BrokenLinesFine - General Broken Lines @ CMS

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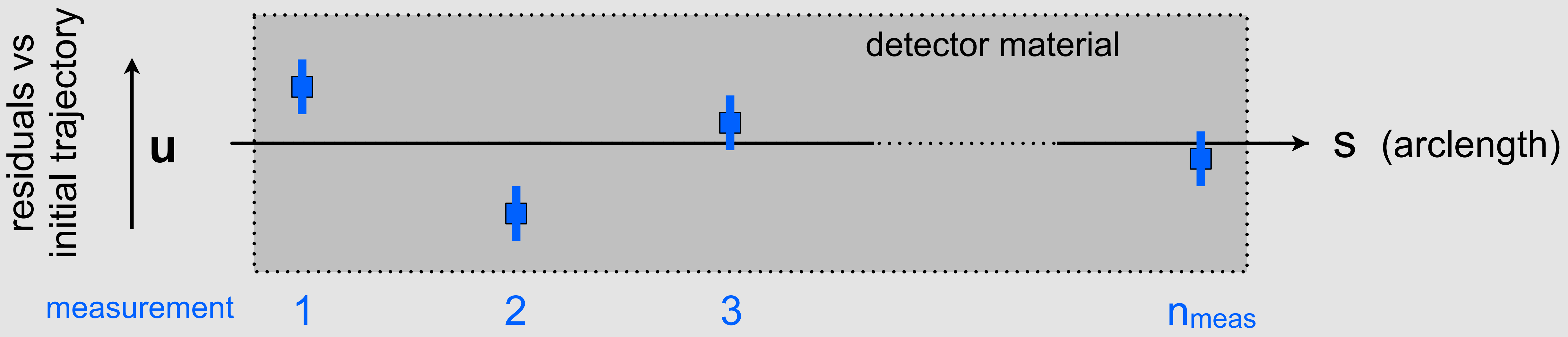
Overview

- General Broken Lines
  - Definition, construction and implementation
  - Parameter transformation
  - Energy loss
  - @CMS
- Comparison with
  - Kalman filter
  - Original Broken Lines
- Summary

Definition

- Trajectory based on 'general broken lines'
  - Track refit to add with local offsets $\mathbf{u}_i(s_i)$ the description of multiple scattering to an initial trajectory based on the propagation in a magnetic field (and energy loss)

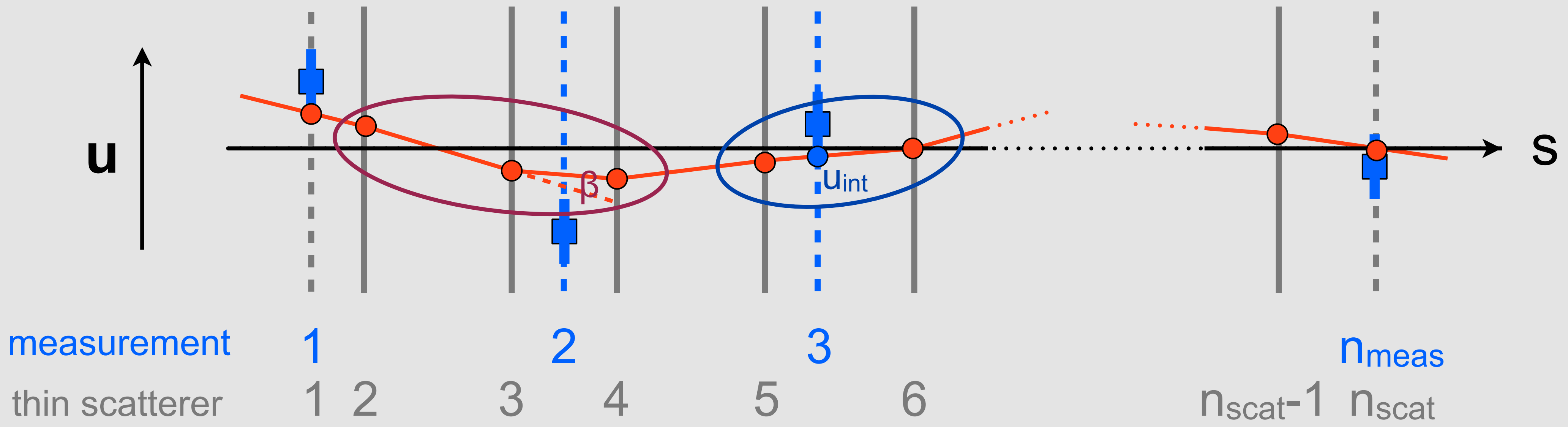


Construction (I)

- Trajectory based on 'general broken lines'
  - Refit for set of (n_{meas}) measurements **m** on a track based on sequence of (n_{scat}) thin scatterers
    - Material between adjacent measurements is in general a thick scatterer, represented by 2 thin scatterers with similar mean and RMS of material
    - Dummy thin scatterers at first and last measurement
    - Offsets $\mathbf{u}=(x_\perp,y_\perp)$ in curvilinear system at each scatterer
  - Track parameters
    - Common 'curvature' correction $\Delta q/p$
    - n_{scat} 2D offsets $\mathbf{u}$, $n_{scat} \leq 2n_{meas}$

Construction (II)

- Prediction $\mathbf{u}_{int}$ at measurement $\mathbf{m}$ from interpolation of adjacent scatterers (➜ residuals $\mathbf{r}_m = \mathbf{m} - \partial\mathbf{m}/\partial\mathbf{u} \cdot \mathbf{u}_{int}$)
- Triplets of consecutive scatterers define kinks $\boldsymbol{\beta}$
  - 2D multiple scattering angles at central scatterers
    - Expectation value zero, variance according to central scatterer
  - Additional n_{scat}-2 2D residuals ($\mathbf{r}_\beta = \boldsymbol{\beta}$)



Implementation (I)

- **Linear least squares fit**
  - Minimize
$$\chi^2(\Delta\tfrac{q}{p},\mathbf{u}_1\ldots\mathbf{u}_{n_{scat}})=\sum_{i=1}^{n_{meas}}(\vec{m}_i-\mathbf{P}_i\mathbf{u}_{\text{int},i})\mathbf{V}^{-1}_{meas,i}(\vec{m}_i-\mathbf{P}_i\mathbf{u}_{\text{int},i})^t \tag{1}$$
$$+\sum_{i=2}^{n_{scat}-1}\vec{\beta}_i(\Delta\tfrac{q}{p},\mathbf{u})\mathbf{V}^{-1}_{\beta,i}\vec{\beta}_i(\Delta\tfrac{q}{p},\mathbf{u})^t \qquad \mathbf{P}=\frac{\partial\vec{m}}{\partial\mathbf{u}}$$
  - Need $\partial \mathbf{u}_{\text{int},i}/\partial \mathbf{x}$, $\partial \boldsymbol{\beta}_i/\partial \mathbf{x}$, $\mathbf{x}=(\Delta q/p,..,\mathbf{u}_j,..)$ to get corresponding linear equation system $\mathbf{A}\cdot\mathbf{x}=\mathbf{b}$
    - $\mathbf{u}_{\text{int},i}$, $\boldsymbol{\beta}_i$ depend only on few (≤3) adjacent $\mathbf{u}_j$
    - Matrix **A** has band structure with band width m≤5
    - Fast solution by root free Cholesky decomposition (**A=LDL**†)
      (effort to calculate **x**: $\sim n_{par}\cdot m^2$, $\mathbf{A}^{-1}$: $\sim n_{par}^2\cdot m$, by inversion: $\sim n_{par}^3$)

Implementation (II)

- Using curvilinear Jacobian

$$\frac{\partial curv}{\partial curv_0} = \frac{\partial\left(\frac{q}{p}, \lambda, \phi, x_\perp, y_\perp\right)}{\partial\left(\frac{q}{p_0}, \lambda_0, \phi_0, x_{\perp 0}, y_{\perp 0}\right)}$$

- Propagation of offset Δu
  - with initial offset $\Delta \mathbf{u}_0$, slope $\Delta \mathbf{a}_0$, curvature $\Delta q/p_0$:

(2)
$$\Delta\mathbf{u} = \frac{\partial\mathbf{u}}{\partial\mathbf{u}_0}\Delta\mathbf{u}_0 + \frac{\partial\mathbf{u}}{\partial\vec{\alpha}_0}\Delta\vec{\alpha}_0 + \frac{\partial\mathbf{u}}{\partial\frac{q}{p_0}}\Delta\frac{q}{p_0} = \mathbf{J}\Delta\mathbf{u}_0 + \mathbf{S}\Delta\vec{\alpha}_0 + \mathbf{d}\Delta\frac{q}{p_0}$$

(3)
$$\frac{\partial\mathbf{u}}{\partial\mathbf{a}_0} = \frac{\partial\mathbf{u}}{\partial curv}\frac{\partial curv}{\partial curv_0}\frac{\partial curv_0}{\partial\mathbf{a}_0}$$

(4)
$$\frac{\partial\mathbf{u}}{\partial curv} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\frac{\partial curv_0}{\partial\mathbf{u}_0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \frac{\partial curv_0}{\partial\vec{\alpha}_0} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ \frac{1}{\cos\lambda_0} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \frac{\partial curv_0}{\partial\Delta\frac{q}{p_0}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(5)
- Solve for Δa:

$$\Delta\vec{\alpha}_0 = \mathbf{S}^{-1}\left(\Delta\mathbf{u} - \mathbf{J}\Delta\mathbf{u}_0 - \mathbf{d}\Delta\frac{q}{p_0}\right)$$

Implementation (III)

- Triplet $\mathbf{u}_-$, $\mathbf{u}_0$, $\mathbf{u}_+$ of offsets

(6) $$\mathbf{u}_+ = \mathbf{J}_+\mathbf{u}_0 + \mathbf{S}_+\Delta\vec{\alpha}_+ + \mathbf{d}_+\Delta\tfrac{q}{p},\quad \Delta\vec{\alpha}_+ = \mathbf{W}_+\left(\mathbf{u}_+ - \mathbf{J}_+\mathbf{u}_0 - \mathbf{d}_+\Delta\tfrac{q}{p}\right),\quad \mathbf{W}_+ = \mathbf{S}_+^{-1}$$

(7) $$\mathbf{u}_- = \mathbf{J}_-\mathbf{u}_0 + \mathbf{S}_-\Delta\vec{\alpha}_- + \mathbf{d}_-\Delta\tfrac{q}{p},\quad \Delta\vec{\alpha}_- = \mathbf{W}_-\left(\mathbf{J}_-\mathbf{u}_0 - \mathbf{u}_- + \mathbf{d}_-\Delta\tfrac{q}{p}\right),\quad \mathbf{W}_- = -\mathbf{S}_-^{-1}$$

- Kink β

(8) $$\vec{\beta} = \Delta\vec{\alpha}_+ - \Delta\vec{\alpha}_- = \mathbf{W}_+\mathbf{u}_+ - \left(\mathbf{W}_+\mathbf{J}_+ + \mathbf{W}_-\mathbf{J}_-\right)\mathbf{u}_0 + \mathbf{W}_-\mathbf{u}_- - \left(\mathbf{W}_+\mathbf{d}_+ + \mathbf{W}_-\mathbf{d}_-\right)\Delta\tfrac{q}{p}$$

- Interpolation (solve (8) for $\mathbf{u}_{int}=\mathbf{u}_0$ with $\beta\equiv0$)

(9) $$\mathbf{u}_{\text{int}} = \mathbf{N}\left(\mathbf{W}_+\mathbf{u}_+ + \mathbf{W}_-\mathbf{u}_-\right) - \mathbf{N}\left(\mathbf{W}_+\mathbf{d}_+ + \mathbf{W}_-\mathbf{d}_-\right)\Delta\tfrac{q}{p},\quad \mathbf{N} = \left(\mathbf{W}_+\mathbf{J}_+ + \mathbf{W}_-\mathbf{J}_-\right)^{-1}$$

Parameter transformation

- From broken lines to (local) curvilinear
  - Point between adjacent scatterers (offsets $\mathbf{u}_-$, $\mathbf{u}_+$)
    - Offset $\mathbf{u}_{int}$ interpolated according to (9)
    - Slope α_{int} from (6) or (7) using $\mathbf{u}_0 = \mathbf{u}_{int}$

(10) $$\vec{\alpha}_{\text{int}} = \mathbf{W}_-\mathbf{J}_-\mathbf{N}\mathbf{W}_+\mathbf{u}_+ - \mathbf{W}_+\mathbf{J}_+\mathbf{N}\mathbf{W}_-\mathbf{u}_- - \left(\mathbf{W}_-\mathbf{J}_-\mathbf{N}\mathbf{W}_+\mathbf{d}_+ - \mathbf{W}_+\mathbf{J}_+\mathbf{N}\mathbf{W}_-\mathbf{d}_-\right)\Delta\tfrac{q}{p}$$

(11)
  - Transformation $$\frac{\partial\left(\frac{q}{p},\lambda,\phi,x_\perp,y_\perp\right)}{\partial\left(\Delta\frac{q}{p},\mathbf{u}_-,\mathbf{u}_+\right)} = \frac{\partial\left(\frac{q}{p},\lambda,\phi,x_\perp,y_\perp\right)}{\partial\left(\Delta\frac{q}{p},\vec{\alpha}_{\text{int}},\mathbf{u}_{\text{int}}\right)}\frac{\partial\left(\Delta\frac{q}{p},\vec{\alpha}_{\text{int}},\mathbf{u}_{\text{int}}\right)}{\partial\left(\Delta\frac{q}{p},\mathbf{u}_-,\mathbf{u}_+\right)}$$

$$\frac{\partial\left(\frac{q}{p},\lambda,\phi,x_\perp,y_\perp\right)}{\partial\left(\Delta\frac{q}{p},\vec{\alpha}_{\text{int}},\mathbf{u}_{\text{int}}\right)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{\cos\lambda} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\frac{\partial\left(\Delta\frac{q}{p},\vec{\alpha}_{\text{int}},\mathbf{u}_{\text{int}}\right)}{\partial\left(\Delta\frac{q}{p},\mathbf{u}_-,\mathbf{u}_+\right)}$$ from (9), (10)

Energy loss

- Implicit in curvilinear Jacobian: $q/p \neq q/p_0$

(12) $$\frac{\partial \frac{q}{p}}{\partial E} = \frac{-q \cdot E}{p^3}, \quad \frac{\partial \frac{q}{p}}{\partial \frac{q}{p_0}} \approx 1 - \frac{E \cdot \overbrace{\left(E - E_0\right)}^{\Delta E}}{p^2}$$

- Explicit
  - Replace 'Δq/p' in (6) by 'Δq/p+δq/p': $\delta \frac{q}{p} = \frac{\partial \frac{q}{p}}{\partial E} \Delta E$
    - Use fixed ΔE ➜ offset in β
    - Use ΔE as local or global fit parameter in MillePede

Broken Lines @ CMS

- CMS silicon tracker: simple case
  - Only thin scatterers
  - Measurement layer = scattering layer (Si sensor)
  - **BrokenLinesFine**
    - AnalyticalCurvilinearJacobian (Helix, see backup)
  - **BrokenLinesCoarse**
    - Limit $q/p \rightarrow 0$, $B=(0,0,B_z)$
    - description of double sided module as one layer
    - to reduce number of parameters (cpu time) and non zero derivatives (size of binary file)

Broken Lines vs Kalman filter

- Equivalent trajectory descriptions

	Kalman filter	Broken Lines
procedure	**progressive**, add measurement or noise one at a time (weighted average)	**one** linear least squares **fit** to all hits and scatterers (Cholesky decomp.)
seeding by	**few** hits	**all** hits (prefit)
covariance matrix	for **local** track parameters only (W. Hulsbergen, LHCb: extended to all)	for **all** track parameters
application (CMS)	general track finding and fitting	local fit for track based alignment with MP-II

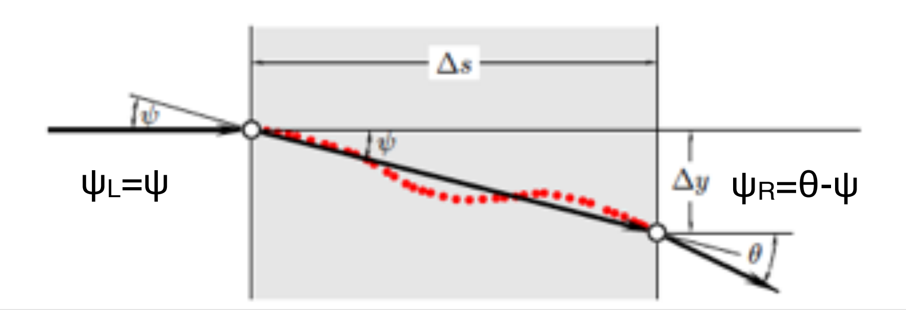
Original vs General Broken Lines

★ Original Broken Lines by V. Blobel (NIM-A, 566:14, 2006)

	Original Broken Lines	General Broken Lines
dimension	**2D** (independent tracking in Rϕ, ZS)	**3D** (include 'stereo' measurements)
trajectory definition	**measurement** planes	**scattering** planes
multiple scattering	**thick** scatterers (ψ_L,ψ_R), neglect correlation of (ψ_L,ψ_R)	**thin** scatterers (ψ_L,ψ_R)=(θ,0) (no correlation), thick = 2 thin
offsets	$u_i(s_i)$: **1D**, measurement **AND** scattering	$u_i(s_i)$: **2D**, measurement **OR** scattering (or both) (scatterer without measurement, meas. without scattering: interpolated between adjacent scatterers)
minimize	$\chi^2(\kappa,u_1,\ldots,u_{nmeas}) = \sum_{i=1}^{nmeas}(m_1-u_1)^2/\sigma_i^2 + \sum_{i=2}^{nmeas-1}\beta_i^2/\sigma_{\beta i}^2$	$\chi^2(\kappa,\vec{u}_1,\ldots,\vec{u}_{nscatt}) = \sum_{i=1}^{nmeas}(m_i-\mathbf{P}_i\bullet\vec{u}_{\mathrm{int},i})^2/\sigma_i^2 + \sum_{i=2}^{nscatt-1}\vec{\beta}_i^2/\sigma_{\beta i}^2$

$$V(\psi_L,\psi_R) = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{pmatrix}\theta_0^2$$

for homogeneous scatterer



$$V(\psi_L,\psi_R) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\theta_0^2$$

for thin scatterer

Summary

- General Broken Lines constructed from curvilinear Jacobian

- General Broken Lines implemented in BrokenLinesFine
  - Improvement in χ^2/ndf of few permille for $p < 10$ GeV vs previous implementation (ideal MC cosmics)

Backup

Helix (const magnetic field) (I)

- **Conventions:** A. Strandlie, W. Wittek, NIM A, 566 (2006) 687-698
  - 3D arclength s, deflection angle θ = Q·s, Q=-|**B**|q/p
  - Direction of Track **T**, magnetic field **H** $\mathbf{T} = \begin{pmatrix} \cos\varphi \sin\vartheta \\ \sin\varphi \sin\vartheta \\ \cos\vartheta \end{pmatrix}$
- **Derivatives**

$$\mathbf{J} = \begin{pmatrix} \mathbf{U}_0 \cdot \mathbf{U} & \mathbf{V}_0 \cdot \mathbf{U} \\ \mathbf{U}_0 \cdot \mathbf{V} & \mathbf{V}_0 \cdot \mathbf{V} \end{pmatrix}$$

(13) $$Q \cdot \mathbf{S} = \sin\theta \cdot \mathbf{J} + \underbrace{(1-\cos\theta)}_{2\sin\left(\frac{\theta}{2}\right)^2} \begin{pmatrix} (\mathbf{H}\times\mathbf{U}_0)\cdot\mathbf{U} & (\mathbf{H}\times\mathbf{V}_0)\cdot\mathbf{U} \\ (\mathbf{H}\times\mathbf{U}_0)\cdot\mathbf{V} & (\mathbf{H}\times\mathbf{V}_0)\cdot\mathbf{V} \end{pmatrix} + (\theta - \sin\theta) \begin{pmatrix} (\mathbf{H}\cdot\mathbf{U}_0)(\mathbf{H}\cdot\mathbf{U}) & (\mathbf{H}\cdot\mathbf{V}_0)(\mathbf{H}\cdot\mathbf{U}) \\ (\mathbf{H}\cdot\mathbf{U}_0)(\mathbf{H}\cdot\mathbf{V}) & (\mathbf{H}\cdot\mathbf{V}_0)(\mathbf{H}\cdot\mathbf{V}) \end{pmatrix}$$

$$\frac{Q^2}{|\mathbf{B}|} \cdot \mathbf{d} = \sin\theta \begin{pmatrix} \mathbf{T}_0 \cdot \mathbf{U} \\ \mathbf{T}_0 \cdot \mathbf{V} \end{pmatrix} + (1-\cos\theta) \begin{pmatrix} (\mathbf{H}\times\mathbf{T}_0)\cdot\mathbf{U} \\ (\mathbf{H}\times\mathbf{T}_0)\cdot\mathbf{V} \end{pmatrix} + (\theta - \sin\theta) \begin{pmatrix} (\mathbf{H}\cdot\mathbf{T}_0)(\mathbf{H}\cdot\mathbf{U}) \\ (\mathbf{H}\cdot\mathbf{T}_0)(\mathbf{H}\cdot\mathbf{V}) \end{pmatrix}$$

(14) - small q/p (limit Q →0) $\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{S} = s \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{d} = -\frac{1}{2}|\mathbf{B}| s^2 \begin{pmatrix} (\mathbf{H}\times\mathbf{T}_0)\cdot\mathbf{U} \\ (\mathbf{H}\times\mathbf{T}_0)\cdot\mathbf{V} \end{pmatrix}$

Helix (II)

- Derivatives for perfect solenoid **(H=(0,0,1))**

$$\mathbf{J} = \begin{pmatrix} \cos\theta & \sin\theta\cos\vartheta \\ -\sin\theta\cos\vartheta & \cos\theta\cos\vartheta^2 + \sin\vartheta^2 \end{pmatrix}$$

(15) $$Q\cdot\mathbf{S} = \sin\theta\cdot\mathbf{J} + (1-\cos\theta)\begin{pmatrix} \sin\theta & -\cos\theta\cos\vartheta \\ \cos\theta\cos\vartheta & \sin\theta\cos\vartheta^2 \end{pmatrix} + (\theta-\sin\theta)\begin{pmatrix} 0 & 0 \\ 0 & \sin\vartheta^2 \end{pmatrix}$$

$$\tfrac{Q^2}{|\mathbf{B}|}\cdot\mathbf{d} = \sin\theta\begin{pmatrix} -\sin\theta\sin\vartheta \\ (1-\cos\theta)\cos\vartheta\sin\vartheta \end{pmatrix} + (1-\cos\theta)\begin{pmatrix} \cos\theta\sin\vartheta \\ -\sin\theta\cos\vartheta\sin\vartheta \end{pmatrix} + (\theta-\sin\theta)\begin{pmatrix} 0 \\ \cos\vartheta\sin\vartheta \end{pmatrix}$$

  - small q/p (limit Q →0)

(16) $$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{S} = s\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{d} = -\tfrac{1}{2}|\mathbf{B}|\, s^2\begin{pmatrix} \sin\vartheta \\ 0 \end{pmatrix}$$ (BrokenLinesCoarse)