epsilon_0} + \frac{q}{e} \frac{n_b}{\epsilon_0}, \quad \text{etc.}$$

Linear plasma wakefield

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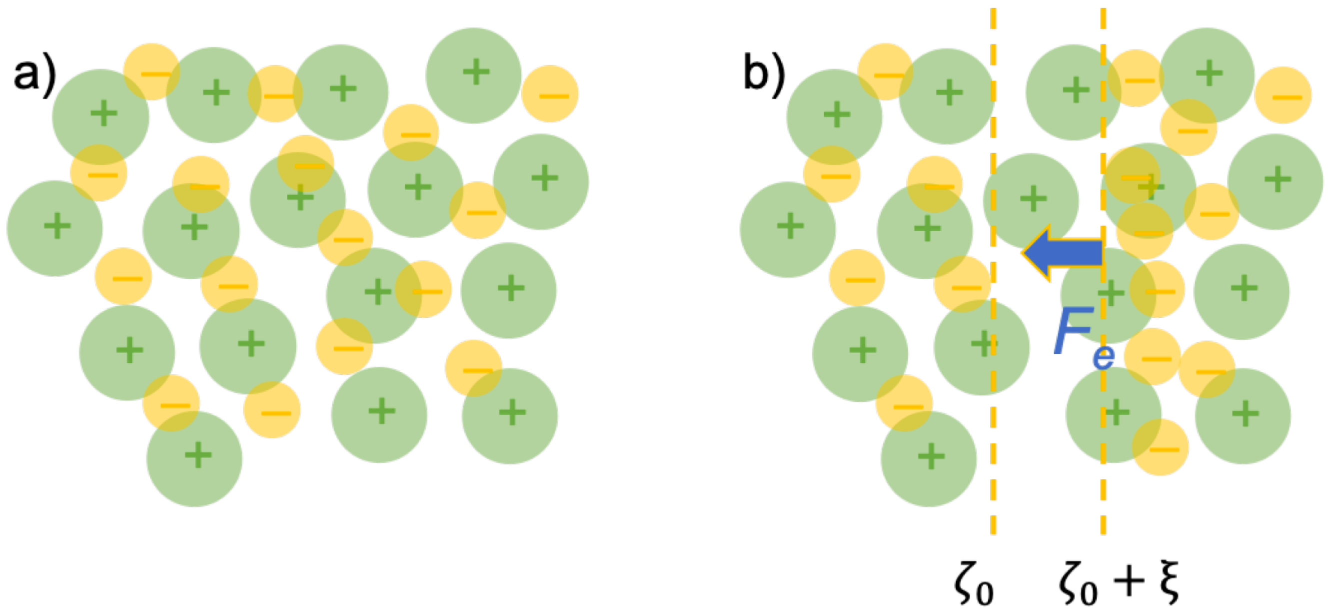

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Maxwell $$\nabla \cdot \vec{E}_1 = -\,e \frac{n_1}{\epsilon_0} + \frac{q}{e}\frac{n_b}{\epsilon_0}, \quad \text{etc}\,.$$

$$\longrightarrow \quad \boxed{\left(\frac{\partial}{\partial t^2} + \omega_p^2\right) n_1 = \omega_p^2 \frac{q}{e} n_b}$$

Linear plasma wakefield

Quasi-static approximation (QSA):

- plasma time scale << beam time scale
- beam considered as « frozen » when plasma response is calculated
- used in most analytical models, and in some particle-in-cell simulation codes.
- particularly powerful for beam-driven plasma acceleration.

QSA in equations:

change of coordinates:

$$(z, t) \longrightarrow (\xi = z - ct, \tau = t)$$

$$QSA : \partial_\tau \ll \partial_\xi$$

$$\Longrightarrow \partial_\tau = 0 \text{ in plasma equations}$$

$$\Longrightarrow \partial_t = -\frac{1}{c}\partial_\xi \quad \text{and} \quad \partial_z = \partial_\xi$$

$$\boxed{\left(\frac{\partial}{\partial \xi^2} + k_p^2\right) n_1 = k_p^2 \frac{q}{e} n_b}$$

➢ This equation is solved using its corresponding Green's function $-\frac{q}{e}\sin(k_p\xi)\Theta(-\xi)$, and the solution reads:

$$n_1(x, y, \xi) = -\frac{q}{e}\int_\xi^\infty n_b(x, y, \xi')\ \sin[k_p(\xi - \xi')]\ k_p d\xi'$$

Note: the plasma density perturbation is local and matches the transverse extent of the beam. This is not true for the fields.

Linear plasma wakefield

What about the fields?

More complicated: use Maxwell-Ampère and Maxwell-Faraday, go to the Fourier space of the ξ coordinate, assume azimuthal symmetry and $v_b \simeq c$, use radial Green's function to solve equation on r, and Fourier back with a contour integral in the complex plane, see Keinigs and Jones, Phys. Fluids 30, 252 (1987).

$$E_z(r,\xi) = \frac{qk_p^2}{\epsilon_0}\int_0^{+\infty} r'dr'K_0(k_pr_>)I_0(k_pr_<)\int_\xi^{+\infty} d\xi' n_b(r',\xi')\cos k_p(\xi-\xi')$$

$$E_r(r,\xi) = -\frac{q}{\epsilon_0}\int_0^{+\infty} r'dr'K_1(k_pr_>)I_1(k_pr_<)\left(\int_\xi^{+\infty} k_p d\xi'\frac{\partial n_b(r',\xi')}{\partial r'}\sin k_p(\xi-\xi') + \frac{\partial n_b(r',\xi)}{\partial r'}\right)$$

$$B_\theta(r,\xi) = -\frac{q}{\epsilon_0}\int_0^{+\infty} r'dr'K_1(k_pr_>)I_1(k_pr_<)\frac{\partial n_b(r',\xi)}{\partial r'}$$

Transverse wakefield: $W = E_r - cB_\theta = -\dfrac{qk_p}{\epsilon_0}\displaystyle\int_0^{+\infty} r'dr'K_1(k_pr_>)I_1(k_pr_<)\int_\xi^{+\infty} d\xi'\frac{\partial n_b(r',\xi')}{\partial r'}\sin k_p(\xi-\xi')$

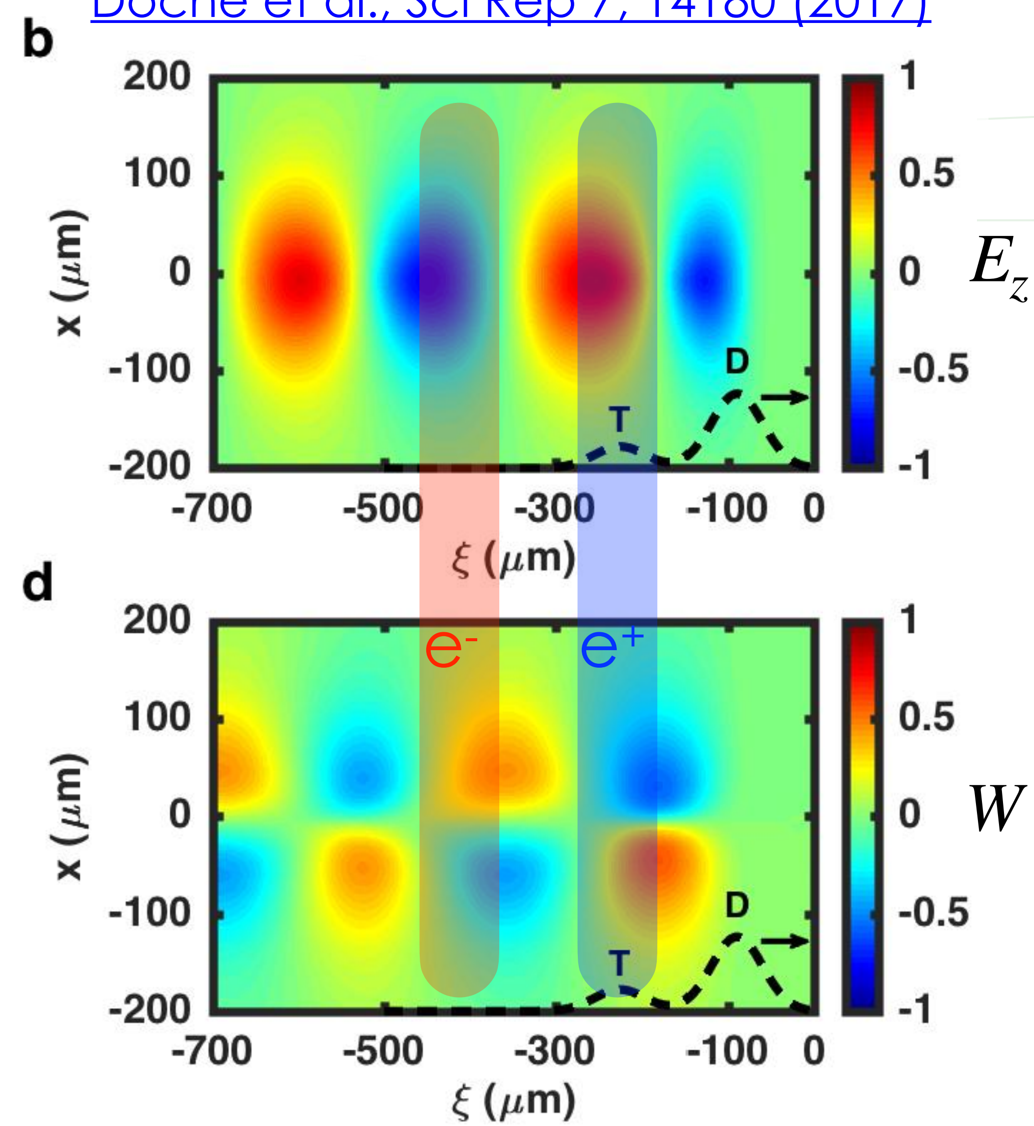
Linear plasma wakefield

Behind the beam:

- E_r and E_z are sinusoidal functions of ξ that are 90° out of phase with each other
- $B = 0$
- Acceleration and focusing: a quarter of the plasma wave

Similar to laser-driven linear plasma wakes, except for the radial profile involving Bessel functions.

[Doche et al., Sci Rep 7, 14180 (2017)](#)



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Linear plasma wakefield

Inside the beam (different from laser-driven case):

- E_r and B_θ are shielded by the plasma, and decays radially over a plasma skin depth $1/k_p$
- $B_\theta \neq 0$ due to beam current and plasma radial and longitudinal currents
- Long beam: $n_1 \simeq \dfrac{q}{e} n_b$ to shield E_r, return current to shield B_θ
- Short beam: there is an instantaneous inductive shielding driven by the fast variation of the plasma radial current

$$E_z(r,\xi) = \frac{qk_p^2}{\epsilon_0}\int_0^{+\infty} r'dr' K_0(k_p r_>) I_0(k_p r_<) \int_\xi^{+\infty} d\xi' n_b(r',\xi') \cos k_p(\xi-\xi')$$

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$k_p\sigma_z = 1$

$k_p\sigma_r = 2$

Going to longer beams

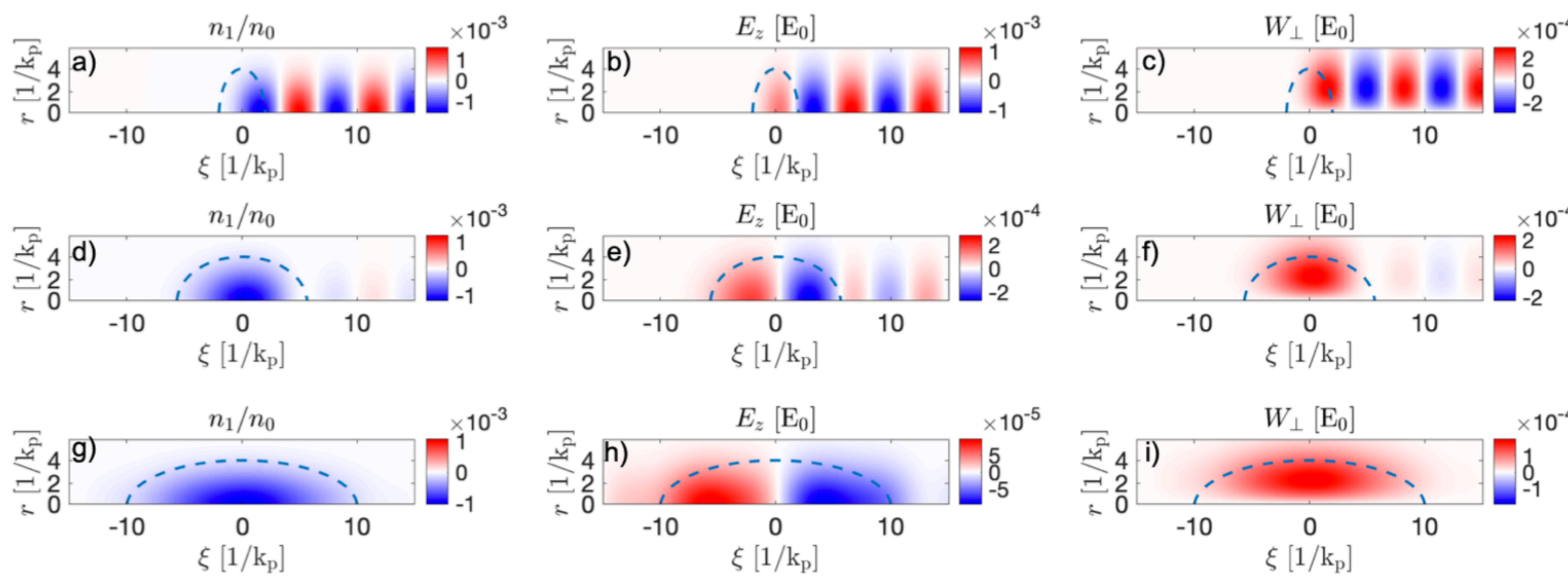
$k_p\sigma_z = 2\sqrt{2}$

$k_p\sigma_r = 2$

$n_{b,\text{peak}} = 10^{-3} n_0$

$k_p\sigma_z = 5$

$k_p\sigma_r = 2$



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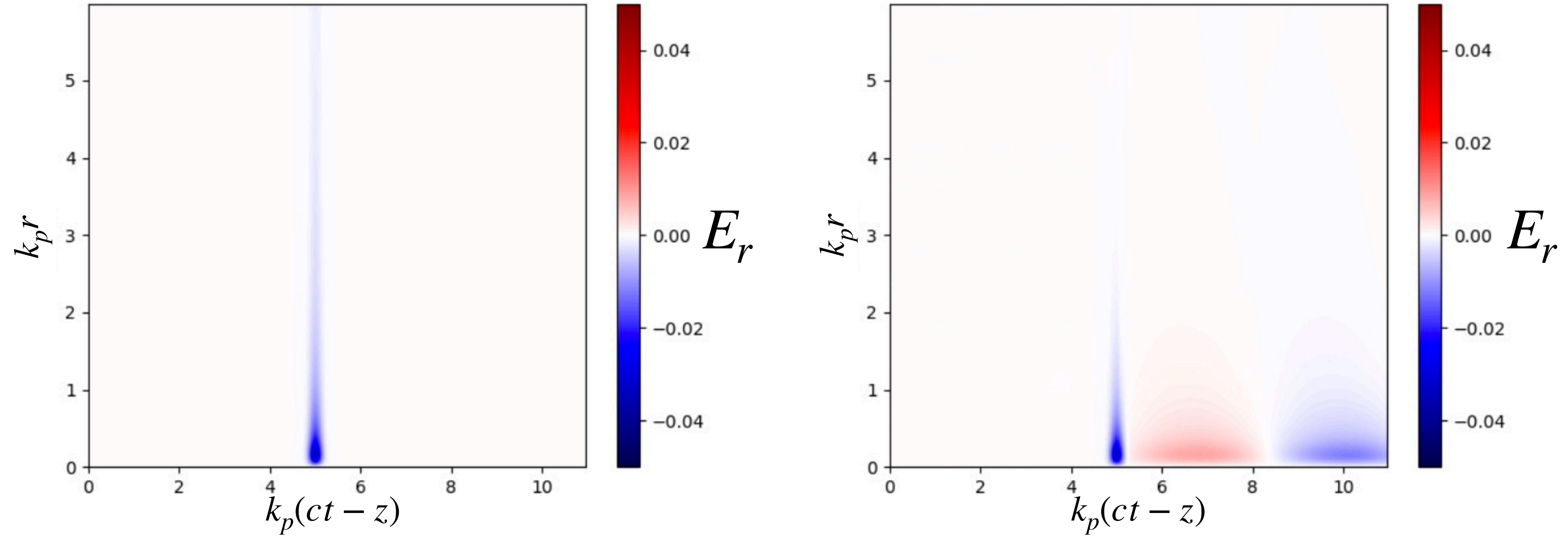
$$\partial_\xi j_r \implies \partial_\xi A_r \implies E_r \text{ (term } \partial_t A_r\text{) and } B_\theta \text{ (term } \partial_z A_r\text{)}$$

Short and small beam with $k_p\sigma_z = k_p\sigma_r = 0.1$



(a) Vacuum (b) Plasma

Linear plasma wakefield

Radial profile:

- Looks like the beam for large beams ($k_p\sigma_r \gg 1$), similar to the laser-driven case, it's local
- Bessel-like radial profiles for small beams ($k_p\sigma_r \ll 1$), extending over a plasma skin depth $1/k_p$, wakefield is not local: it extends outside the beam
- In sharp contrast to laser-driven case for which radially $\phi, E_z, E_r \propto I, \partial_r I$, in other words a Gaussian laser drives a Gaussian wake radially (it's local), no Bessel involved.

$$E_z(r,\xi) = \frac{qk_p^2}{\epsilon_0}\int_0^{+\infty} r'dr'K_0(k_pr_>)I_0(k_pr_<)\int_\xi^{+\infty} d\xi' n_b(r',\xi')\cos k_p(\xi-\xi')$$

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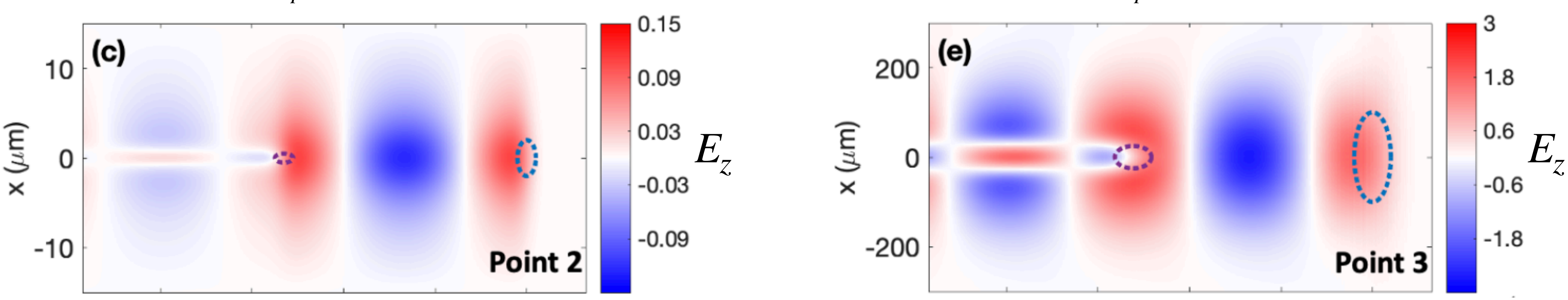
Small beams ($k_p\sigma_r \ll 1$) | Large beams ($k_p\sigma_r \gg 1$)



Hue et al., PRR 3, 043063 (2021)

moving to nonlinear wakes…

Nonlinear plasma wakefield

If $n_b > n_0$:

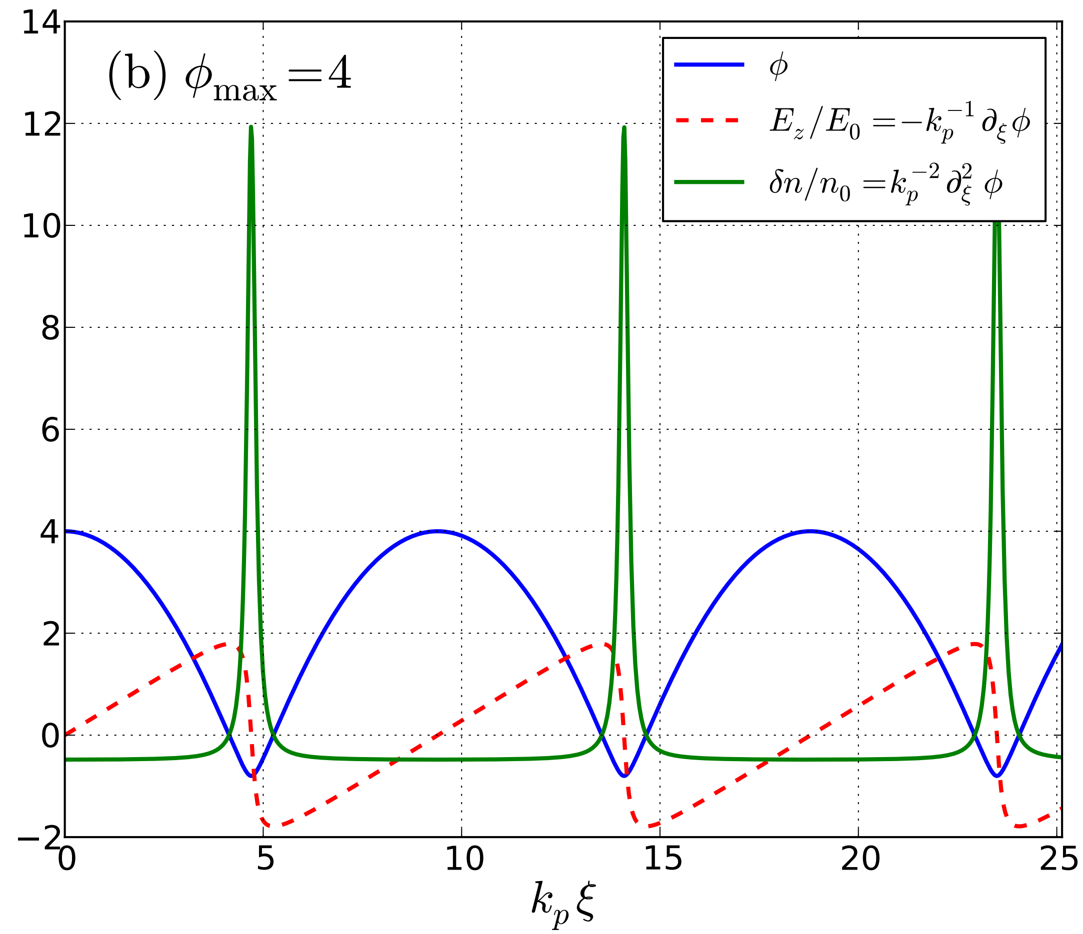
- the plasma is no longer able to screen the beam
- the perturbation theory, that assumes $n_1 \ll n_0$, is no longer valid

In 1D and using QSA, it's purely electrostatic and it's analytical:
[Krall et al., PRA 44, 6854 (1991); Rosenzweig, PRL 58, 555 (1987)]

$$\frac{d^2\phi}{d\xi^2} = \frac{k_p^2}{2}\left[\frac{2n_b}{n_0} + \frac{1}{(1+\phi)^2} - 1\right]$$

$$\frac{E_z}{E_0} = -\frac{1}{k_p}\frac{d\phi}{d\xi}$$

with $E_0 = mc\omega_p/e$ the nonrelativistic wavebreaking field, or equivalently the field of a 1D sinusoidal density perturbation with $n_1 = n_0$.



QSA $\Longrightarrow$ stationary plasma flow in moving window

$\Longrightarrow \; n\,|d_t\xi| = n_0 c \;\Longrightarrow\; n = n_0 c/(c - v_z)$

Nonlinear plasma wakefield

What about nonlinear 3D wakes?

- no complete and self-consistent analytical solution, but phenomenological models and numerical modelling
- possible to have rather good physical intuition

In addition to the dimensionless parameter n_b/n_0, the nonlinear 3D wakes also depend on:

$$\Lambda = 2I/I_A = k_p^2 \sigma_r^2 n_b/n_0$$

normalized current
relevant for moderately-long beams ($k_p\sigma_z \gtrsim 0.2$)

$$\tilde{Q} = k_p^3 N_b/n_0 = (2\pi)^{3/2} k_p \sigma_z \Lambda$$

normalized charge
relevant for short beams with $k_p\sigma_z \ll 1$

(with $I_A = 4\pi m_e \epsilon_0 c^3/e \simeq 17$ kA the Alvén current)

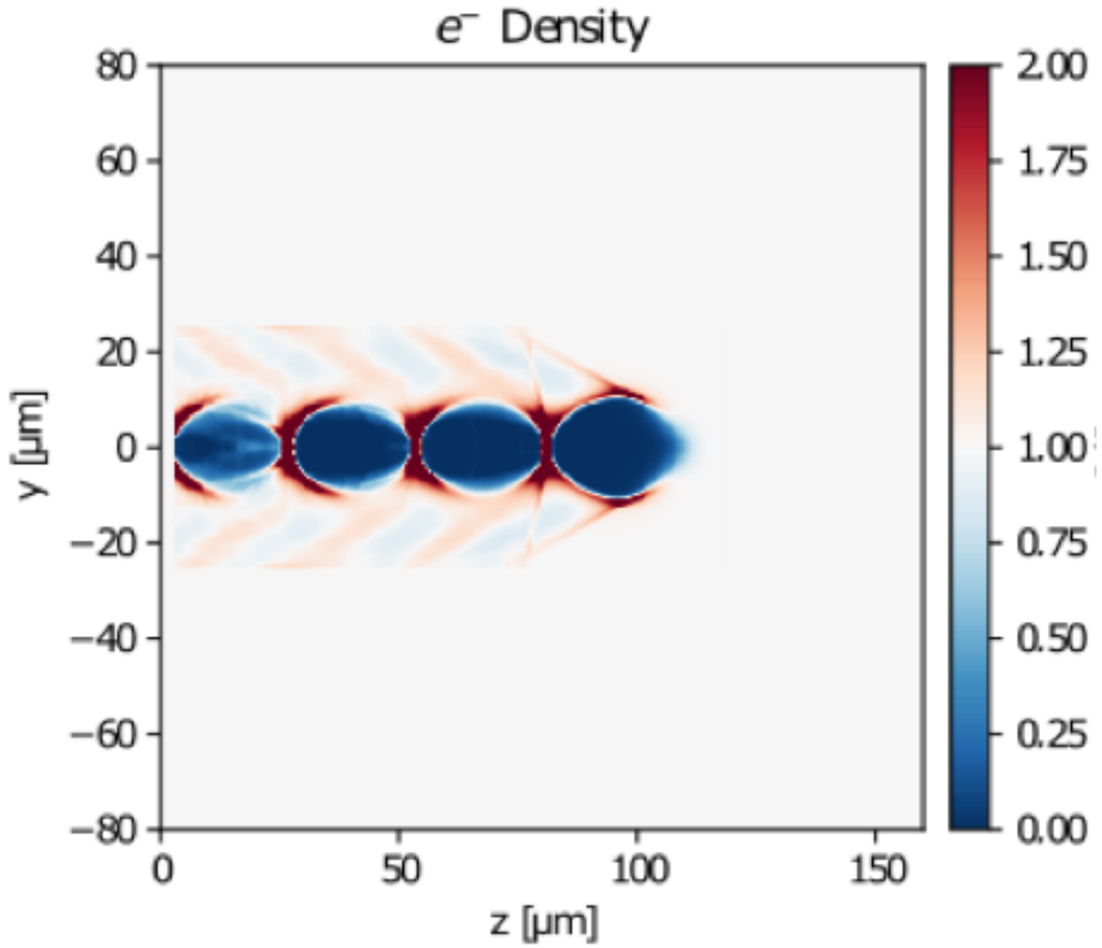
Three regimes:

- $n_b/n_0 \ll 1$: linear regime
- $n_b/n_0 \gtrsim 1$ and ($\Lambda < 1$ or $\tilde{Q} < 1$): nonrelativistic and nonlinear regime
- $n_b/n_0 \gtrsim 1$ and ($\Lambda > 1$ or $\tilde{Q} > 1$): relativistic and nonlinear regime

Nonlinear plasma wakefield - electron driver

Electron-driven nonlinear 3D wakes

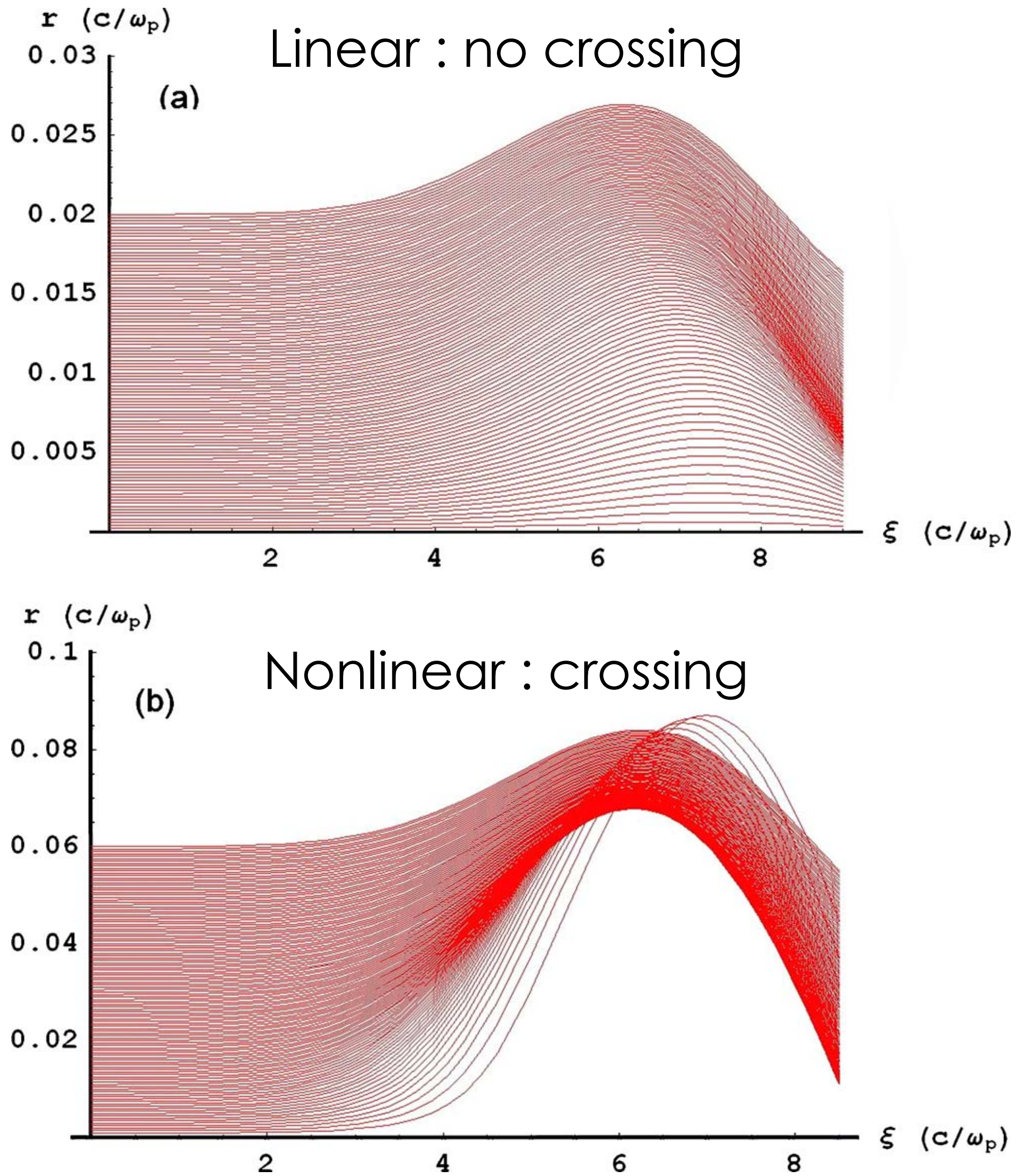
- plasma electrons are expelled/blown out of the propagation axis, thus forming an ion cavity
- plasma electrons are then pulled back by plasma ions, overshooting the propagation axis and setting up a nonlinear plasma oscillation
- this is the so-called blowout regime, where the plasma wave takes the form of ion cavities surrounded by thin electron sheaths



Whether it's a relativistic or nonrelativistic blowout, a key property of these wakes is that particle crossing (or transverse wave breaking) occurs, and the fluid theory becomes inadequate to describe the response of plasma electrons. A fully kinetic approach is required.

Nonlinear plasma wakefield - electron driver

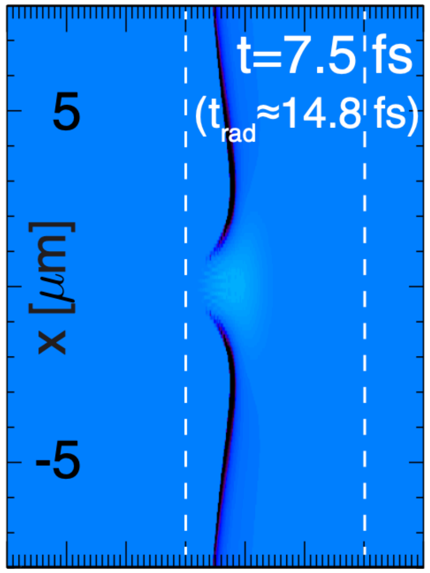
Crossing of plasma electron trajectories:



Lu et al., PoP 13, 056709 (2006)

Note: particle crossing is a necessary condition for the blowout regime, where a cavity void of plasma electrons is formed.

Relativistic blowout: plasma electrons are initially pushed forward instead of backward as observed in linear and nonlinear 1D regimes. This is due to the $\vec{v} \times \vec{B}$ force that can overcome the decelerating field.



Xu et al., PRL 126, 094801 (2021)

Nonlinear plasma wakefield - electron driver

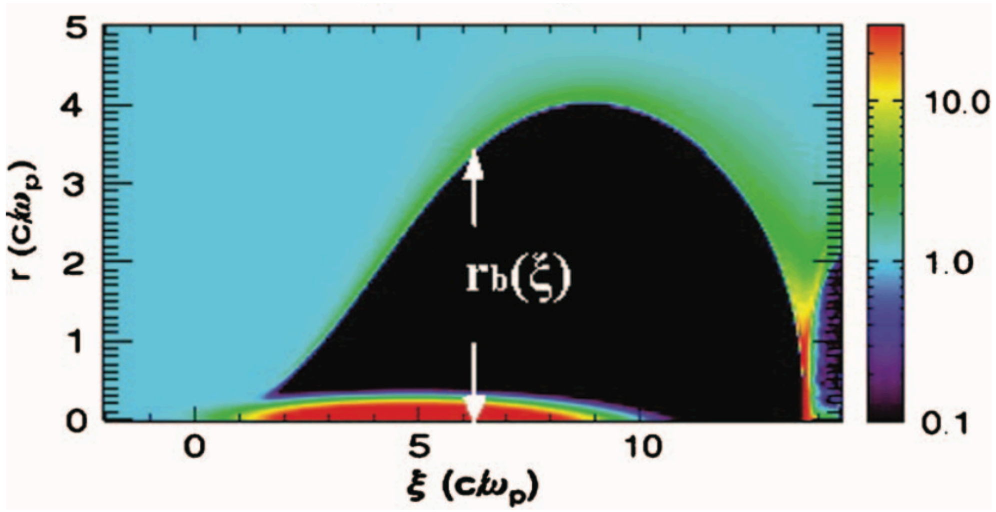
An example of phenomenological model for the blowout regime:

[Lu et al., PoP 13, 056709 (2006)]

[Lu et al., PRL 96, 165002 (2006)]

Two coupled systems:

- the electron sheath circulating around the blowout cavity, described by a trajectory $r_b(\xi)$ that depends on the fields
- the fields that depend on the source terms which can be expressed from $r_b(\xi)$ using Maxwell.



Three parts in the plasma:

- the blowout cavity
- the electron sheath
- the plasma outside, for which the response is nearly linear (weak density perturbation) and extends transversely over a plasma skin depth

Using pseudo-potential $\psi = \phi - cA_z$, we have:

$$\nabla_\perp^2 \psi = -(\rho - j_z/c)/\epsilon_0$$ 2D Poisson-like equation

$$\gamma mc^2 - p_z c = mc^2 + e\psi$$ Conserved quantity for a plasma electron in QSA

Note: ψ is very useful for the electron beam dynamics as

$$F_\parallel = -eE_z = e\nabla_\parallel \psi$$

$$F_\perp = -e(E_r - cB_\theta) = -eW = e\nabla_\perp \psi$$

It thus determines both the longitudinal and transverse wakefields.

Nonlinear plasma wakefield - electron driver

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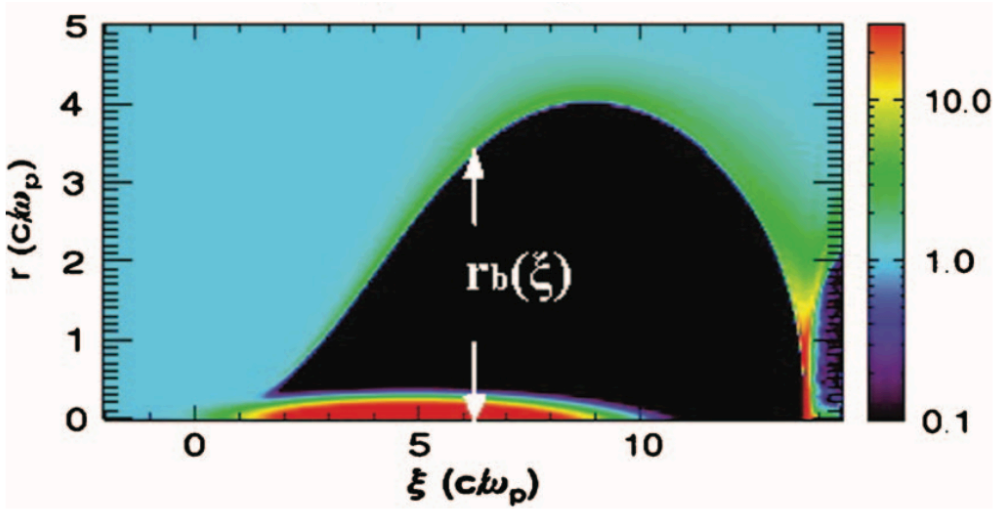
Lu et al., PoP 13, 056709 (2006)

Lu et al., PRL 96, 165002 (2006)



Three parts in the plasma:

- the blowout ion cavity
- the electron sheath
- the plasma outside, for which the response is nearly linear (weak density perturbation) and extends transversely over a plasma skin depth

$$\psi(r=0,\xi) = \int_0^\infty \frac{dr}{r}\int_0^r r'dr'(\rho - j_z/c)/\epsilon_0 = \left\{\int_0^{r_b} + \int_{r_b}^{r_b+\Delta} + \int_{r_b+\Delta}^{\infty}\right\}\frac{dr}{r}\int_0^r r'dr'(\rho - j_z/c)/\epsilon_0 = \psi_{\text{ion}} + \psi_{\text{sheath}} + \psi_{\text{linear}}$$

$\tilde{\psi}_{\text{ion}} = (k_p r_b)^2/4$

$\tilde{\psi}_{\text{sheath}} = \text{small fraction of } \tilde{\psi}_{\text{ion}}$

$\tilde{\psi}_{\text{linear}}$ dominates for nonrelativistic blowout $(k_p r_b \ll 1)$

$\tilde{\psi}_{\text{linear}}$ negligible for relativistic blowout $(k_p r_b \gg 1)$

$\Longrightarrow$ $E_z = -\partial_z \psi$ is close to linear scalings for nonrelativistic blowout

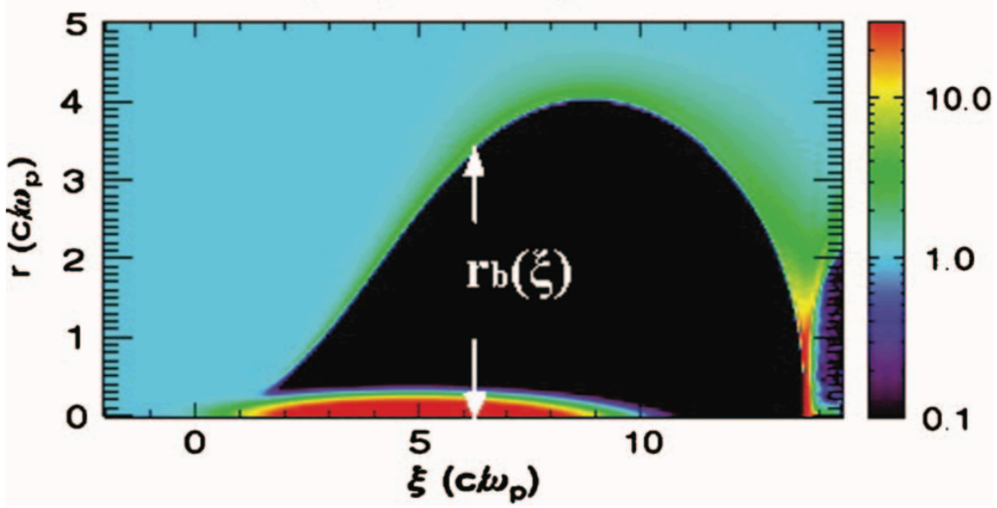
$E_z = -\partial_z \psi$ has completely different scalings for relativistic blowout

Nonlinear plasma wakefield - electron driver

An example of phenomenological model for the blowout regime:

Lu et al., PoP 13, 056709 (2006)

Lu et al., PRL 96, 165002 (2006)



In the ultrarelativistic blowout, the coupled sheath-field system simplifies to a single differential equation on $r_b(\xi)$:

$$r_b \frac{d^2 r_b}{d\xi^2} + 2\left[\frac{dr_b}{d\xi}\right]^2 + 1 = \frac{4\lambda(\xi)}{k_p^2 r_b^2}$$

$$E_z(r=0,\xi) = -\frac{1}{2} k_p r_b \frac{dr_b}{d\xi} E_0$$

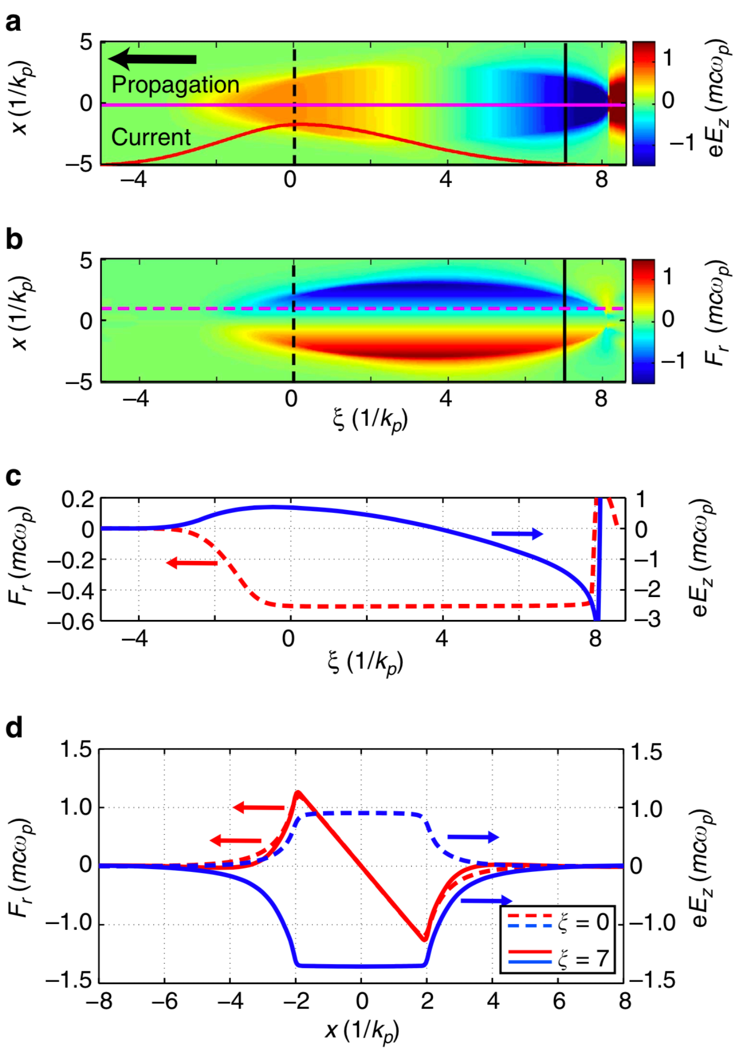
with $\lambda(\xi) = k_p^2 \int_0^{+\infty} rdr\, n_b(\xi, r)/n_0$

For moderately-long beam, $R_b = \max r_b$ can be found by balancing the repulsing beam space-charge force and the restoring wakefield force, or by assuming $\lambda(\xi) = \Lambda = \text{const}$ in the equation above.

$$k_p R_b = 2\sqrt{\Lambda}$$

Nonlinear plasma wakefield - electron driver

Key properties of the blowout regime:



EM fields inside cavity:

$$\mathbf{E}/E_0 = \frac{1}{2}k_p\xi\,\mathbf{e}_z + \frac{1}{4}k_p r\,\mathbf{e}_r$$

$$c\mathbf{B}/E_0 = -\frac{1}{4}k_p r\,\mathbf{e}_\theta$$

Transverse force experienced by an e-:

$$F_r = -e(E_r - cB_\theta) = -\frac{eE_0k_p}{2}r$$

→ Focusing force linear in r

Additional properties:

$$\partial_\xi F_r = 0 \qquad \partial_r F_z = 0$$

The blowout regime has ideal field properties for e-:

→ emittance preservation is expected to be achievable.

→ beam loading allow for high efficiency, flat E_z field and therefore low energy spread.

→ most studied regime for electron acceleration, in both LWFA and PWFA.

→ hosing instability may be an important limitation for collider beam parameters.

→ ion motion may lead to emittance growth.

Clayton et al., Nat Comm 7, 12483 (2016)

Nonlinear plasma wakefield - electron driver

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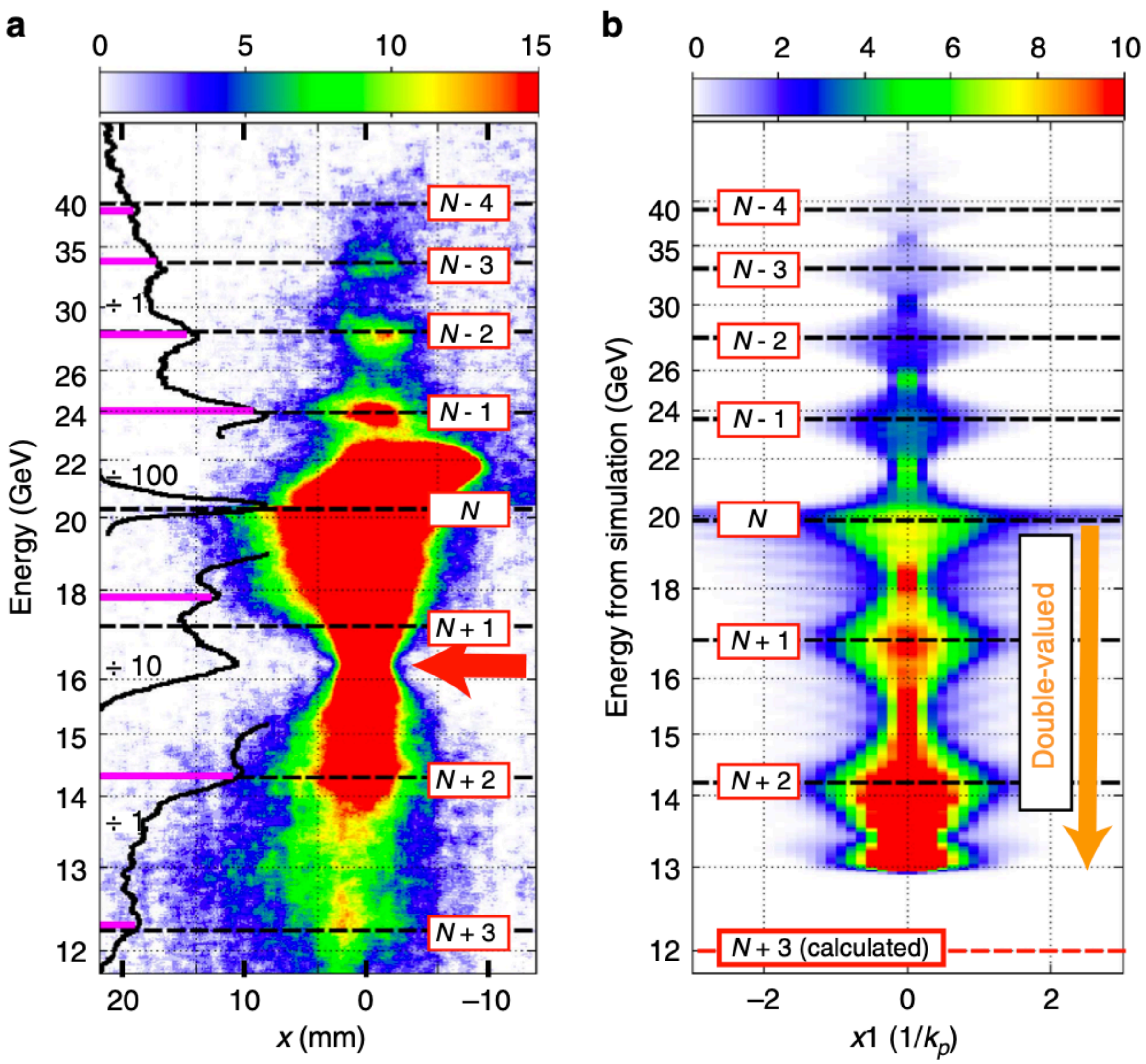


Experimental data in the blowout regime:

confirmed that the radial force is uniform longitudinally within 3% accuracy

$$\partial_\xi F_r \simeq 0$$

Panofsky-Wenzel theorem:

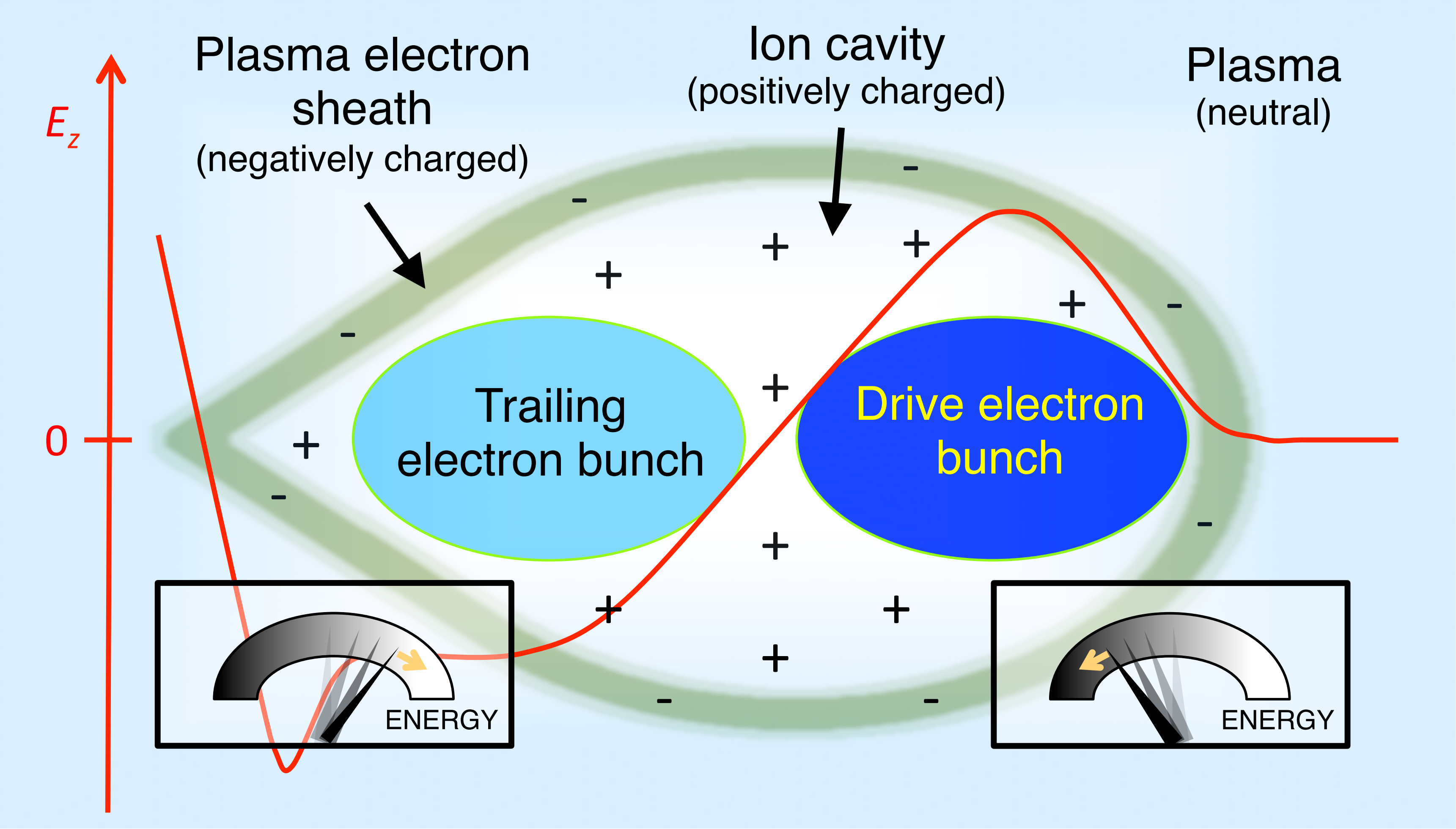
$$\partial_r F_z = \partial_\xi F_r$$

$$\Longrightarrow \quad \partial_r F_z \simeq 0$$

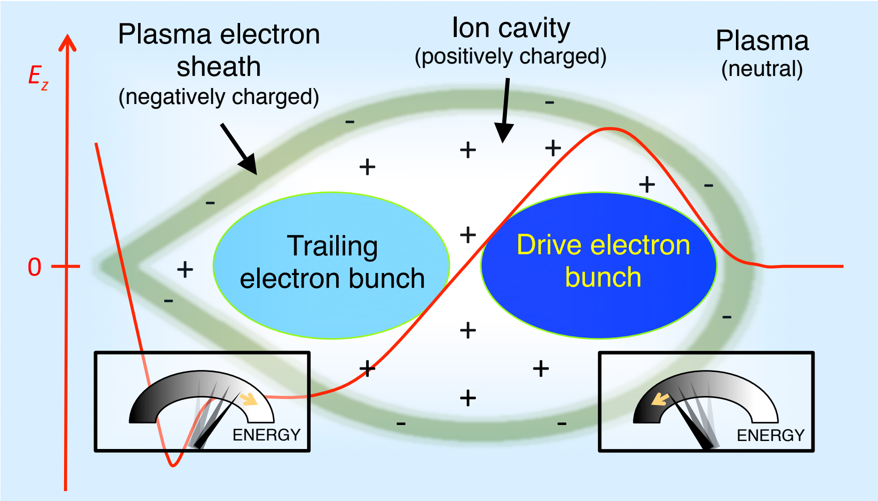
Clayton et al., Nat Comm 7, 12483 (2016) moving to the two-bunch configuration…

The two-bunch configuration

How do we accelerate a beam?



The two-bunch configuration



<u>Why using a beam to accelerate a beam?</u>

We already have the beam…

- The drive beam provides the energy to the plasma, does not necessarily need to be of extremely high quality. Advantage: efficiency of RF accelerators.
- The drive particles can have a reasonable energy, e.g. 1 or 10 GeV, which is reasonable for RF accelerators, while the main trailing beam could go to much higher energy (because of high transformer ratio and/or multistage). The energy of many drive beams can be transferred to a single trailing bunch.
- Beam-driven wakefields are dephasingless, and very stable over large propagation distances.
- For a brightness transformer: generate a high-brightness beam from a low-brightness one.

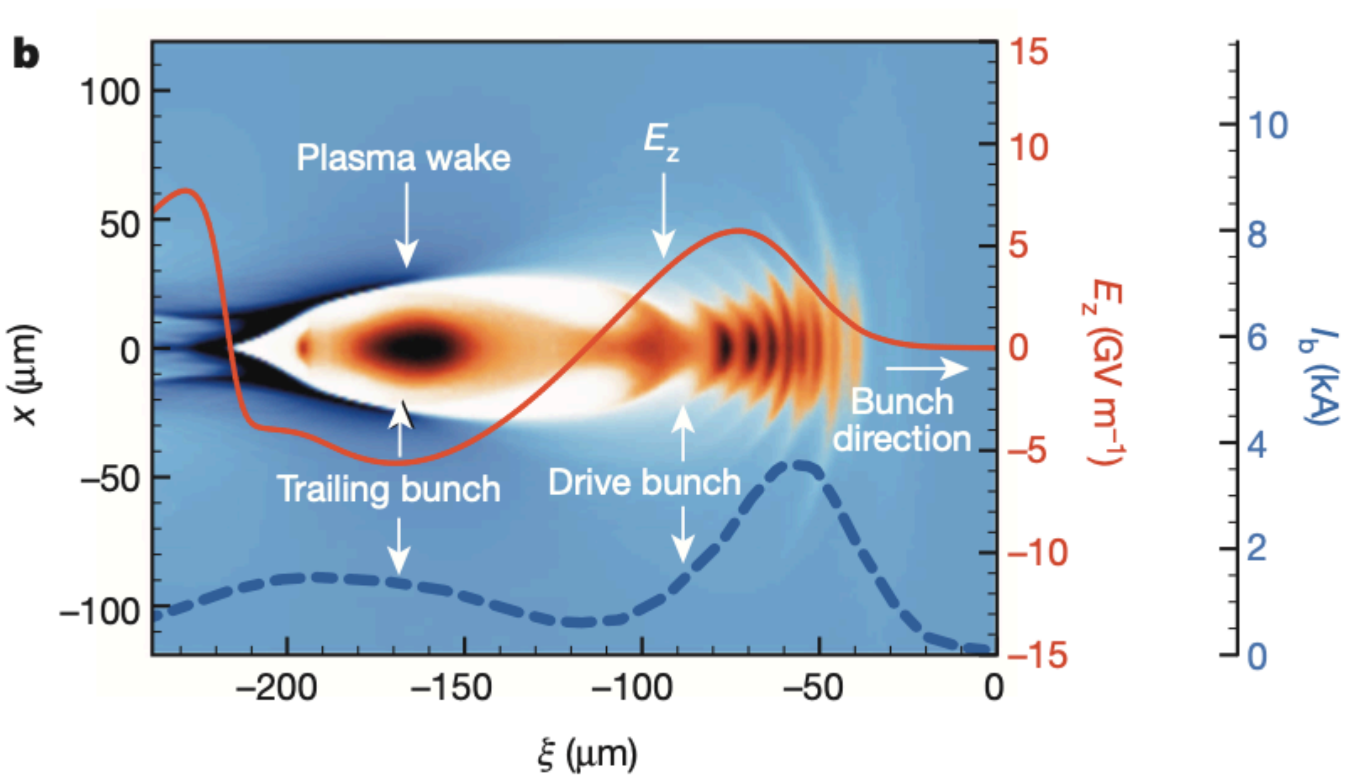
<u>Alternative?</u>

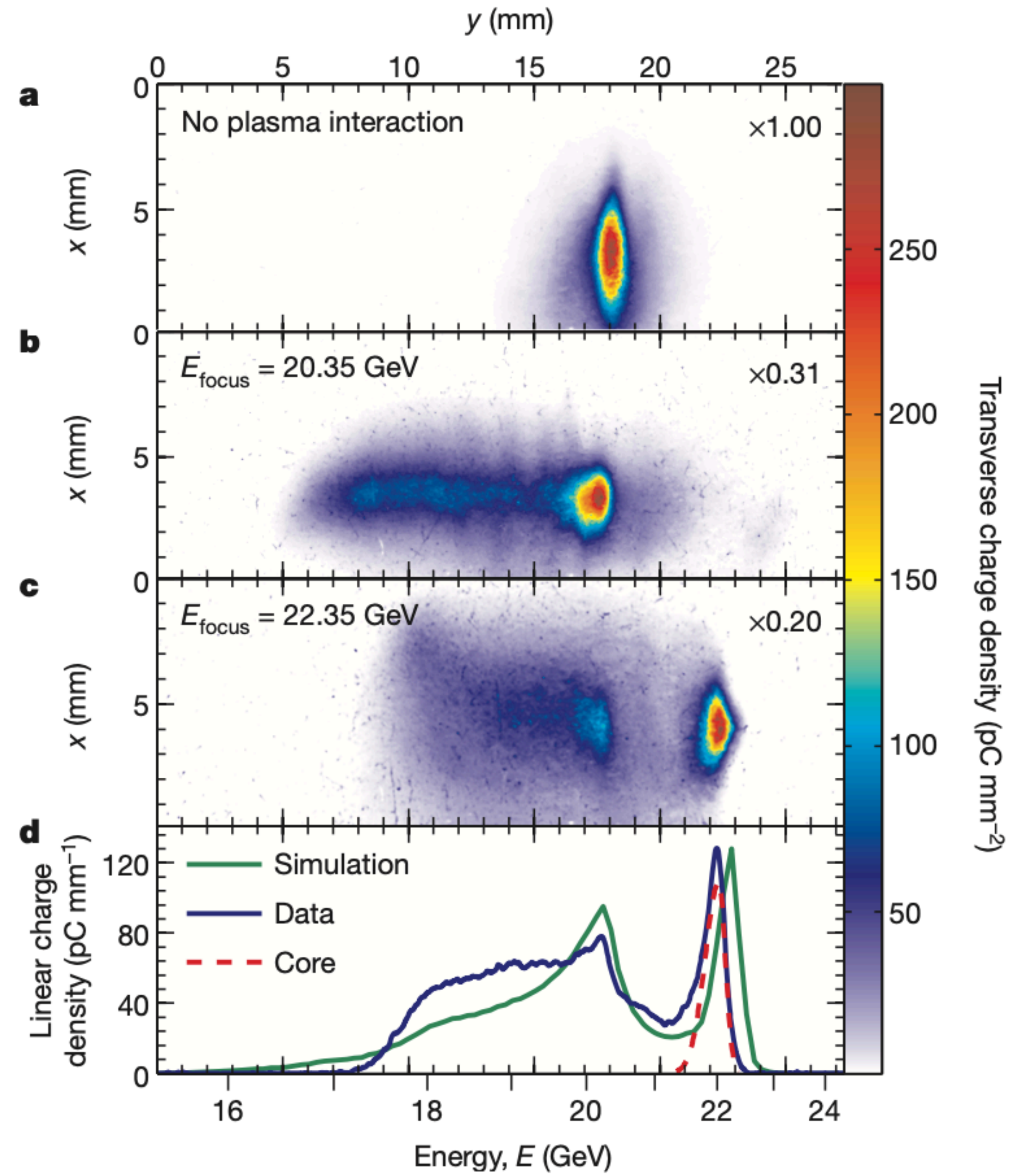
- proton drivers: can be multi-bunch (e.g. after self-modulation)
- advantage: enormous amount of energy stored into a high-energy proton beams, so that a trailing electron bunch could be accelerated to very high energy in a single stage

The two-bunch configuration

<u>Examples of experimental results</u> (blowout and/or two-bunch and/or multi-bunch):

not exhaustive!





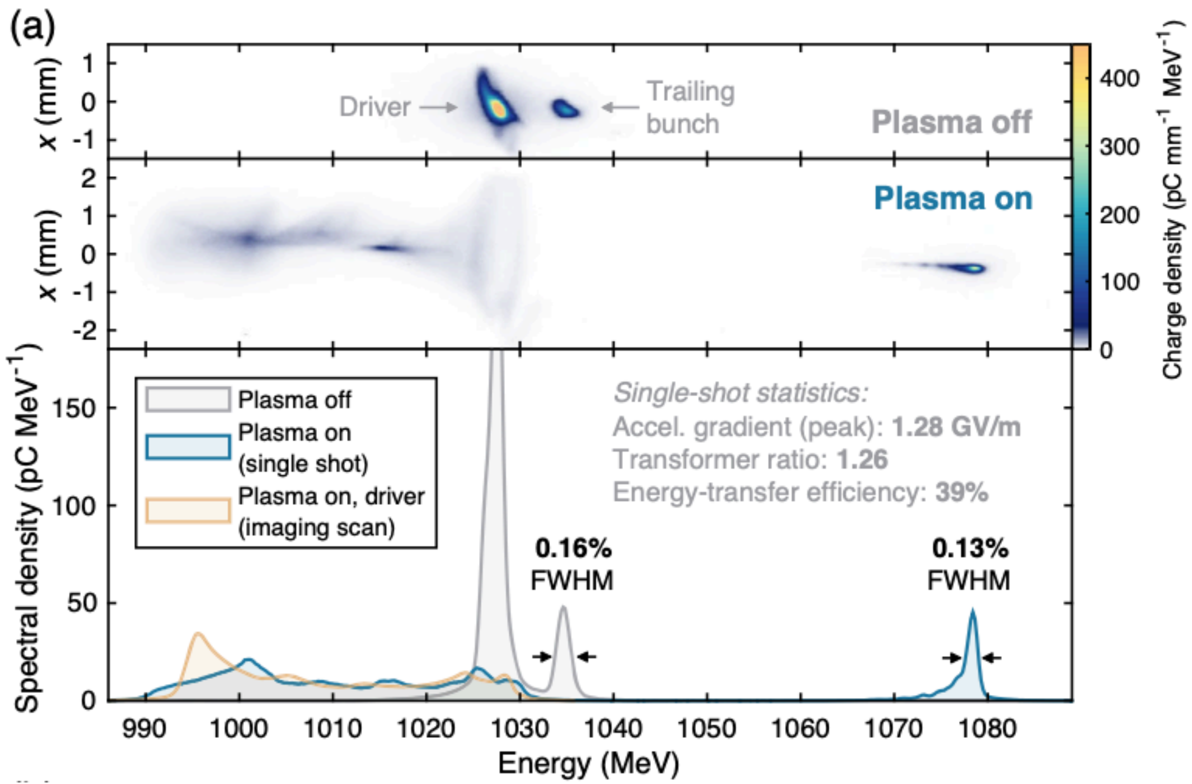
➢ Two-bunch PWFA demonstrated with GeV-scale energy gain and acceleration gradient of ~5 GeV/m

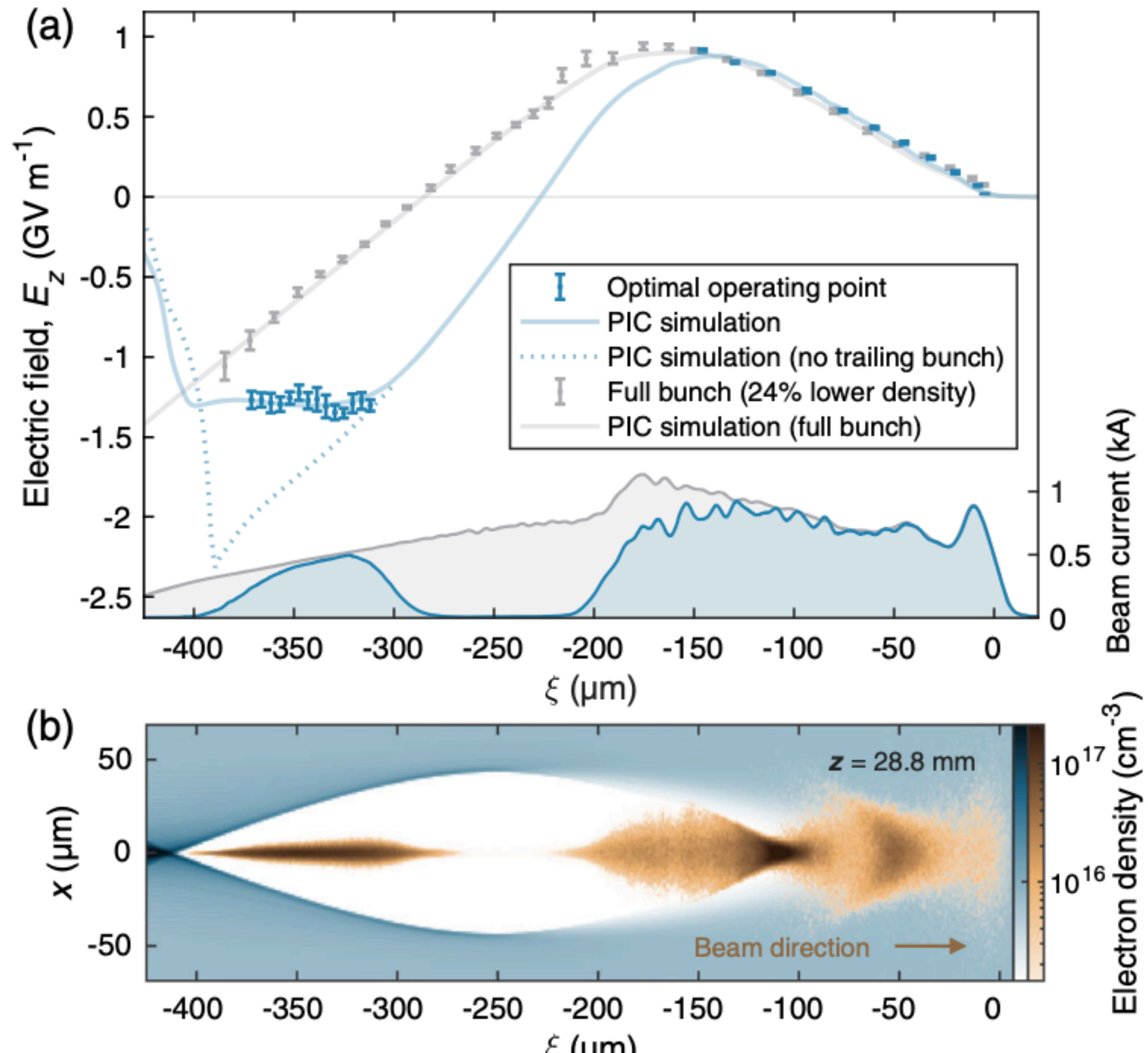
➢ 20% energy efficiency from plasma to trailing bunch

Litos et al., Nature 515, 92 (2014)

The two-bunch configuration

Examples of experimental results (blowout and/or two-bunch and/or multi-bunch):



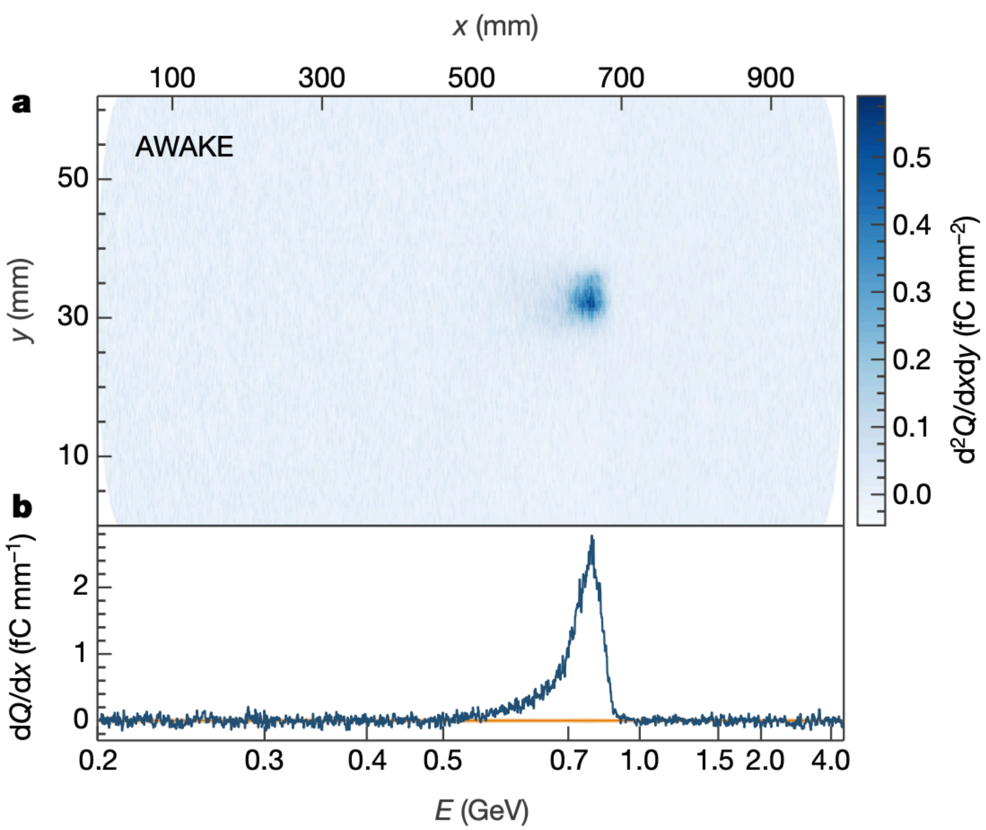


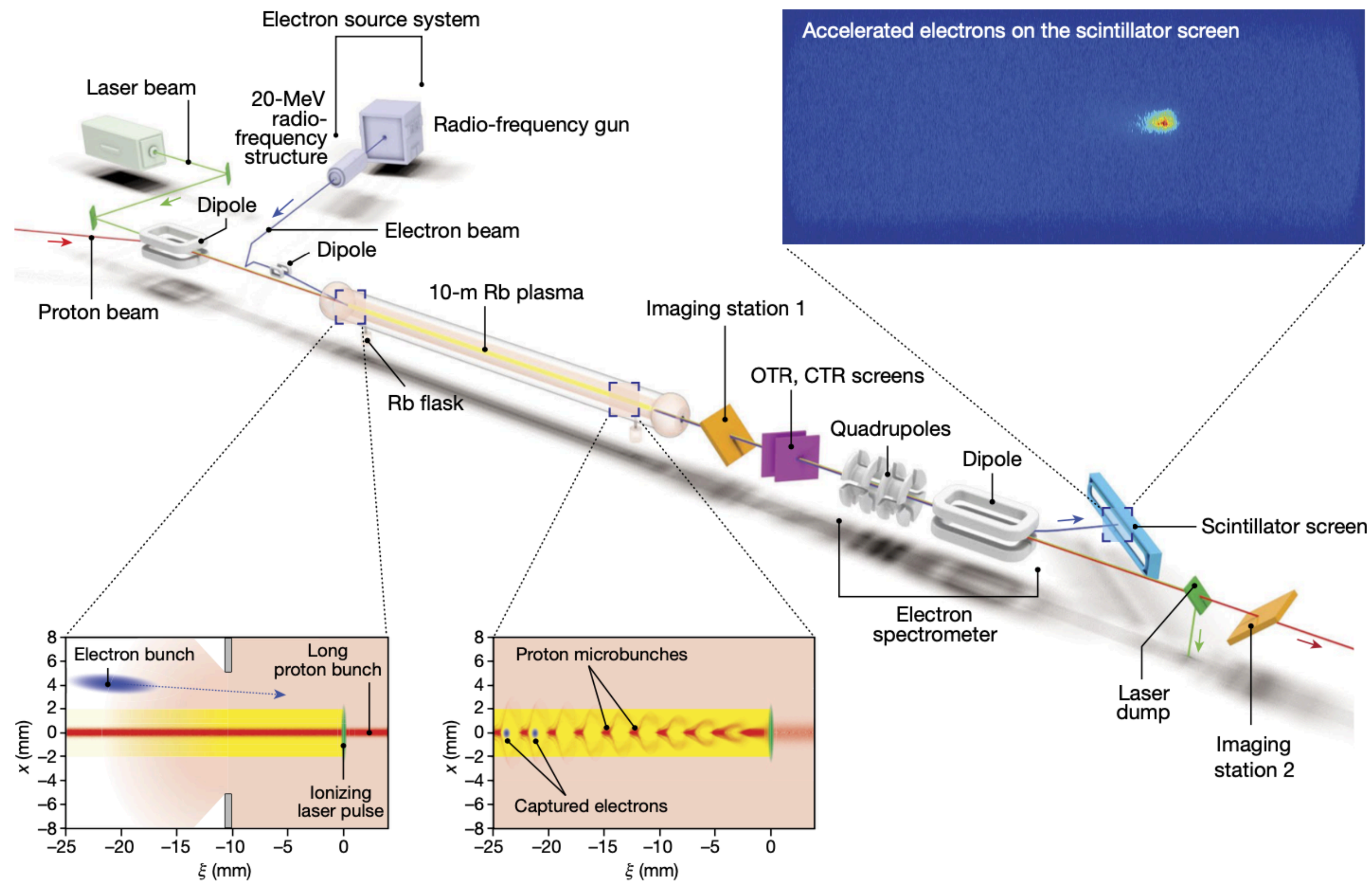
- ➢ 45 MeV energy gain with acceleration gradient of ~1.3 GeV/m
- ➢ 100% charge coupling and per-mile energy-spread preservation
- ➢ 42% energy efficiency from plasma to trailing bunch
- ➢ 2.8% field variation across 60 um of the trailing bunch

Lindstrøm et al., PRL 126, 014801 (2021)

The ~~two-bunch~~ AWAKE configuration

Examples of experimental results (blowout and/or two-bunch and/or multi-bunch):





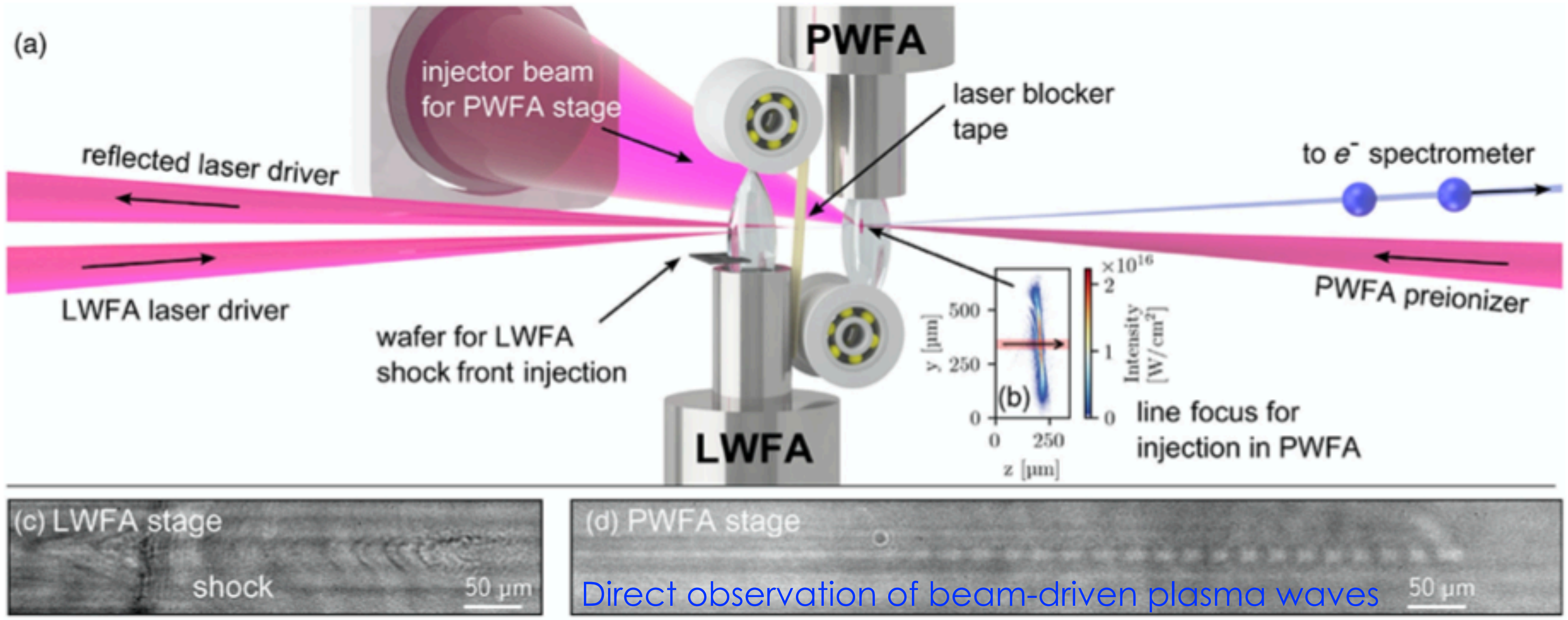
- ➢ 400 GeV proton driver is self-modulated into microbunches in a 10-m-long Rb plasma
- ➢ 20 MeV electrons injected with a small angle into the plasma wakefield
- ➢ Observation of electron acceleration up to 2 GeV for a small fraction of the initial charge

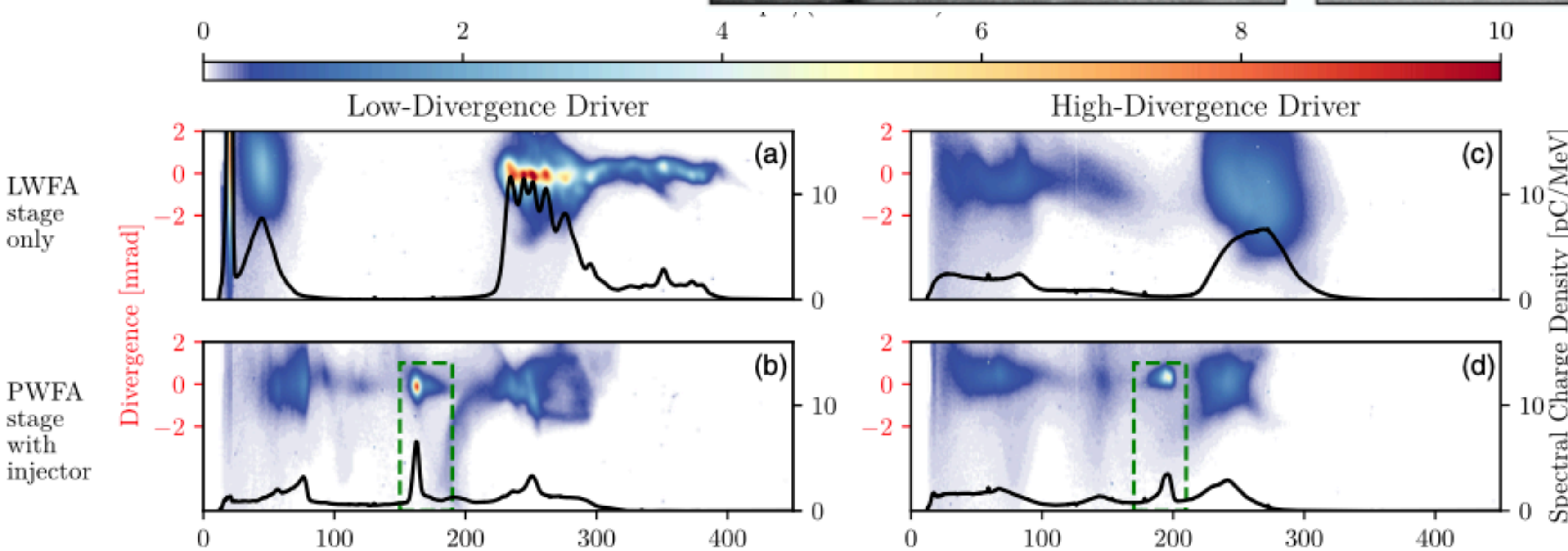
Adli et al., Nature 561, 363 (2018)

The two-bunch configuration

<u>Examples of experimental results</u> (blowout and/or two-bunch and/or multi-bunch):

Staging LWFA and PWFA and injecting a new beam in the PWFA stage, thus forming a two-bunch configuration





from 5 to 7 pC MeV^{-1} μsr^{-1} Energy [MeV] from 0.4 to 6 pC MeV^{-1} μsr^{-1}

Experimental proof-of-principle demonstration of a brightness transformer:

angular-spectral charge density beyond the initial drive beam parameters

Foerster et al., PRX 12, 041016 (2022)

Basic concepts for particle beam evolution in plasma

Envelope equation and matching

Transverse dynamics of a single beam particle

- We assume a linear transverse focusing force (can be an approximation, or be exact for blowout with immobile ions):

$$F_x \simeq -gx$$ with g the gradient of the focusing force,

Example for the blowout: $$g = \frac{eE_0k_p}{2} = \frac{1}{2}m_e\omega_p^2$$

- Assuming no acceleration, we have for individual beam particles:

$$\frac{d}{dt}\left(\gamma m_e \frac{dx}{dt}\right) = F_x \simeq -gx \quad \Longrightarrow \quad \frac{d^2x}{dz^2} = -k_\beta^2 x \quad \text{with} \quad k_\beta = \sqrt{g/\gamma m_e c^2}$$

for the blowout: $k_\beta = k_p/\sqrt{2\gamma}$

Transverse dynamics of the whole beam

- Twiss or Courant-Snyder parameters as beam moments in the trace space $(x, x' = p_x/p_z)$:

$$\alpha = -\langle xx'\rangle/\varepsilon$$

$$\beta = \langle x^2\rangle/\varepsilon \qquad \varepsilon = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

$$\gamma_{\mathrm{twiss}} = \langle x'^2\rangle/\varepsilon = (1+\alpha^2)/\beta$$

- From the single-particle equation of motion, we get:

$$\frac{d\alpha}{dz} = -\frac{1+\alpha^2}{\beta} + k_\beta^2\beta$$

$$\frac{d\beta}{dz} = -2\alpha$$

and

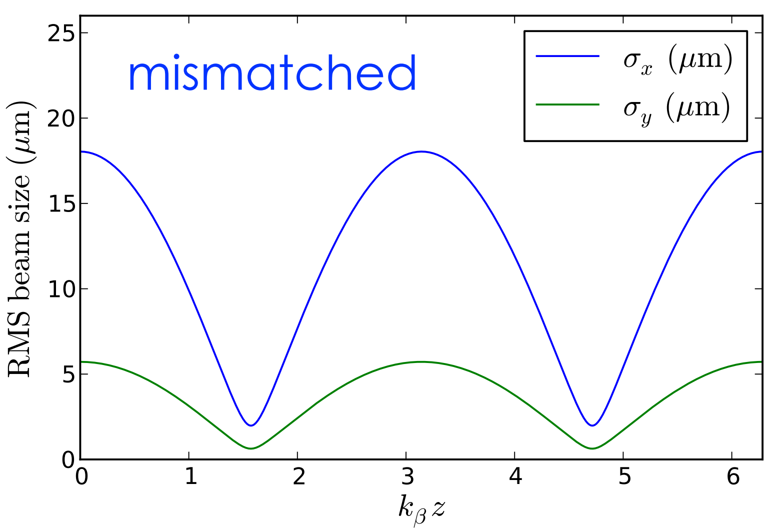
$$\boxed{\frac{d^2\sigma_x}{dz^2} = -k_\beta^2\sigma_x + \frac{\varepsilon^2}{\sigma_x^3}}$$

envelope equation

with $\sigma_x = \sqrt{\langle x^2\rangle} = \sqrt{\varepsilon\beta}$

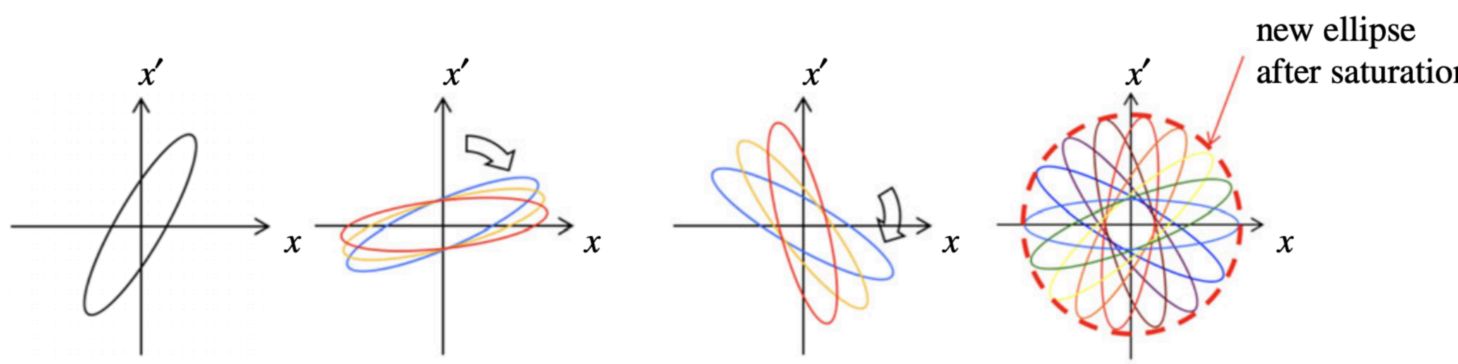
Envelope equation and matching

➢ If the focusing term is not balanced by the emittance term, we observe betatron envelope oscillations, the beam is said to be mismatched to the plasma.

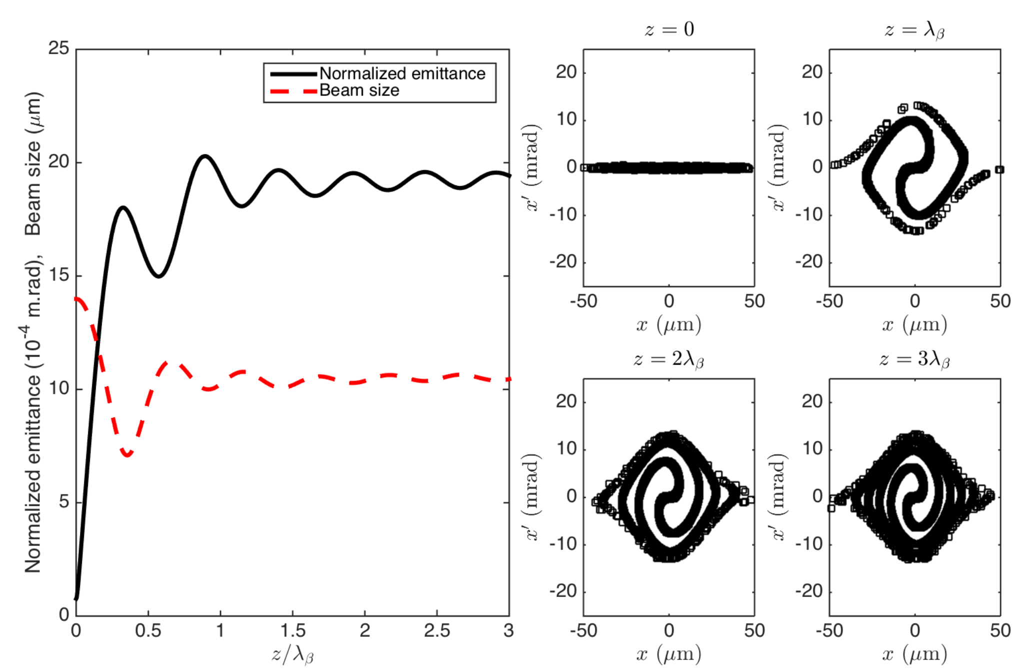


envelope oscillates at twice the frequency of single-particle betatron oscillations

not good because:



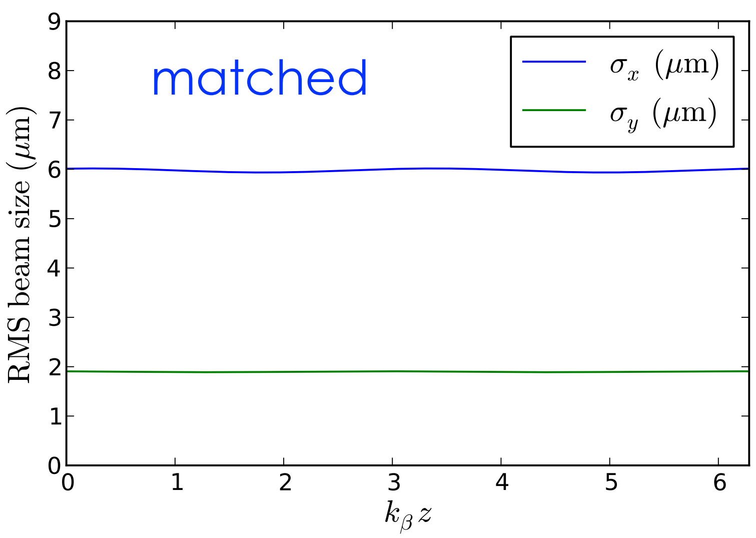
in the presence of energy spread



in the presence of nonlinearities in focusing force

Envelope equation and matching

- The beam is said to be matched in the absence of beam envelope oscillations, which represents the best conditions for quality preservation. When neglecting variation in energy and density, this corresponds to $\sigma_x = \mathrm{const}$ and thus:



no envelope oscillations

$$\frac{d\sigma_x}{dz} = 0 \implies \alpha_{\mathrm{matched}} = 0$$

$$\frac{d^2\sigma_x}{dz^2} = 0 \implies -k_\beta^2\sigma_x + \frac{\varepsilon^2}{\sigma_x^3} = 0$$

$$\implies k_\beta^2\sigma_x^4 = \epsilon^2 \implies \boxed{\beta_{\mathrm{matched}} = \frac{\sigma_x^2}{\epsilon} = 1/k_\beta}$$

Thank you for your attention