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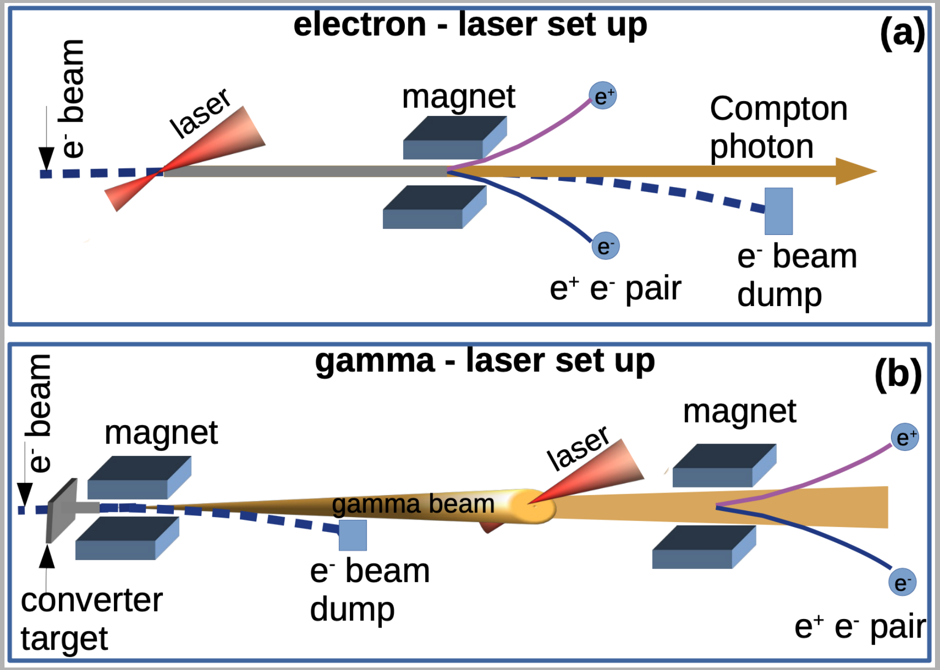
Beam-Tracking and position of the BPMs in the TL/T20 beamline (up to the IP)

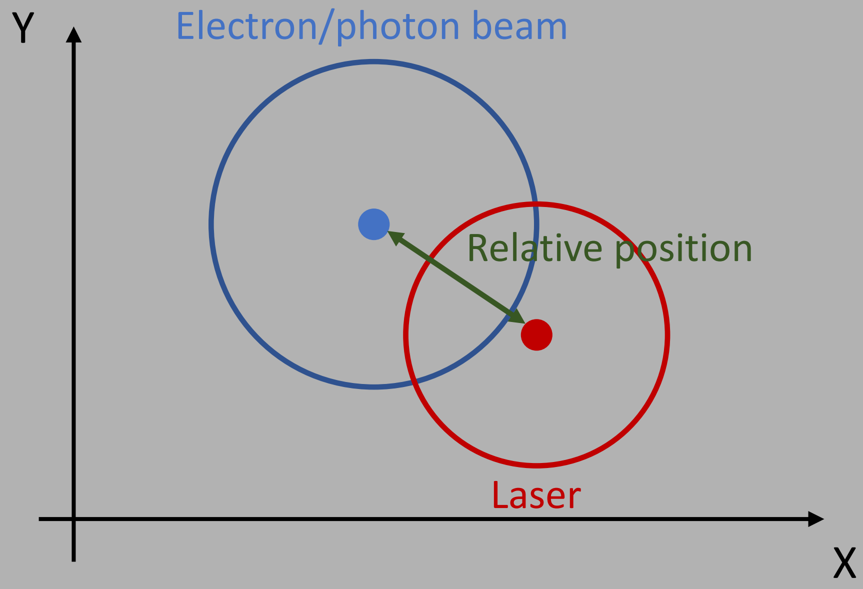
DENIAUD Marin - 20/02/2023

1. Objectives of this study
2. Tracking and Correlation
3. Singular Value Decomposition
4. Resolution at the IP
5. Next Objectives

1. Objectives of this study



Transverse view of the beams at the IP



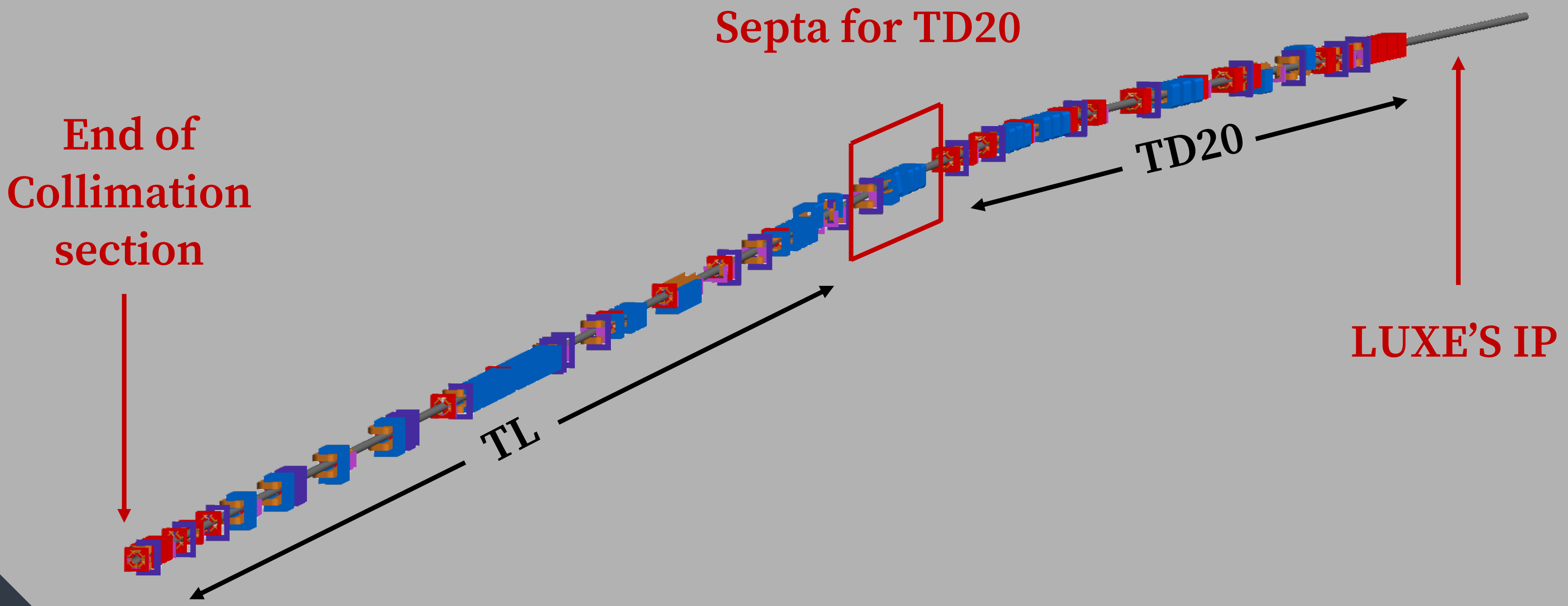
- The goal of LUXE in both configurations is to make 2 beams interact
- The smaller the relative position, the higher is the number of interactions

- We can't put some position measurements at the IP so we have to search for correlation points upstream

How can we put BPMs upstream to measure the positions and angles at the IP ?

And what resolution can we get at this IP ?

1. Objectives of this study



- The lattice simulated is composed of the TL and the TD20
- This lattice TWISS and RMAT files come from Mad8 and all the beam tracking is done using custom python tools

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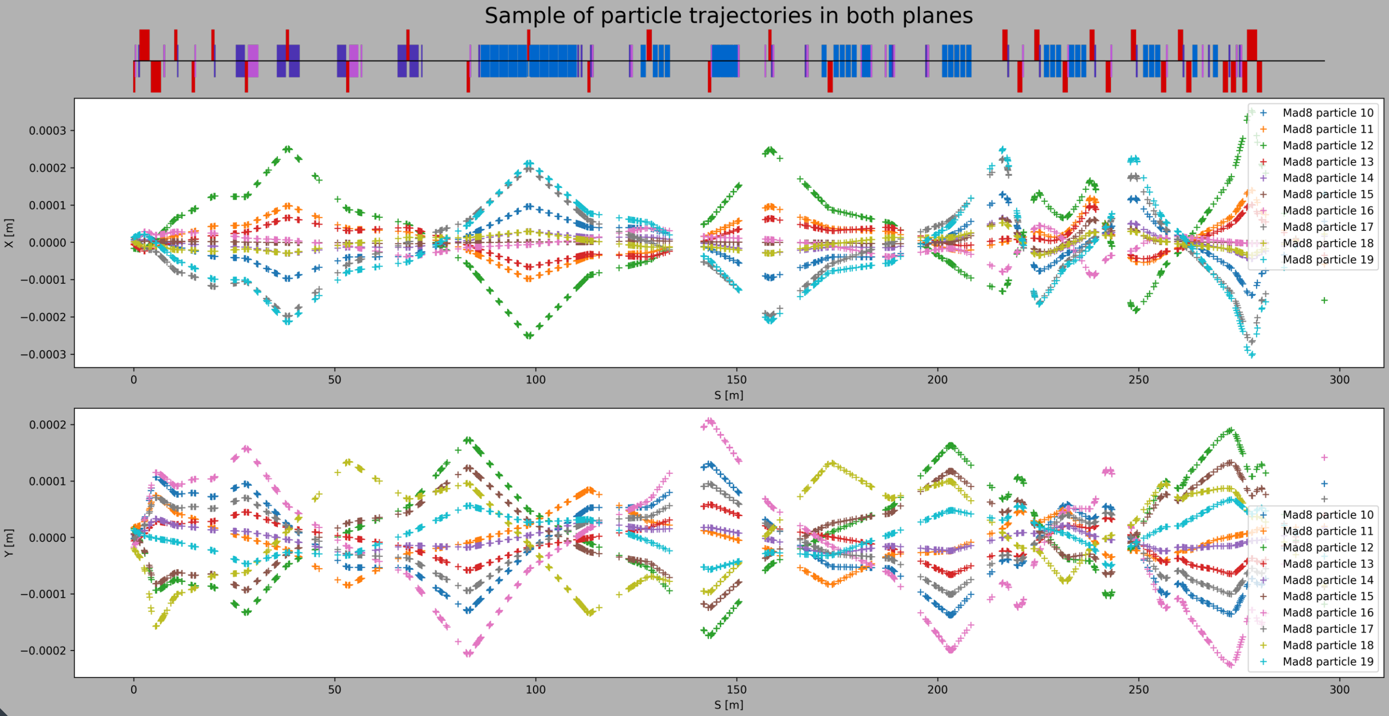
2. Tracking and Correlation



INITIAL BEAM :

- 10 000 particles
- X/Y = 10e-6 m std
- X'/Y' = 10e-6 rad std
- Z = 30e-6 m std
- DE = 3 MeV std

- In this study there is no X/Y coupling
- We want to extract the position data to identify correlation points :

2. Tracking and Correlation

- Mathematically, in x for example, a point at position s is correlated if :

$$x_{IP} = a(s)x(s)$$

- From Hill's equations we know that :

$$x(s) = \sqrt{\varepsilon_x \beta_x(s)} \cos(\mu_x(s) + \varphi)$$

- Then using both equations above we can show that we have correlation when :

$$x_{IP} = a(s)x(s) \Rightarrow \sin\left(\mu_{IP} - \mu(s)\right) = 0 \Rightarrow \mu(s) = \mu_{IP} - n\pi$$

- All the correlation positions can be predicted using the phase advance data which is already given by our Mad8 simulations.

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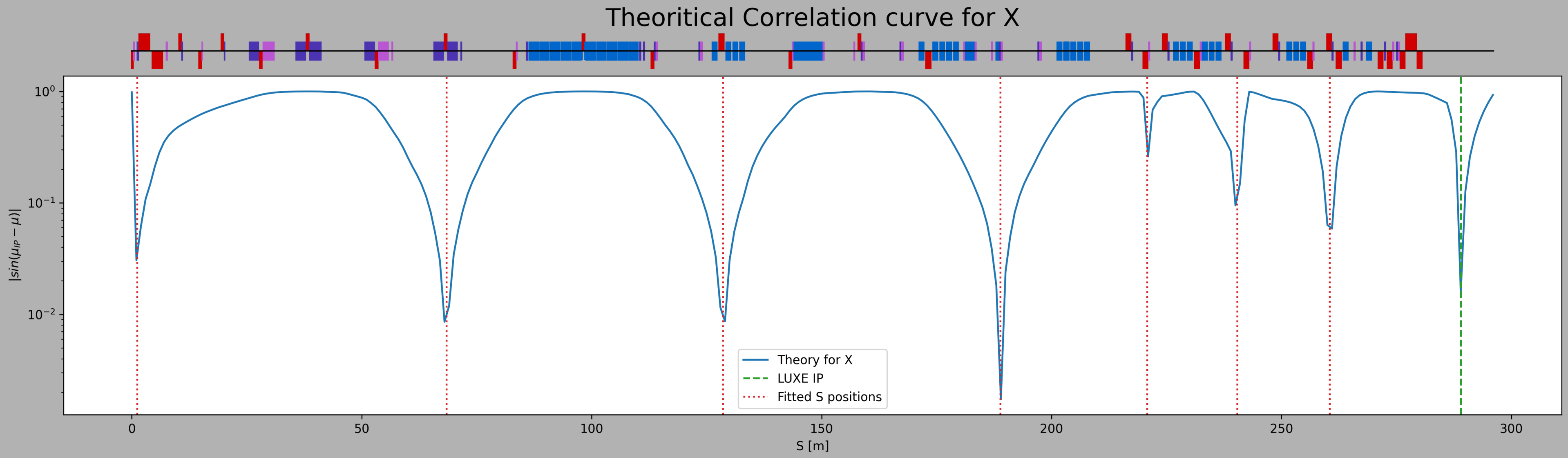
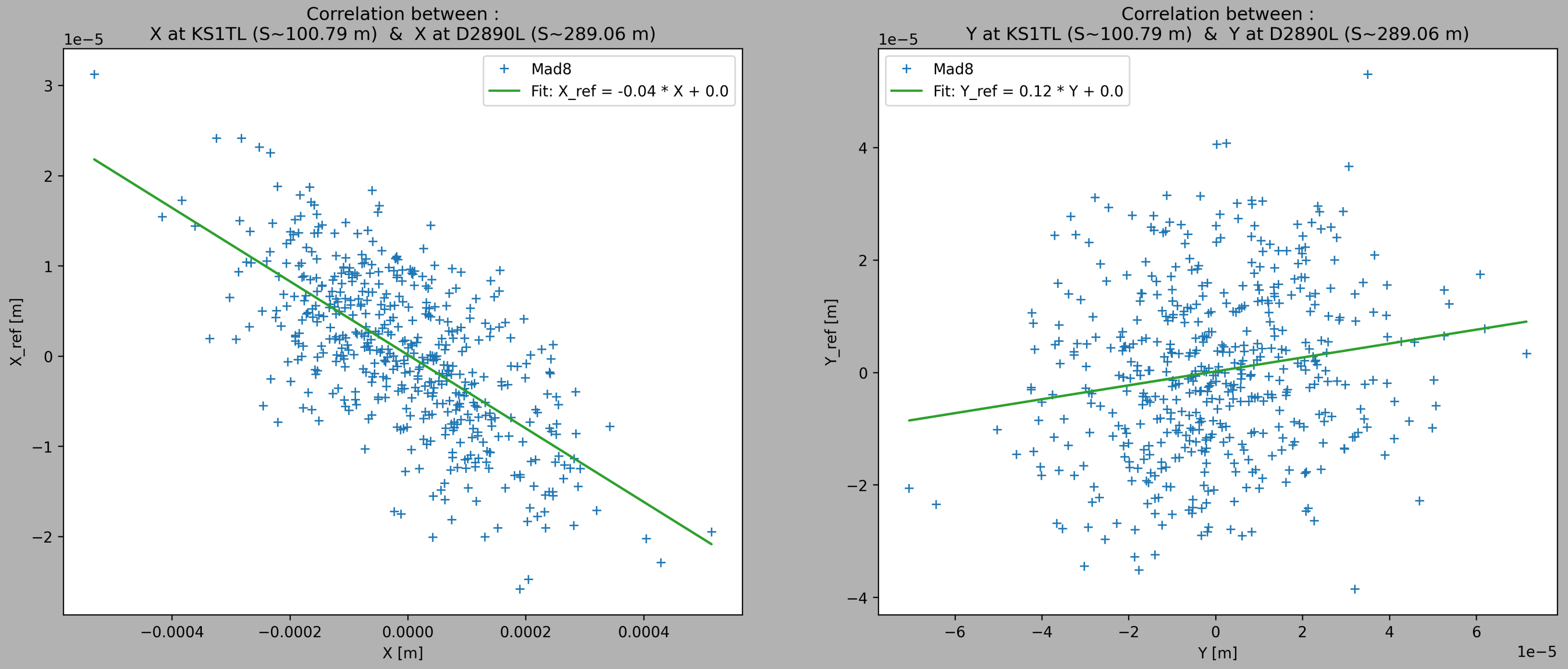
2. Tracking and Correlation

$$|\sin(\mu_{IP} - \mu(s))| \ll 0$$



- Plotting this value along the lattice, we can see and fit a list of S positions that are in theory good candidates for a BPM

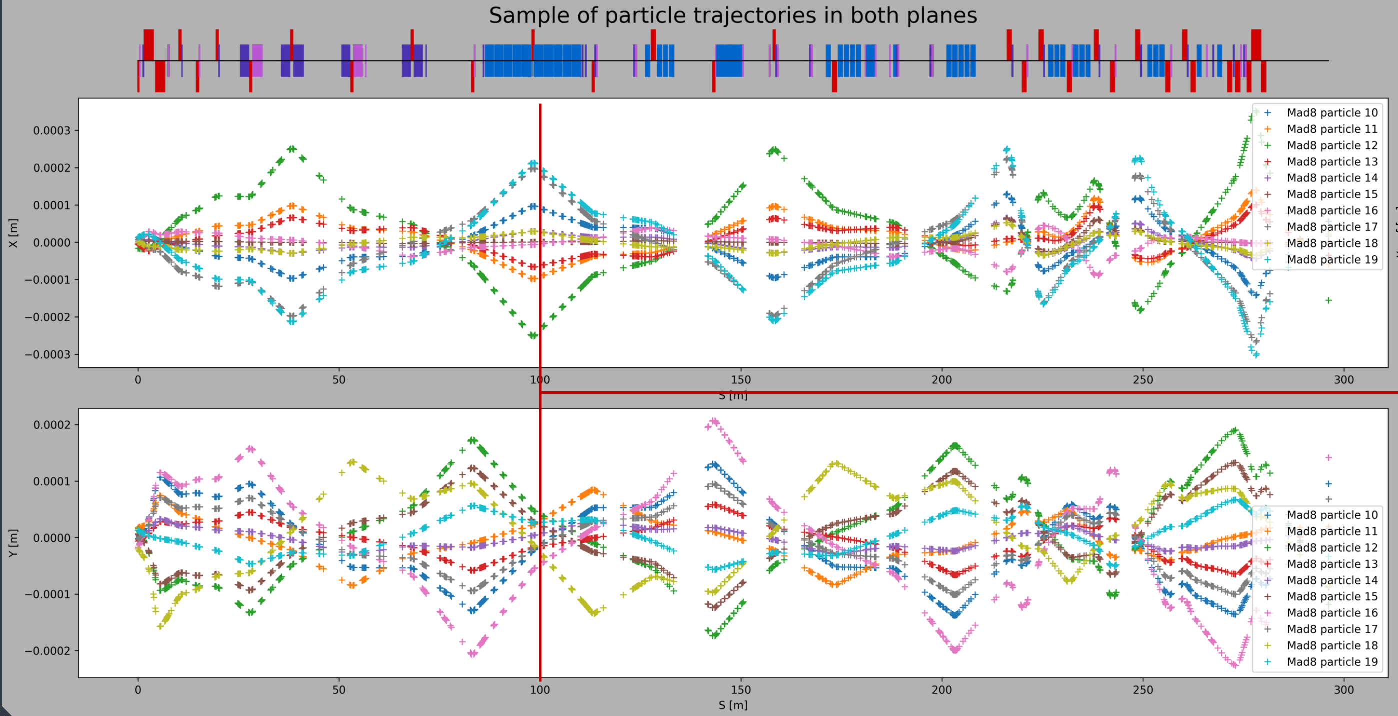
2. Tracking and Correlation

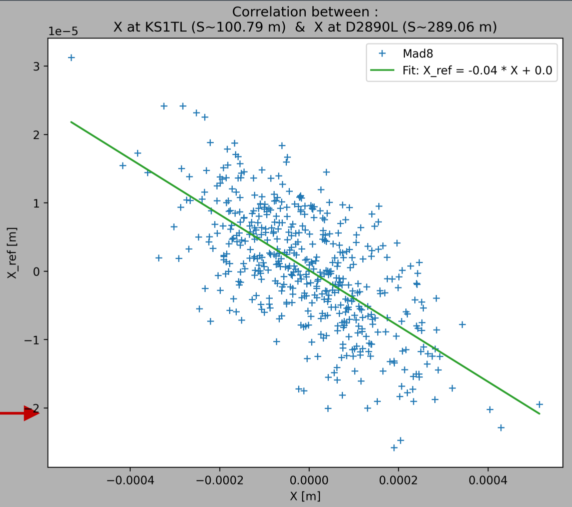


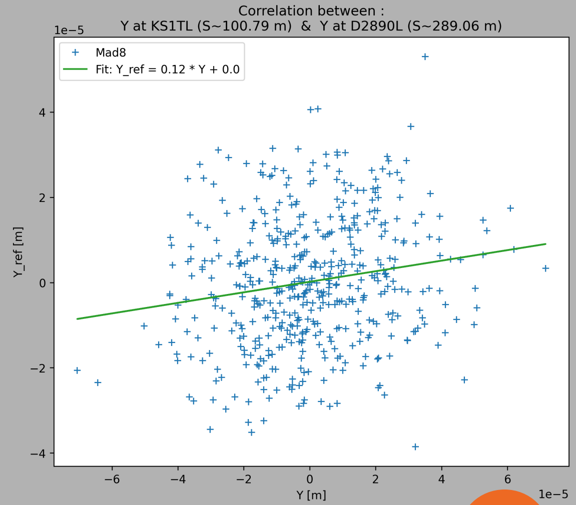
- **Using the tracking data we can plot the correlation between the IP and any point along the lattice**
- **Each of these correlation plots can be linearly fitted, for a particle i :**

$$x_{ref,i} = a x_i - b$$

2. Tracking and Correlation

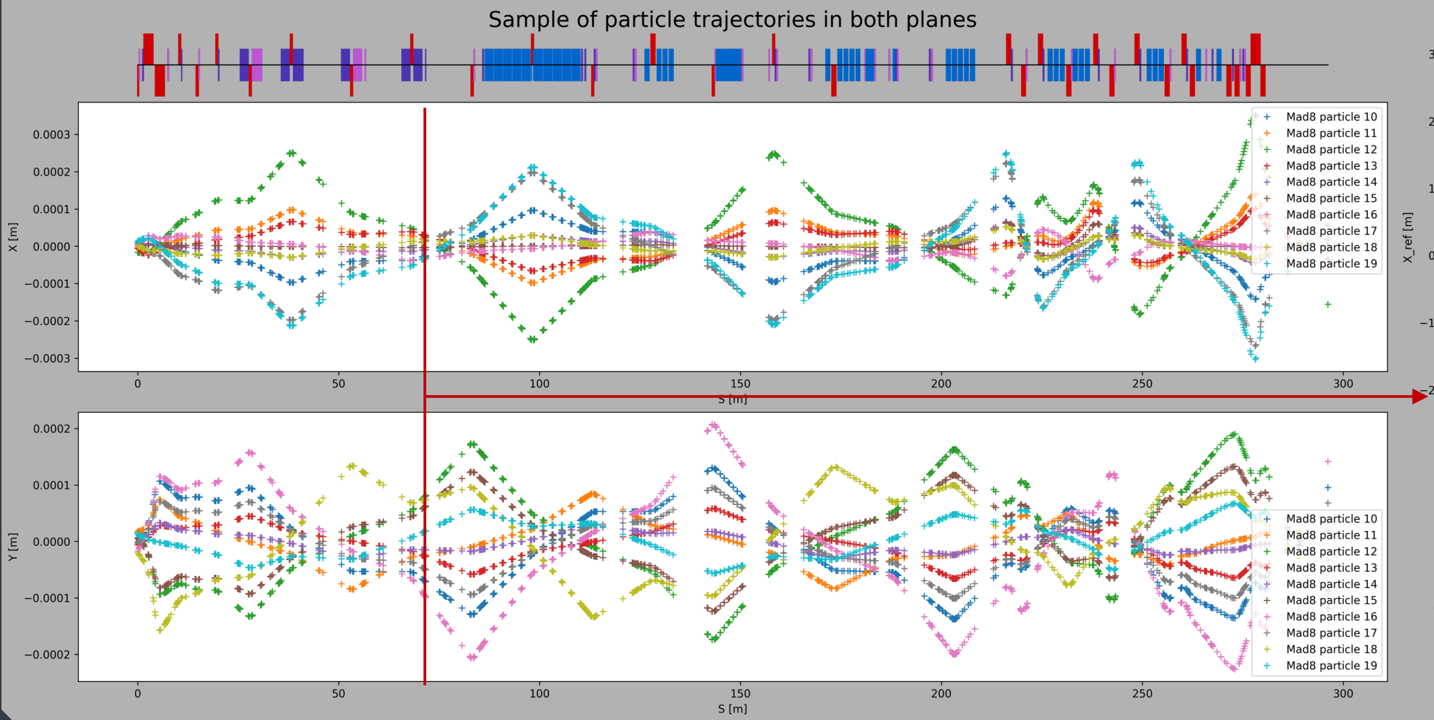


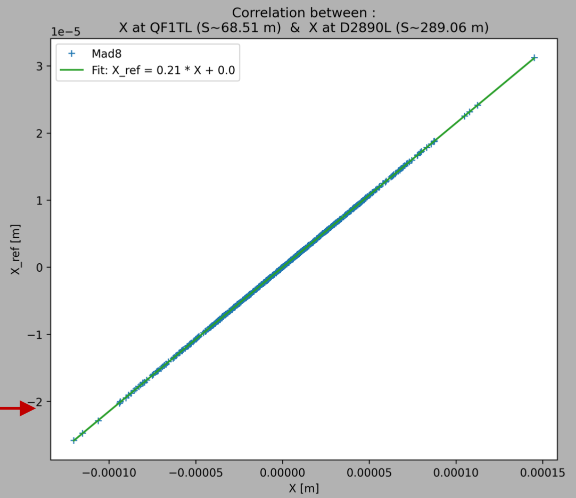


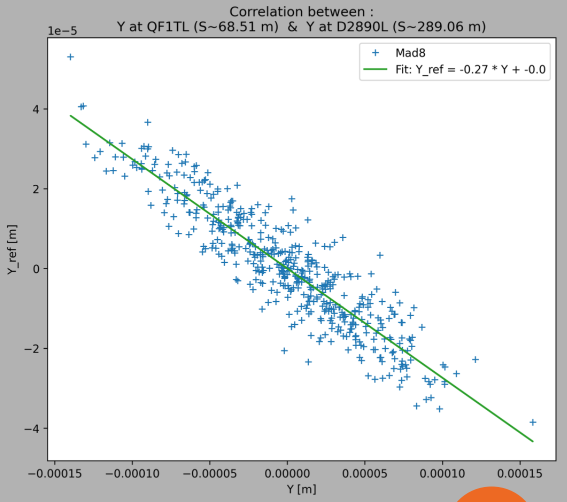


- Scanning along the lattice we can find correlation points in both planes

2. Tracking and Correlation







- Scanning along the lattice we can find correlation points in both planes

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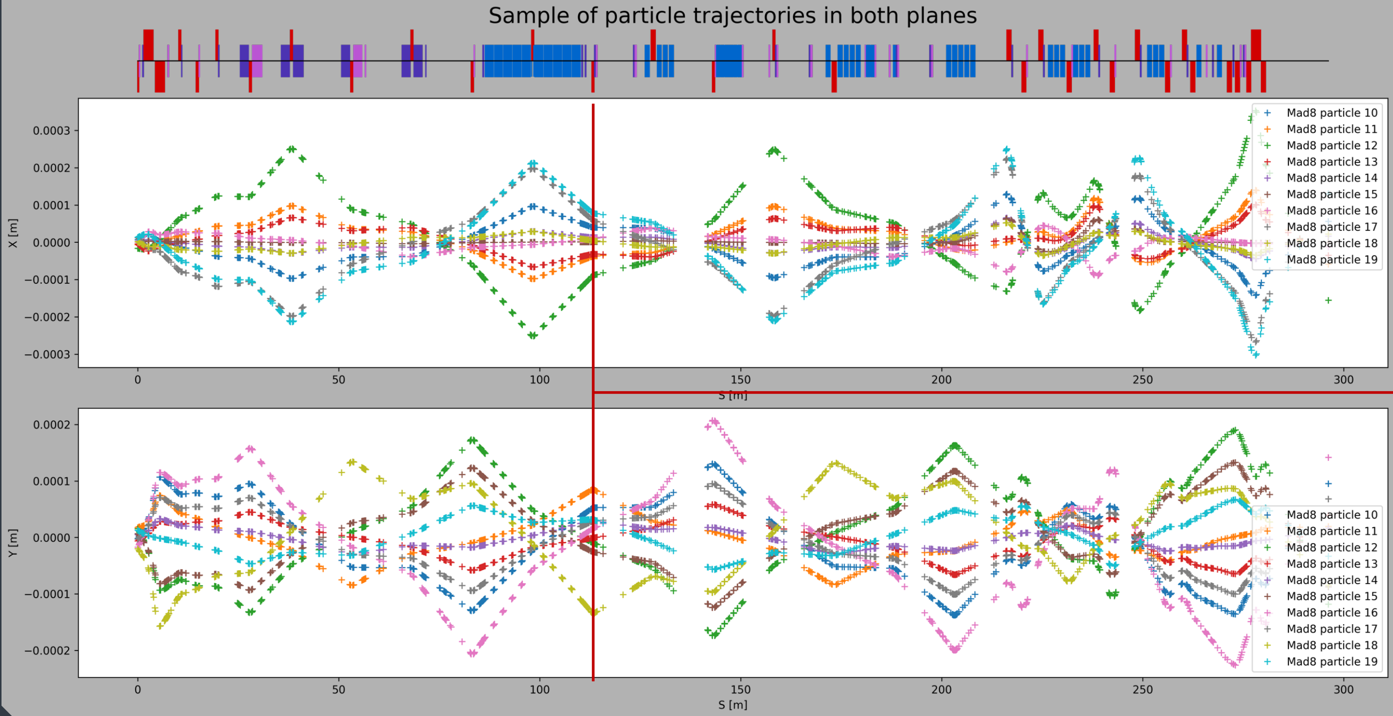
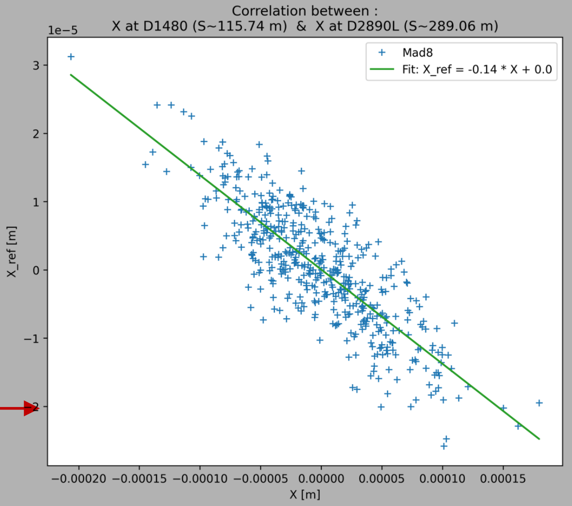
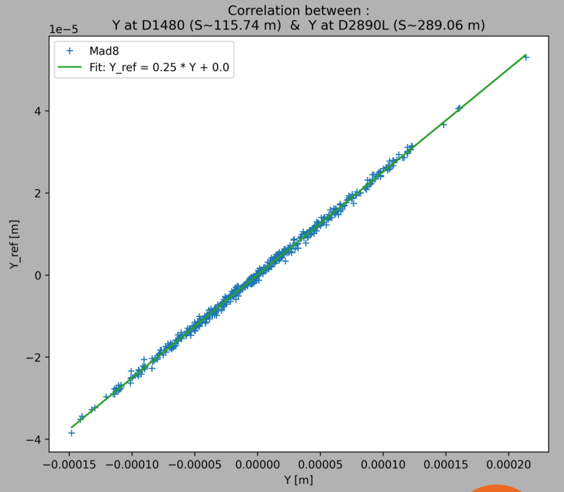
2. Tracking and Correlation





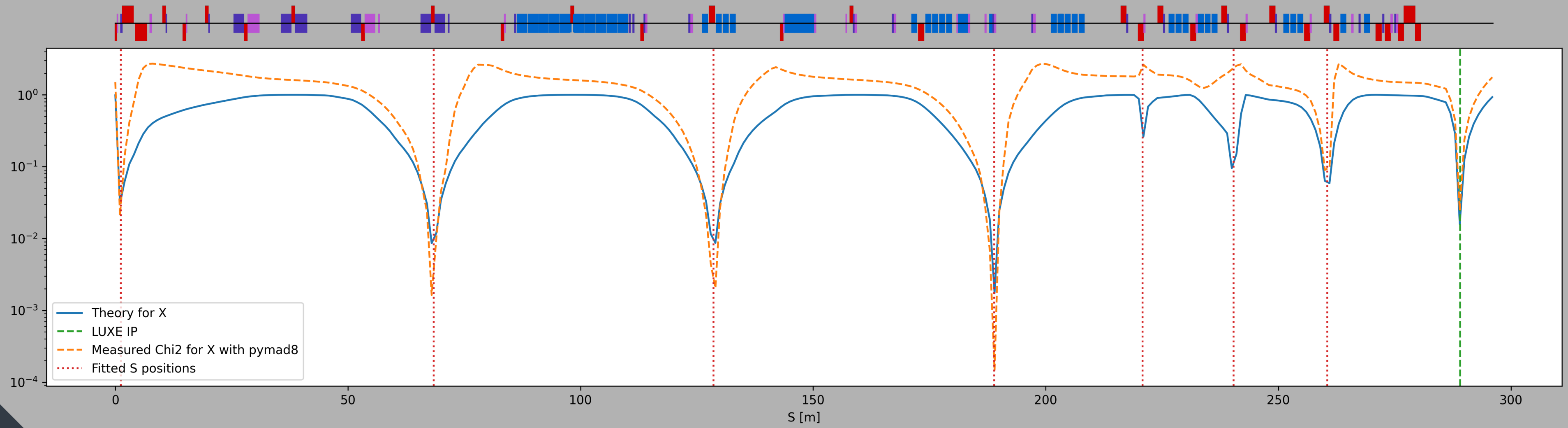


- Scanning along the lattice we can find correlation points in both planes

2. Tracking and Correlation

$$\chi^2 = \sum_{i=1}^{N} (x_{ref,i} - a x_i - b)^2$$

Correlation curves and fit for X



- Those qualitative curves are matching pretty well on the S positions

- **Previously used equations for the position**

$$\begin{cases} x_{IP} = a(s)x(s) \\ x(s) = \sqrt{\varepsilon_x \beta_x(s)} \cos(\mu_x(s) + \varphi) \end{cases} \Rightarrow \sin\big(\mu_{IP} - \mu(s)\big) = 0 \Rightarrow |\sin\big(\mu_{IP} - \mu(s)\big)| \ll 0$$

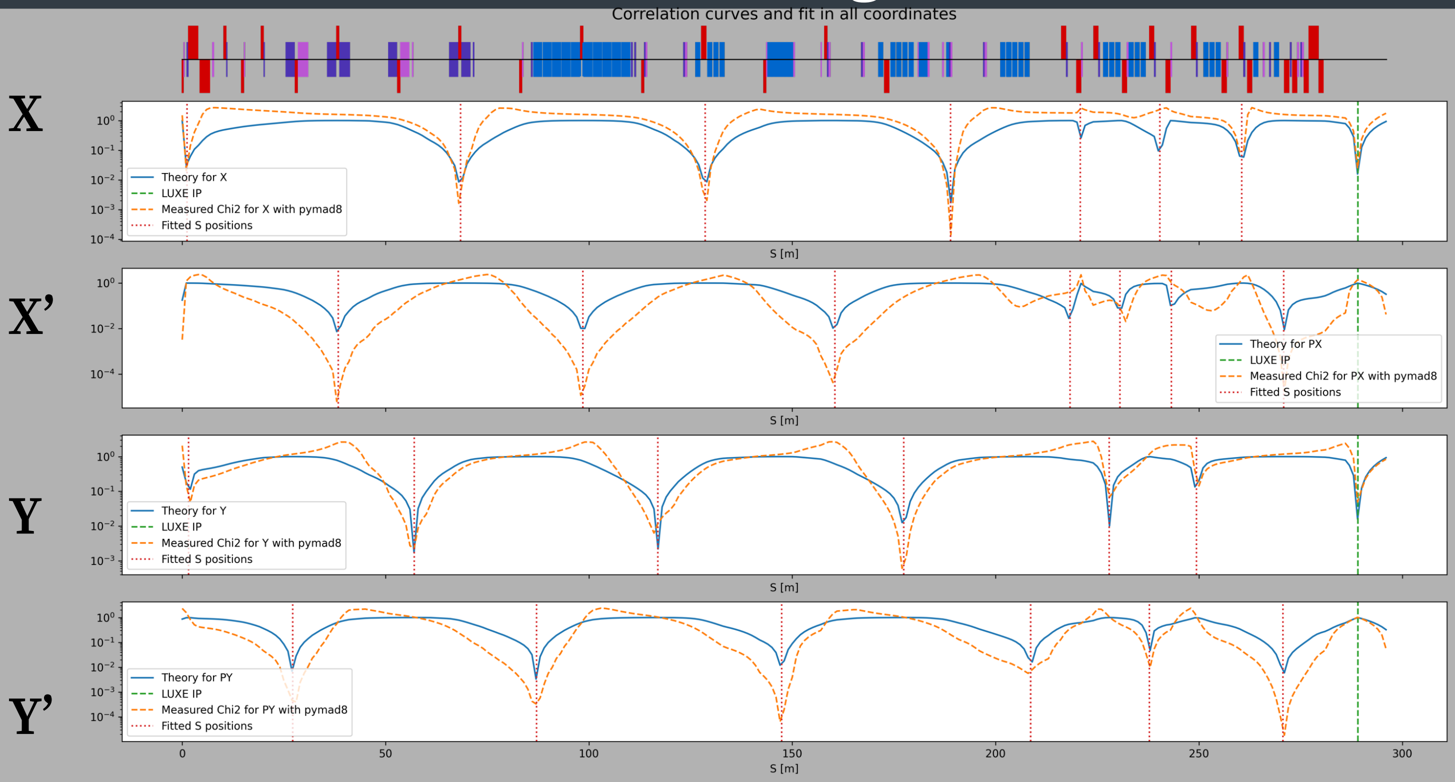
- **We can use the same reasoning for the angle correlation**

$$\begin{cases} x'_{IP} = a(s)x(s) \\ x'(s) = \sqrt{\dfrac{\varepsilon_x}{\beta_x(s)}} (\alpha_x(s) \cos(\mu_x(s) + \varphi) + \sin(\mu_x(s) + \varphi)) \end{cases}$$

$$\Rightarrow \sin\left(\mu_{x,IP} - \mu_x(s) + \arctan(\frac{-1}{\alpha_{x,IP}})\right) = 0 \Rightarrow |\sin\left(\mu_{x,IP} - \mu_x(s) + \arctan(\frac{-1}{\alpha_{x,IP}})\right) \ll 0$$

- **We are then able to extract correlation data for X, Y, X’ and Y’ at the IP**

2. Tracking and Correlation



- For each coordinates we found 7 possible locations for BPMs (except in Y where we found only 6)

- But we need a quantitative method to find what are the most efficient BPMs if we take the given S positions !

1. Objectives of this study
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3. Singular Values Decomposition

- The previous method only shows the position of high correlation and not the correlation coefficients.
- We want to extract, from all these positions, the corresponding coefficients c_s to measure the beam coordinates at the IP

$$x_{IP} = \sum_{s \neq s_{IP}}^{S} x_s c_s$$

- We can resolve this problem with a matrix formulation in which X_{IP} contain all the positions information at each BPM and for each particle

$$\underline{x_{IP}} = \underline{\underline{X_{IP}}}.\underline{c_{IP}} \Rightarrow \underline{c_{IP}} = \underline{\underline{X_{IP}{}^{-1}}}.\underline{x_{IP}}$$

- Inverting the matrix $\underline{\underline{X_{IP}{}^{-1}}}$ require that we use a SVD !

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3. Singular Values Decomposition

- The Singular Value Decomposition is a method that allow the inversion of a non trivial matrix by decomposing it into 3 matrices

$$\underline{\underline{X_{IP}}} \stackrel{\text{def}}{=} \underline{\underline{U}}.\underline{\underline{D}}.\underline{\underline{V^T}} \qquad \begin{cases} D \text{ is diagonal} \\ U, V \text{ are unitary} \end{cases}$$

- These matrices are easily invertible or transposable so we can re-arrange our previous equation to get the correlation coefficients

$$\underline{c_{IP}} = \underline{\underline{V}}.\underline{\underline{D^{-1}}}.\underline{\underline{U^T}}.\underline{x_{IP}}$$

- WARNING : This method works here because in this beamline the relationship between two positions is linear or quasi-linear (we don't have any sextupole)

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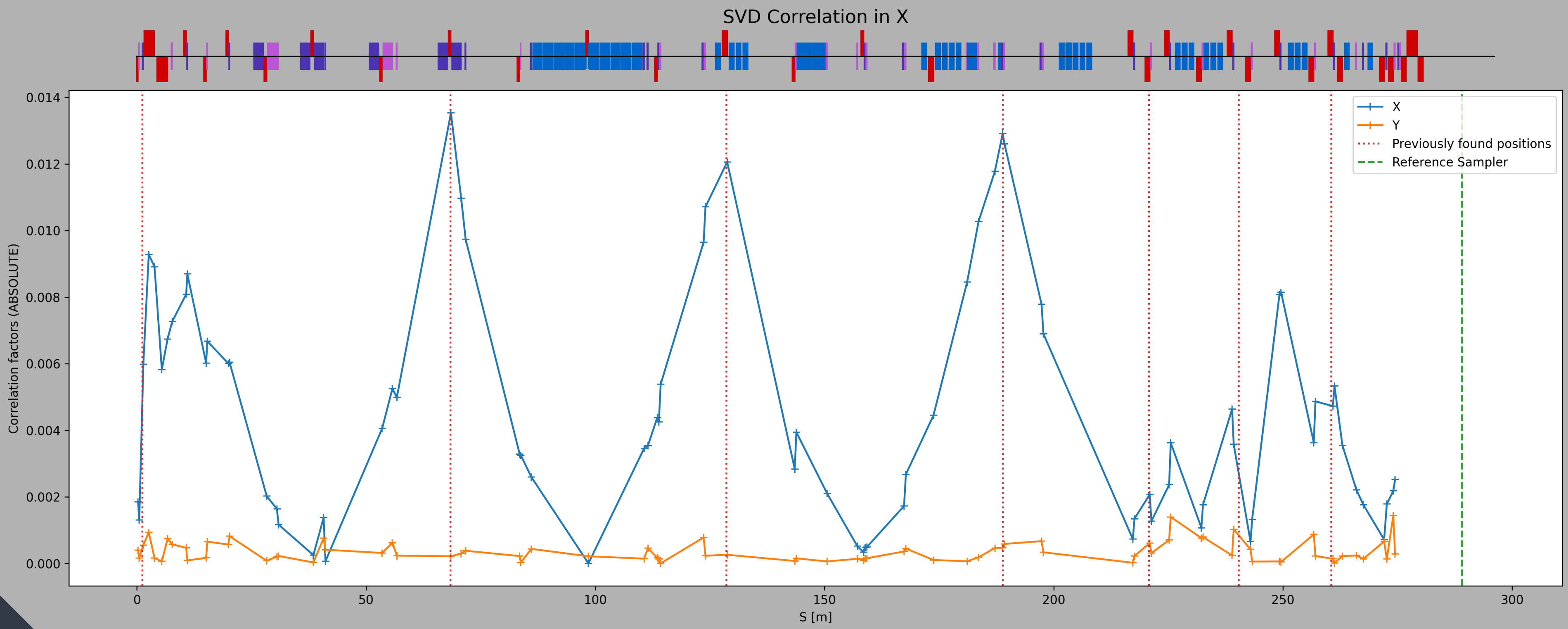
3. Singular Values Decomposition



- Attempt to find the same S position that we found with Chi2 method
- Only the QUADs and KICKs are registered as samplers

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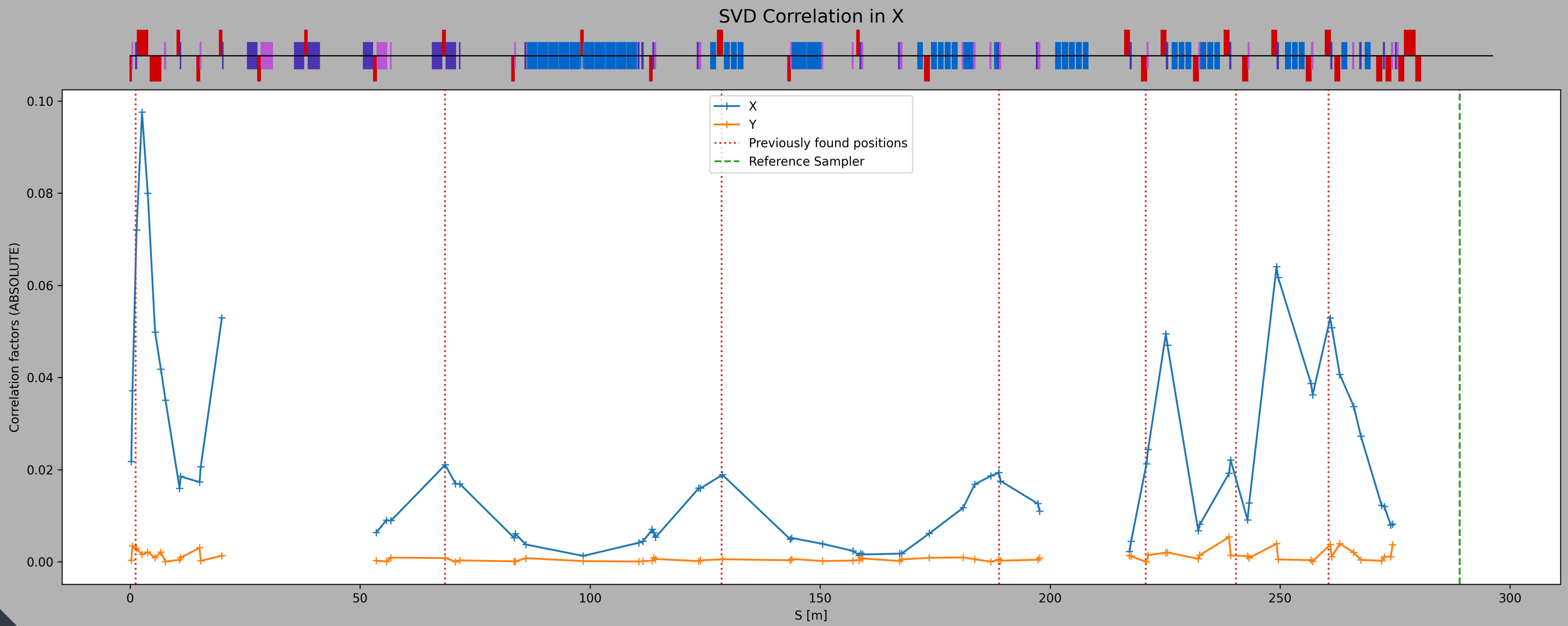
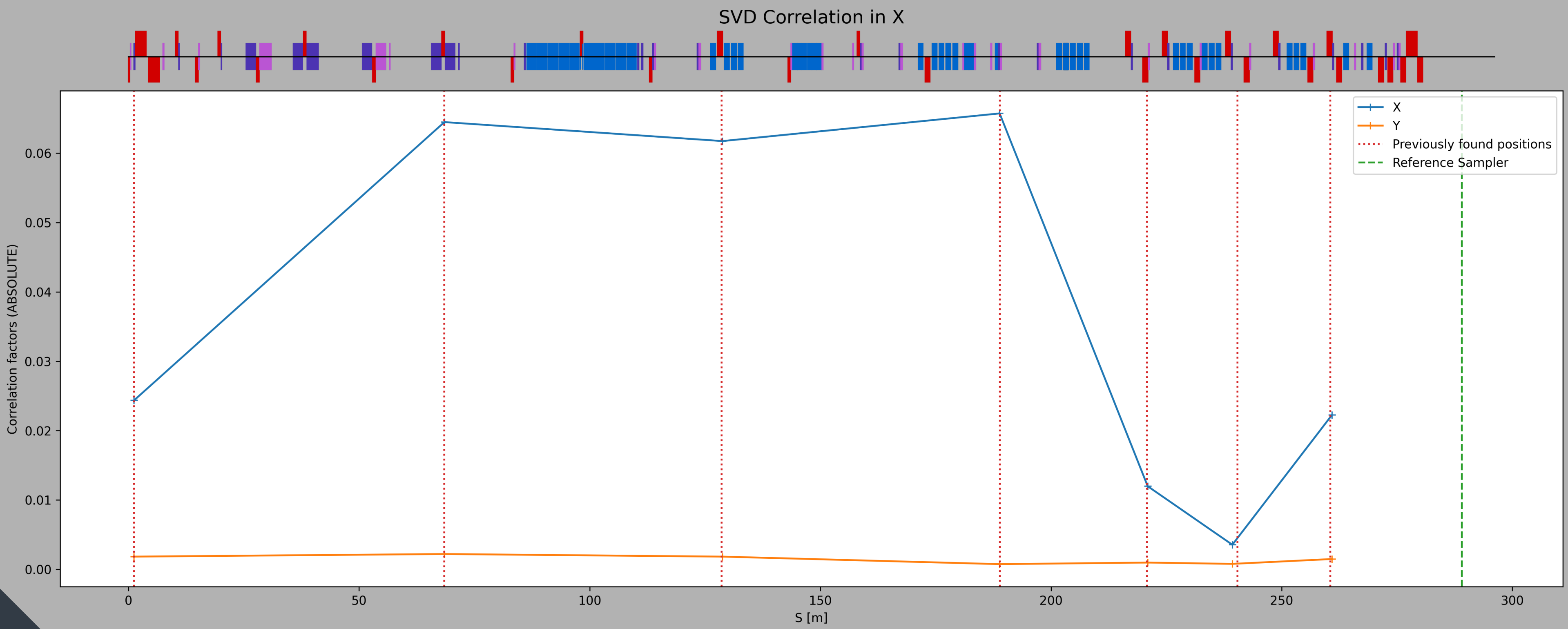
3. Singular Values Decomposition



- **Easier when doing SVD on less elements. Just a qualitative check.**

3. Singular Values Decomposition



- Now we want to extract the coefficients for each previously found S position
- We can now use them to simulate a measurement of the beam at the IP !

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4. Resolution at the IP

- Now we have all the coefficients c_s and we can flip the calculation in order to get a “measure” of a given coordinate at the IP

$$x_{IP,meas} = \sum_{s \neq s_{IP}}^{S} x_s c_s$$

- Doing this with all 10 000 particles, we can now get a histogram of the resolution at the IP :

$$x_{IP,real} - x_{IP,meas}$$

- To have an accurate simulation, we put 10 um of noise at each BPM

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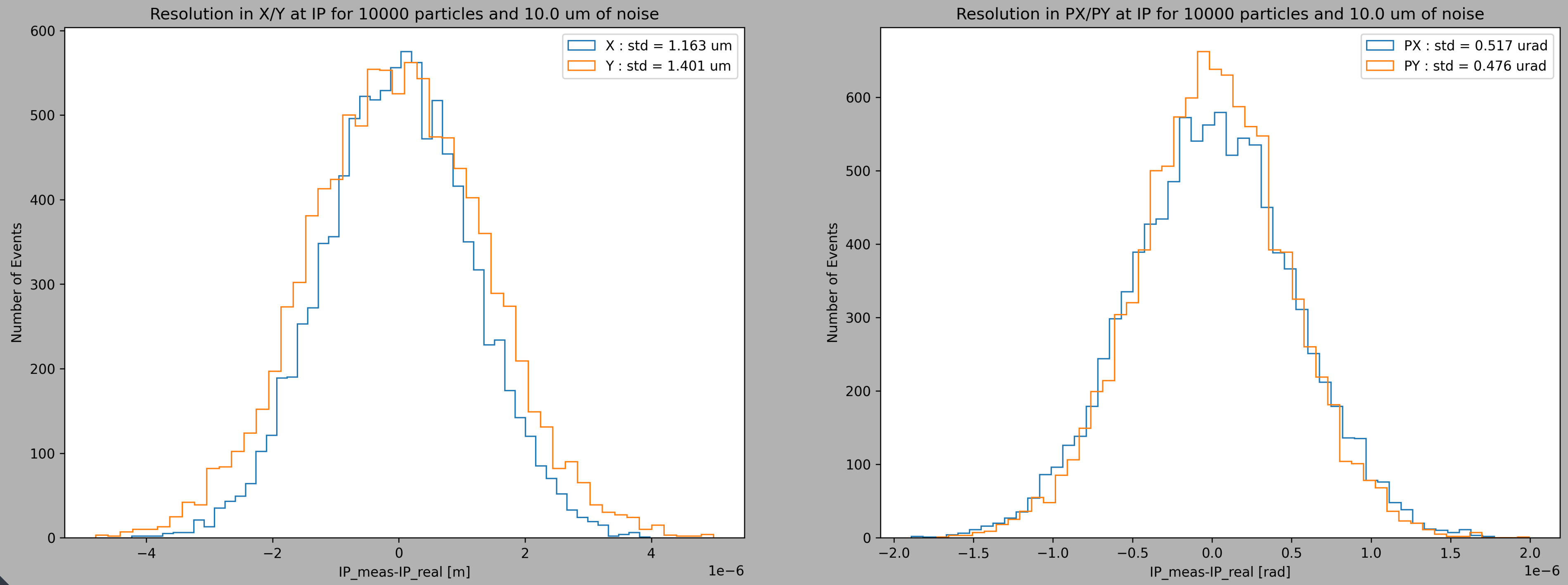
4. Resolution at the IP



- **In X/Y we have ~1-1.5 um of resolution**
- **In X'/Y' we have ~0.5 urad of resolution**

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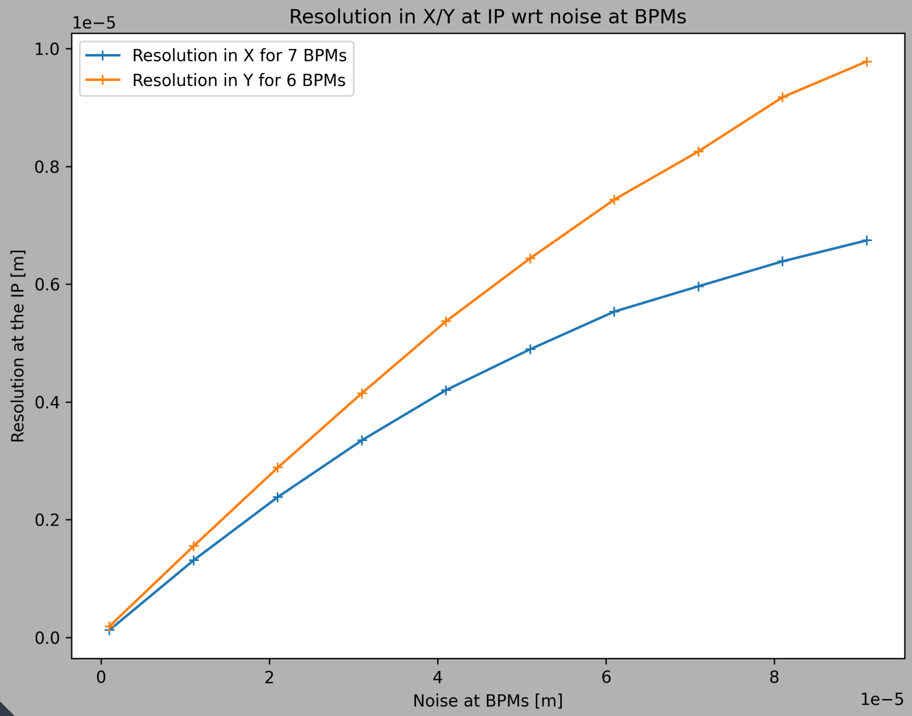
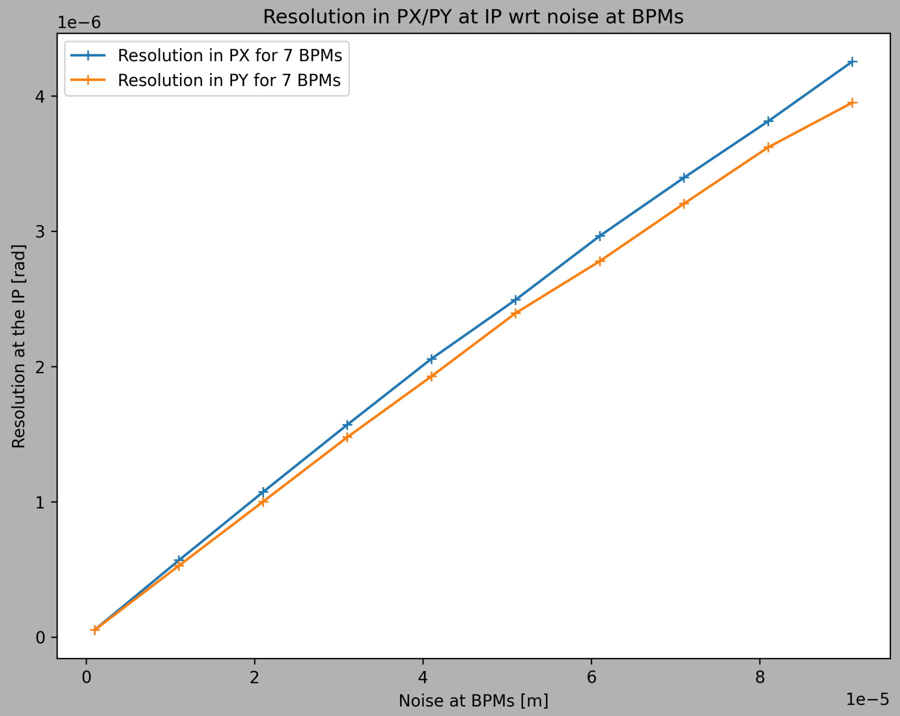
4. Resolution at the IP





- We can look at how this resolution vary w.r.t. the BPM noise

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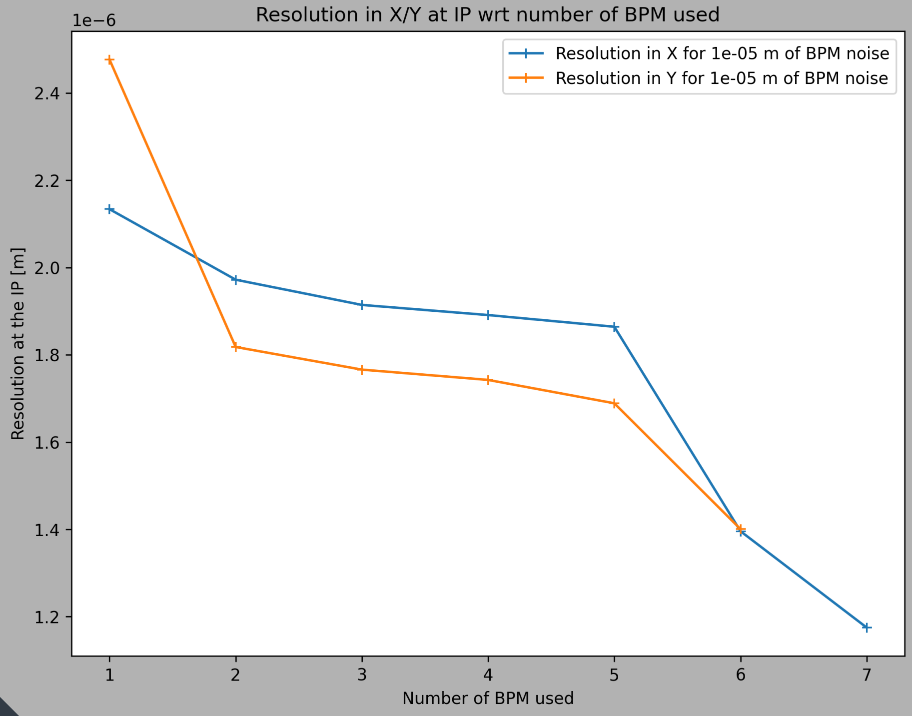
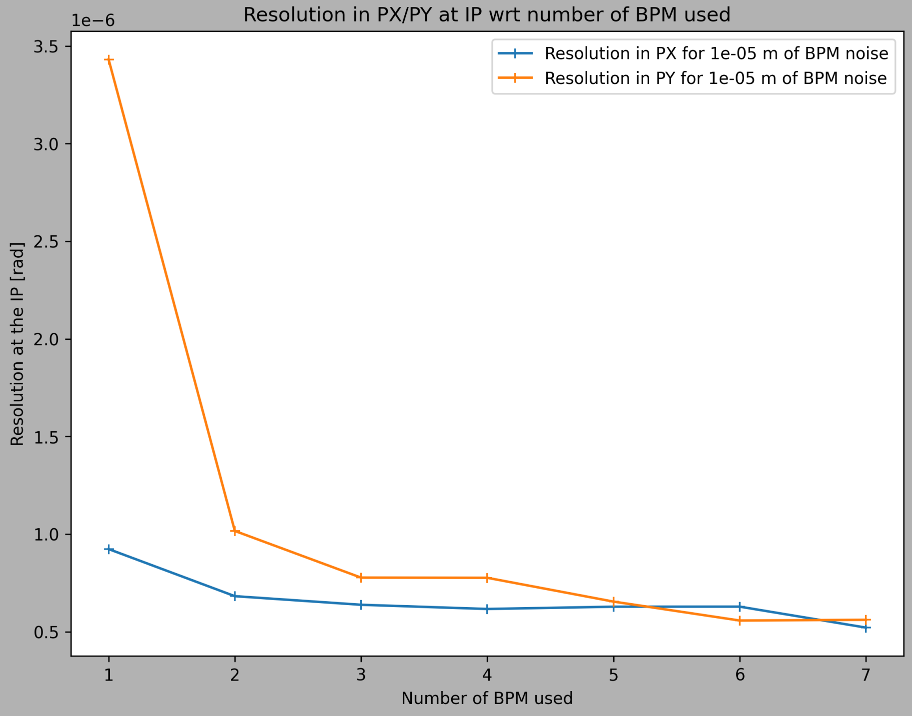
4. Resolution at the IP





- Or w.r.t the number of BPMs used for the SVD (by order of coefficients)

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5. Next objectives

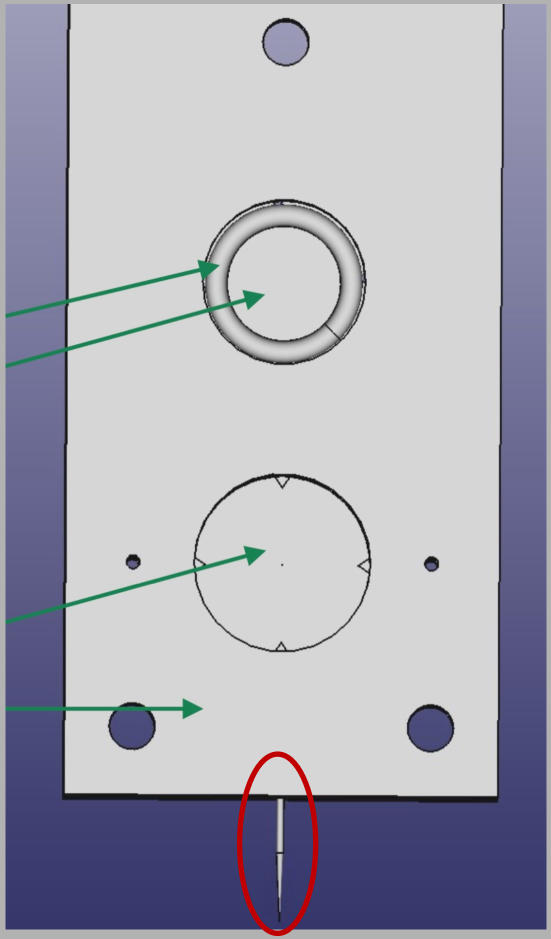
- To get the correlation coefficients from the SVD we need the positions and angles at the IP :

$$\underline{c_{IP}} = \underline{\underline{X_{IP}}}^{-1} . \underline{x_{IP}}$$

- Since we can't put a BPM at this point, we need to think about another method to make this measurement.

- Then after we've calculated the coefficients once, we only need the BPM data to calculate the beam coordinates at the IP.

5. Next objectives



- We could move a tiny needle in front of the beam.
- When in contact with the electron beam it will produce some photons
- These photons could be detected with a photodetector

5. Next objectives

TO SUMMARIZE :

- Mad8 tools for tracking studies in TL/TD20
- Chi2 method to confirm the theoretical analysis on correlation points in the lattice
- The SVD can find the corresponding coefficients in order the do a measurement at the IP
  - 1-1.5 um res in X/Y
  - 0.5 urad res in X'/Y'
- We need some measurement device at the IP for the initial calculation of the coefficients

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LUXE

John Adams Institute

BDSIM Beam Delivery Simulation

Thank you for your attention !

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BDSIM
Beam Delivery Simulation