Beam-driven plasma acceleration II

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Thanks to my colleagues R. D’Arcy, S. Gessner, C. Lindstrøm, and J. Osterhoff
from which the lecture is inspired.

Outline

Beam-driven plasma acceleration I

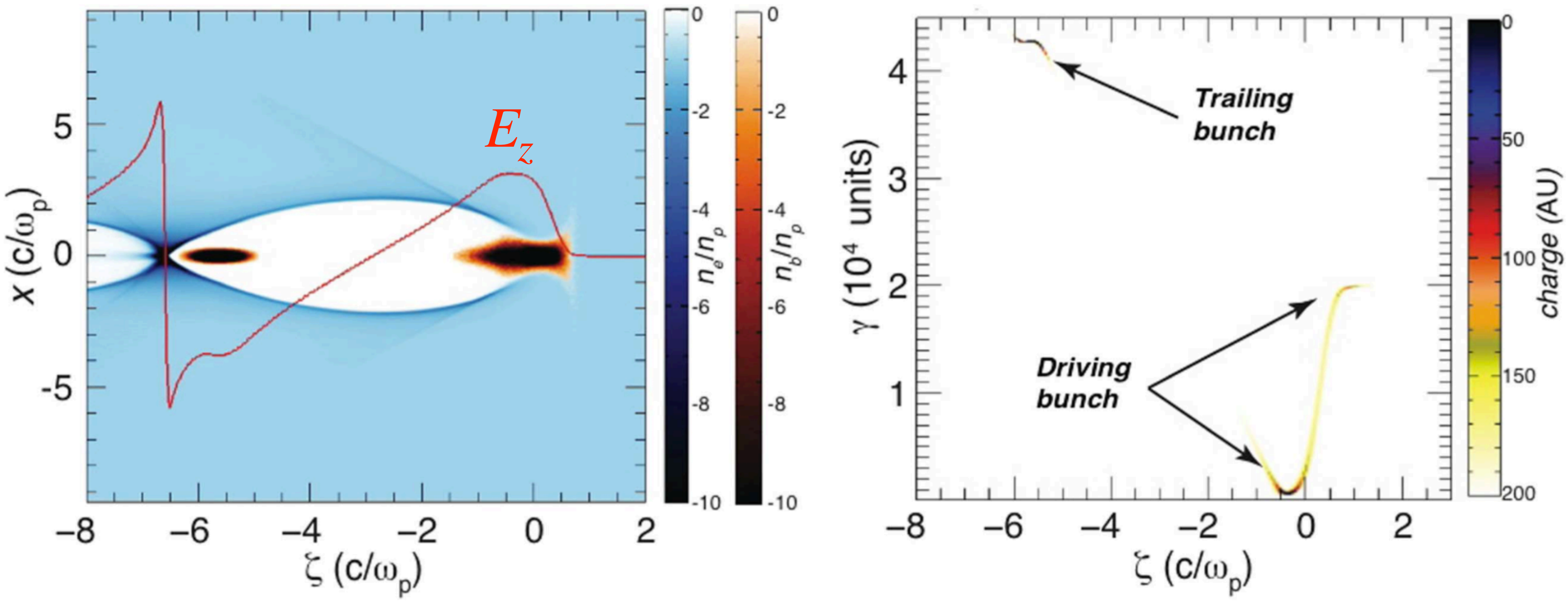
- ~~Exciting plasma wakefields with particle beams~~
  - ~~Linear wakes~~
  - ~~Nonlinear wakes~~
  - ~~How to accelerate: the two-bunch configuration~~
- ~~Basic concepts for particle beam evolution in plasma~~
  - ~~Envelope equation and matching~~
  - Evolution of longitudinal phase space

Beam-driven plasma acceleration II

- Beam loading and energy-transfer efficiency
  - In 1D and 3D linear wakes
  - In nonlinear wakes
- Advanced concepts for particle beam evolution in plasma
  - Head erosion of drive beam
  - Instabilities, ion motion and emittance of trailing beam

Evolution in the longitudinal phase space

[Joshi et al., PPCF 60, 034001 (2018)](#)



➢ Beam particles are ultrarelativistic: their ξ do not change, no slippage and no dephasing!

➢ Non-uniformities of E_z along ξ induces a correlated energy spread: final energy is ξ-dependent.

➢ Driver: plasma acceleration needs to be stopped just before drive particles get to rest. This distance corresponds to the depletion length. Drive to plasma energy efficiency is best when the decelerating field is as uniform as possible along the drive bunch.

$$\text{depletion length} = \frac{\text{beam energy}}{\text{peak decelerating field}}$$

$$\text{drive-to-plasma energy efficiency} = 1 - \frac{\text{final drive beam energy (average)}}{\text{initial drive beam energy}} \simeq \frac{\text{average decelerating field}}{\text{peak decelerating field}}$$

Evolution in the longitudinal phase space

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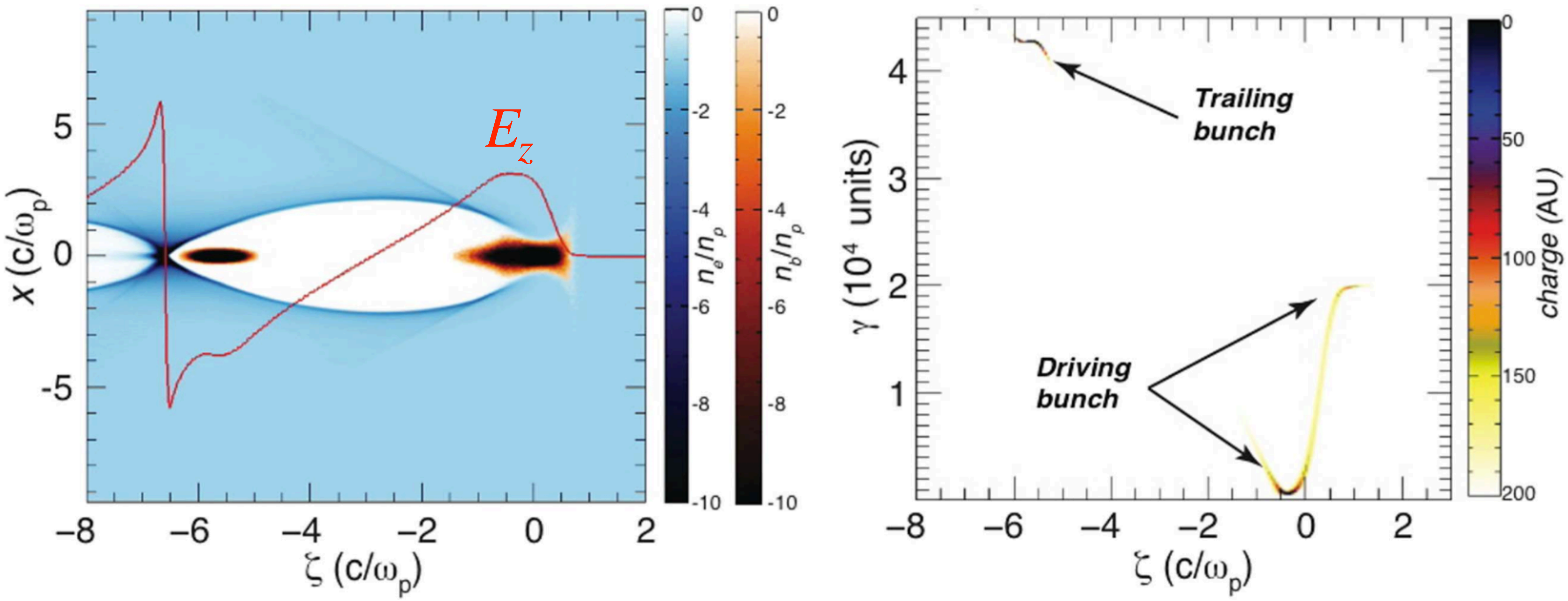

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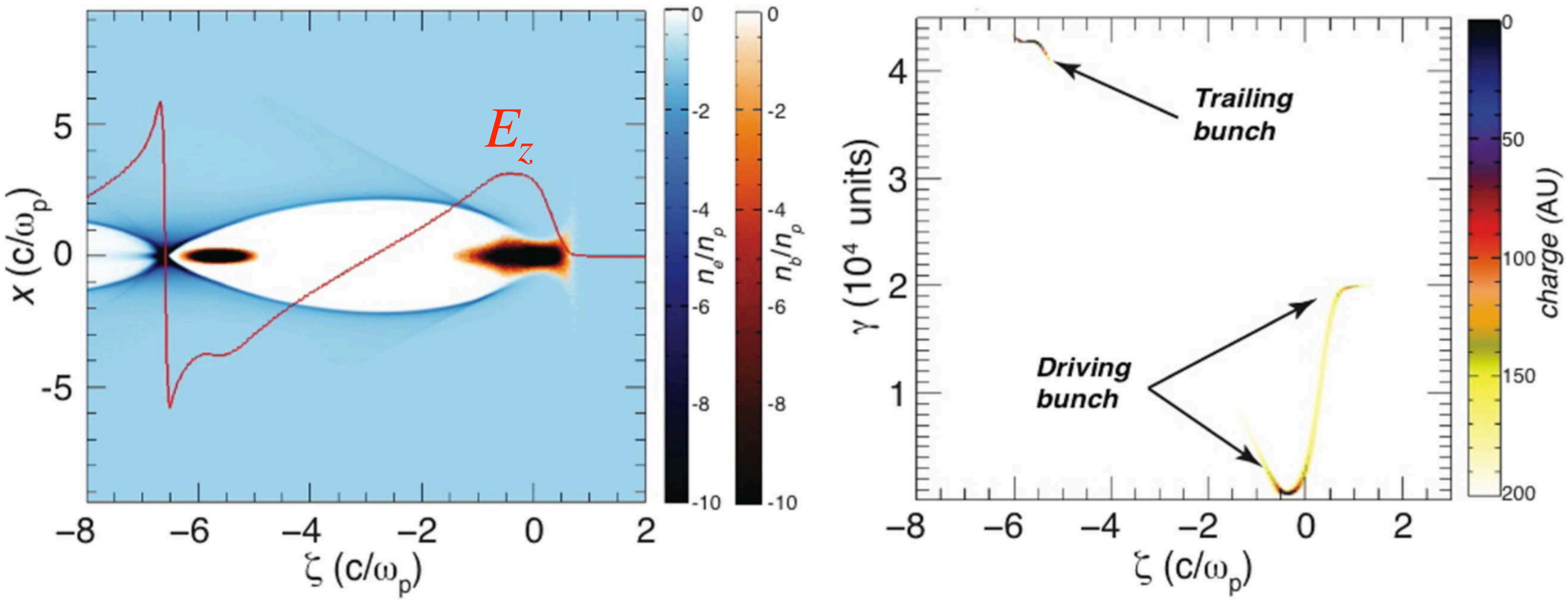
Drive-to-plasma efficiency in the experiment?

50% reported in DESY experiment, poster "Large Energy Depletion of a Beam Driver in a Plasma-Wakefield Accelerator" by Felipe Peña

Evolution in the longitudinal phase space

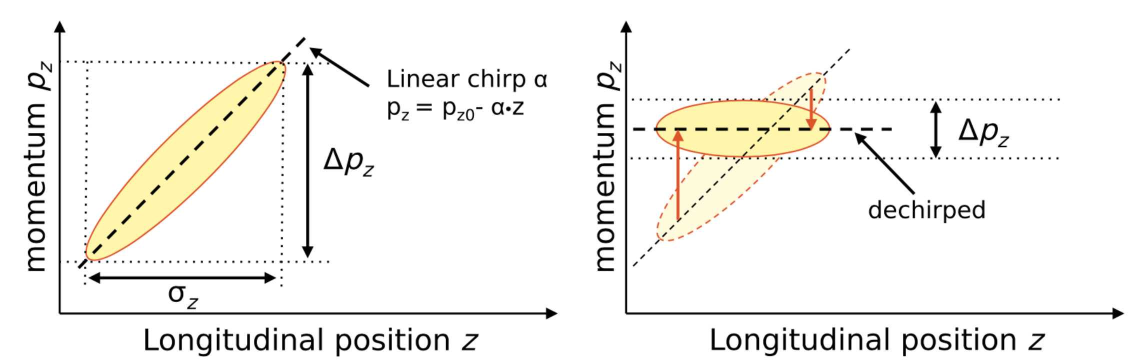
Joshi et al., PPCF 60, 034001 (2018)



- Beam particles are ultrarelativistic: their ξ do not change, no slippage and no dephasing!
- Non-uniformities of E_z along ξ induces a correlated energy spread: final energy is ξ-dependent.
- Trailing: E_z to be as uniform along the bunch/along ξ as possible to minimize correlated energy spread. Otherwise, one can also rely on a dechirping approach:

Note: a chirped beam of low longitudinal emittance can also be used to be compressed to MA current with attosecond spike.

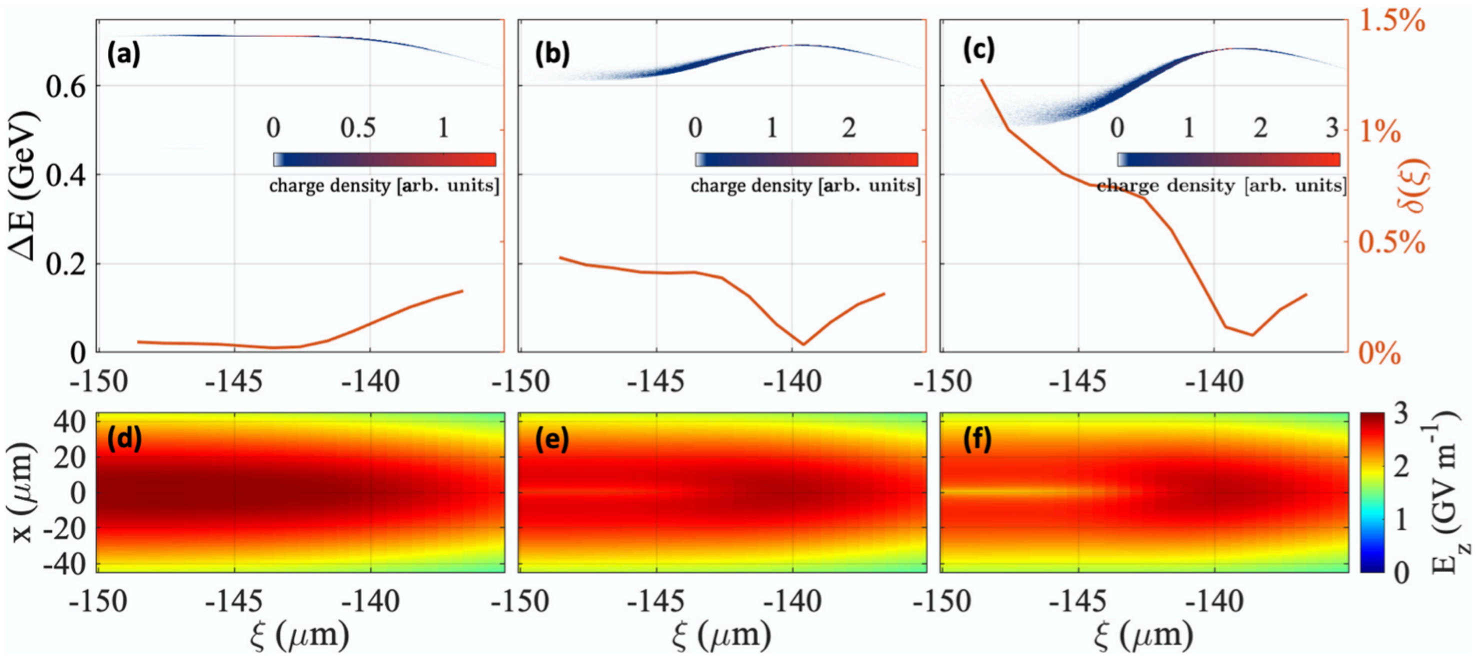
Poster "Radiation detection and coherent harmonic generation for the PAX Experiment at FACET-II" by Rafi Hessami.

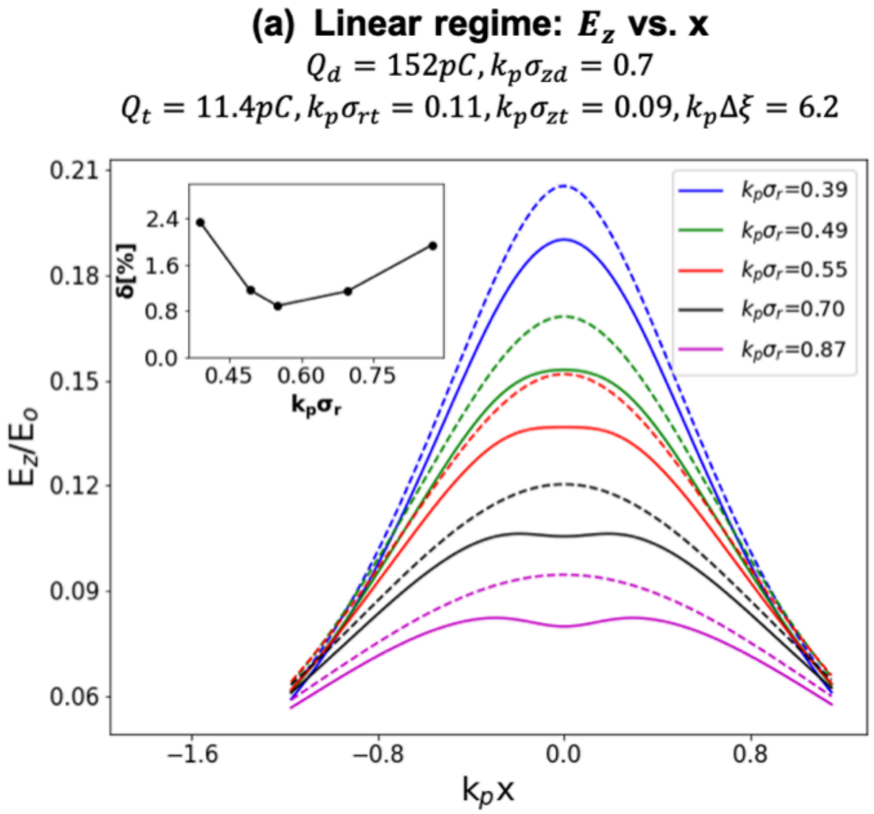


Döpp et al., PRL 121, 074802 (2018)

Evolution in the longitudinal phase space

➢ Trailing: E_z to be as transversely uniform as possible to minimize slice energy spread. This is a fundamental limit, as it cannot be removed (in contrast to the chirp). Example with linear wakefields:





Hue et al., PRR 3, 043063 (2021)

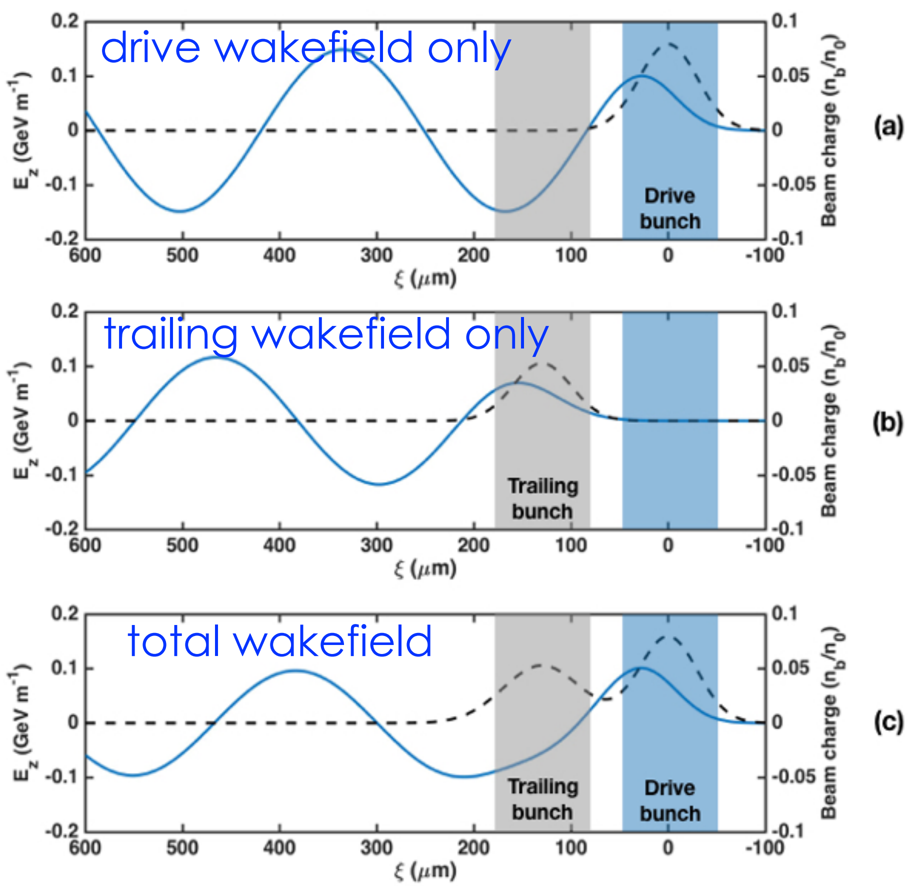
Beam loading and energy-transfer efficiency

Beam loading and energy-transfer efficiency

➢ When the trailing beam is sent into the plasma wakefield to be accelerated, it modifies the wakefield in a way that makes it weaker. This is beam loading.

➢ It's most easily understood in the linear regime for which the principle of superposition applies:

$$\text{total wakefield} = \text{drive wakefield} + \text{trailing wakefield}$$



Drive bunch: transfers its energy to the plasma.

Trailing bunch: located at an accelerating phase of the drive wakefield.

Total wakefield: reduced/loaded inside the trailing bunch. Loaded accelerating field smaller than unloaded one.

Behind trailing: sinusoidal wave with reduced amplitude. Energy was removed from the plasma by the trailing bunch. Consistent with energy being gained by the trailing bunch.

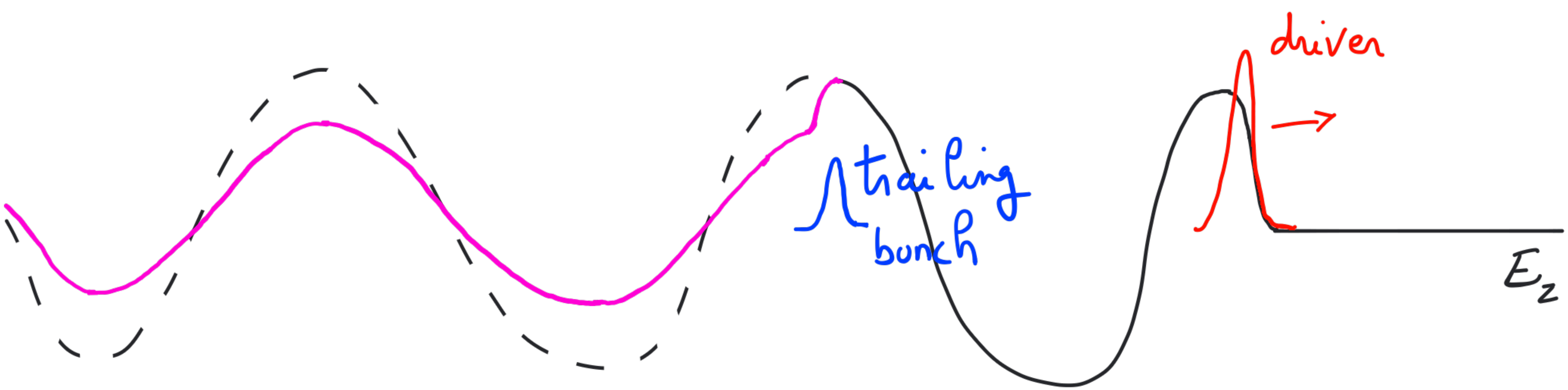
Beam loading and energy-transfer efficiency

➢ In the linear regime, one can also think of the problem using the language of wave interference.

Plasma waves from drive and trailing (or more) interfere:

- constructively if the waves are in phase or equivalently, when bunches are located at a position where the field is decelerating; this is the case of resonant excitation by a bunch train (e.g. AWAKE self-modulated proton beam); all bunches decelerate as they lose energy by reinforcing the plasma wave; deceleration understood as constructive wave interference.

- destructively if the waves are out of phase or equivalently when the trailing is located at a position where the field is accelerating; acceleration understood as destructive wave interference.

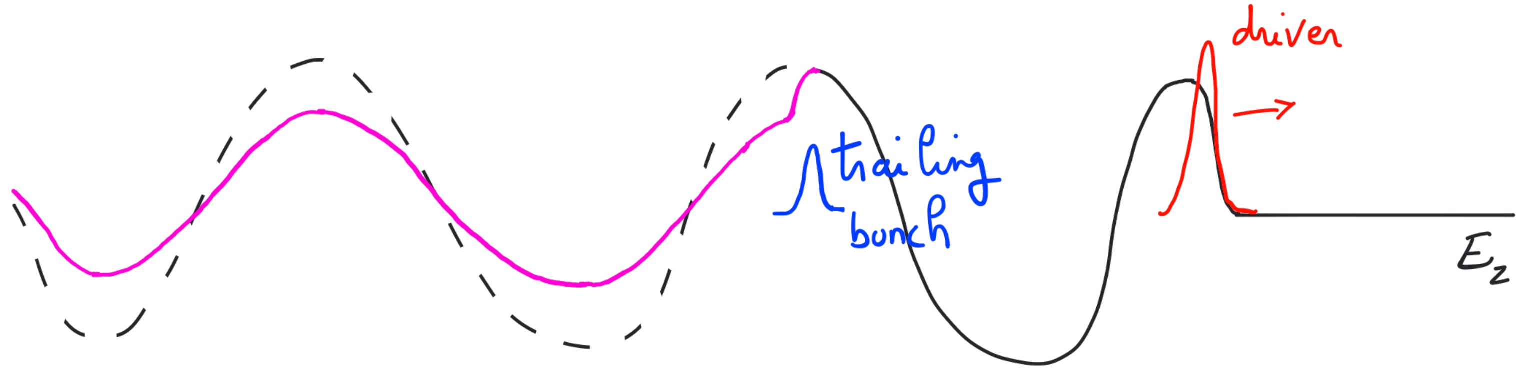


$\Longrightarrow$ Acceleration/deceleration and destructive/constructive interference provide two complementary points of view of energy transfers in linear beam-plasma systems.

Beam loading and energy-transfer efficiency

The 1D case with short bunches:

$$n_d = \sigma_d \delta(\xi - \xi_d)$$
$$n_t = \sigma_t \delta(\xi - \xi_t)$$



Total electric field (superposition):

$$E_z(\xi) = -\frac{q_d \sigma_d}{\epsilon_0} \cos[k_p(\xi - \xi_d)]\Theta(\xi_d - \xi) - \frac{q_t \sigma_t}{\epsilon_0} \cos[k_p(\xi - \xi_t)]\Theta(\xi_t - \xi)$$

Energy-transfer efficiency (from plasma to trailing bunch):

$$\eta_{p\to t} = \frac{W_{\text{gain}}}{W_{\text{loss}}} = \left| \frac{N_t \langle E_z \rangle_t}{N_d \langle E_z \rangle_d} \right| \qquad \eta_{p\to t} = \frac{\int E_d^2 d^2\mathbf{r}_\perp - \int E_{\text{tot}}^2 d^2\mathbf{r}_\perp}{\int E_d^2 d^2\mathbf{r}_\perp}$$

$$\Longrightarrow \quad \eta_{p\to t,\text{1D linear}} = \frac{N_t}{N_d}\left(2 - \frac{N_t}{N_d}\right)$$

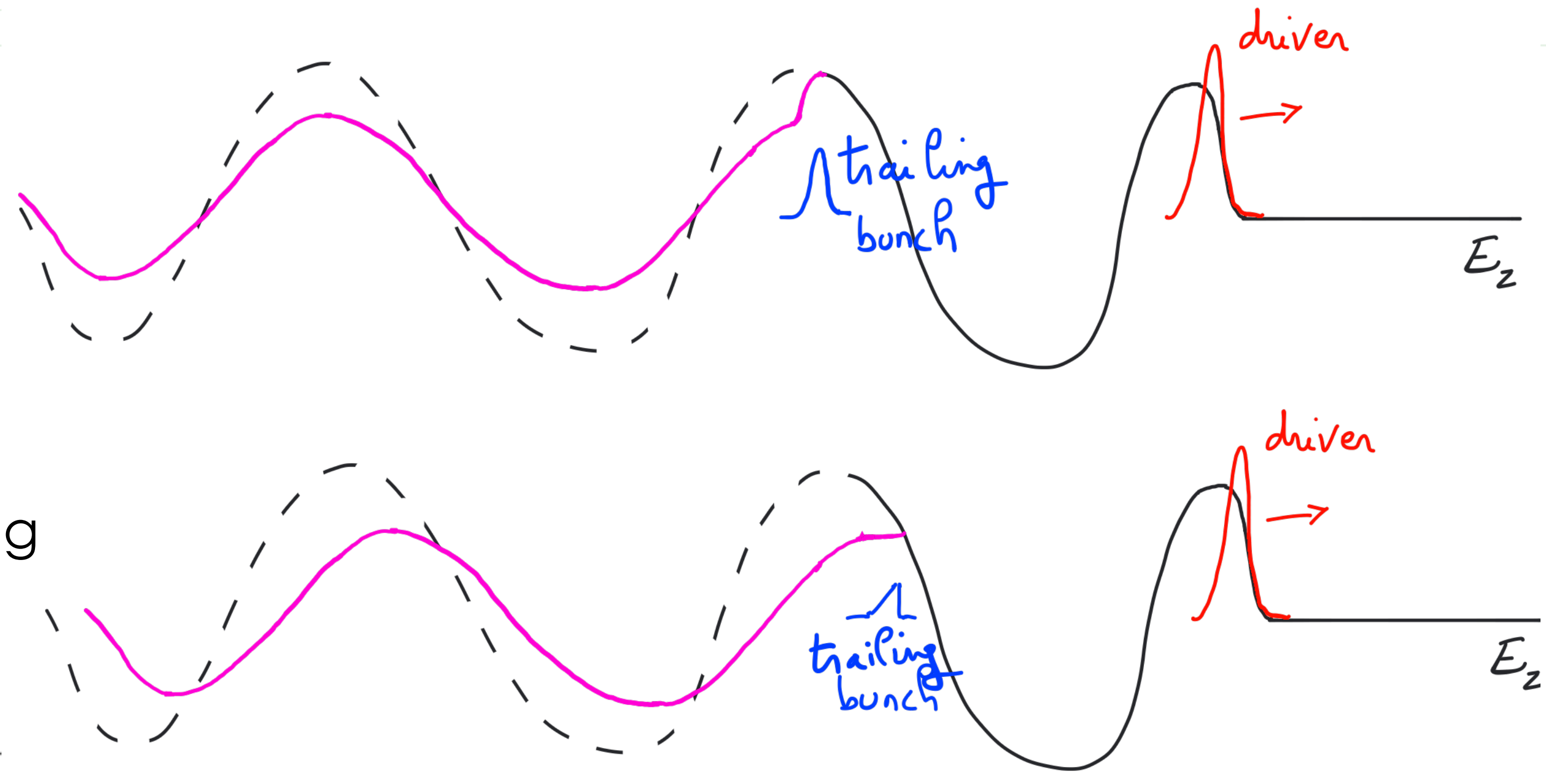
when $\Delta\xi = \lambda_p/2$ for $q_t = q_d$
when $\Delta\xi = \lambda_p$ for $q_t = -q_d$

Required: $\Theta(0) = 1/2$

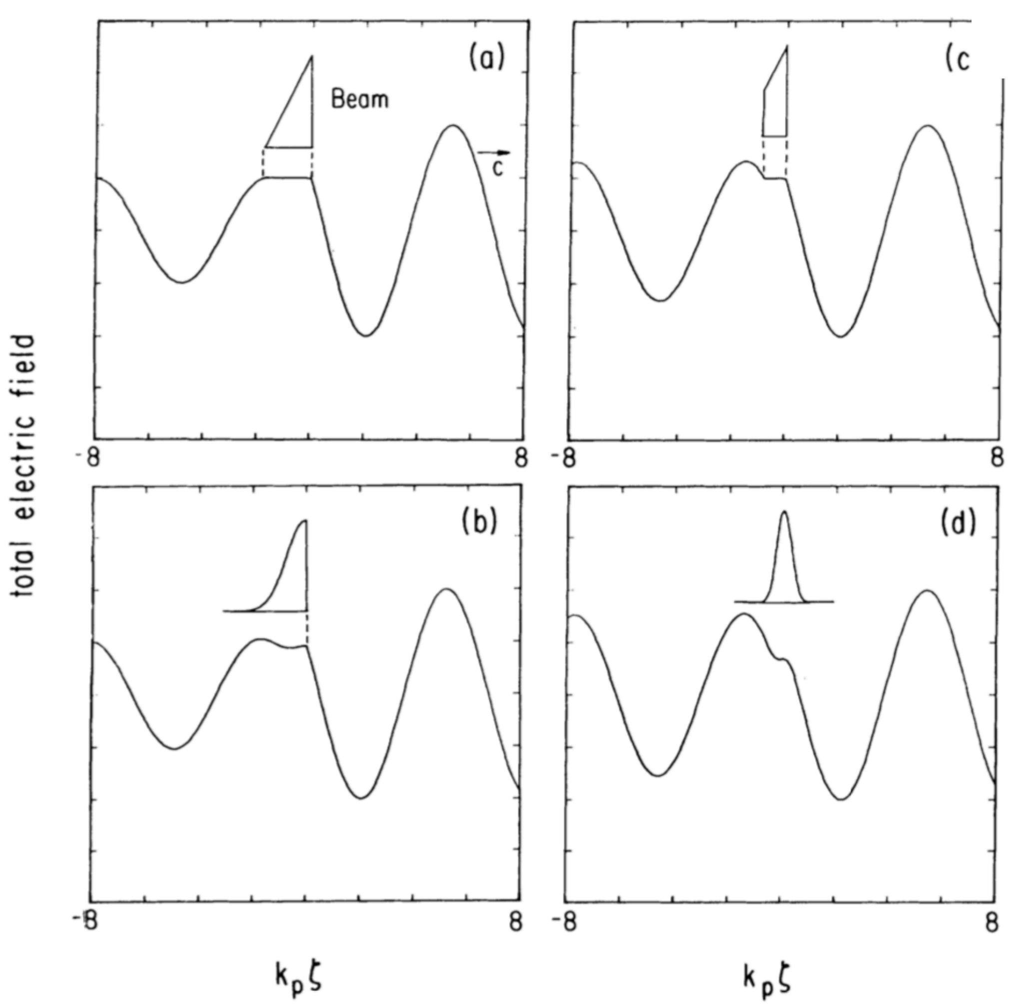
fundamental theorem of beam loading
(otherwise energy conservation is broken)

Beam loading and energy-transfer efficiency

<u>Optimized beam loading (linear):</u>



Uniform E_z field experienced by trailing bunch: low final energy spread.



[Katsouleas et al., Part. Accel. 22, 81 (1987)]

Perfect solution is a triangular current profile.

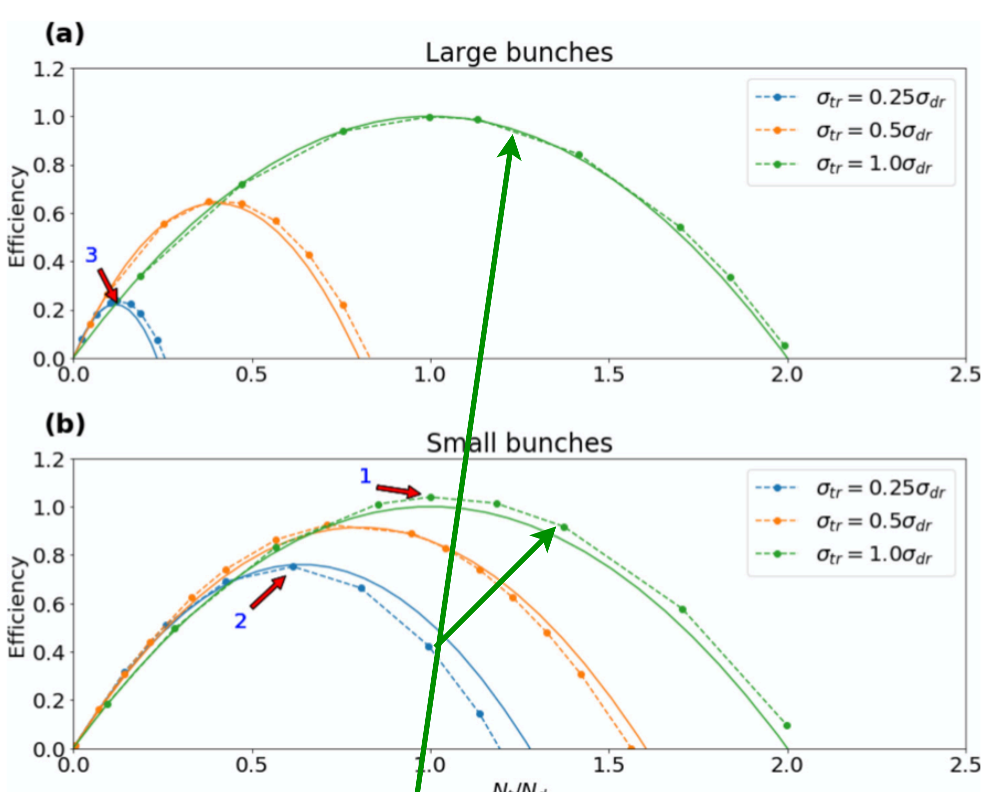
Gaussian profiles can only flatten approximately, but with reasonable residual energy spread.

$\Longrightarrow$ Beam loading is a key process for both efficiency and energy spread

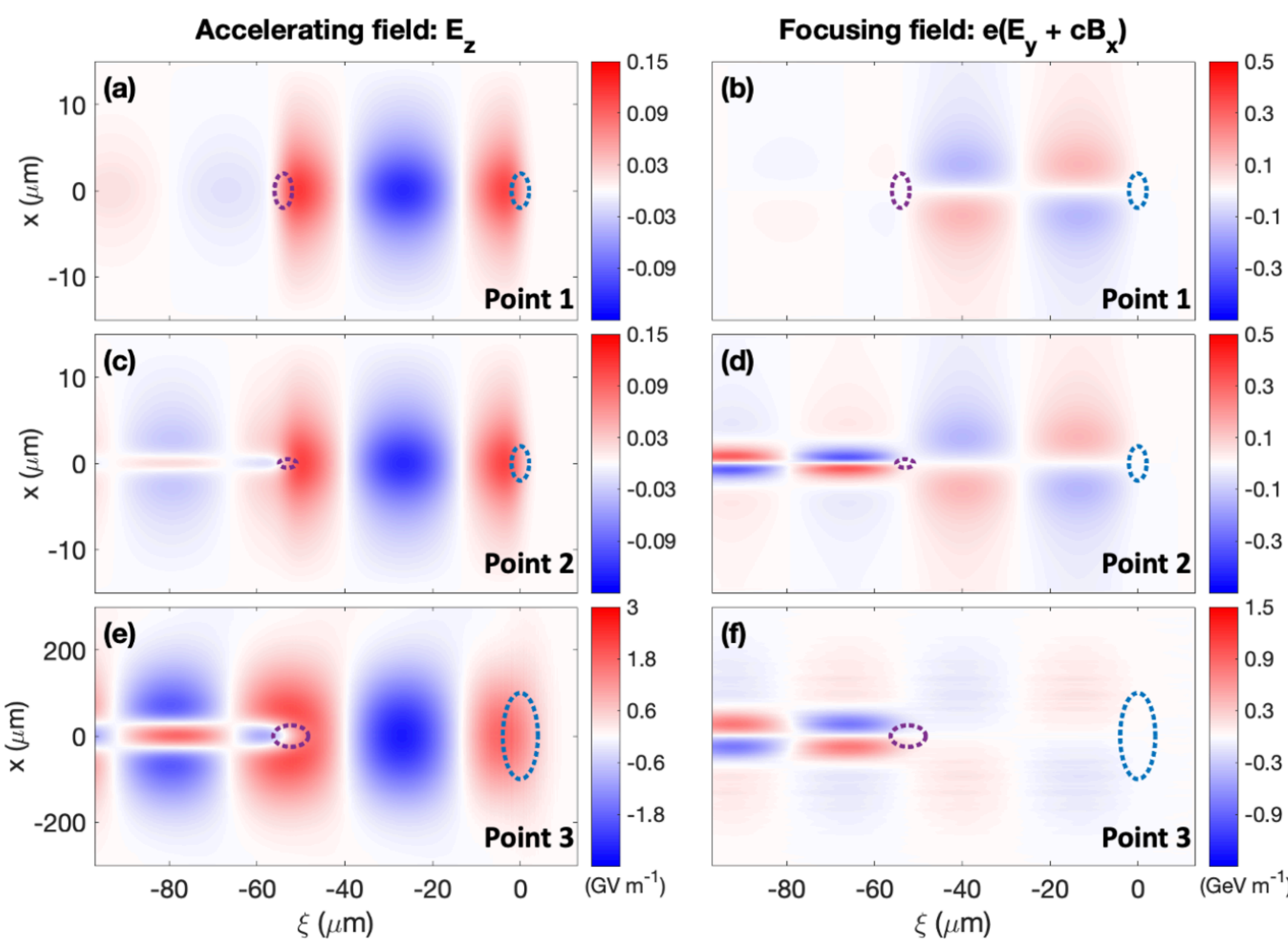
Beam loading and energy-transfer efficiency

Linear 3D case:

- Same shape for drive and trailing bunches: linear 3D = linear 1D.
- Highest efficiency: smallest fields left behind the trailing.
- Different shape for drive and trailing bunches: small beams are better because the fields extend over a plasma skin depth regardless of beam size



$$\eta_{p\to t,\mathrm{1D\ linear}} = \frac{N_t}{N_d}\left(2 - \frac{N_t}{N_d}\right)$$



Hue et al., PRR 3, 043063 (2021)

moving to nonlinear wakes…

Beam loading and energy-transfer efficiency

Nonlinear 3D case:

- Beam loading also describes how the wakefield is modified by the presence of the main beam
- But the principle of superposition doesn't hold.

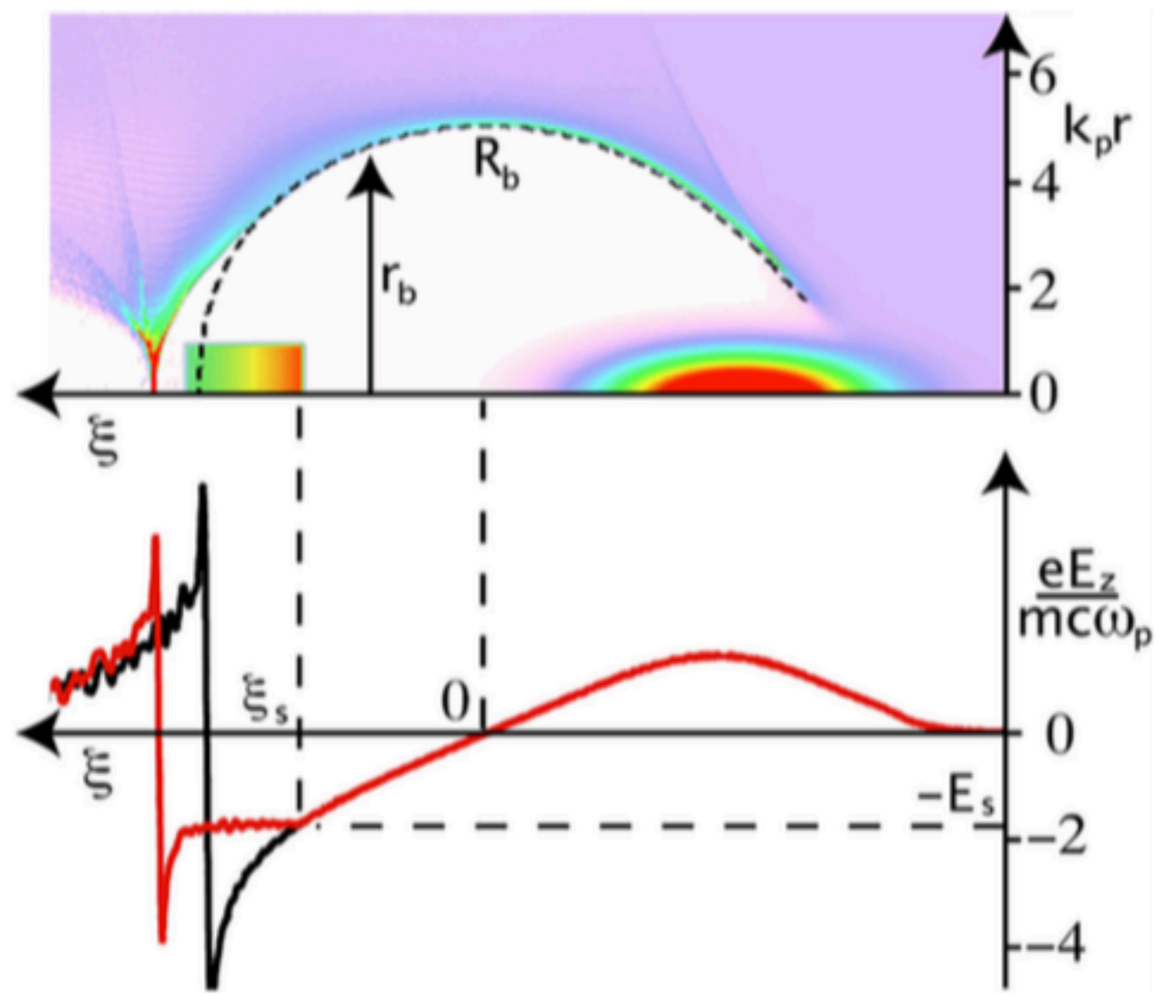
Electron-driven blowout regime:

- Sheath-field model: trailing bunch acts on sheath electron trajectory (repulsive action), which then modifies the longitudinal electric field

$$r_b \frac{d^2 r_b}{d\xi^2} + 2\left[\frac{dr_b}{d\xi}\right]^2 + 1 = \frac{4\lambda(\xi)}{k_p^2 r_b^2}$$

$$\lambda(\xi) = \lambda_d(\xi) + \lambda_t(\xi)$$

- Optimized beam loading: also for triangular current profile. Gaussian provides only approximate flattening, with residual energy spread.



Tzoufras et al., PRL 101, 145002 (2008)

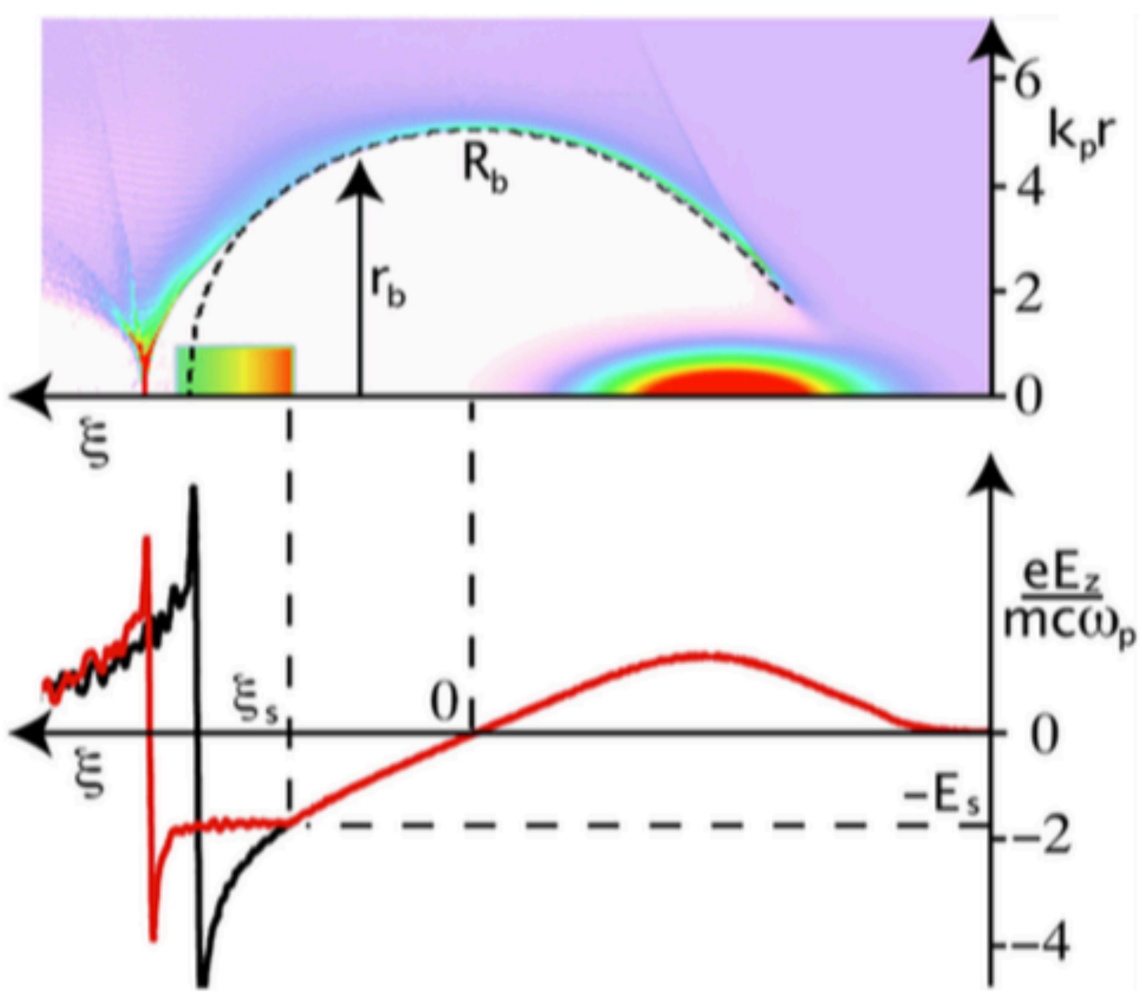
Beam loading and energy-transfer efficiency

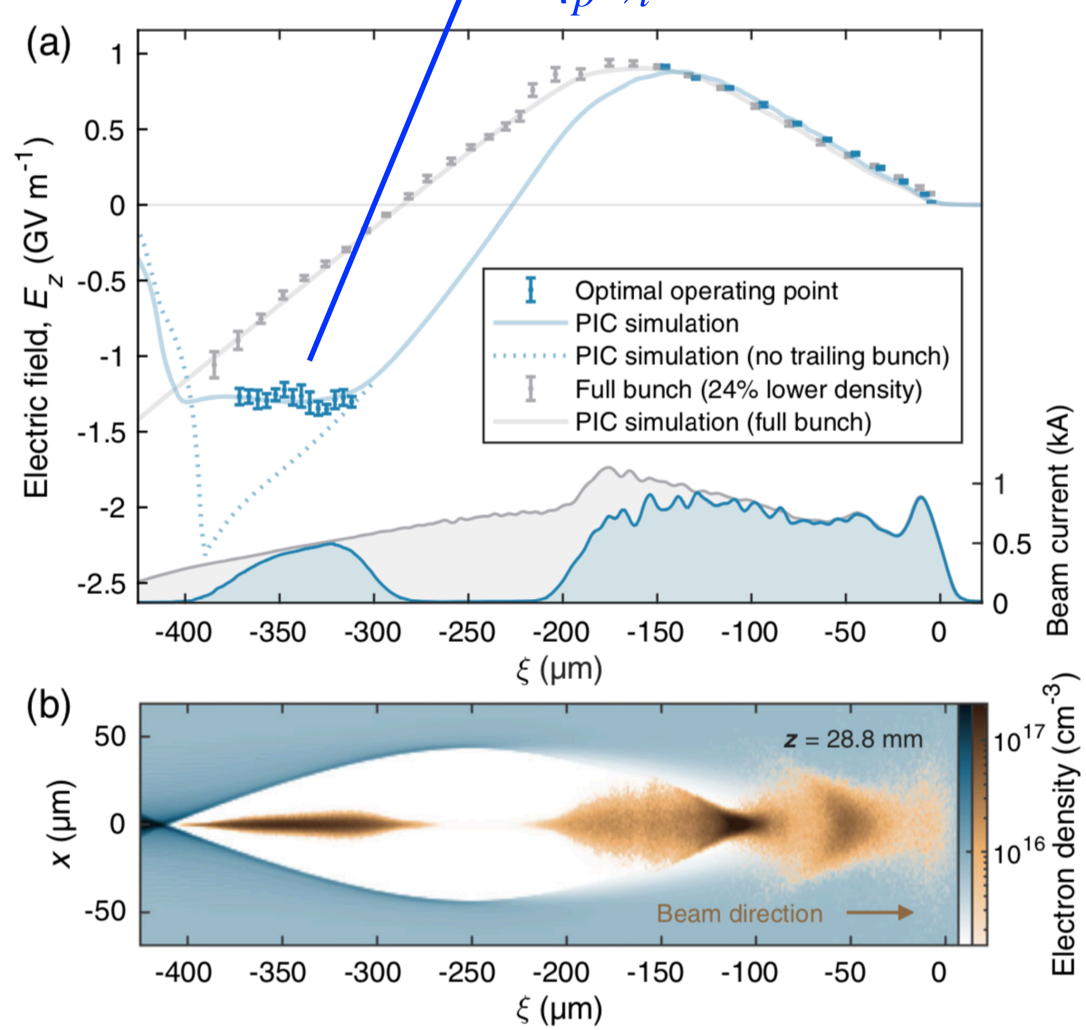
Electron-driven blowout regime:

experimental evidence that beam loading provides simultaneously high efficiency and low energy spread

field variation $= 2.8\,\%$

$\eta_{p\to t} = 42\,\%$





Tzoufras et al., PRL 101, 145002 (2008)

Lindstrøm et al., PRL 126, 014801 (2021)

Advanced concepts for particle beam evolution in plasma

Head erosion of drive beam

Electron-driven blowout regime:

allow to guide the drive beam over a distance much larger than β^*

- From first PWFA lecture, in blowout we have:

$$F_x = -gx \qquad g = \frac{1}{2} m_e \omega_p^2 \qquad \boxed{\frac{d^2\sigma_x}{dz^2} = -k_\beta^2 \sigma_x + \frac{\varepsilon^2}{\sigma_x^3}} \qquad k_\beta = k_p / \sqrt{2\gamma}$$

- True for all drive particles? for most, but not all

- At the very head of the drive beam: no focusing force because the plasma wakefield hasn't been established or self-ionization hasn't occurred yet.

- Beam head expands due to its finite emittance, and becomes effectively lost when too large to contribute to the plasma wakefield excitation or ionization.

- Once a beam segment is lost through this process, a new one becomes the head and suffers the same fate, leading to a continuous erosion of the beam with an erosion front moving backward along the beam.

$\Longrightarrow$ emittance-driven head erosion

Head erosion of drive beam

Head erosion in pre-ionized plasma:

- Not exactly true for pre-ionized plasma: head erosion is important during a relaxation distance after which a quasi-equilibrium is established even near the beam head and erosion occurs at a much slower rate

- Why?

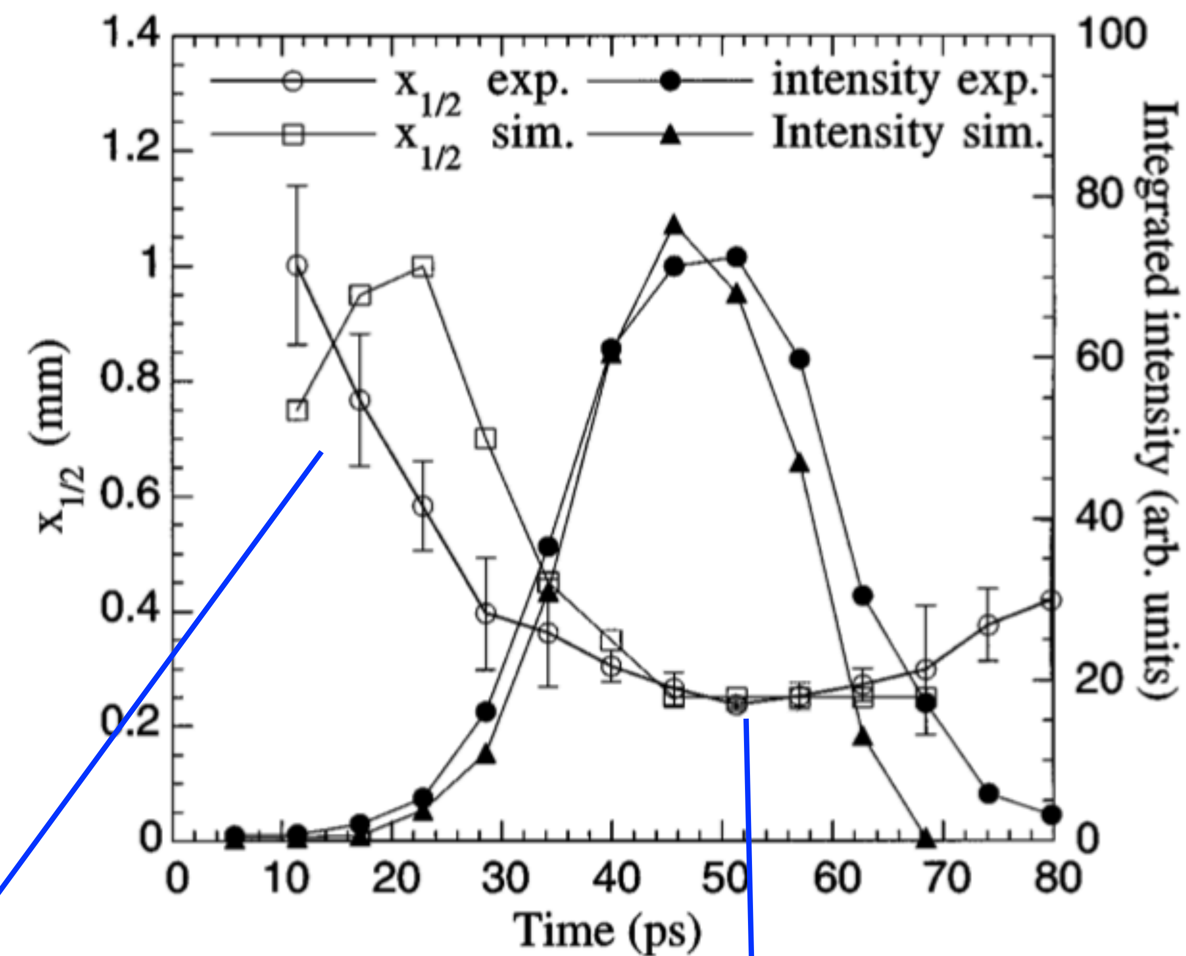
when a new segment becomes the head, it evolves adiabatically from a matched state

adiabaticity ensures it stays matched with $\beta = 1/k_\beta$, that becomes larger as the focusing is decreased.

the new head has a much larger β than the original head

strong reduction of the radial expansion of the head and of the erosion rate

Barov et al., PRL 80, 81 (1998)



trumpet-shaped head with a size increasing towards the head

matched equilibrium body with constant size

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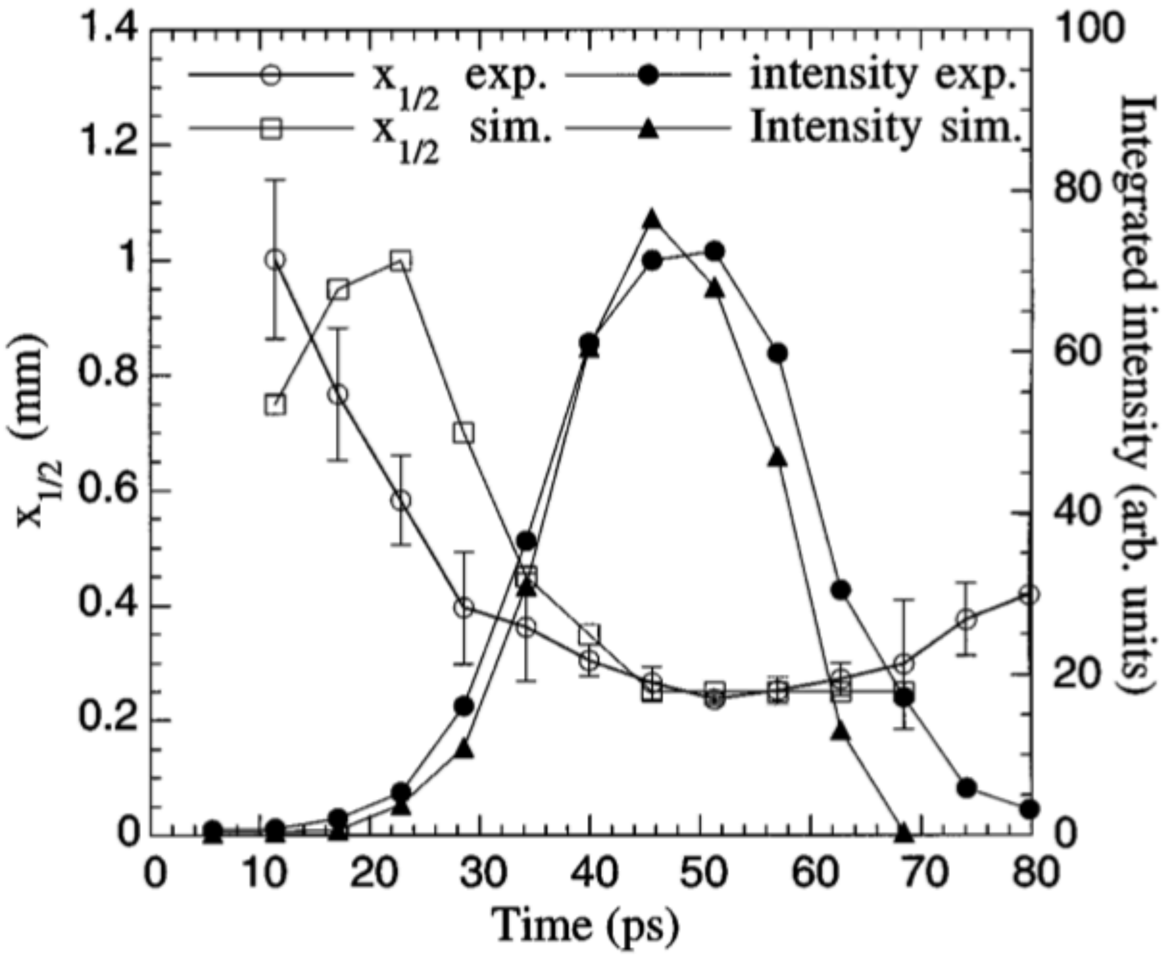


Barov et al.: demonstration of near-steady-state propagation in second half of plasma (relaxation during first half).

Head erosion of drive beam

Head erosion in self-ionized plasma:

- Head erosion is exacerbated when the plasma is field-ionized by the beam, because ionization leads to a sudden transition between guided and vacuum propagation, and starts off axis.
- Model for matched beam:

$$V_{\text{erosion}} = \frac{\Delta\xi}{\Delta z}$$

$\Delta\xi$ being the length of head segment being lost
Δz being the distance over which it is lost

$\Delta\xi$ is the ramp length over which the focusing force is established:

$$\Delta\xi \sim \frac{1}{k_b} = \frac{c}{\sqrt{n_b e^2/m_e\epsilon_0}} = \sqrt{\frac{n_0}{n_b}}\frac{1}{k_p} \propto \frac{\sigma_{\text{matched}}}{\sqrt{I}}$$

$\Delta z \sim$ distance from σ_{matched} to $\sigma_{\text{ionization}} \sim \beta^* \left(\frac{\sigma^2_{\text{ionization}}}{\sigma^2_{\text{matched}}} - 1\right)^{1/2} \sim \frac{\beta^*\sigma_{\text{ionization}}}{\sigma_{\text{matched}}}$

$$\sigma_{\text{ionization}} \propto \frac{I}{\epsilon_i^{1.73}} \implies V_{\text{erosion}} \propto \frac{\sigma^2_{\text{matched}}\epsilon_i^{1.73}}{\beta^* I^{3/2}} \boxed{\propto \frac{\epsilon_n \epsilon_i^{1.73}}{\gamma I^{3/2}}} = 3.7\ 10^4 \frac{\epsilon_n[\text{mm . mrad}]\ \epsilon_i^{1.73}[\text{eV}]}{\gamma I^{3/2}[\text{kA}]}$$

ionisation energy

empirical from simulations

empirical from simulations

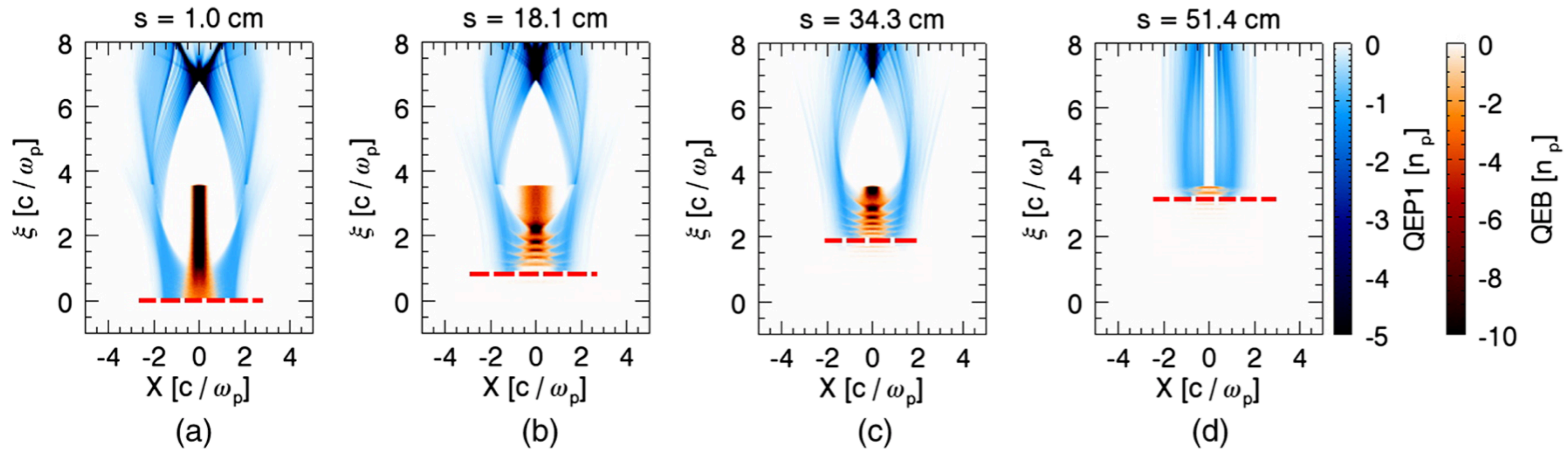
Blumenfeld, Ph.D. thesis (Stanford University), 2009

Head erosion of drive beam

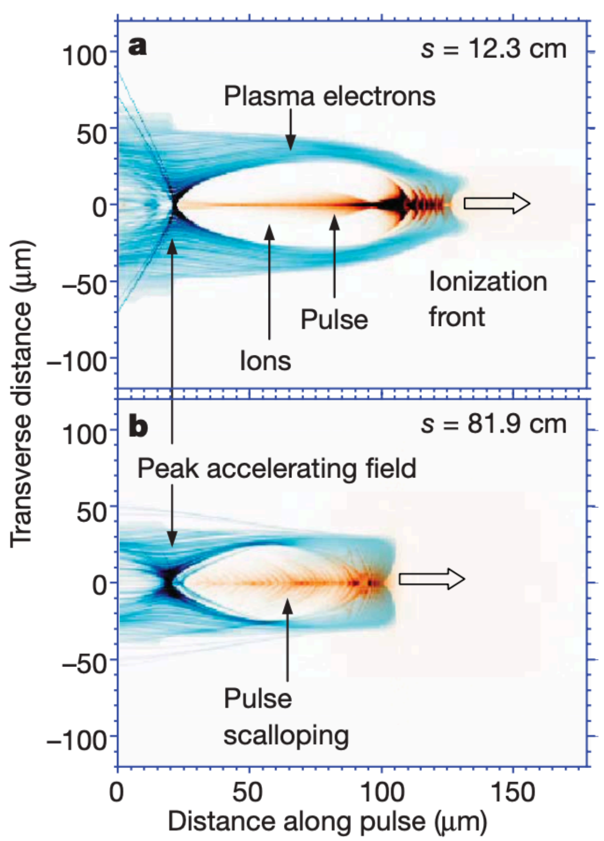
Head erosion in self-ionized lithium plasma:

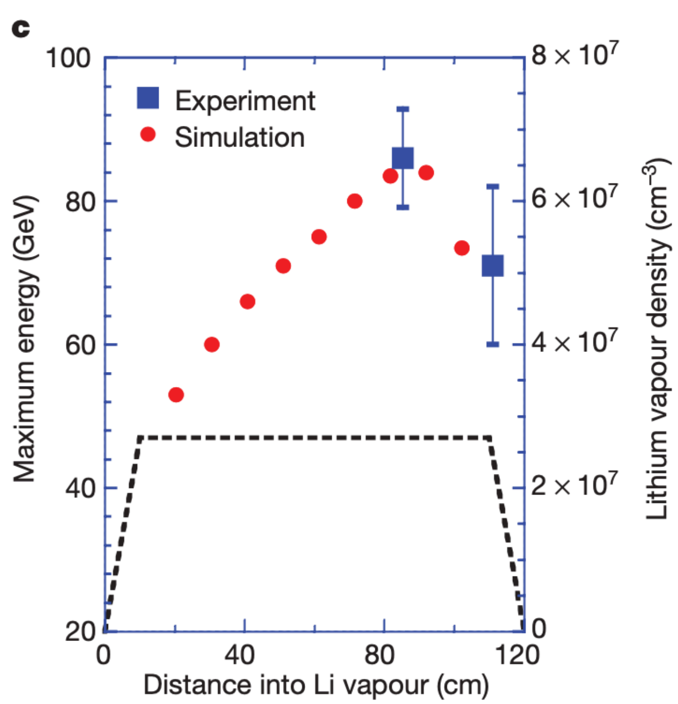
- Simulation with clearly visible head erosion

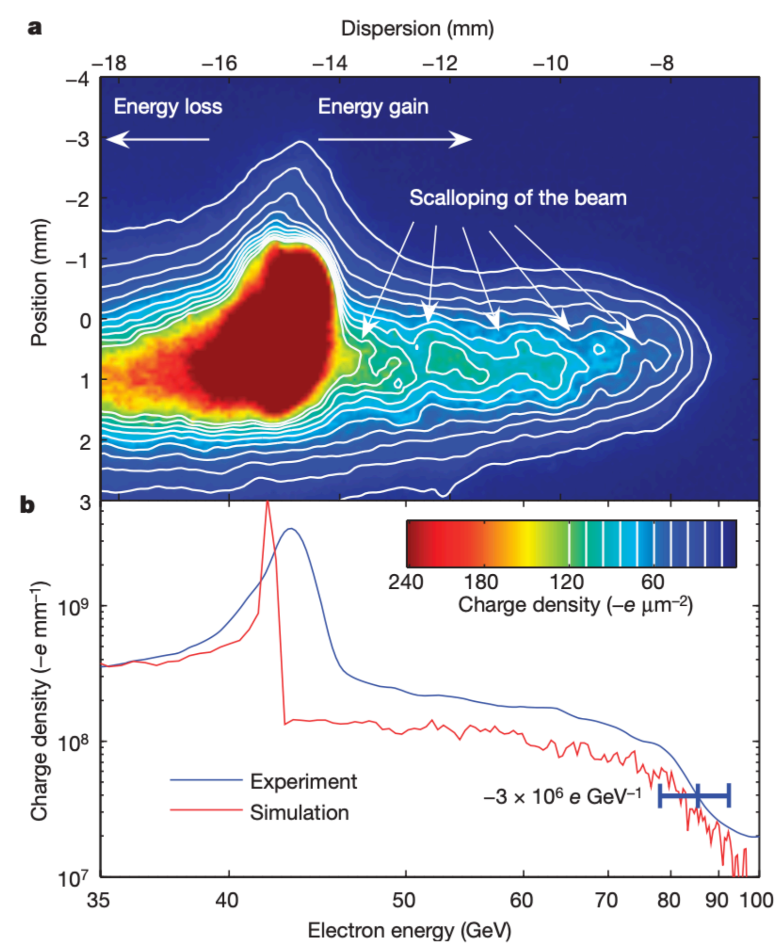
An et al., PRAB 16, 101301 (2013)



- Head erosion as a limit for PWFA experiments







Blumenfeld et al., Nature 16, 101301 (2007)

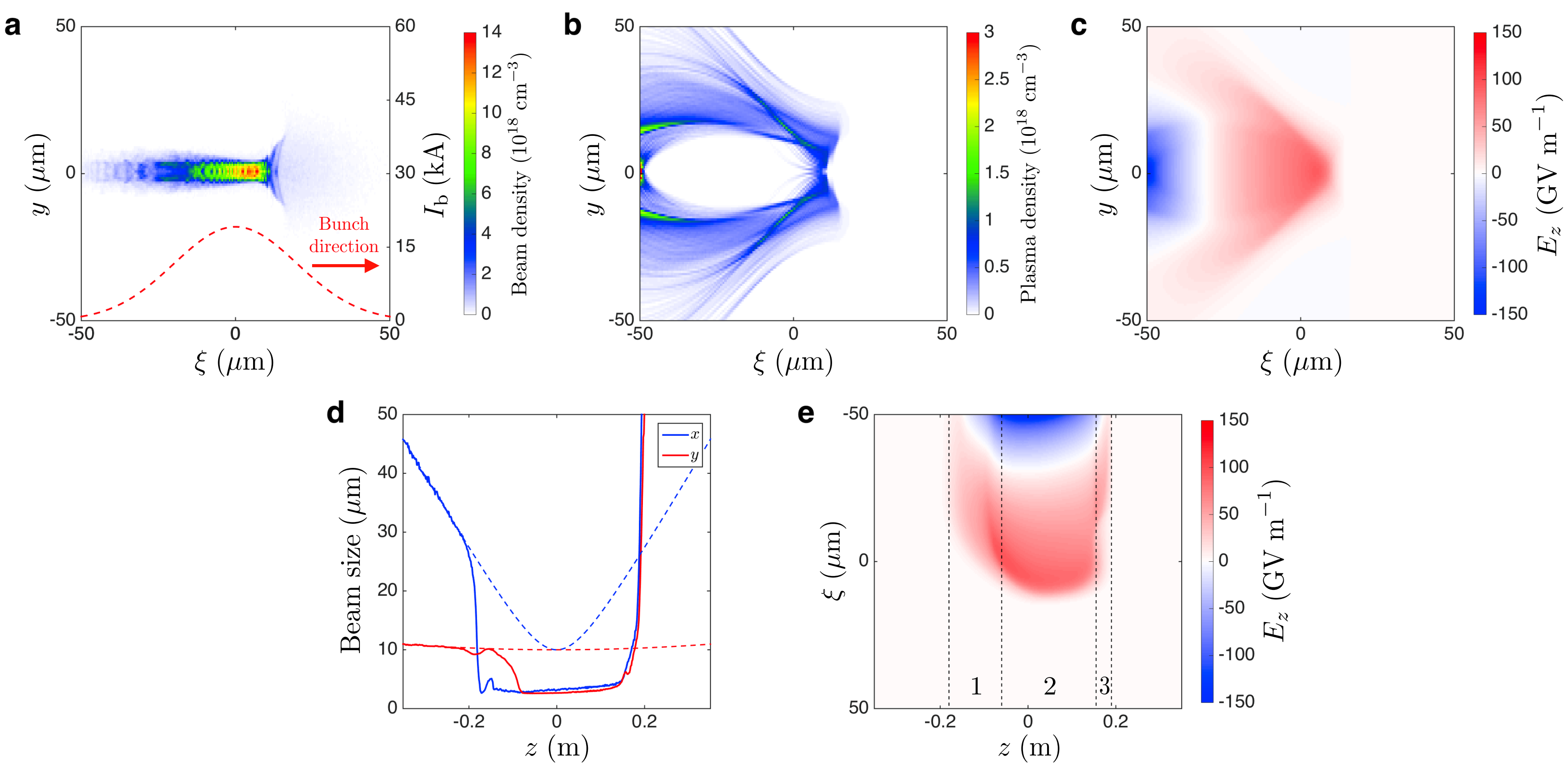
Head erosion of drive beam

<u>Head erosion in self-ionized plasma with mismatched drive beam:</u>

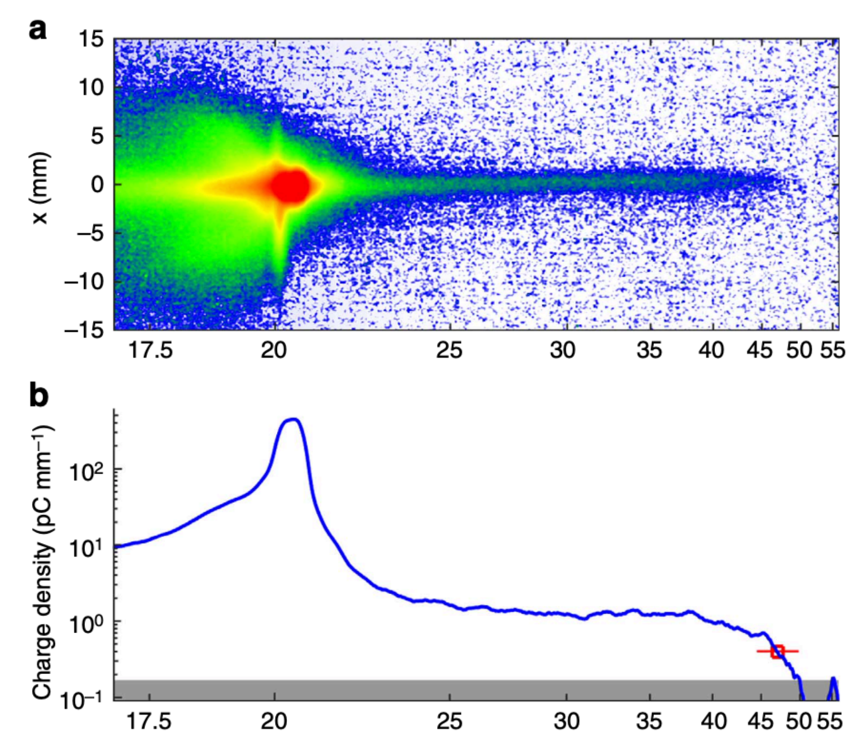
- Mistmatched 20 GeV electron beam into Ar gas

- Standard reasoning: emittance growth due to mismatch leading to very fast head erosion, in the cm scale

simulation



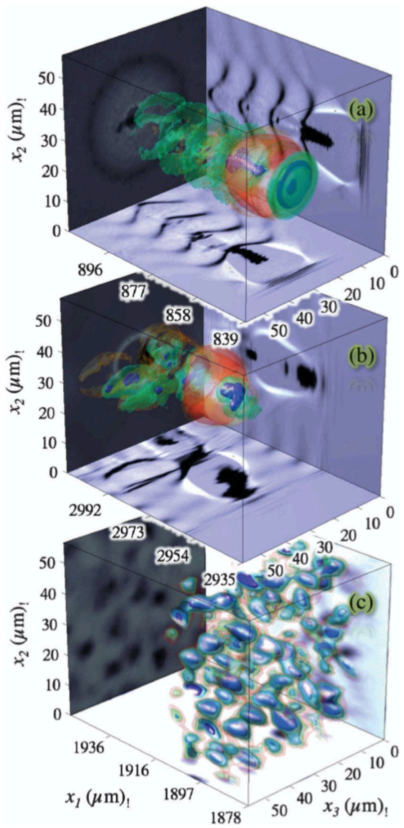
experiment



[Corde et al., Nat Comm 7, 11898 (2016)] moving to instabilities, ion motion and emittance of trailing beam…

Instabilities

Large class of instabilities in beam-plasma systems:



current filamentation instability

Huntington et al., PRL 106, 105001 (2011)

$$k_p\sigma_r \gg 1$$

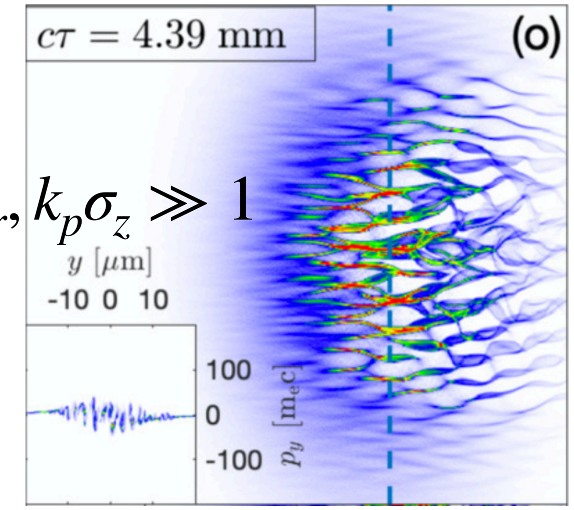
$$k_p\sigma_z < 1 \text{ or } k_p\sigma_z \gg 1$$

$$\downarrow$$

$$\left[\partial_\tau^2 - \Gamma_{\mathrm{CFI}}^2\right]\delta\tilde{n}_b = 0$$

$$\delta\tilde{n}_b \propto \exp(\Gamma_{\mathrm{CFI}}\tau)$$

$$k_p\sigma_r, k_p\sigma_z \gg 1$$



oblique two-stream instability

San Miguel Claveria et al., PRR 4, 023085 (2022)

$$\left(\partial_\tau^3 + v_b\partial_\tau^2\partial_\xi + \frac{8i}{3^{3/2}}\Gamma_{\mathrm{OTSI}}^3\right)\delta\tilde{n}_p = 0$$

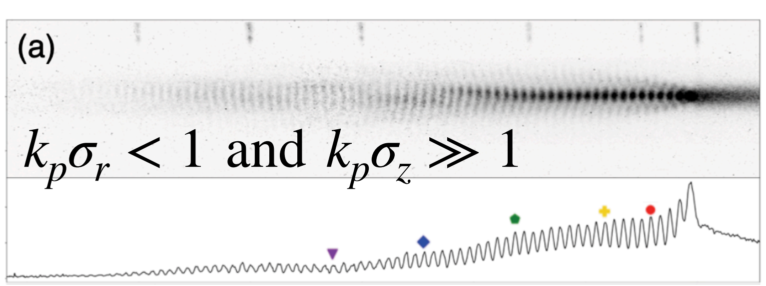
$$\delta\tilde{n}_p \propto \exp(a\xi^{1/3}\tau^{2/3})$$

self-modulation instability

Schroeder et al., PRL 107, 145002 (2011)

Pukhov et al., PRL 107, 145003 (2011)

$$\left(\partial_\tau^2\partial_\xi + i\nu\right)\tilde{r} = 0 \qquad \tilde{r} \propto \exp(a\xi^{1/3}\tau^{2/3})$$



$$k_p\sigma_r < 1 \text{ and } k_p\sigma_z \gg 1$$

Braunmüller et al., PRL 125, 264801 (2020)

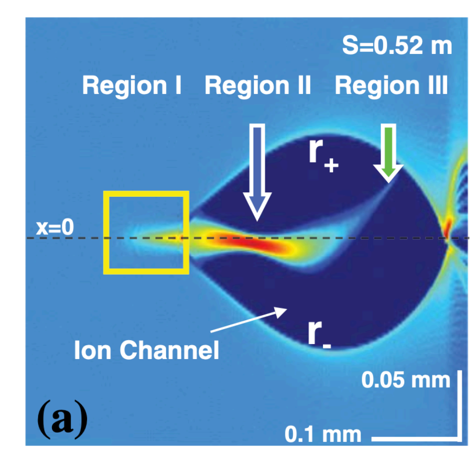
hosing/beam break-up instability

$$k_p\sigma_r, k_p\sigma_z < 1$$

$$(\partial_\tau^2 + \omega_\beta^2)x_b = \omega_\beta^2 x_c$$

$$(\partial_\xi^2 + c_r c_\psi k_p^2/2)x_c = c_r c_\psi k_p^2 x_b/2$$

$$\tilde{x}_b \propto \exp(a\xi^{2/3}\tau^{1/3})$$

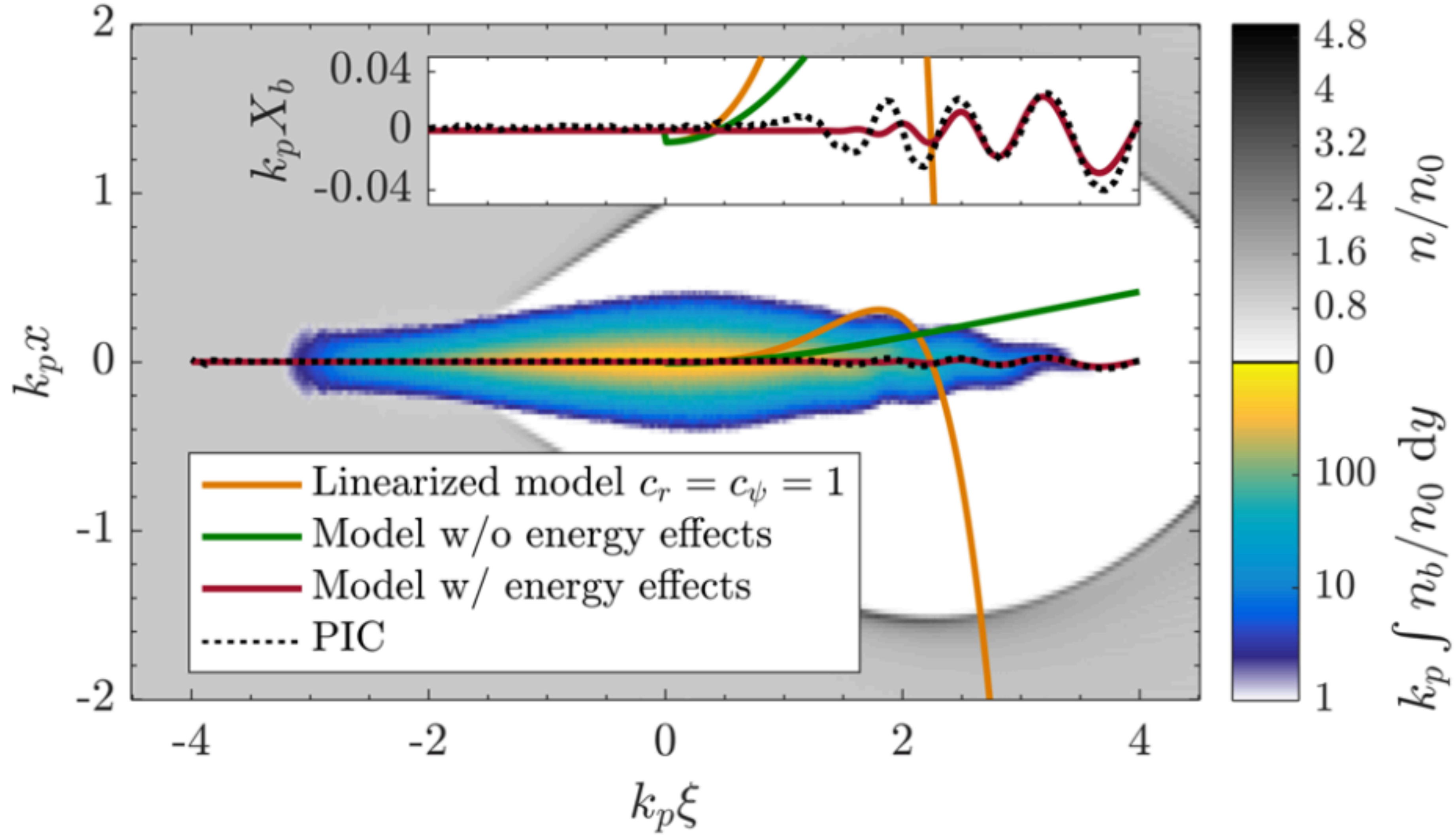


Huang et al., PRL 99, 255001 (2007)

Hosing

Hosing of drive beam:

Mehrling et al., PRL 118, 174801 (2017)



- Easily mitigated, by energy chirp or more simply from natural deceleration; works by detuning betatron oscillations of different slices
- Similar detuning principle as other mitigations: e.g. in linear regime that presents a head-to-tail variation of focusing force [Lehe et al., PRL 119, 244801 (2017)] or using wide drive beams [Martinez de la Ossa et al., PRL 121, 064803 (2018)]
- Also by slice energy spread, reducing betatron oscillations by decoherence

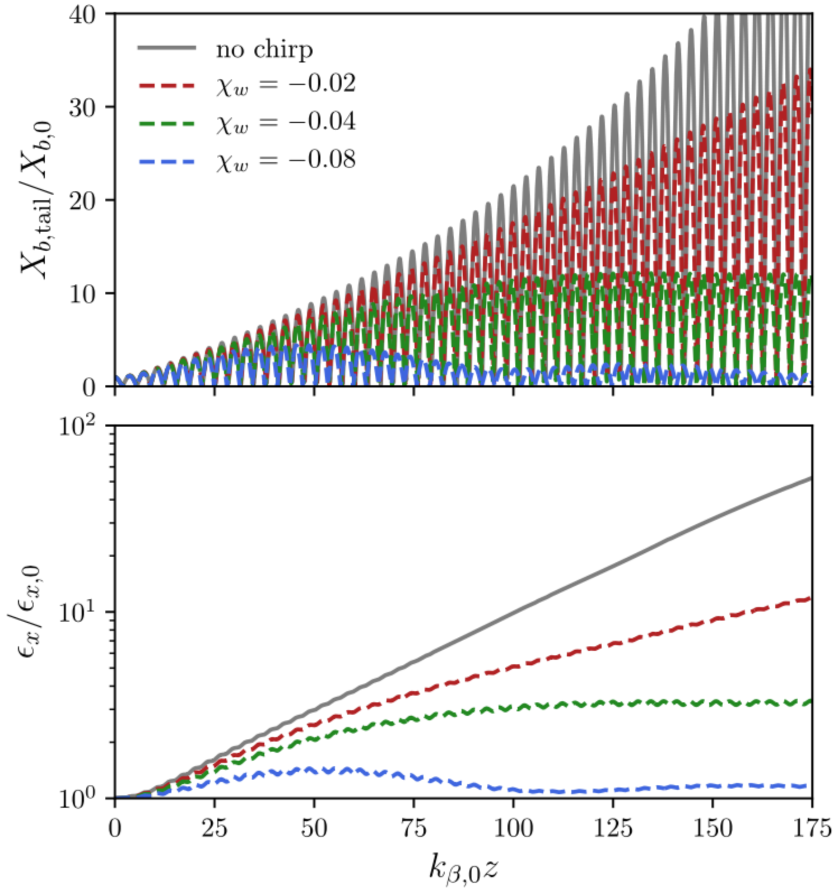
Hosing

Hosing/beam break-up of trailing bunch:

- Much more challenging, may be a critical limitation! Efficiency vs instability?

- Energy chirp can help:

realistic?



Mehrling et al., PRAB 22, 031302 (2019)

$$\frac{\text{wake-deflecting force}}{\text{focusing force}} \simeq \frac{\eta_{p\to t}^2}{4(1-\eta_{p\to t})}$$

Lebedev et al., PRAB 20, 121301 (2017)

- Use quasilinear regime with head-to-tail variation of focusing force [Lehe et al., PRL 119, 244801 (2017)]

- Ion-motion induced head-to-tail decoherence?

Ion motion

When ion motion should be accounted for?

$$k_{b,i} = \frac{1}{c}\sqrt{\frac{n_b e^2}{m_i \epsilon_0}} = \sqrt{\frac{n_b}{n_0}}\sqrt{\frac{m_e}{m_i}}k_p$$

$1/k_{b,i}$ is the typical scale in ξ over which ions are pinched on axis

Ion motion is important if: $k_{b,i}\sigma_{z,t} \gtrsim 1$

Or simpler, for $k_p\sigma_{z,t} \sim 1$, ion motion is important if: $\boxed{\frac{n_b}{n_0} \gtrsim \frac{m_i}{m_e}}$

Ion motion

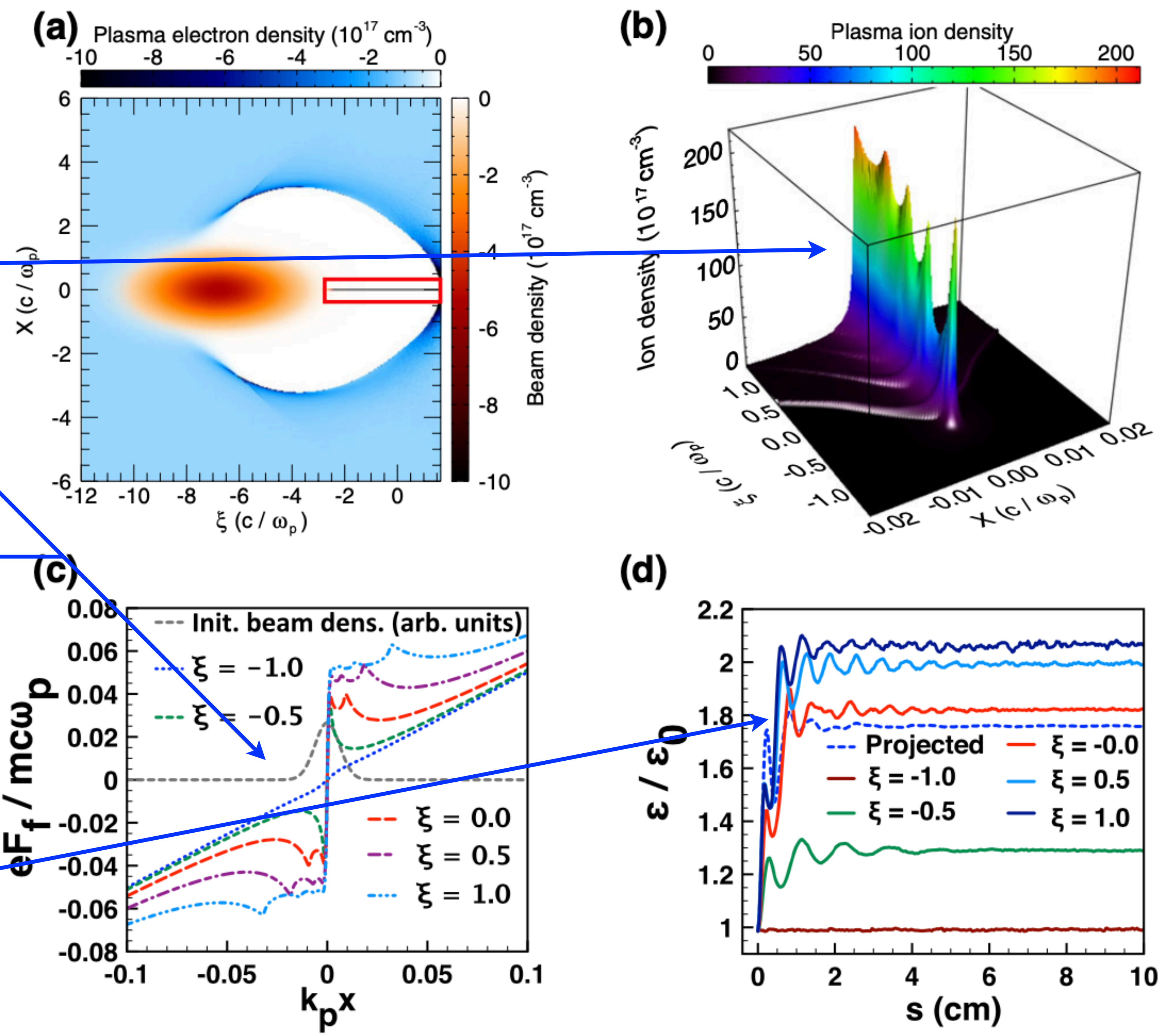
<u>How does ion motion look?</u>

Ion density spikes on axis

Nonlinear focusing force

Head-to-tail variation of focusing force

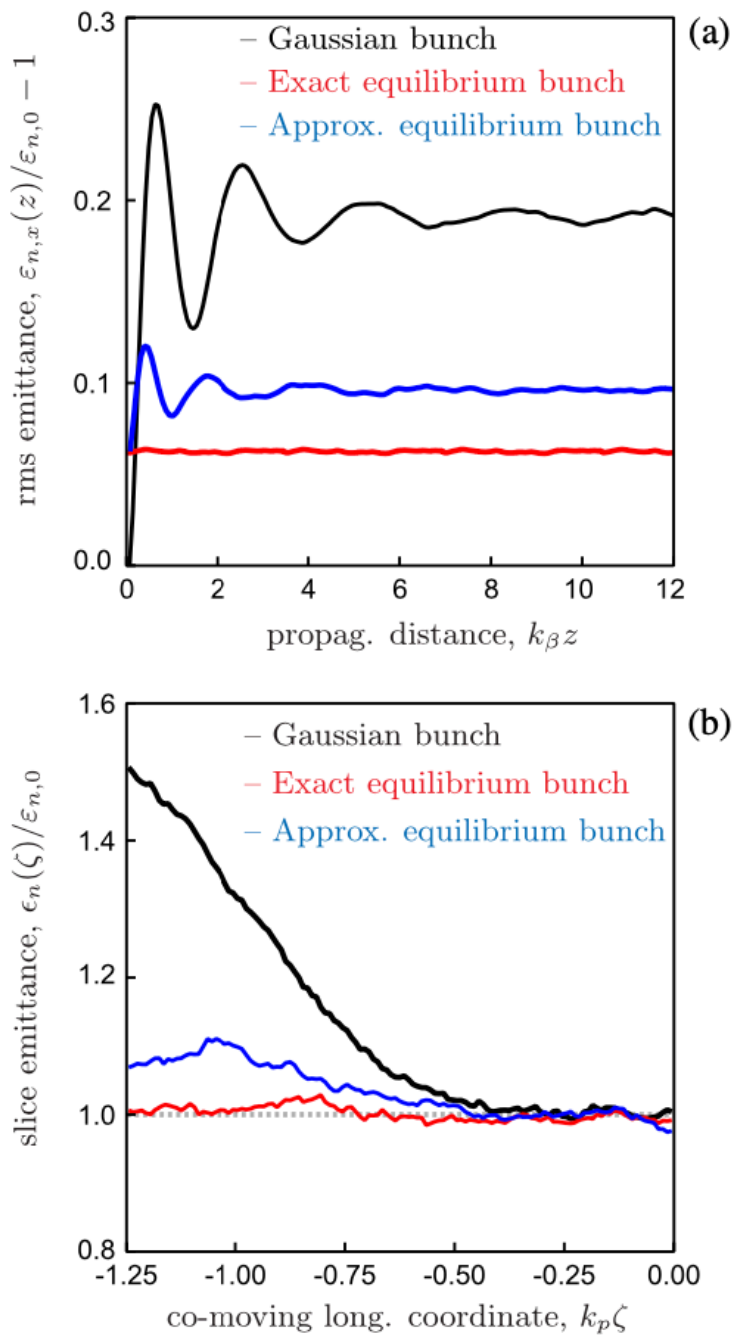
Some emittance growth

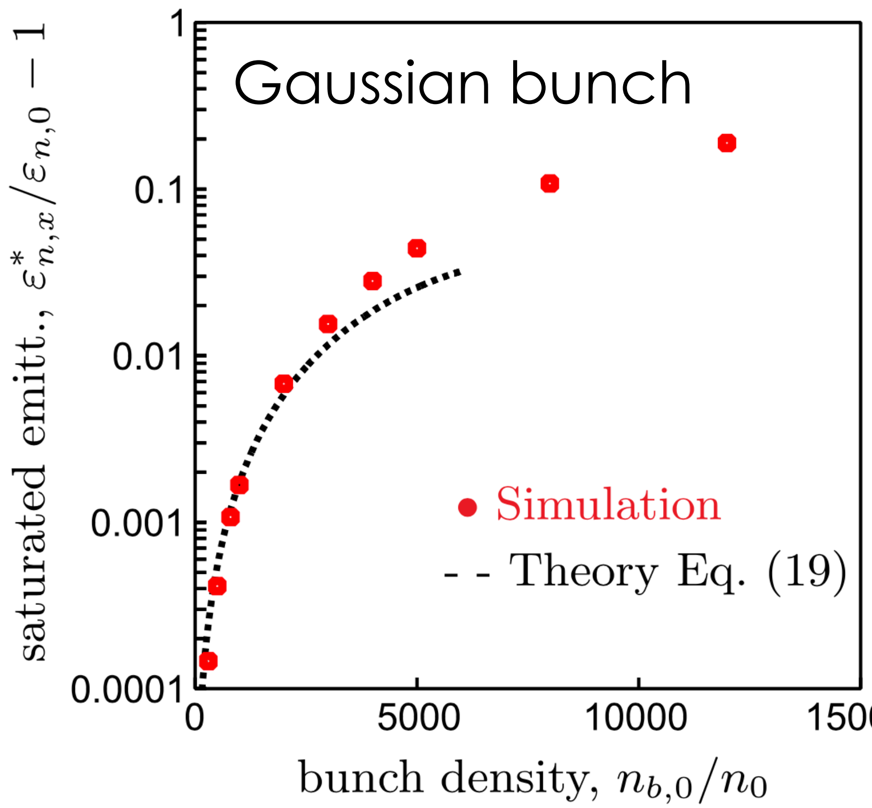


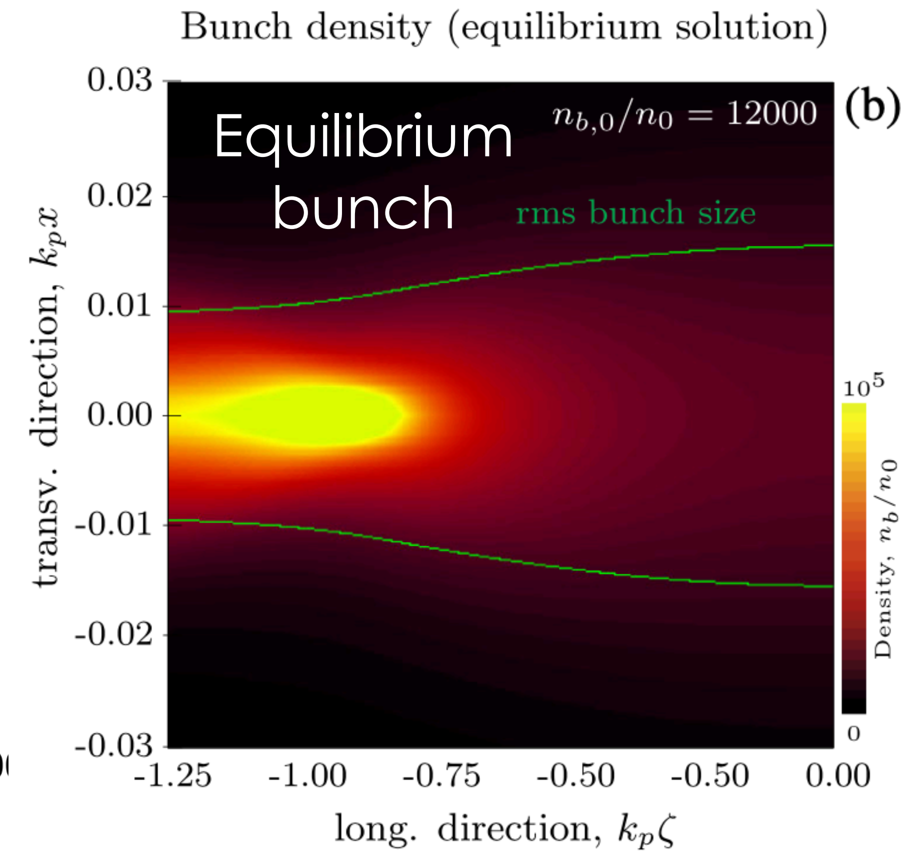
[An et al., PRL 118, 244801 (2017)](#)

Ion motion

Can it be optimized?







Head-to-tail shaping can considerably reduce emittance growth.

Beam size smaller at the rear than the front

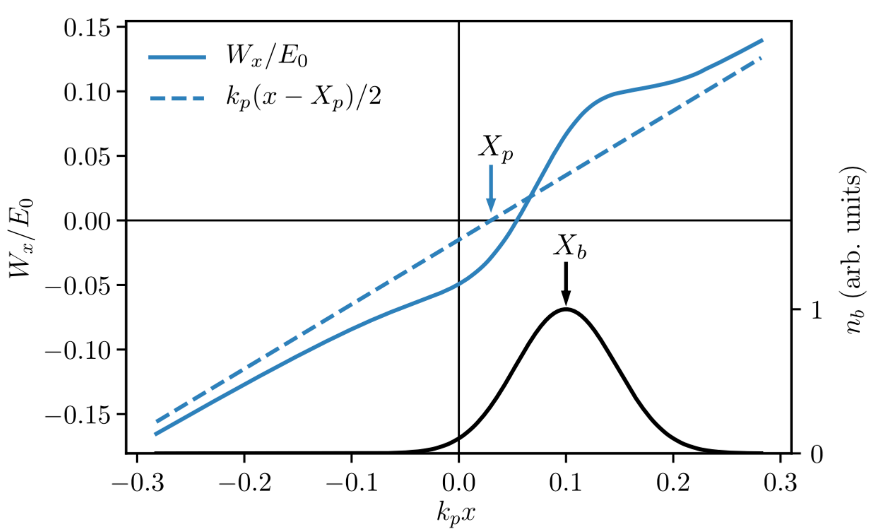
[Benedetti et al., PRAB 20, 111301 (2017)]

Hosing and ion motion

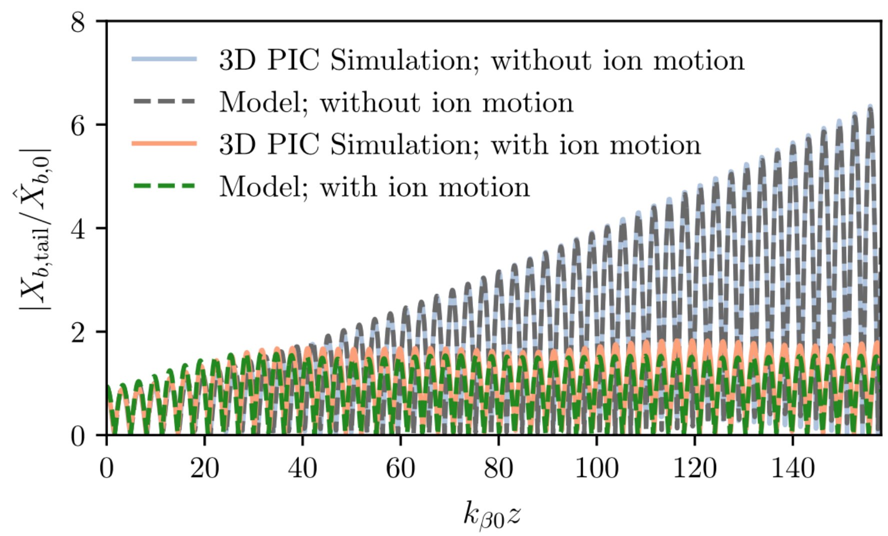
PHYSICAL REVIEW LETTERS **121,** 264802 (2018)

Suppression of Beam Hosing in Plasma Accelerators with Ion Motion

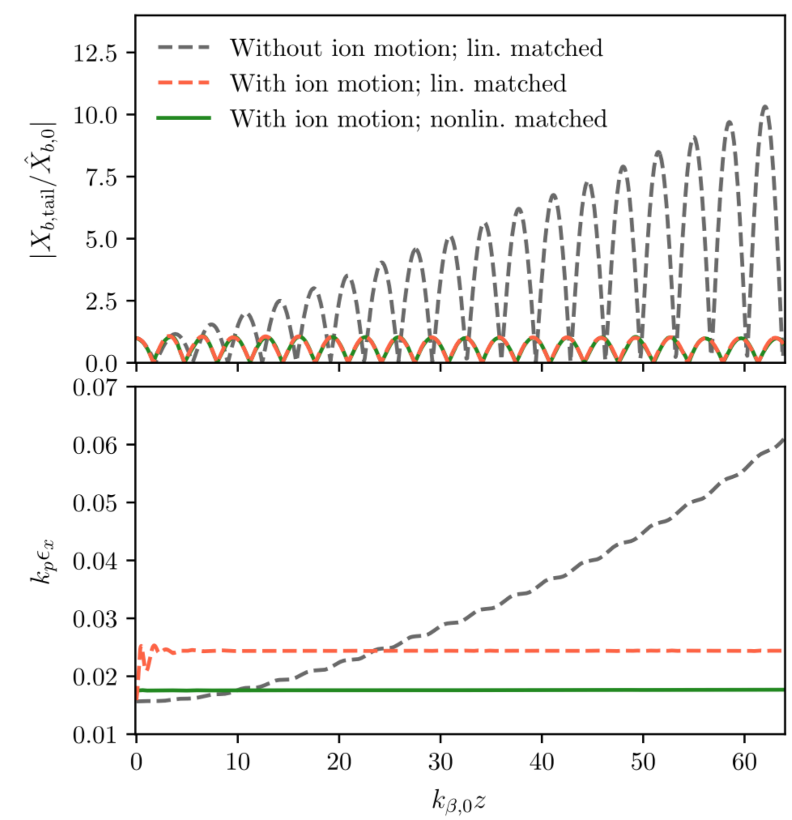
T. J. Mehrling,* C. Benedetti, C. B. Schroeder, E. Esarey, and W. P. Leemans
Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA



Transverse force better centered on beam with ion motion



ion motion induces head-to-tail variation of focusing force, suppressing hosing



promising for beam emittance

emittance can be preserved by performing a slice-by-slice matching of the transverse beam distribution to the nonlinear focusing force

Mehrling et al., PRL 121, 264802 (2018)

Other considerations for trailing emittance

mismatch

already discussed, extremely important otherwise:

$$\frac{\epsilon_{\text{sat}}}{\epsilon_0} = \frac{1}{2}\left(\beta_m \frac{1+\alpha^2}{\beta} + \frac{\beta}{\beta_m}\right)$$

Mehrling et al., PRAB 15, 111303 (2012)

Coulomb scattering

$$\frac{d\epsilon_n}{ds} \propto \frac{\beta}{\gamma} n_i Z_i^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

$$\Delta\epsilon_n \sim \sqrt{\gamma_{\text{final}}} - \sqrt{\gamma_{\text{initial}}}$$

Schroeder et al., PRAB 13, 101301 (2010)

staging

need to match and couple in and out of plasma for emittance preservation

challenge: achromatic transport, otherwise chromatic emittance growth

interstage distance is getting large at high energies

Lindstrøm, PRAB 24, 014801 (2021)

misalignment

lead to emittance growth by betatron decoherence

tight tolerances: pointing < angular spread
offset < beam size

Or negligible decoherence: $\delta\gamma/\gamma \ll \lambda_\beta / L_{\text{plasma}}$

Thévenet et al., PRAB 22, 051302 (2019)

radiative cooling

radiation = loss of momentum in the direction of emission
acceleration = forward momentum
net effect = damping of transverse momentum

can be important for betatron radiation in plasma accelerators , reducing emittance but also increasing energy spread

Michel et al., PRE 74, 026501 (2006)

Deng et al., PRAB 15, 081303 (2012)

Kostyukov et al., PRAB 15, 111001 (2012)

Thank you for your attention