QUBO partitioning and choice of quantum device for charged particle track reconstruction at LUXE

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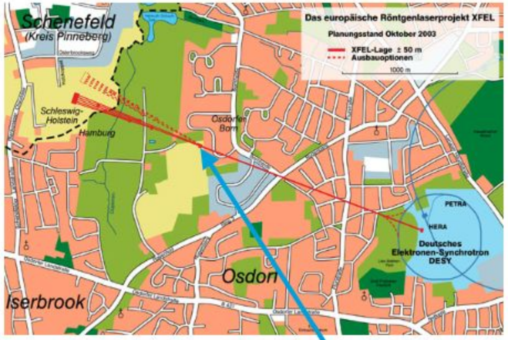
HELMHOLTZ LUXE

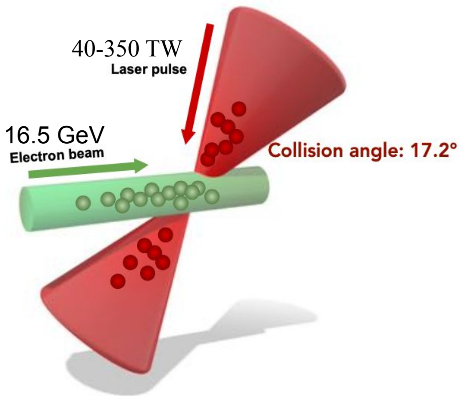
LUXE Laser und XFEL Experiment

The experiments primary aim is to investigate the transition from the perturbative to the non-perturbative regime of QED → **not probed yet!**

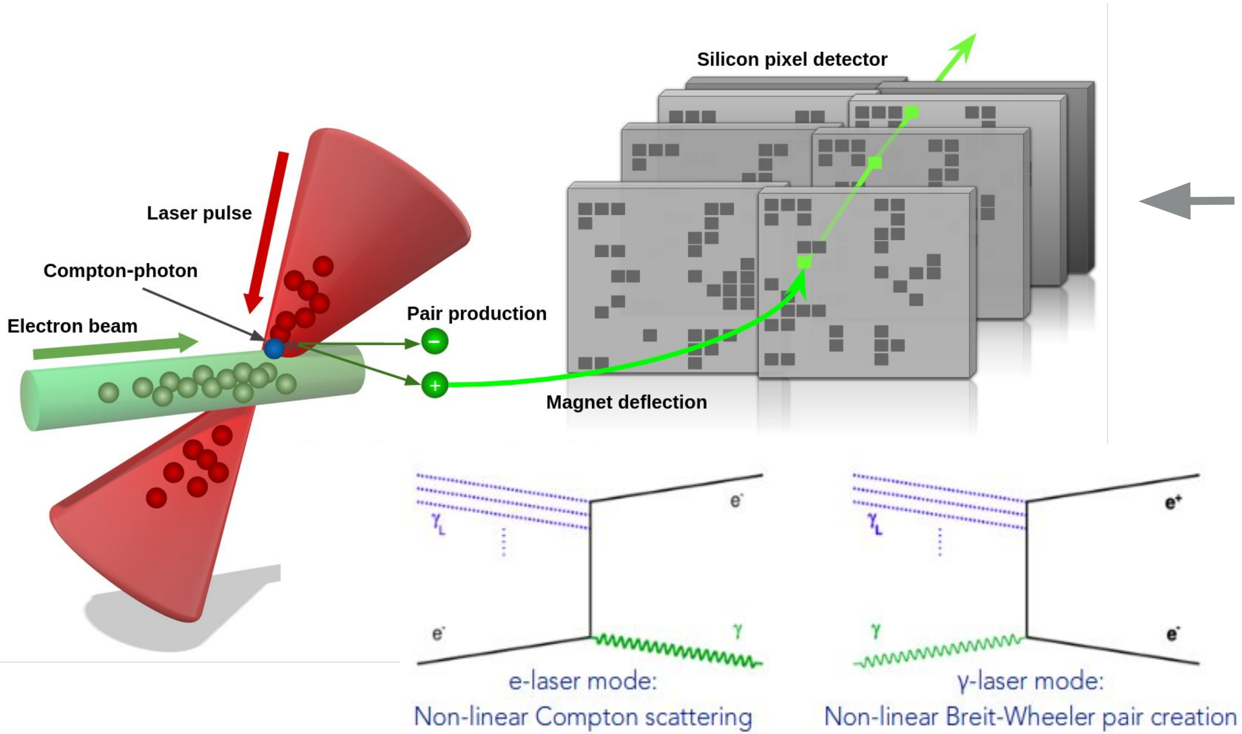
Transition happens at the **Schwinger Limit:**

$$\epsilon_{crit} = \frac{m_e^2 c^3}{\hbar e} \simeq 1.3 \cdot 10^{18}\,\mathrm{V/m}$$



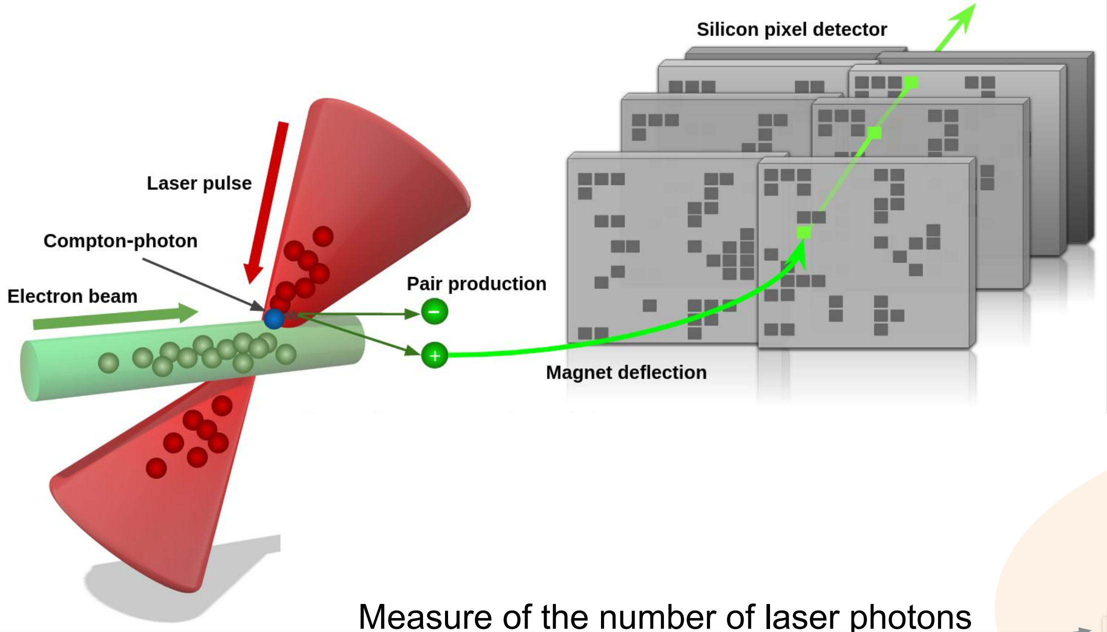


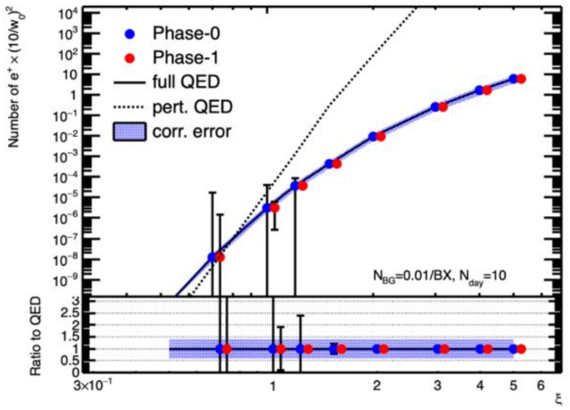
Positron tracking with a quantum computer



Silicon Pixel detector with occupancy of 100 hits per square millimeter

Positron tracking with a quantum computer





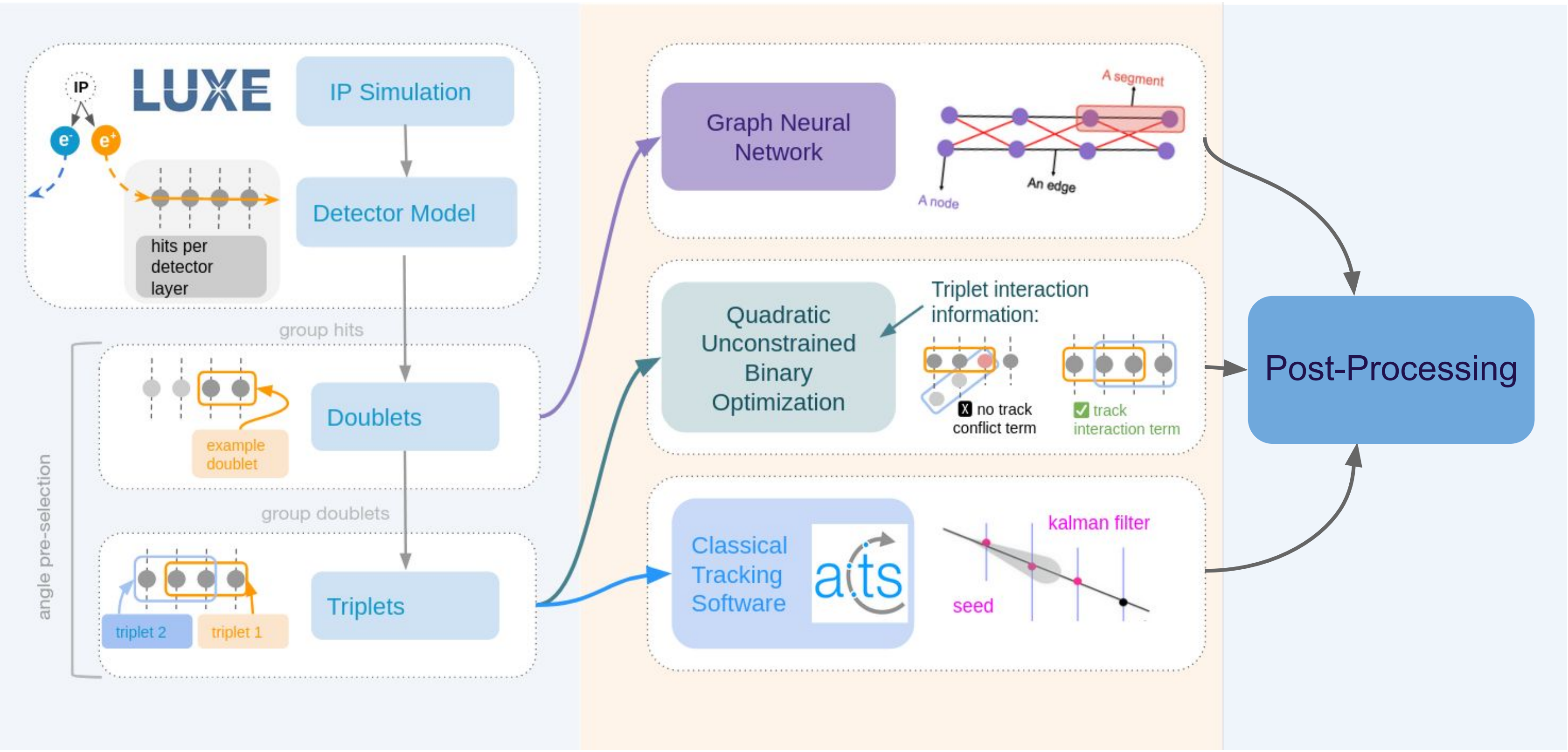
Intensity parameter

$$\xi = \frac{m_e \epsilon_L}{\omega_L \epsilon_{cr}}$$

m_e : *electron mass*
ω_L: *laser frequency*
$\epsilon_{L,cr}$: *laser/critical field strength*

Measure of the number of laser photons interacting with the electron

Overview: full project



QUBO Quadratic Unconstrained Binary Optimization

$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \quad T_i, T_j \in \{0,1\}$$

Binary value:

0: discarded

1: kept

QUBO Quadratic Unconstrained Binary Optimization

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Weight triplets by a_i

QUBO Quadratic Unconstrained Binary Optimization

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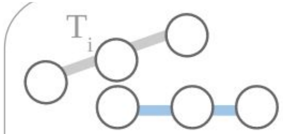
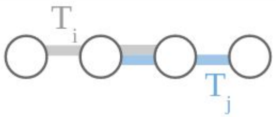
Assign each triplet pairs connectivity b_{ij}

$$b_{ij} = \begin{cases} -S(Ti,Tj), & \text{if } (T_i,T_j) \text{ form a quadruplet,} \\ \zeta & \text{if } (T_i,T_j) \text{ are in conflict,} \\ 0 & \text{otherwise.} \end{cases}$$

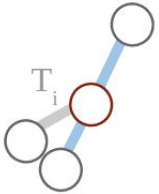


no shared hit:
$b_{ij} = 0$

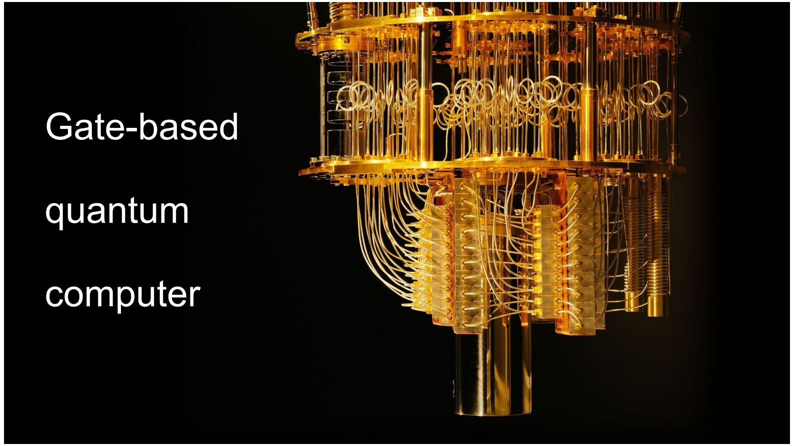


quadruplet:
$b_{ij} < 0$

conflict:
$b_{ij} > 0$

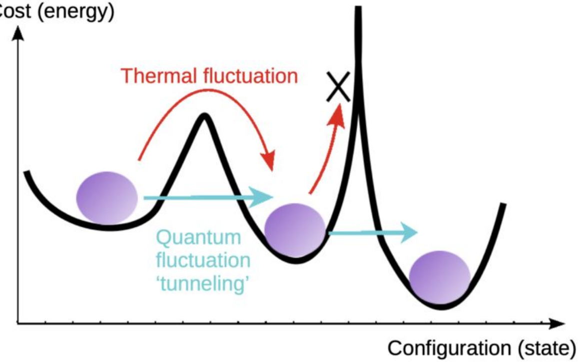


Choice of quantum device



Quantum

Annealer



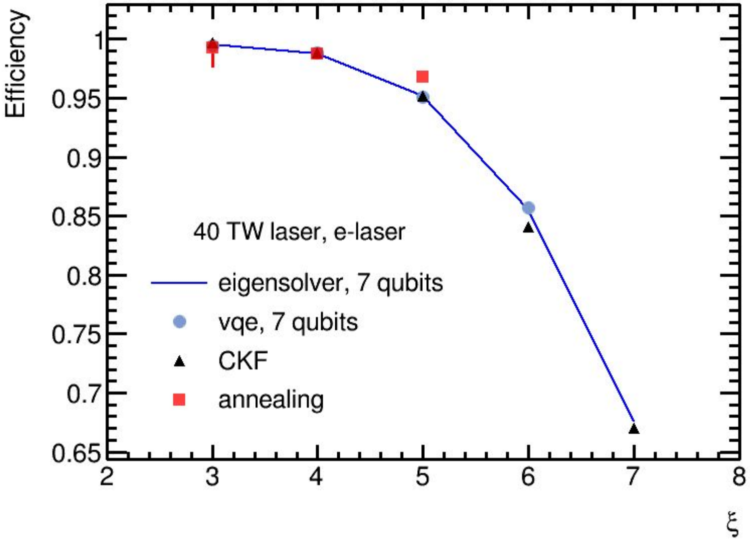
Gate-based quantum computers: Utilize quantum logic gates

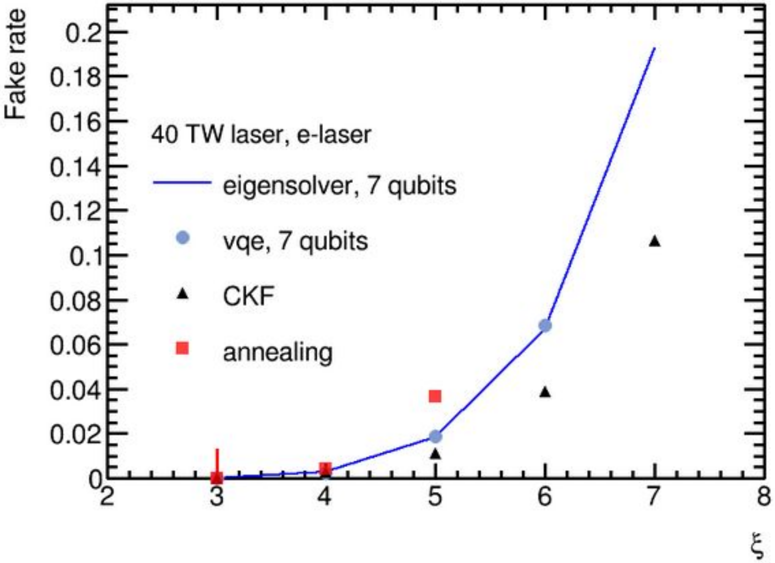
- Offer more control and precision than quantum annealers
- Have highly connected qubits, allowing for entanglement

Quantum annealers: Find the global minimum of a cost function

- Specialized for solving optimization problems (Limited to specific types of computations)
- Have fewer connections between qubits

Annealing vs gate-based Simulators: Results



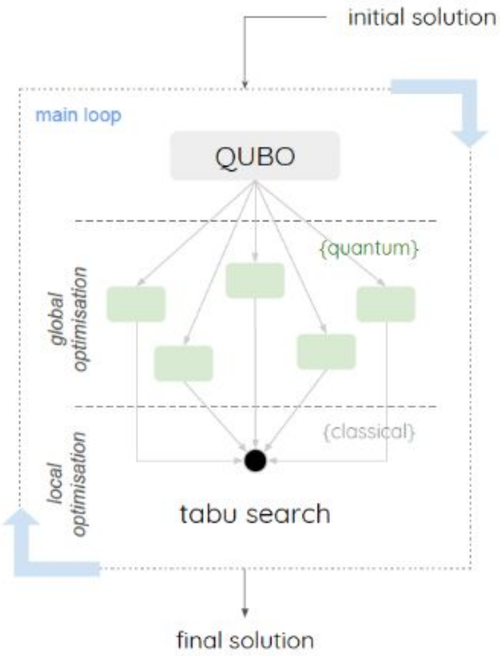


$$\text{Efficiency} = \frac{N_{\text{tracks}}^{\text{matched}}}{N_{\text{tracks}}^{\text{generated}}}$$

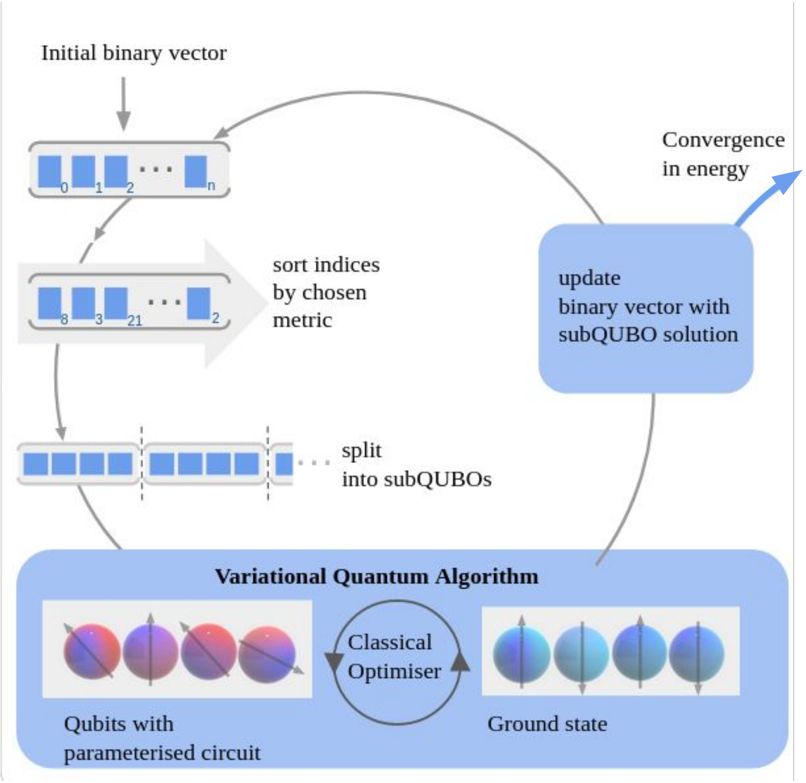
$$\text{Fake rate} = \frac{N_{\text{tracks}}^{\text{fake}}}{N_{\text{tracks}}^{\text{reconstructed}}}$$

QUBO Partitioning: subQUBOs

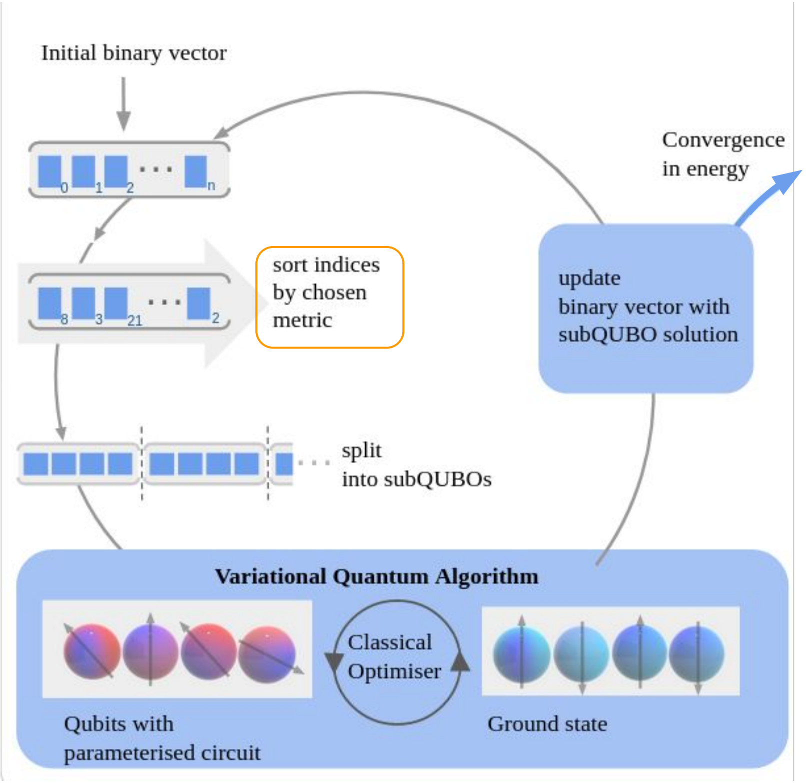
- Each triplet is mapped onto a qubit
  - QUBO is too big to be mapped onto a quantum device!
- Choice of partitioning influences result quality
- Ground state of subQUBO is found using quantum algorithm
- Goal: Sum of sub-solutions converges against overal solution



QUBO Partitioning Approaches



QUBO Partitioning Approaches



$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \quad T_i, T_j \in \{0,1\}$$

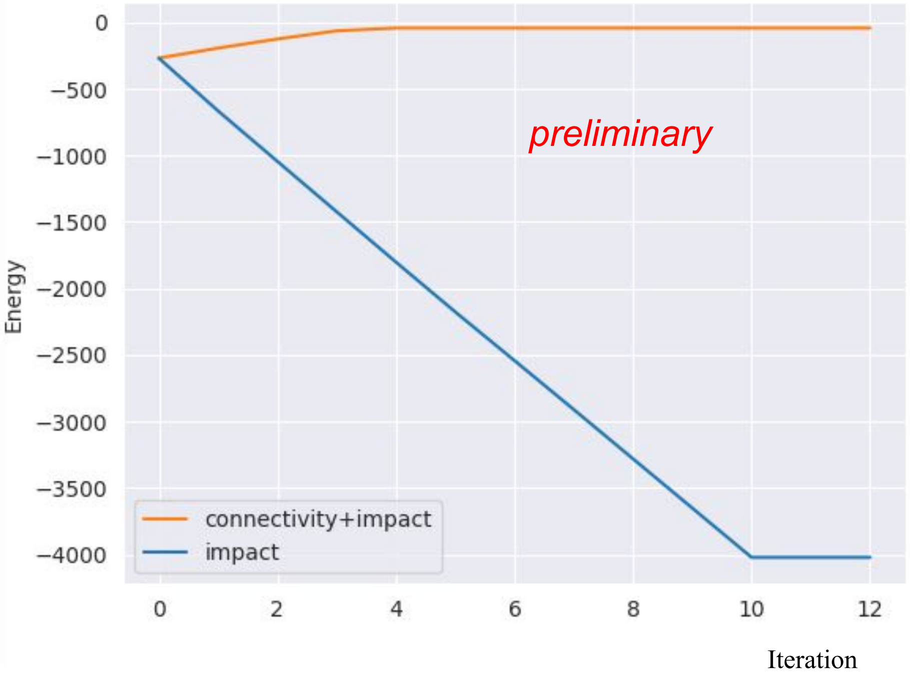
Impact of triplet T_i:

$\Delta O(T_i \rightarrow 1 - T_i)$

Partitioning types:

1. Partition using impact only
2. Partition using impact with additional constraints that triplets have to be connected

QUBO Partitioning: Results



- Study: Annealing with subsize of 7 qubits
- Sorting using only impact has advantage over including connectivity between triplets
- Same triplets always get grouped together→stuck in local minimum

Summary and Outlook

- ❖ Positron track reconstruction for LUXE is studied using a QUBO encoding and quantum simulators
- ❖ Gate-based quantum computing with (7 qubits) performs similar to quantum annealing
- ❖ Quantum annealing is a good tool to study partitioning and scaling
- ❖ Partitioning challenge: Allow for enough fluctuation without making it random

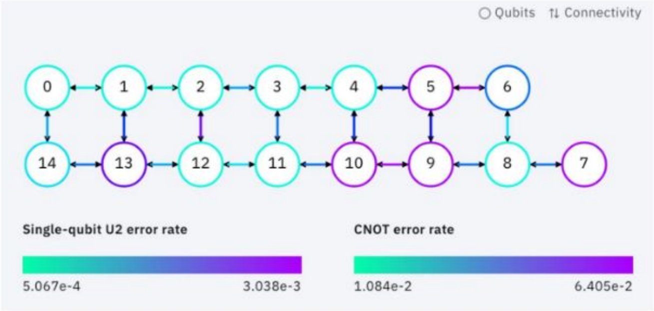
- ❖ Important measure for comparing annealer with gate-based QC: real devices with noise!
- ❖ Other methods of partitioning need to be studied

Thank you

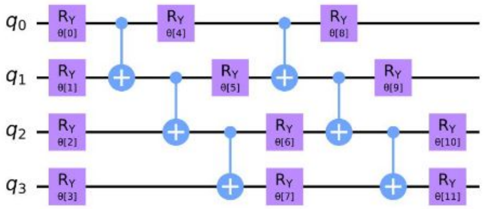
Designing a Quantum Circuit II

Two Local configurations as benchmarks

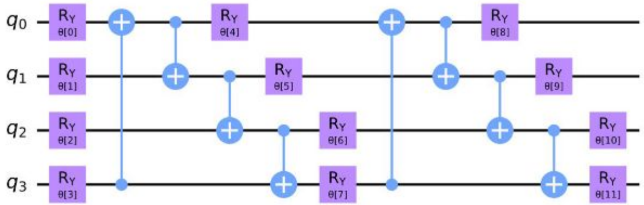
- Direct entanglements **only** possible if qubits on devices are connected, otherwise, one has to propagate values through the circuit
- Error rates of qubits and gates vary



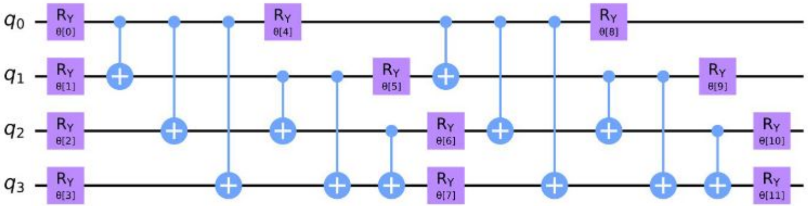
[5]



linear entanglement



circular entanglement



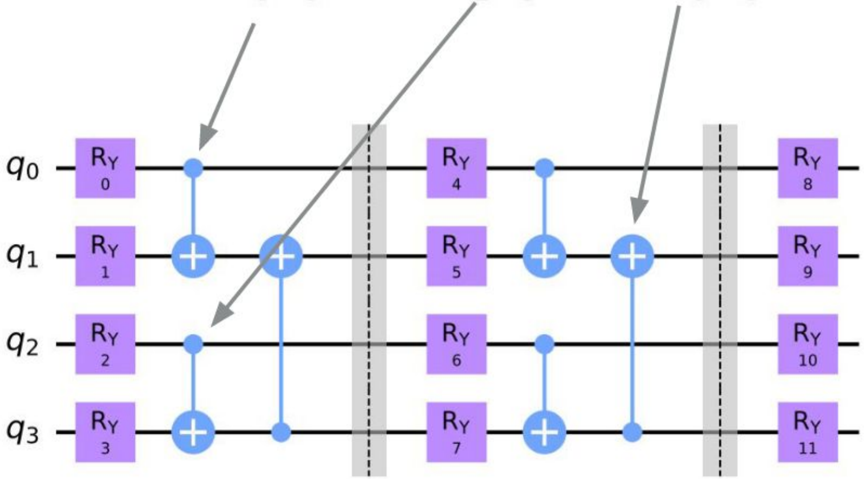
full entanglement

Designing a Quantum Circuit III

Dynamically created hamiltonian-aware ansatz

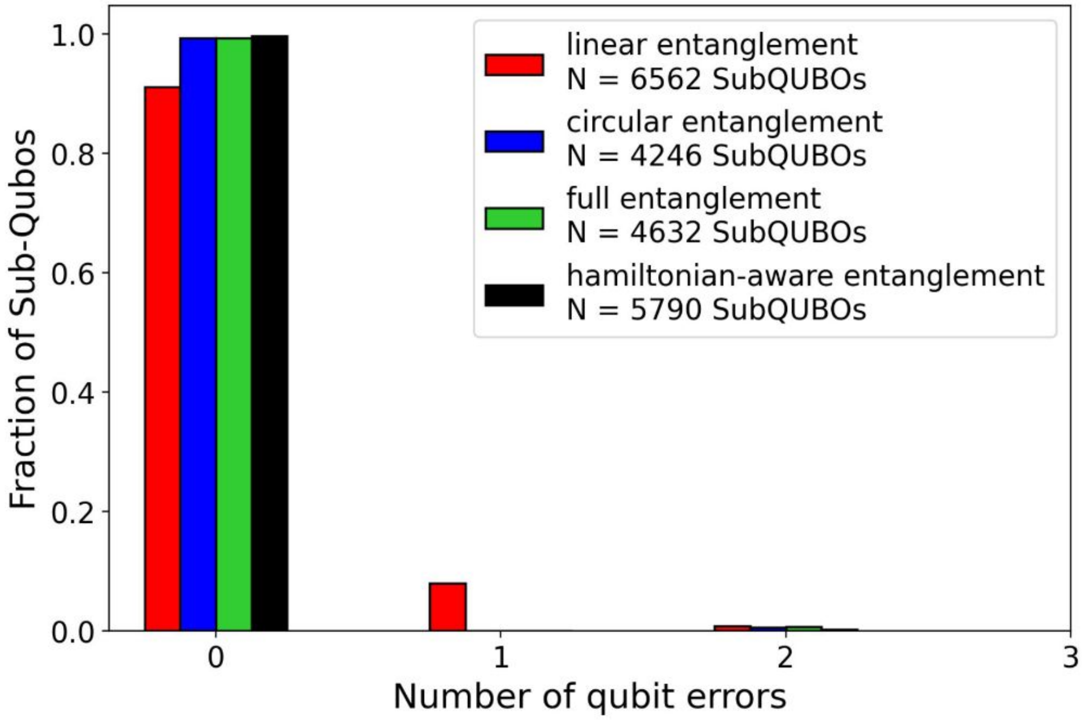
- Structure of the ansatz resembles structure of the hamiltonian
- CX - gates have a high error probability → use as few controlled CX - gates as possible

$$\hat{H} = 0.5x_0 - 0.3x_1 + 0.1x_2 - 0.3x_3 - 1.1x_0x_1 + 1.2x_2x_3 - 1.3x_3x_1$$



Performance on ideal simulation

Solving success and time performance



Solving time / SubQUBO:

- Linear: 2.17 ± 0.31s
- Circular: 3.62 ± 0.14s
- Full: 4.79 ± 0.54s
- custom: 3.35 ± 0.33s

DESY.

QAOA

Solving the subQubo

-QAOA can be viewed as a special case of VQE.

- Hamiltonian contains only Z terms, we do not need to change the basis for measurements.

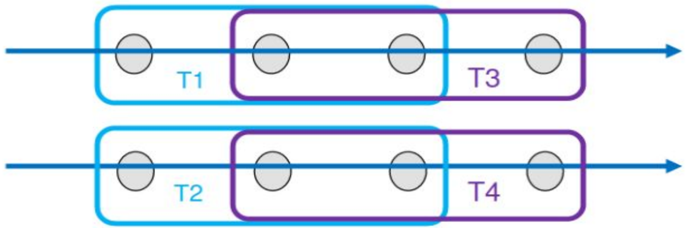
Differences to VQE:

- The form of the ansatz is limited
- Restricted to Ising Hamiltonians
- In QAOA our goal is to find the solution to the problem. To do that we don't need to find the ground state.

QUBOs

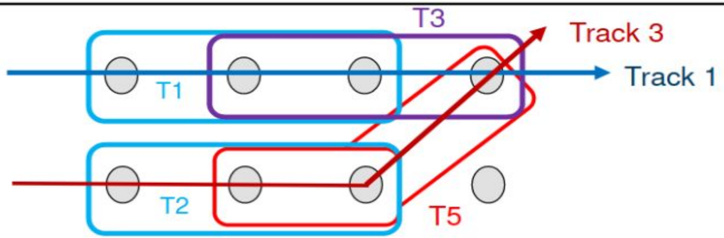
$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \quad T \in \{0,1\}$$

$$b_{ij} = \begin{cases} -S(Ti,Tj), & \text{if } (T_i,T_j) \text{ form a quadruplet,} \\ \zeta & \text{if } (T_i,T_j) \text{ are in conflict,} \\ 0 & \text{otherwise.} \end{cases}$$



[T1, T2, T3,T4]→combinations:

T1T2	T1T3	T1T4	T2T3	T2T4	T3T4
+0	-S	+0	+0	-S	+0



[T1, T2, T3,**T5**] → combinations:

T1T2	T1T3	T1**T5**	T2T3	T2**T5**	T3**T5**
+0	-S	+0	+0	-S	conflict

7

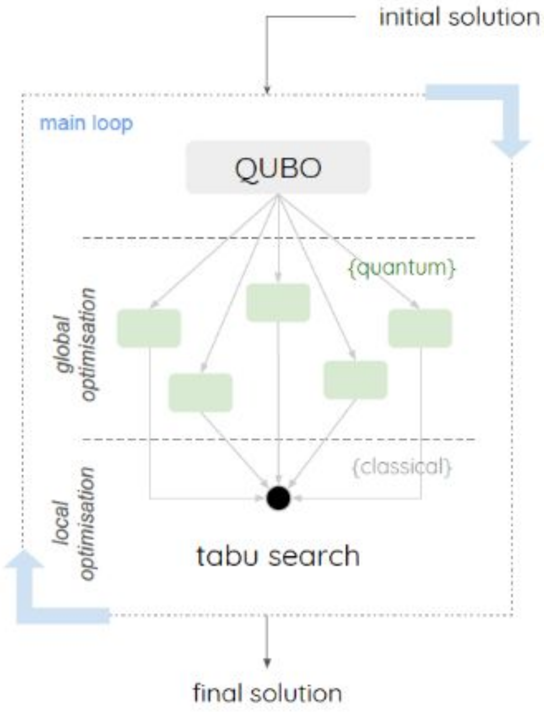
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

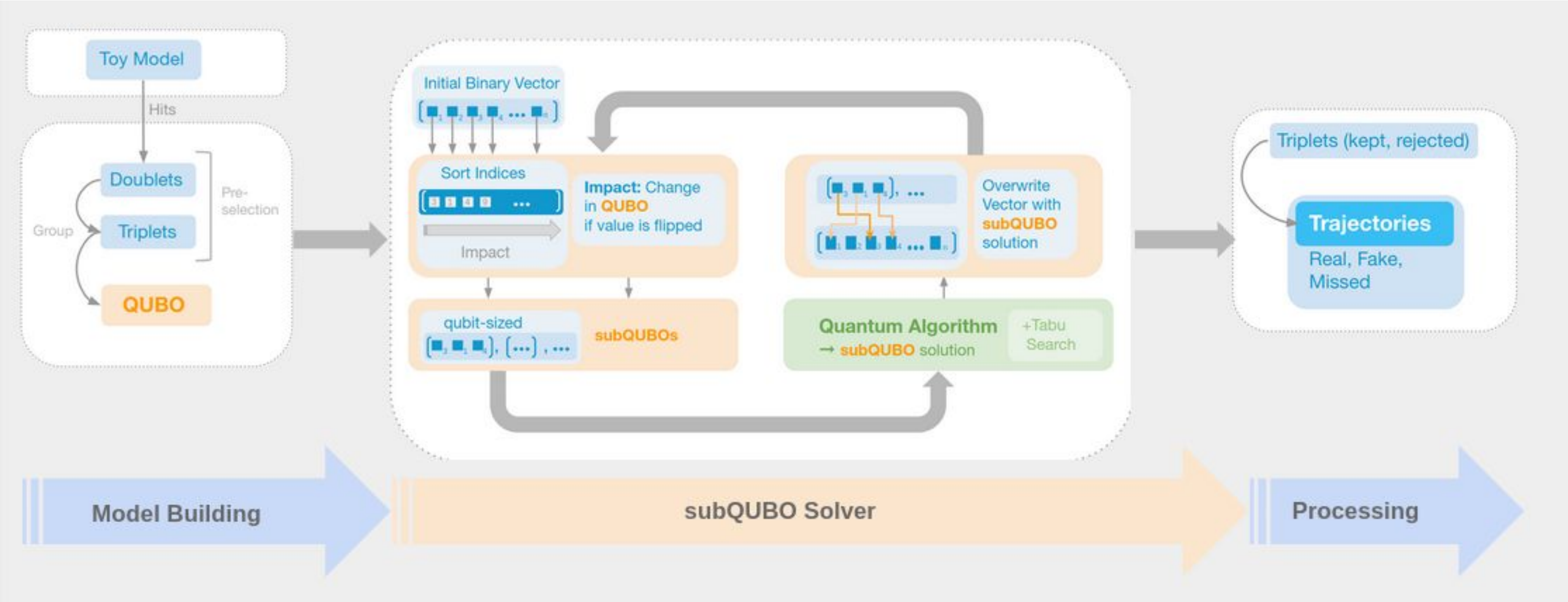
SubQubos

Problem: Devices restricted to small number of qubits

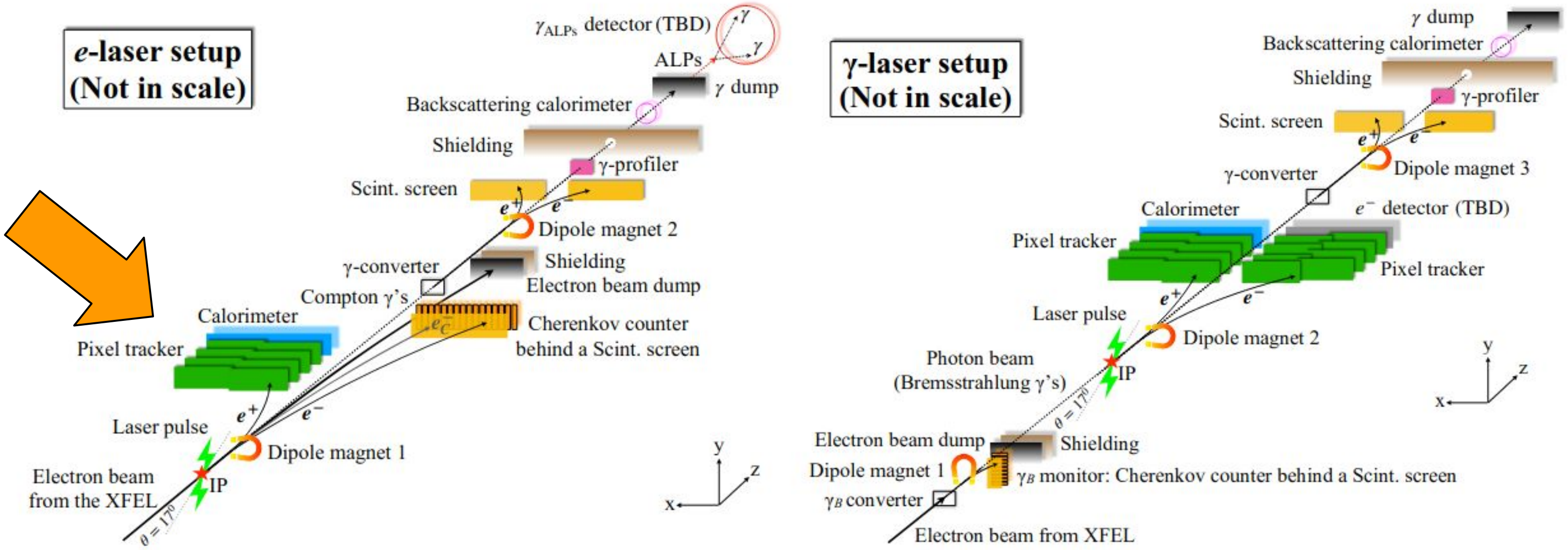
- Big QUBOS cannot be simulates
- Break QUBO into subsets→ subQUBOS!
- Iterated vector converges to solution vector



HELMHOLTZ RESEARCH FOR GRAND CHALLENGES


Toy Model
Hits
Doublets
Triplets
Group
Pre-selection
QUBO
Initial Binary Vector
Sort Indices
Impact
Impact: Change in QUBO if value is flipped
qubit-sized
subQUBOs
Overwrite Vector with subQUBO solution
Quantum Algorithm
→ subQUBO solution
+Tabu Search
Triplets (kept, rejected)
Trajectories
Real, Fake, Missed
Model Building
subQUBO Solver
Processing

LUXE setup



LUXE CDR: arXiv:2102.02032