N3LO Drell-Yan cross-sections confronts data

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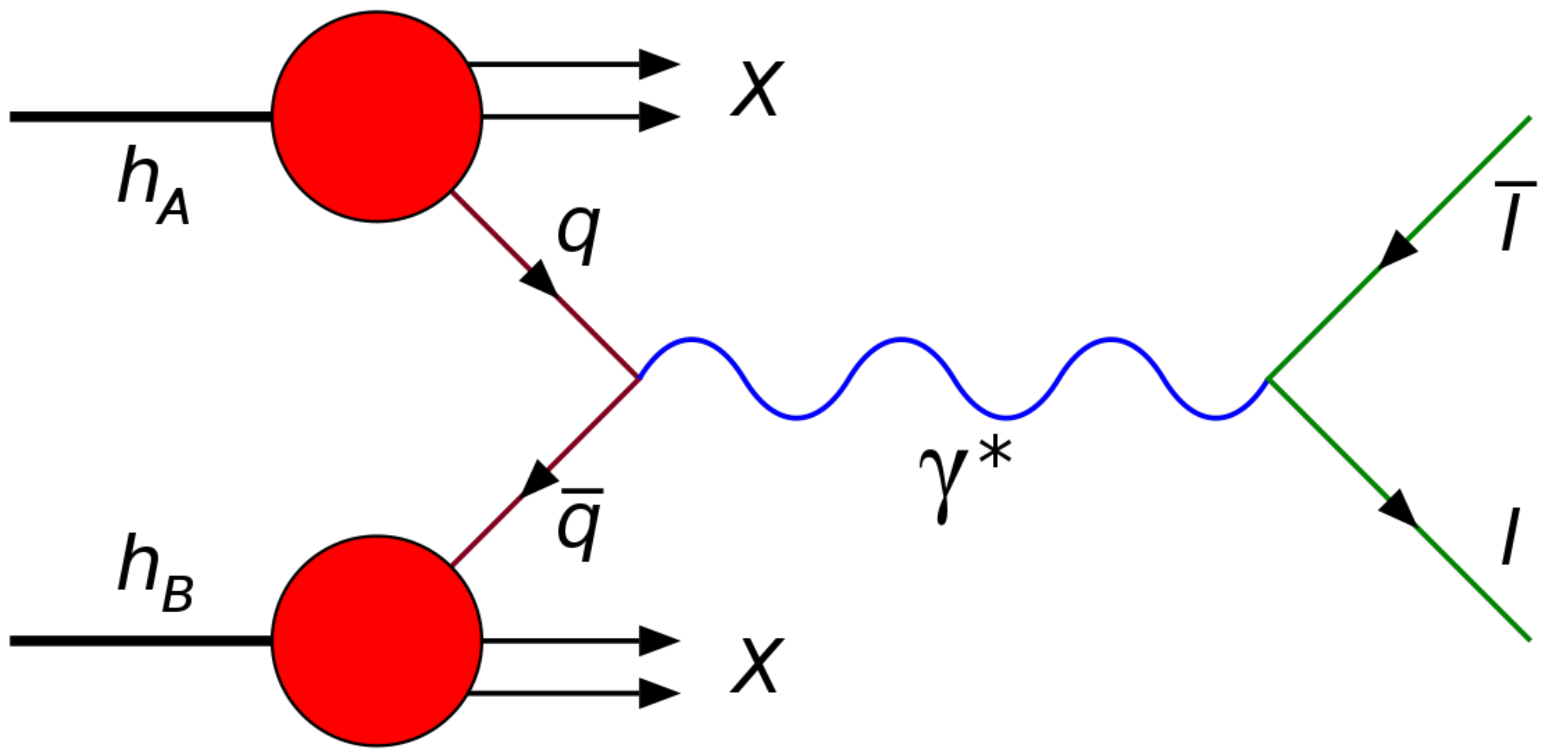
New perspectives in Conformal Field Theory and Gravity - DESY

27th September 2023

Outline

- The Drell-Yan processes Historical intro
- Bleeding-edge Drell-Yan predictions
- Scope of this work
- Simulation setup
- Data confronting theory
- Outlook

The Drell-Yan process



Drell-Yan processes play a fundamental role in advancing our understanding of subatomic particles and their interactions:

- **Test theoretical predictions** + **determination** of **fundamental parameters** of the SM: α_s, m_W, $\sin^2\theta_W$, …
- **Probing PDFs**: increased data/prediction accuracy → increased PDFs determination
- **Constraint New Physics**: extremely tiny deviations could be visible from extremely precise measurements, EFT approach

Precise Drell-Yan predictions

$$\frac{d\sigma}{dQ^2} = \sum_{i,j} \int_0^1 dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \cdot \hat{\sigma}_{i,j}(z, \mu_F, \mu_R)$$

- Increase the precision of our predictions → reduce theoretical uncertainties

- Factorisation theorem:

  - **Parton Distribution Functions (PDFs)**: non-computable in pert. theory (fits to data)

  - **Partonic cross-section**: missing higher-order contributions

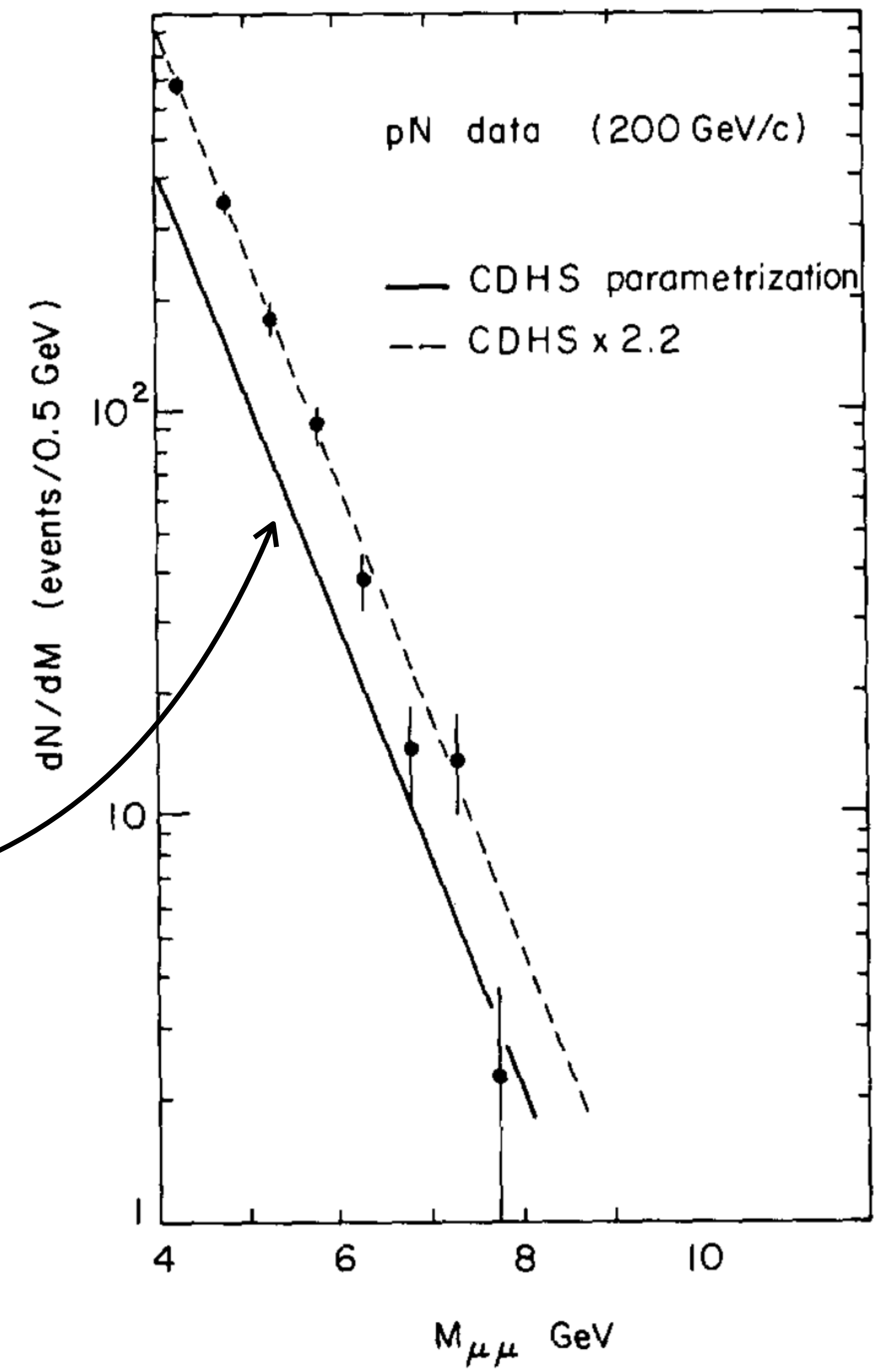
    - QCD: “captured” by μ_R and μ_F scale variations

    - EW: include weak and QED corrections

Drell-Yan: data and K-factors

- Observation from the beginning: data lay above the naive DY prediction → introduction of a "K-factor"

$$\left(\frac{d\sigma}{dQ^2}\right)_{\text{meas.}} = k\cdot\left(\frac{d\sigma}{dQ^2}\right)_{\text{pred.}}$$

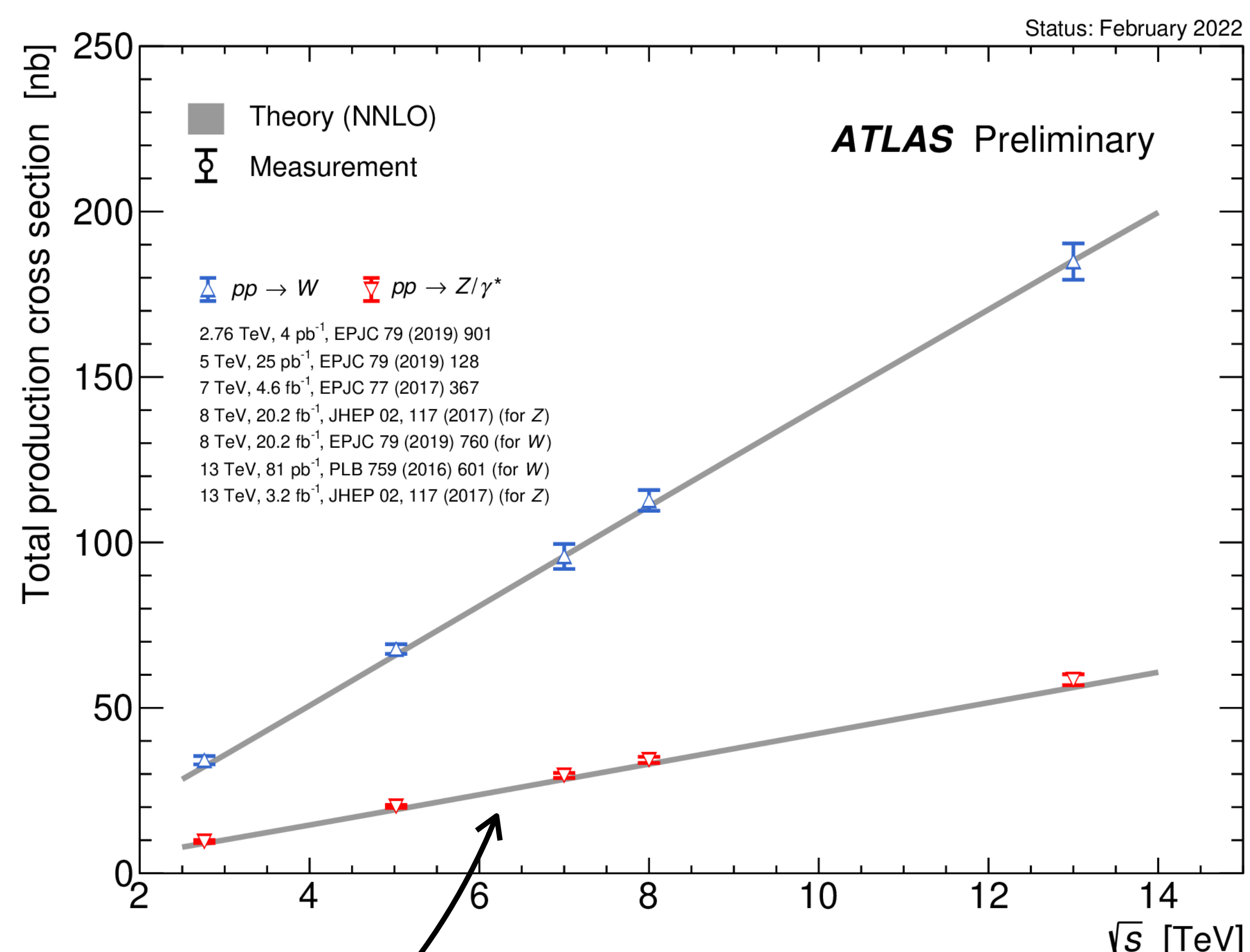


- "Simple" tree-level predictions are not enough to describe the data!

- What happens when including higher-order corrections (e.g. beyond LO)?

Drell-Yan: data and K-factors

- State-of-the-art: NNLO QCD fiducial predictions
- First NNLO QCD predictions for the Drell-Yan process were published by Petriello et al in 2004 [hep-ph/0404184]
- Nowadays widely used by experimental collaborations (e.g. at the LHC)
- Several publicly available PDF sets available at NNLO → fully consistent predictions
- Good agreement with experimental data



Bleeding-edge predictions

Partonic cross-section:

$$\frac{d\sigma}{dQ^2} = \sum_{i,j} \int_0^1 dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \cdot \boxed{\hat{\sigma}_{i,j}(z, \mu_F, \mu_R)}$$

(Inclusive) N^{3}LO QCD
computations recently available

C. Duhr, F. Dulat, and B. Mistlberger, "The Drell-Yan cross section to third order in the strong coupling constant". Phys. Rev. Lett. 125 (2020) 172001. DOI: 10.1103/PhysRevLett.125.172001, arXiv:2001.07717.

C. Duhr, F. Dulat, and B. Mistlberger, "Charged Current Drell-Yan Production at N3LO". JHEP 11 (2020) 143. DOI: 10.1007/JHEP11(2020)143, arXiv:2007.13313.

C. Duhr and B. Mistlberger, "Lepton-pair production at hadron colliders at N3LO in QCD". JHEP 03 (2022) 116. DOI: 10.1007/JHEP03(2022)116, arXiv:2111.10379.

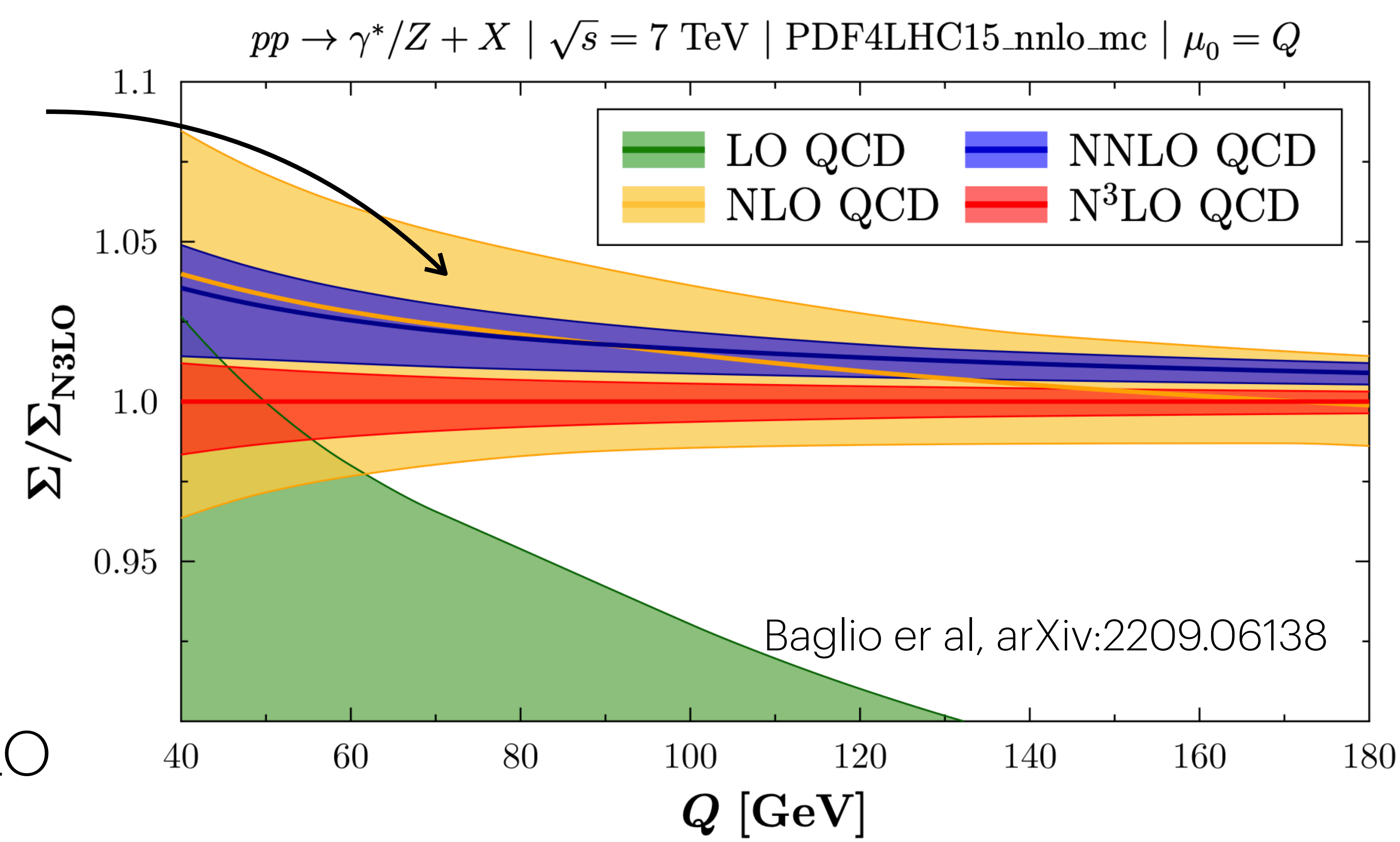
+ Others …

Bleeding-edge predictions

Partonic cross-section:

$$\frac{d\sigma}{dQ^2} = \sum_{i,j} \int_0^1 dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \cdot \boxed{\hat{\sigma}_{i,j}(z, \mu_F, \mu_R)}$$

- **No overlap** between NNLO and N^3LO bands:
  - NNLO PDFs?
  - Missing contributions at ME?
- Scale unc. seems not to decrease from NNLO to N^3LO



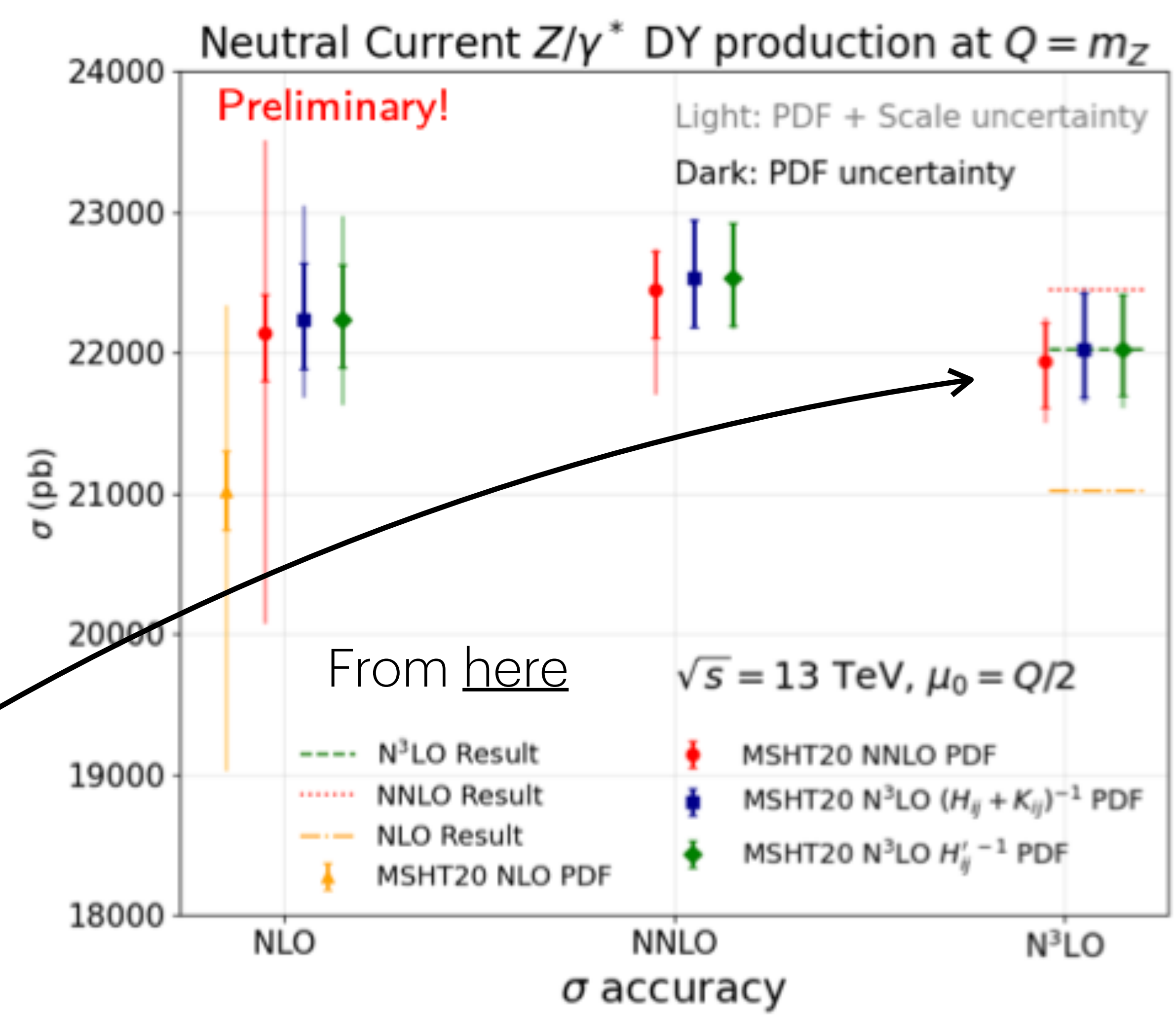
Bleeding-edge predictions

Parton distribution functions:

$$\frac{d\sigma}{dQ^2} = \sum_{i,j} \int_0^1 dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \cdot \hat{\sigma}_{i,j}(z, \mu_F, \mu_R)$$

(approx.) N^{3}LO PDFs from the MSHT group recently available

T. Cridge et al, arXiv:2207.04739



Small change in using aN^{3}LO PDFs relative to NNLO PDFs

Scope of this work

- Compare inclusive DY cross-section measurements to bleeding-edge predictions (N^3LO QCD + NLO EW)

aN^3LO PDFs sets + N^3LO ME $\rightarrow$ fully consistent theoretical prediction

- LHC offers precise measurements at different energies (probe different x)
- An important benchmark for (new) ME and PDFs at N^3LO
- Inspired by similar work by S. Moch et. al arXiv:1011.6259 (data compared to NNLO predictions)

Why inclusive cross-sections?

Experimental cross-sections are measured in a fiducial volume (e.g. by applying kinematic requirements on the lepton(s) kinematics variable)

Fiducial predictions at N^{3}LO are possible within the q_T subtraction framework (remove divergences associated with small transverse momenta) $\rightarrow$ DYTurbo, Radish+NNLOJet, MCFM, $\ldots$

But:

- **Very easy to compute inclusive predictions**: fiducial predictions require (very) slow NNLO V + Jet(s) piece
- **Inclusive predictions are theoretically "cleaner"**: no need to care for fiducial power corrections, q_T resummation effects, $\ldots$

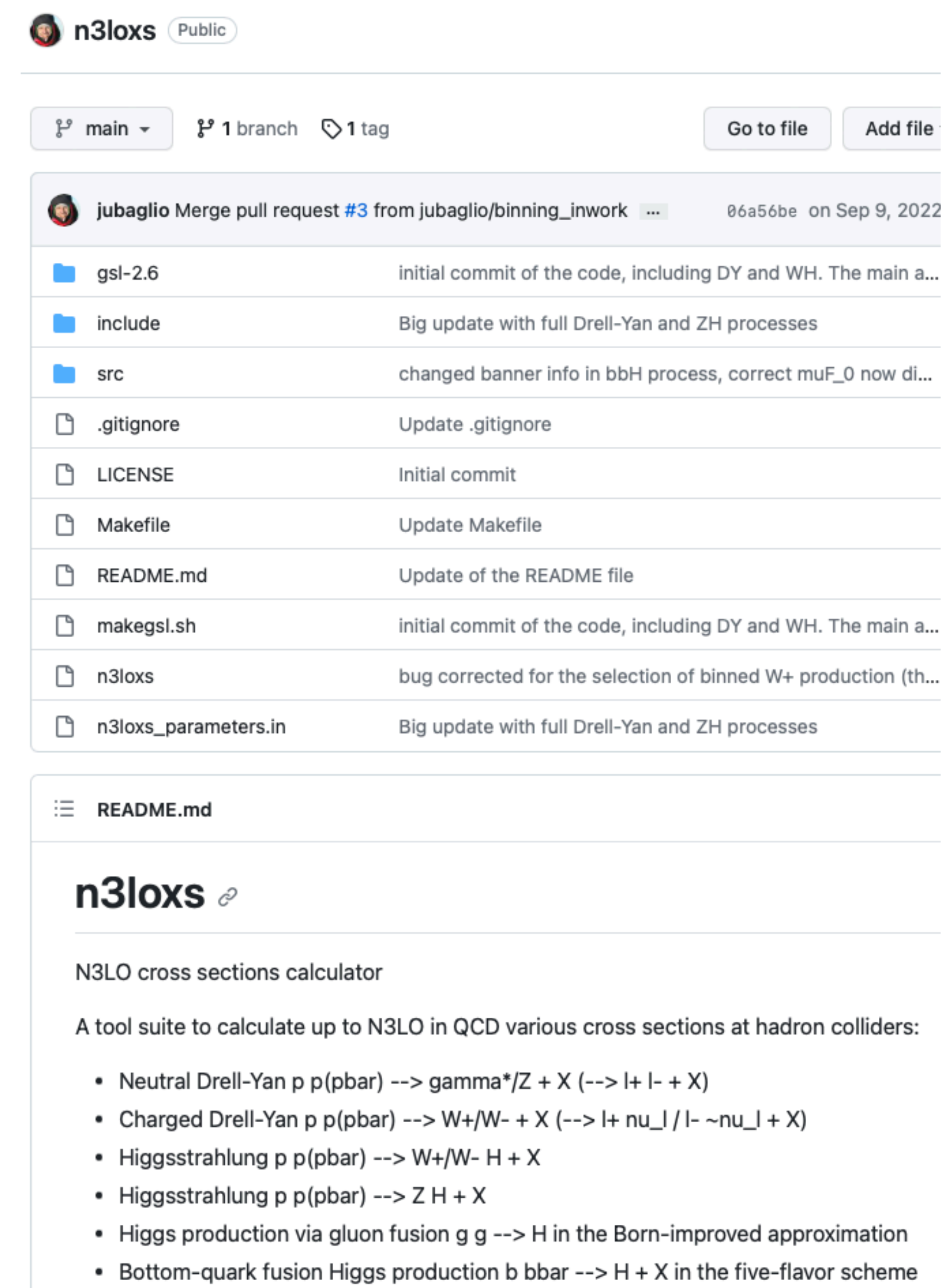
The N3LOXS Matrix Element computation tool

- n3loxs tool (Baglio, Duhr, Mistlberger and Szafron, arXiv:2209.0613)

- Calculate cross sections up to N^{3}LO in QCD at hadron colliders (NC and CC Drell-Yan but also Higgsstrahlung, Higgs production via gluon fusion …)

- Drell-Yan calculation based on:
  - “The Drell-Yan cross section to third order in the strong coupling constant” arXiv:2001.07717
  - “Charged Current Drell-Yan Production at N3LO” arXiv:2007.13313
  - “Lepton-pair production at hadron colliders at N3LO in QCD” arXiv:2111.10379

- Quasi-Monte-Carlo (QMC) integration as implemented by arXiv:1811.11720



Simulation setup

- Drell-Yan processes calculated for an off-shell gauge boson at a given virtuality $\mu_R^0 = \mu_F^0 = Q$ (= off-shellness of the intermediate vector boson)

- Envelope of 7-point μ_R and μ_F scales (scales uncertainties)

- PDFs uncertainties included

- Integrated cross section calculated in Q bin range:

  - **NCDY**: $60 < Q < 120\ \mathrm{GeV}$

  - **CCDY**: $10 < Q < 200\ \mathrm{GeV}$ (ideally not needed, used to minimise execution code time. Predictions practically insensitive to larger intervals)

Simulation setup

- EW scheme used: $(\alpha_{\mathrm{EM}}(0), m_{\mathrm{Z}}, m_{\mathrm{W}})$

- Input Electroweak parameters:

```
MW        = 80.377
GammaW    = 2.085
MZ        = 91.1876
GammaZ    = 2.4952
MH        = 125.09
Mt        = 172.5
Mb        = 4.58
mt(mt)    = 162.7
mb(mb)    = 4.18
vev       = 246.221
1/alpha =  132.23229791
```

PDFs sets considered

- Only available N^3LO PDF from the MSHT group → from the same group sets at every order included ...
- ... + all other available PDFs at NNLO
- Include NNPDF N^3LO PDF set once available

- MSHT20lo_as130
- MSHT20nlo_as118
- MSHT20nnlo_as118
- MSHT20an3lo_as118
- NNPDF31_nnlo_as_0118_hessian
- NNPDF40_nnlo_as_01180_hessian
- CT18NNLO
- CT18ANNLO
- ABMP16als118_5_nnlo
- ATLASpdf21_T3

A word on NLO EW corrections

- NLO EW corrections must be included to have the full picture → $\alpha_S^2 \sim \alpha_{\mathrm{EW}}$
- NLO EW corrections not yet integrated into our work → **crucial step forward**
- Experimental measurements → leptons are corrected to the Born level (pre-FSR)
- Simulation → separates ISR, weak and IFI contributions from FSR
  - Gauge invariance helps separate ISR+weak+IFI components from FSR for the Z boson
  - Not gauge invariant for the W (FSR interplay) but can be approximated
- Employed powheg-ew for estimating (preliminary) EW corrections
  - ~6% effects on inclusive cross-sections
  - K factors almost energy-independent
- Plan to utilise ReneSANCe for W boson

Inclusive cross-sections: predictions

$$pp \to Z/\gamma^* + X$$

$\sqrt{s}$ [TeV]	σ_{LO} [pb]	σ_{NLO} [pb]	K^{NLO}	σ_{NNLO} [pb]	K^{NNLO}	σ_{N^3LO} [pb]	K^{N^3LO}
2.76	231.5	301.5	1.3	311.1	1.03	313.3	1.01
5.02	503.9	637.2	1.26	661.7	1.04	659.1	1.0
7.0	754.1	938.5	1.24	977.7	1.04	968.0	0.99
8.0	882.6	1091.4	1.24	1138.2	1.04	1124.4	0.99
13.0	1536.9	1853.1	1.21	1936.6	1.05	1901.9	0.98
13.6	1616.2	1943.9	1.2	2031.4	1.05	1994.3	0.98

~ 20% $\longrightarrow$ ~ 5% $\longrightarrow$ ~ 2%

Inclusive cross-sections: predictions

$$pp \to W^+ + X$$

$\sqrt{s}$ [TeV]	σ_{LO} [pb]	σ_{NLO} [pb]	K^{NLO}	σ_{NNLO} [pb]	K^{NNLO}	σ_{N^3LO} [pb]	K^{N^3LO}
2.76	1645.9	2150.2	1.31	2207.6	1.03	2217.4	1.0
5.02	3301.3	4198.3	1.27	4335.0	1.03	4314.8	1.0
7.0	4730.8	5959.1	1.26	6177.8	1.04	6115.0	0.99
8.0	5447.5	6835.5	1.25	7096.4	1.04	7010.7	0.99
13.0	8998.1	11096.7	1.23	11555.7	1.04	11358.0	0.98

~ 20% → ~ 5% → ~ 2%

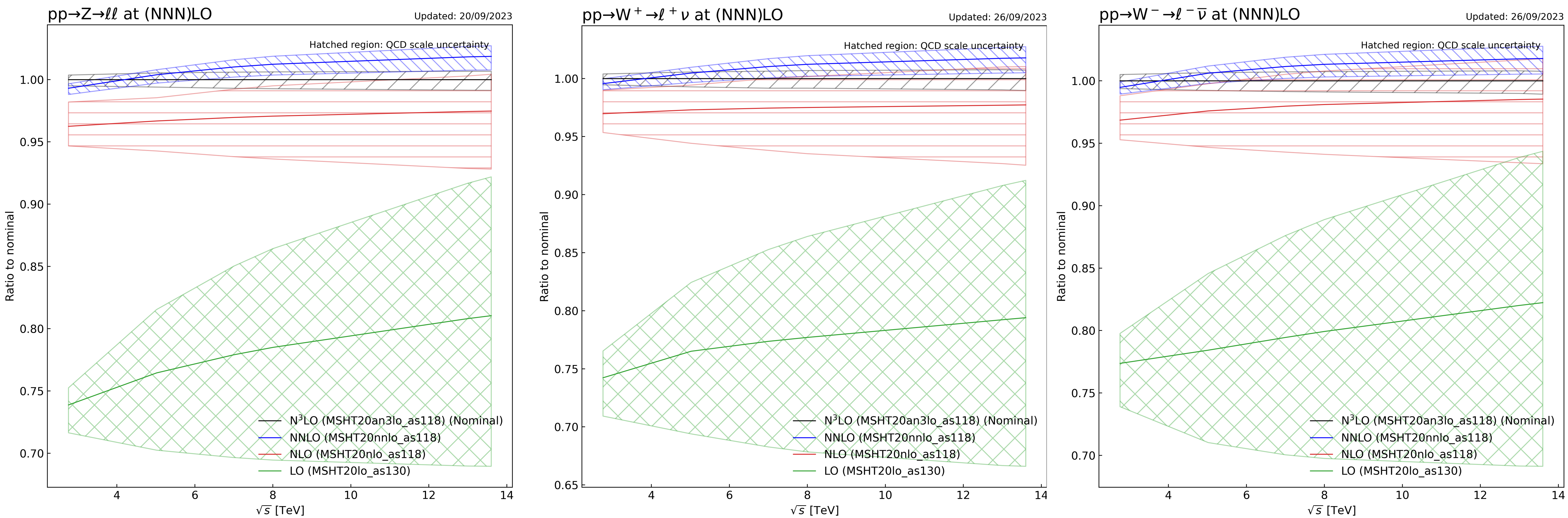
Inclusive cross-sections: predictions

$$pp \to W^- + X$$

$\sqrt{s}$ [TeV]	σ_{LO} [pb]	σ_{NLO} [pb]	K^{NLO}	σ_{NNLO} [pb]	K^{NNLO}	σ_{N^3LO} [pb]	K^{N^3LO}
2.76	1018.7	1275.2	1.25	1309.8	1.03	1316.7	1.01
5.02	2226.2	2770.1	1.24	2855.9	1.03	2838.6	0.99
7.0	3346.4	4126.7	1.23	4260.5	1.03	4212.2	0.99
8.0	3924.6	4817.1	1.23	4975.2	1.03	4910.3	0.99
13.0	6880.7	8264.4	1.2	8537.6	1.03	8390.0	0.98

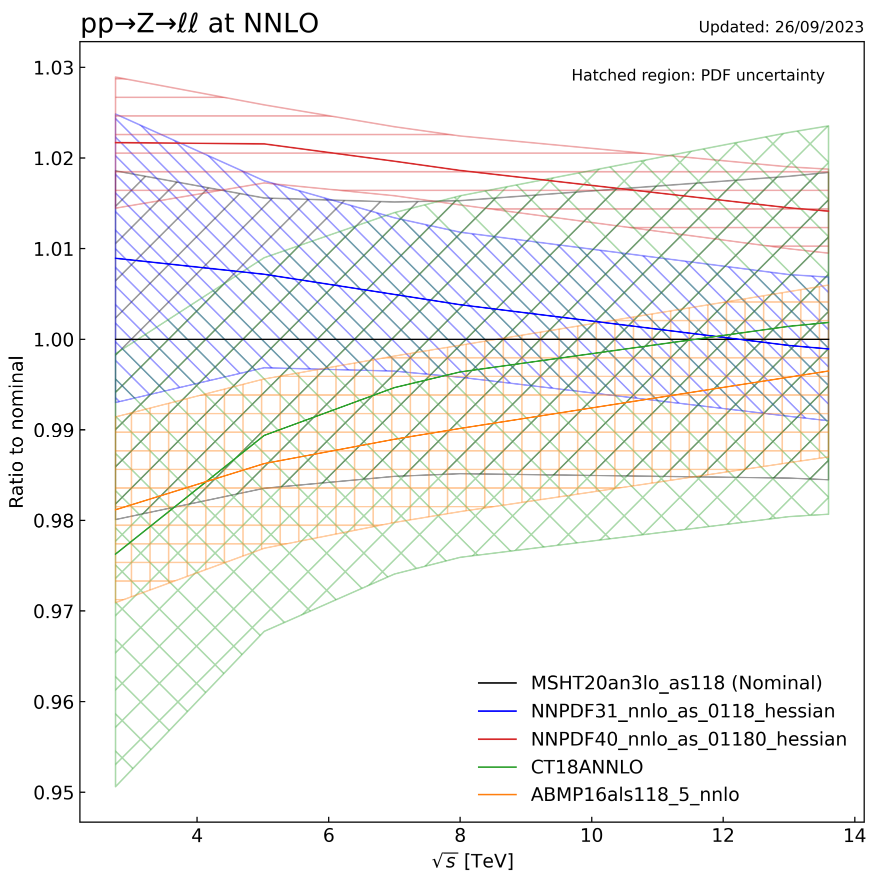
~ 20% → ~ 5% → ~ 2%

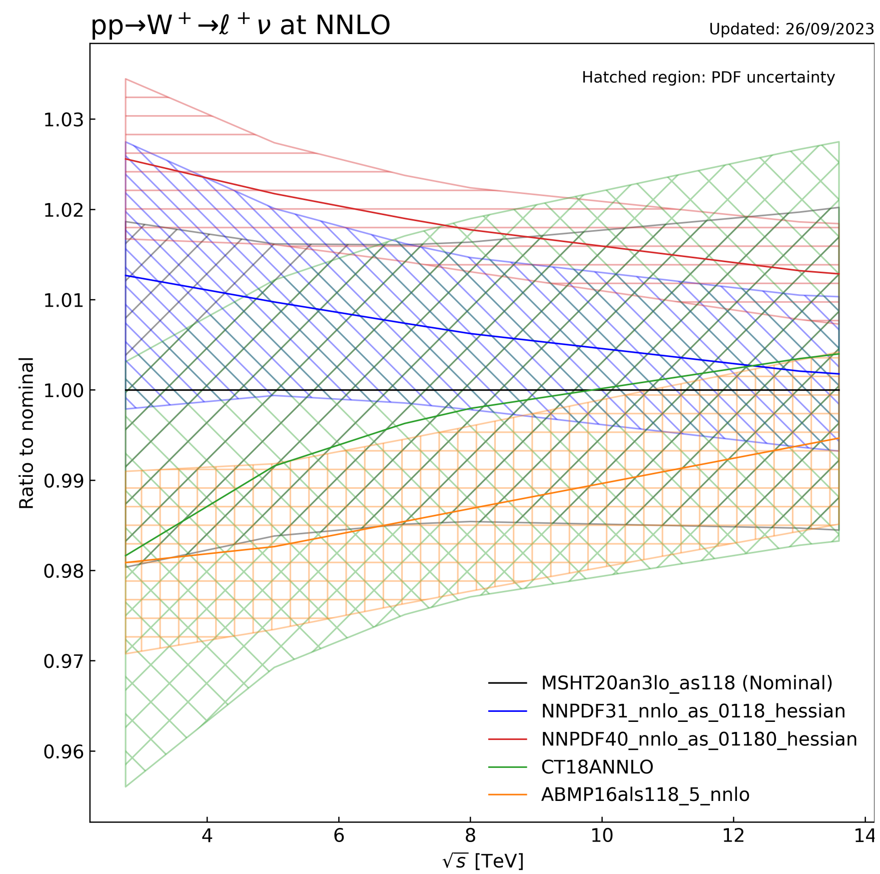
Inclusive cross-sections: predictions

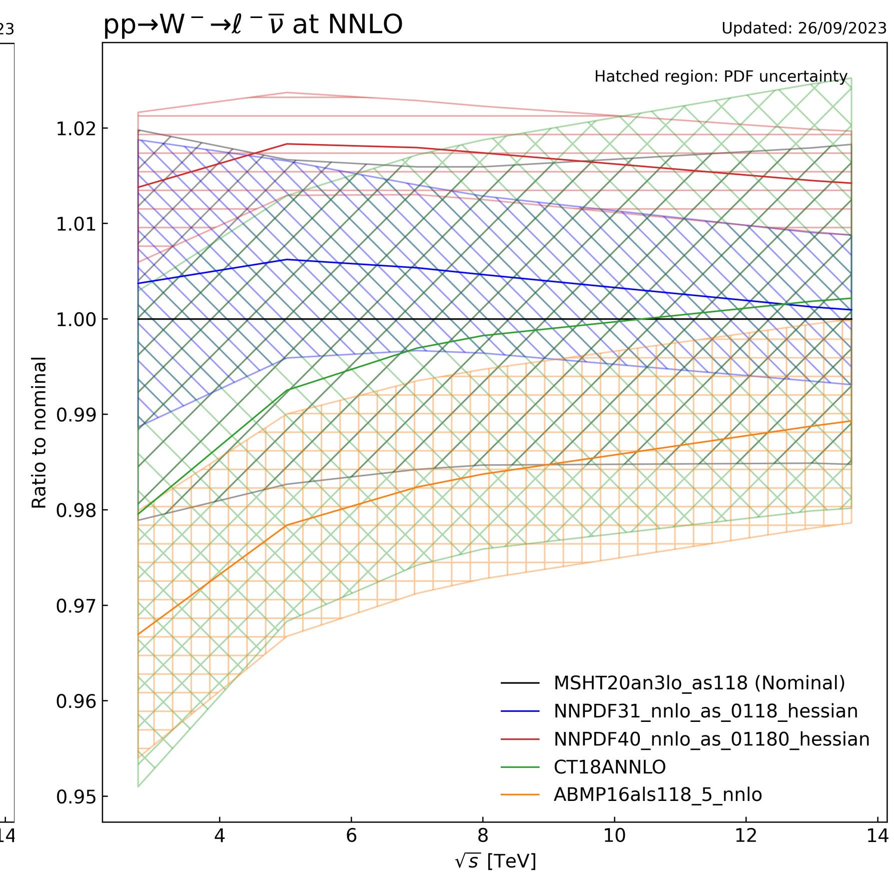


- Consistent PDF set order and ME computations
- Low $\sqrt{s}$ values → N^{3}LO comparable to NNLO + bands overlapping
- High $\sqrt{s}$ values → N^{3}LO < NNLO + bands not overlapping

Inclusive cross-sections: predictions







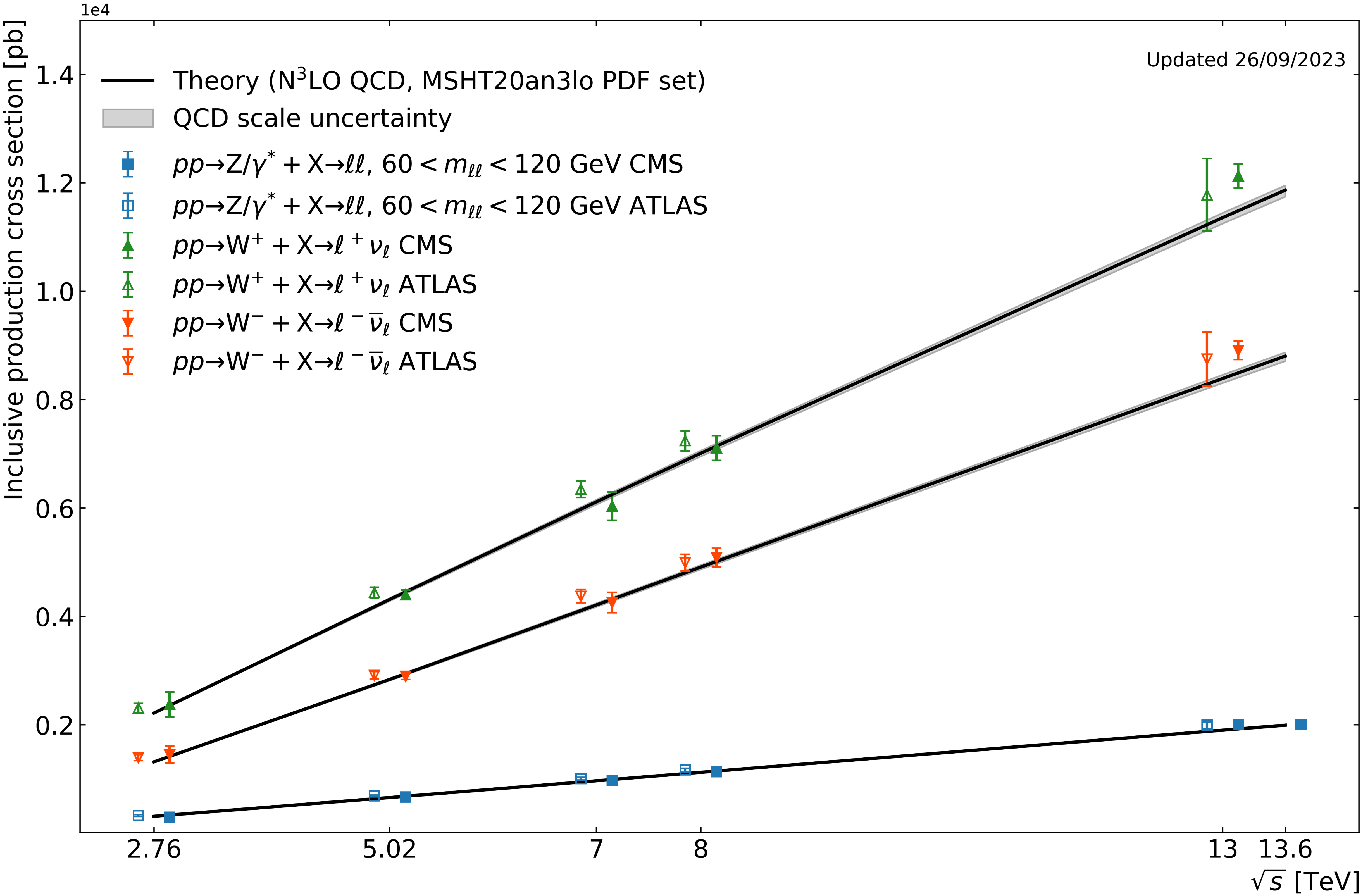
- Predictions at NNLO obtained with different PDFs sets compared to the aN^{3}LO MSHT set
- Nothing unexpected → would be interesting to have another N^{3}LO PDF set to compare with

Inclusive cross-sections: measurements

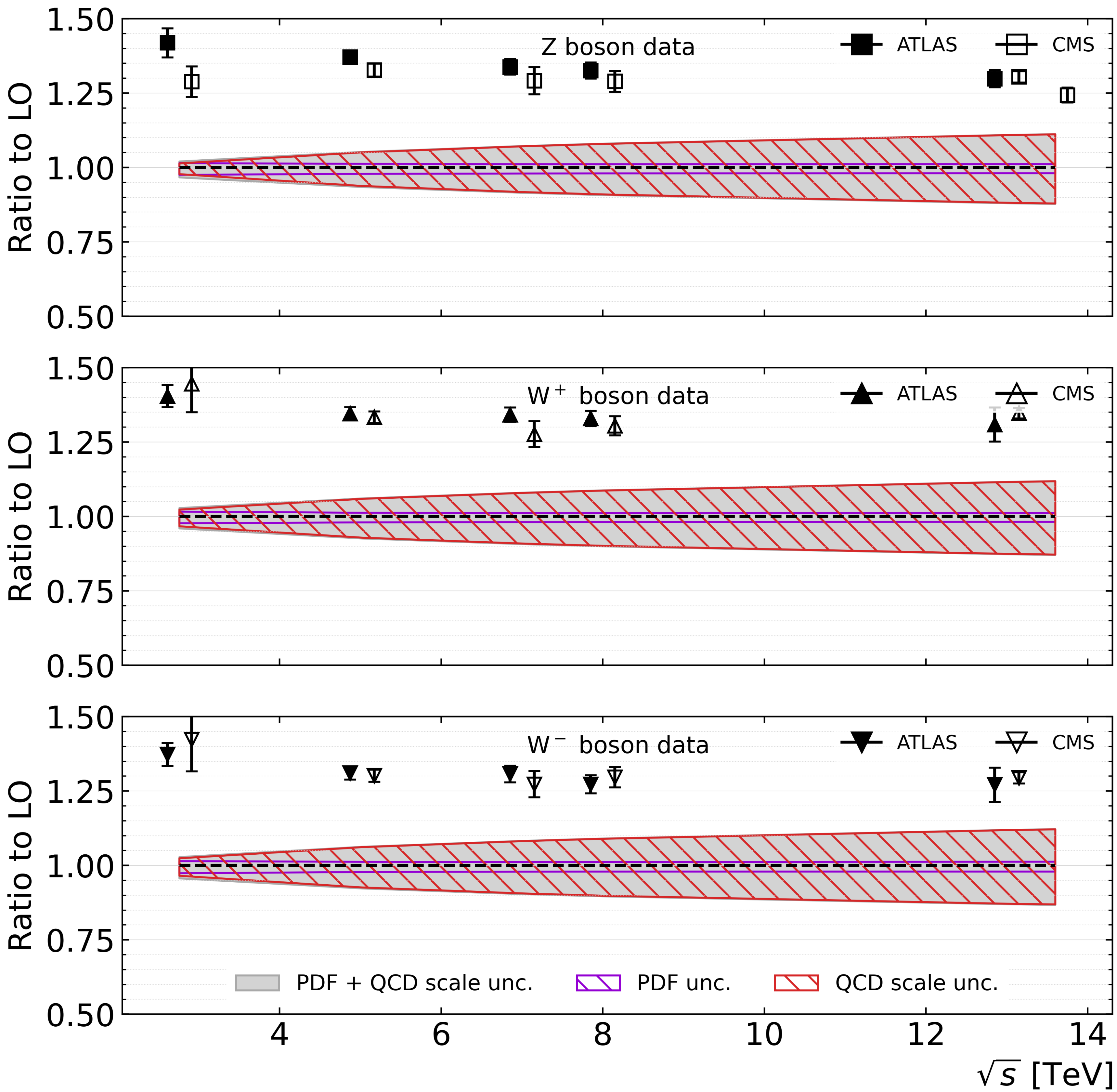
Input experimental data considered: $\mathrm{pp} \rightarrow \mathrm{W}^{\pm}$ and $\mathrm{pp} \rightarrow \mathrm{Z}$ inclusive cross sections from ATLAS and CMS

- $\sqrt{s} = 2.76$, 5.02, 7, 8, 13 and 13.6 TeV
- Electron and muon decay channels combined
- Measurements are extrapolated to inclusive phase-space by means of NLO PS predictions (extrapolation unc. included in the total meas. unc.)
- Some (potentially more) precise measurements not included atm. $\rightarrow$ focussed on measurements extrapolated by experiments
- Total unc. $\delta^{\mathrm{Tot.}}$ ranges from 1.8 to 10 %
- We extrapolated ATLAS Z boson data to a common $m_{\ell\ell}$ range ($60 < m_{\ell\ell} < 120\,\mathrm{GeV}$) at N^{3}LO

Measurements v.s. N^{3}LO predictions

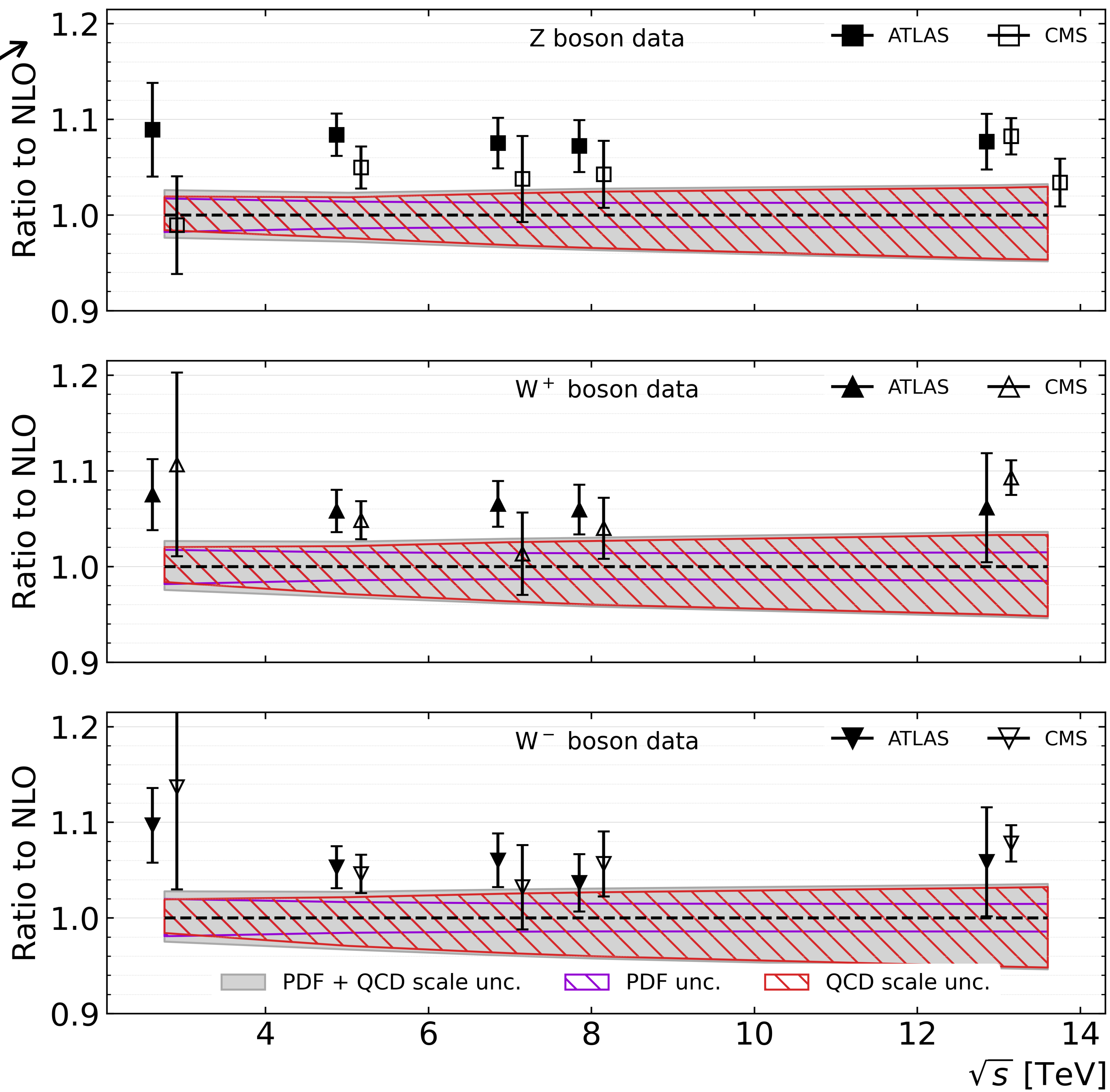


Are LO predictions able to describe the data?

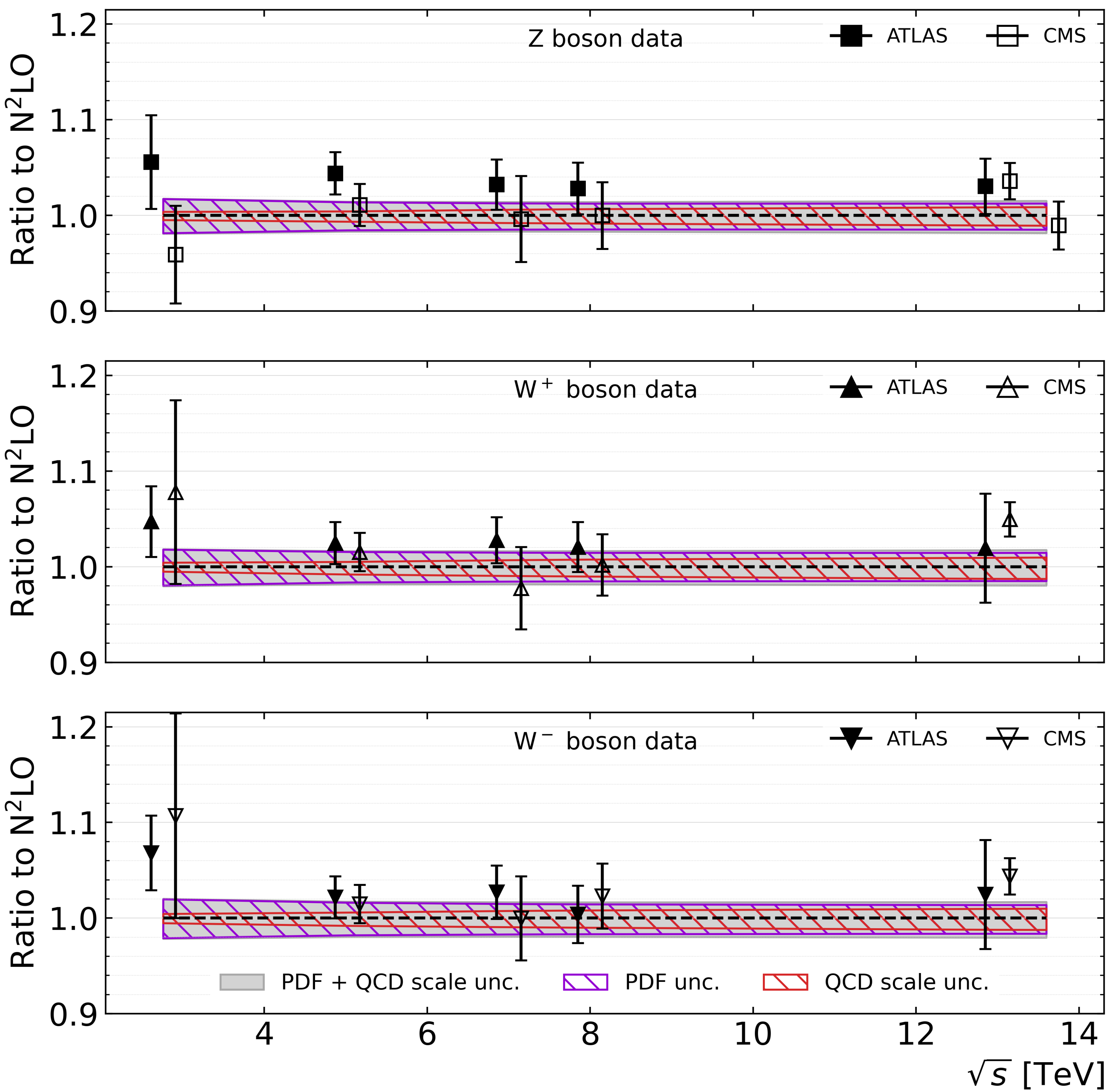


Are NLO predictions able to describe the data?

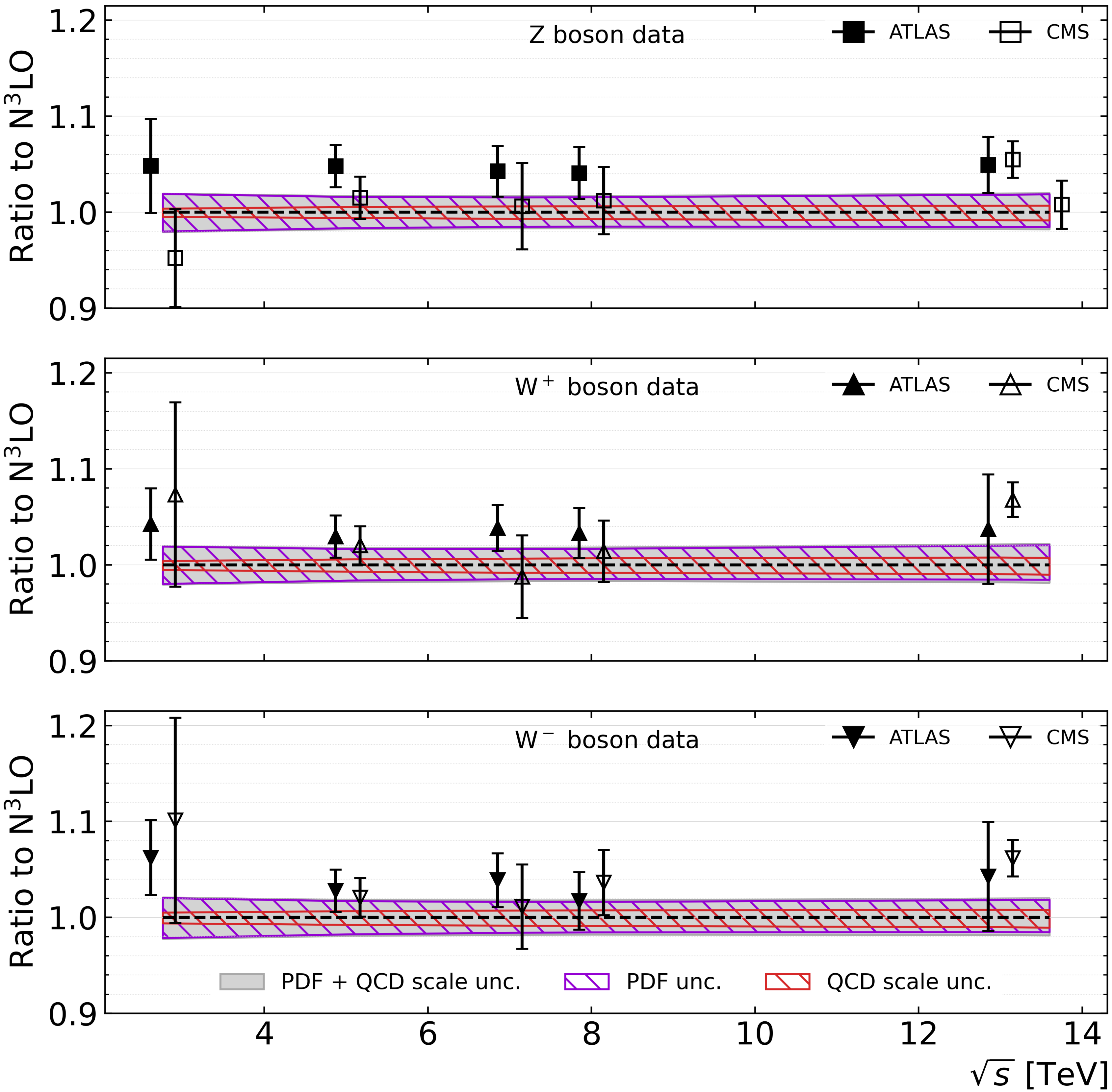
The scale has changed!



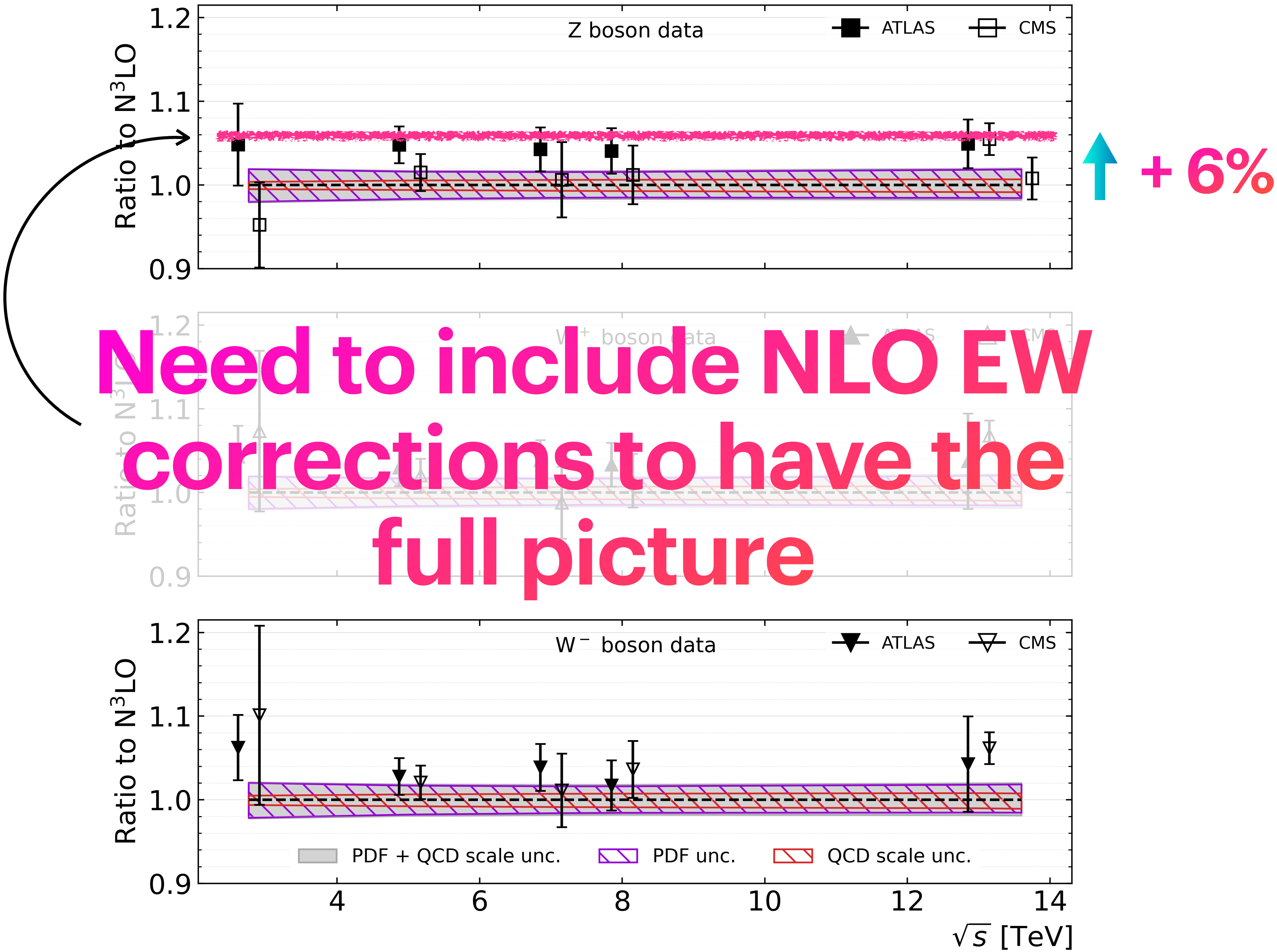
Are NNLO predictions able to describe the data?



Are N^{3}LO predictions able to describe the data?

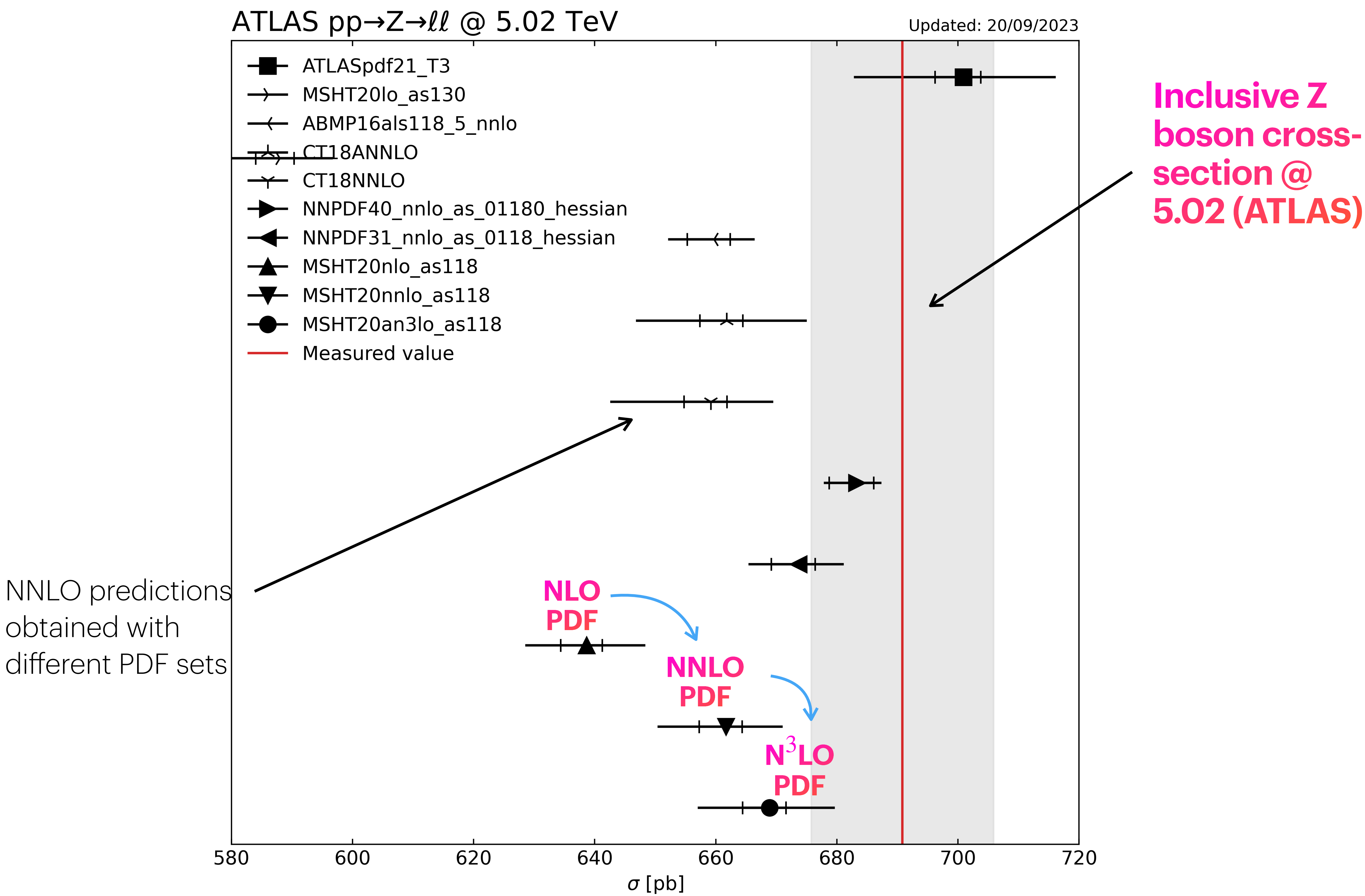


Are N^{3}LO predictions able to describe the data?

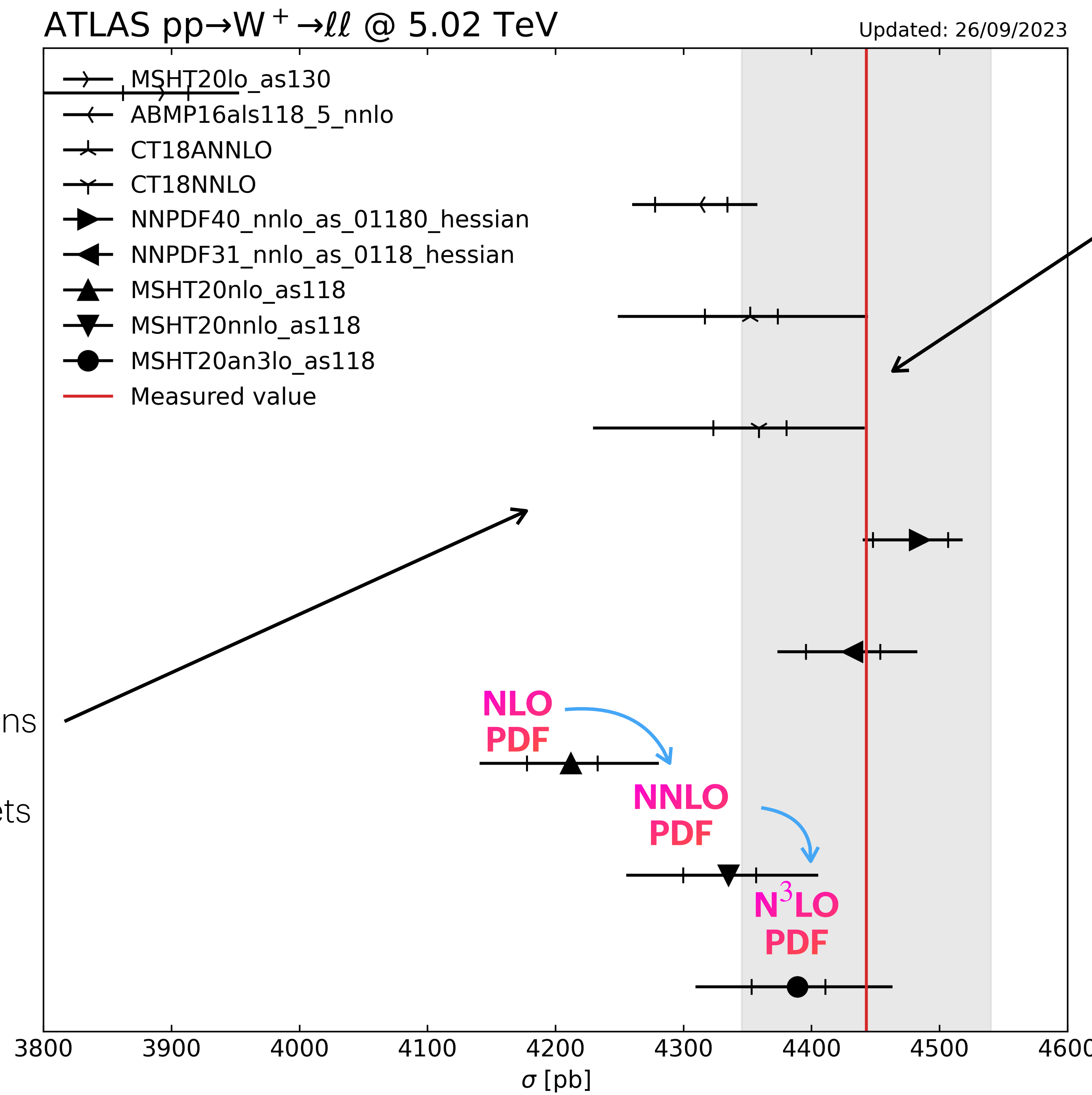


Need to include NLO EW corrections to have the full picture

Impact of different PDF sets



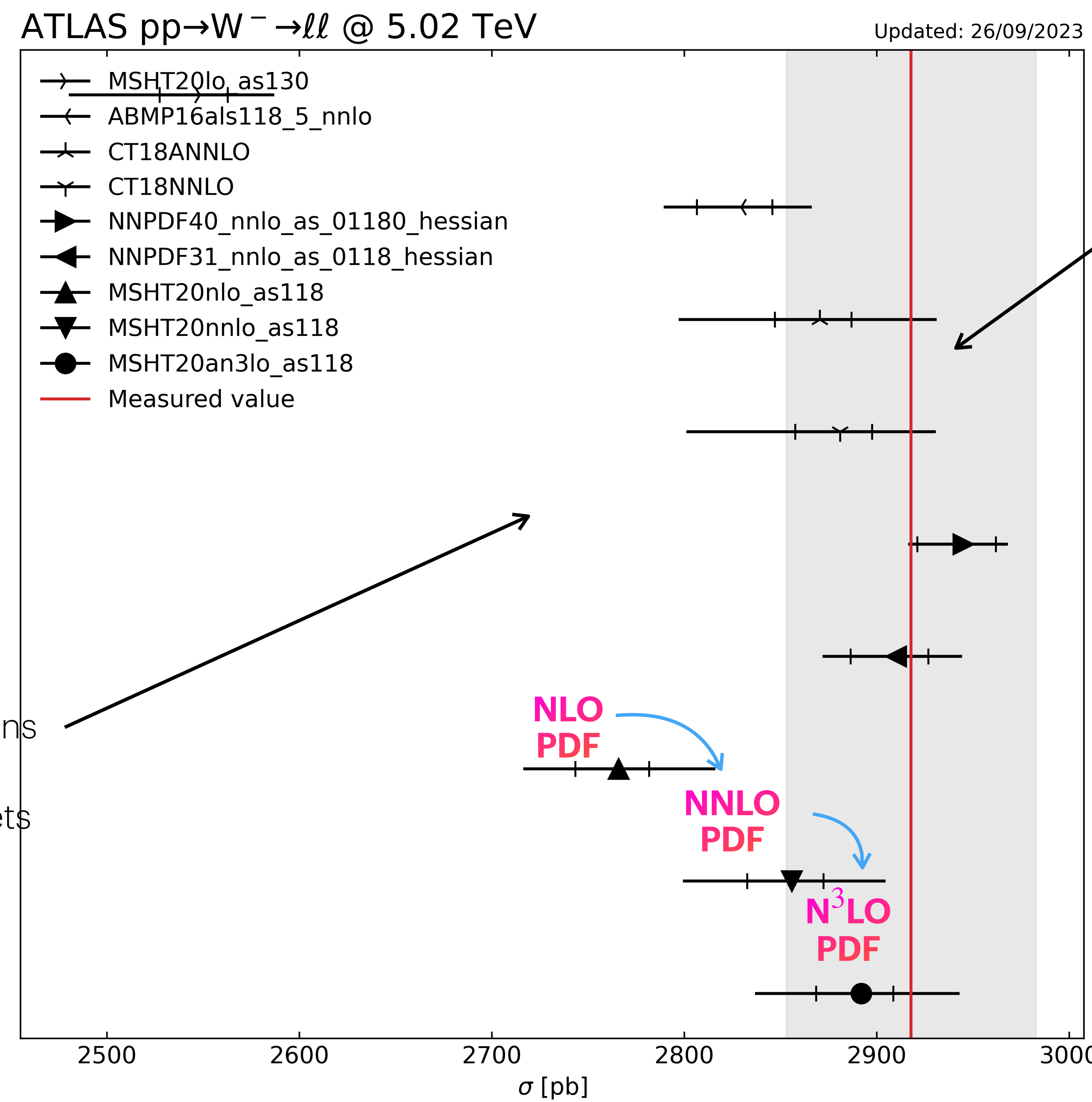
Impact of different PDF sets



Inclusive W+ boson cross-section @ 5.02 (ATLAS)

NNLO predictions obtained with different PDF sets

Impact of different PDF sets



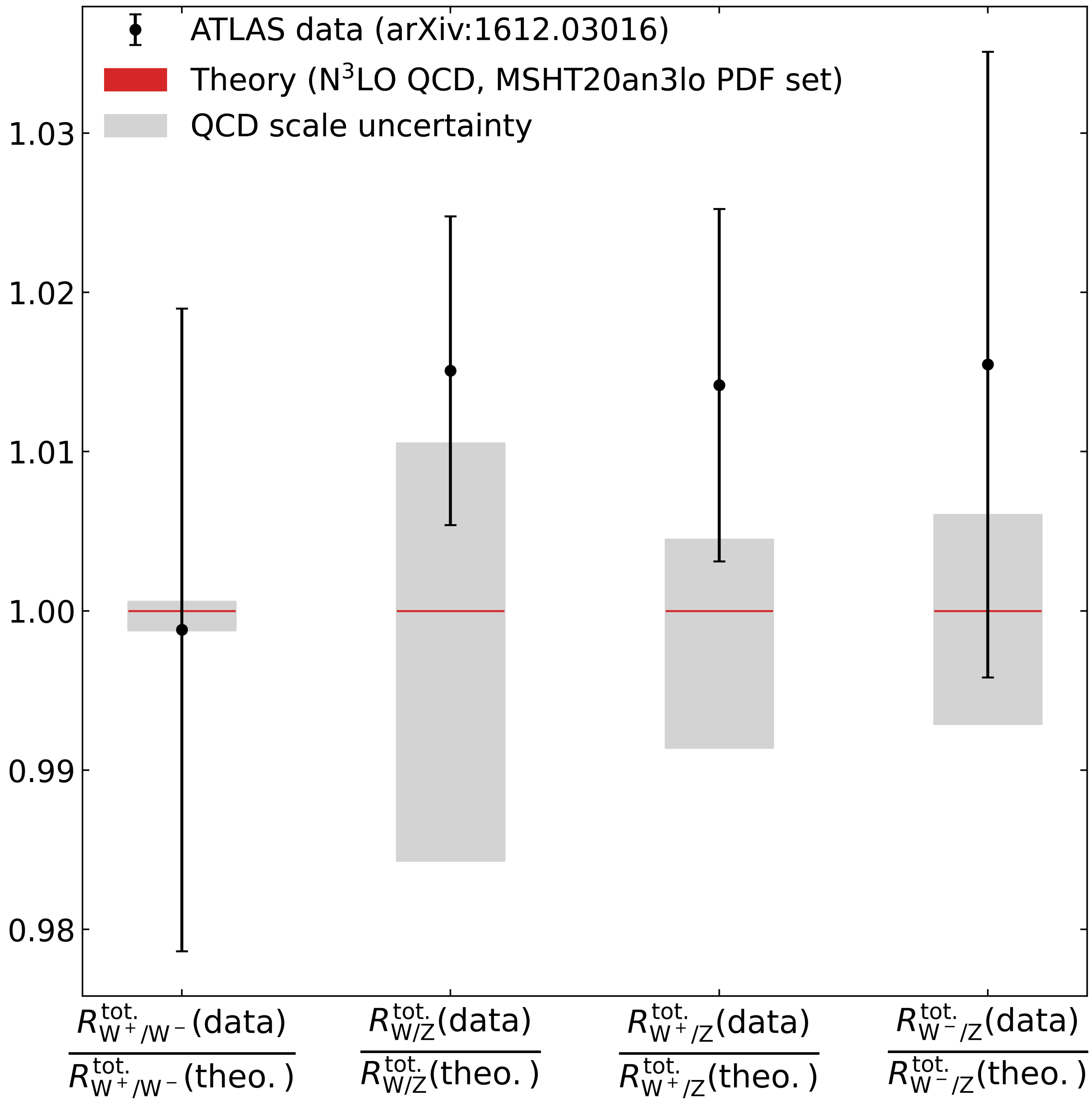
Inclusive W-boson cross-section @ 5.02 (ATLAS)

NNLO predictions obtained with different PDF sets

Inclusive cross-section ratios

- Cross-section ratios (e.g. $\sigma_{W^+ \to \ell^+ \nu} / \sigma_{Z \to \ell^+ \ell^-}$) excellent tool to probe predictions
- Double cross-section ratios: $R_{\text{data}} / R_{\text{theo.}}$ considered
- R_{data} from ATLAS 7 TeV measurements
- $R_{\text{theo.}}$ computed at N^{3}LO (only scale unc. included)
- Good description of the W^+/W^- ratio
- For the $W^\pm/Z$ ratios N^{3}LO predictions tend to undershoot the data

Inclusive cross-section ratios



Outlook

- Presented first steps for a comprehensive comparison of inclusive DY cross-section measurements (CMS + ATLAS) to bleeding-edge predictions (N^{3}LO QCD)

- This study will help benchmark precision QCD calculations and phenomenological studies at the LHC

- Still a lot of room for improvement:

  - Incorporate Electroweak (EW) Corrections

  - Perform a quantitative comparison between data and predictions

  - Include recently available data from both 5 and 13 TeV collision energies with low pile-up conditions

  - Investigate differential cross-sections as a function of $m_{\ell\ell}$

  - Full N^{3}LO PDF fit (?)

Additional material

Inclusive cross-sections: measurements

$$\mathrm{pp} \rightarrow \mathrm{Z}/\gamma^* + \mathrm{X}$$

$\sqrt{s}$	$\sigma^{\mathrm{CMS}}_{\mathrm{meas.}}$	$\delta^{\mathrm{Tot.}}\ \%$	$\sigma^{\mathrm{ATLAS}}_{\mathrm{meas.}}$	$\delta^{\mathrm{Tot.}}\ \%$	σ_{LO}	σ_{NLO}	σ_{NNLO}	$\sigma_{\mathrm{N^3LO}}$
2.76	298.4	5.1	328.5	4.9	231.5	301.5	311.1	313.3
5.02	669.0	2.2	690.8	2.2	503.9	637.2	661.7	659.1
7.0	974.0	4.5	1009.2	2.6	754.1	938.5	977.7	968.0
8.0	1138.0	3.5	1170.2	2.7	882.6	1091.4	1138.2	1124.4
13.0	2006.0	1.9	1995.6	2.9	1536.9	1853.1	1936.6	1901.9
13.6	2010.0	2.5	N.A.	N.A.	1616.2	1943.9	2031.4	1994.

Inclusive cross-sections: measurements

$$pp \to W^+ + X$$

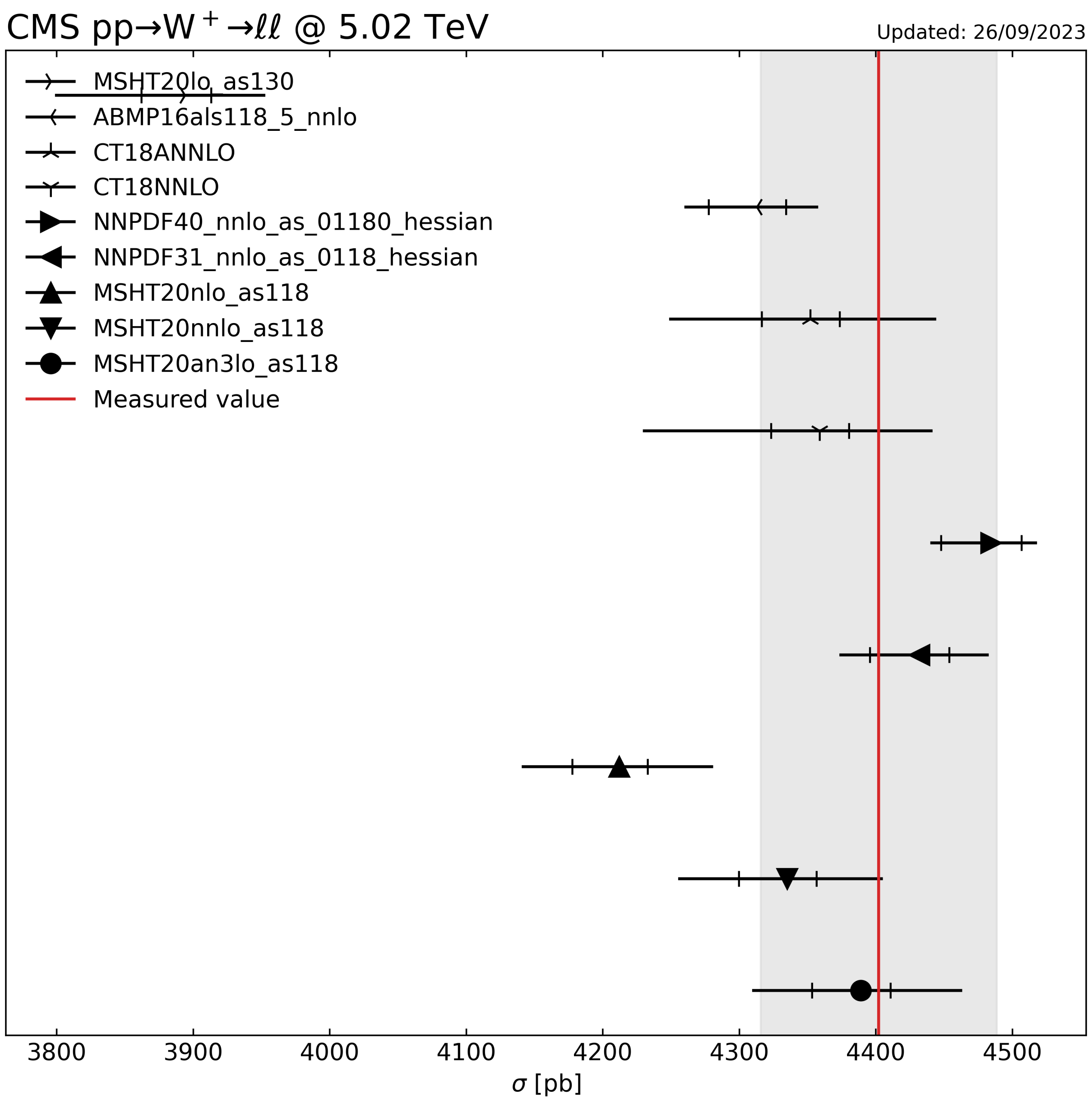
$\sqrt{s}$	$\sigma^{\text{CMS}}_{\text{meas.}}$	$\delta^{\text{Tot.}}\ \%$	$\sigma^{\text{ATLAS}}_{\text{meas.}}$	$\delta^{\text{Tot.}}\ \%$	σ_{LO}	σ_{NLO}	σ_{NNLO}	$\sigma_{\text{N}^3\text{LO}}$
2.76	2380.0	9.6	2312.0	3.7	1645.9	2150.2	2207.6	2217.4
5.02	4402.0	2.0	4442.7	2.2	3301.3	4198.3	4335.0	4314.8
7.0	6040.0	4.3	6350.0	2.4	4730.8	5959.1	6177.8	6115.0
8.0	7110.0	3.2	7243.0	2.6	5447.5	6835.5	7096.4	7010.7
13.0	12130.0	1.8	11780.0	5.7	8998.1	11096.7	11555.7	11358.0

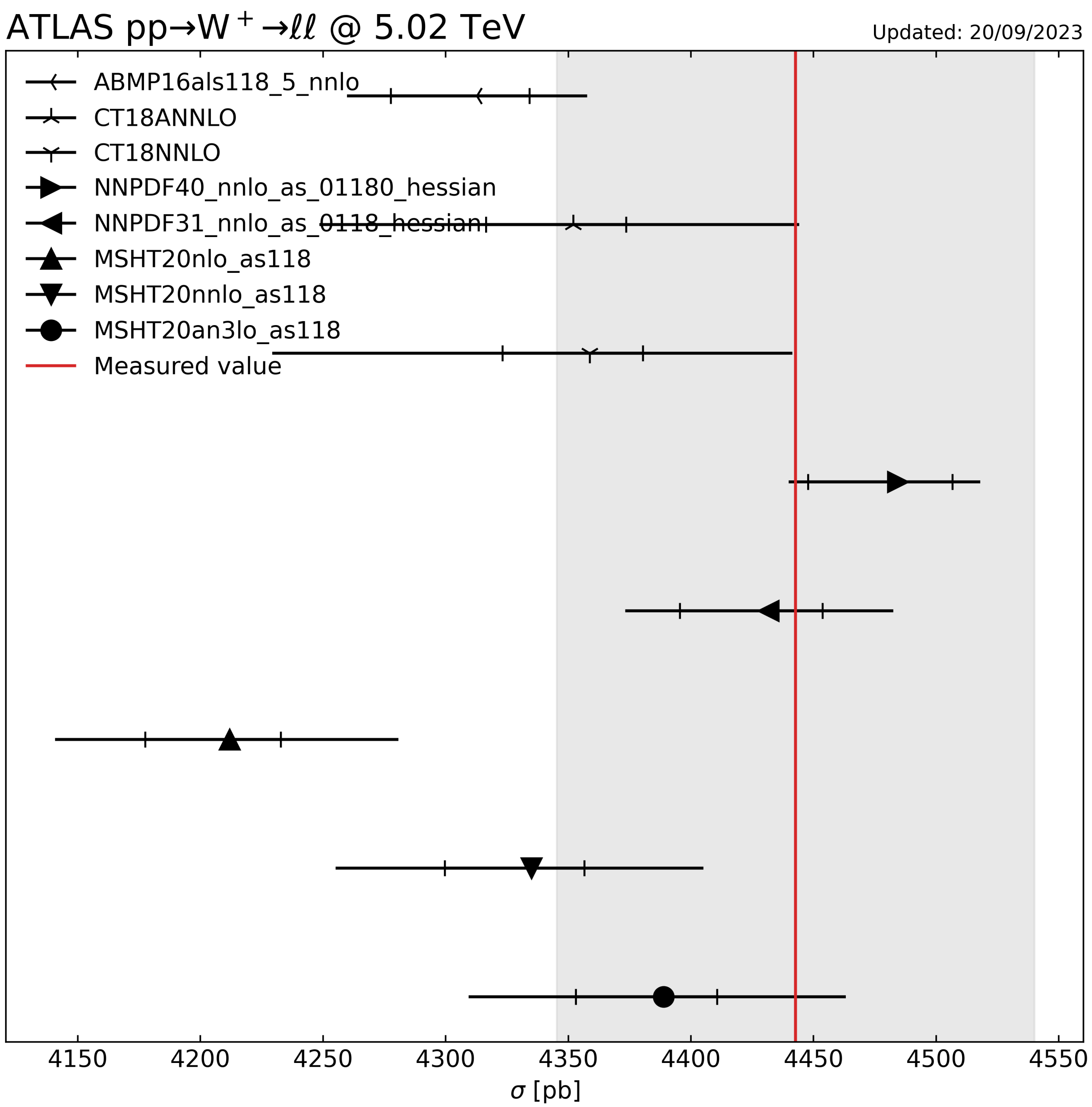
Inclusive cross-sections: measurements

$$\mathrm{pp} \to \mathrm{W}^- + \mathrm{X}$$

$\sqrt{s}$	$\sigma^{\mathrm{CMS}}_{\mathrm{meas.}}$	$\delta^{\mathrm{Tot.}}\ \%$	$\sigma^{\mathrm{ATLAS}}_{\mathrm{meas.}}$	$\delta^{\mathrm{Tot.}}\ \%$	σ_{LO}	σ_{NLO}	σ_{NNLO}	$\sigma_{\mathrm{N^3LO}}$
2.76	2.76	1450.0	10.7	1399.0	3.9	1018.7	1275.2	1309.8
5.02	5.02	2898.0	2.0	2917.8	2.2	2226.2	2770.1	2855.9
7.0	7.0	4260.0	4.4	4376.0	2.8	3346.4	4126.7	4260.5
8.0	8.0	5090.0	3.4	4995.0	3.0	3924.6	4817.1	4975.2
13.0	13.0	8910.0	1.9	8750.0	5.7	6880.7	8264.4	8537.6

Some selected results...





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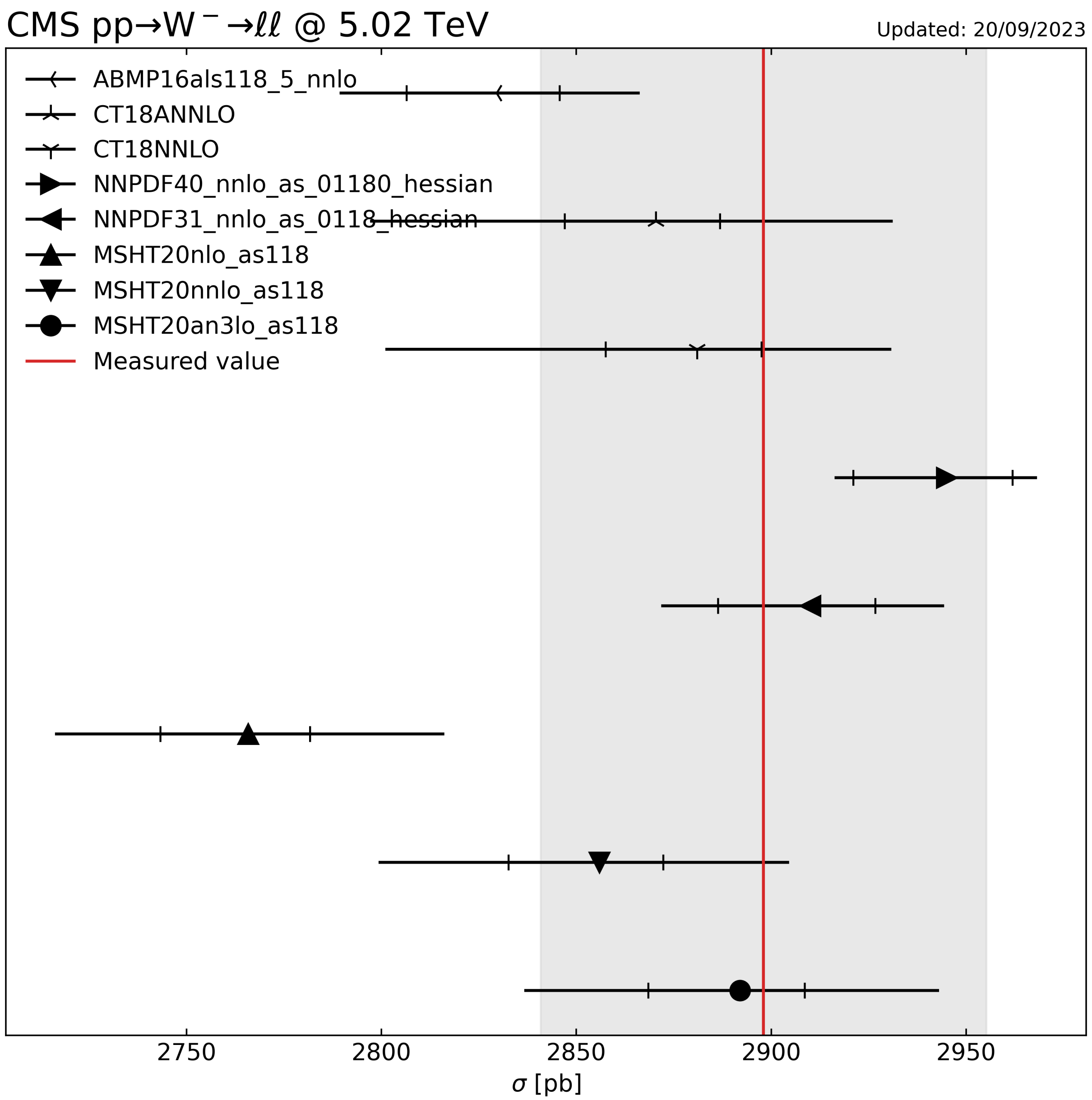

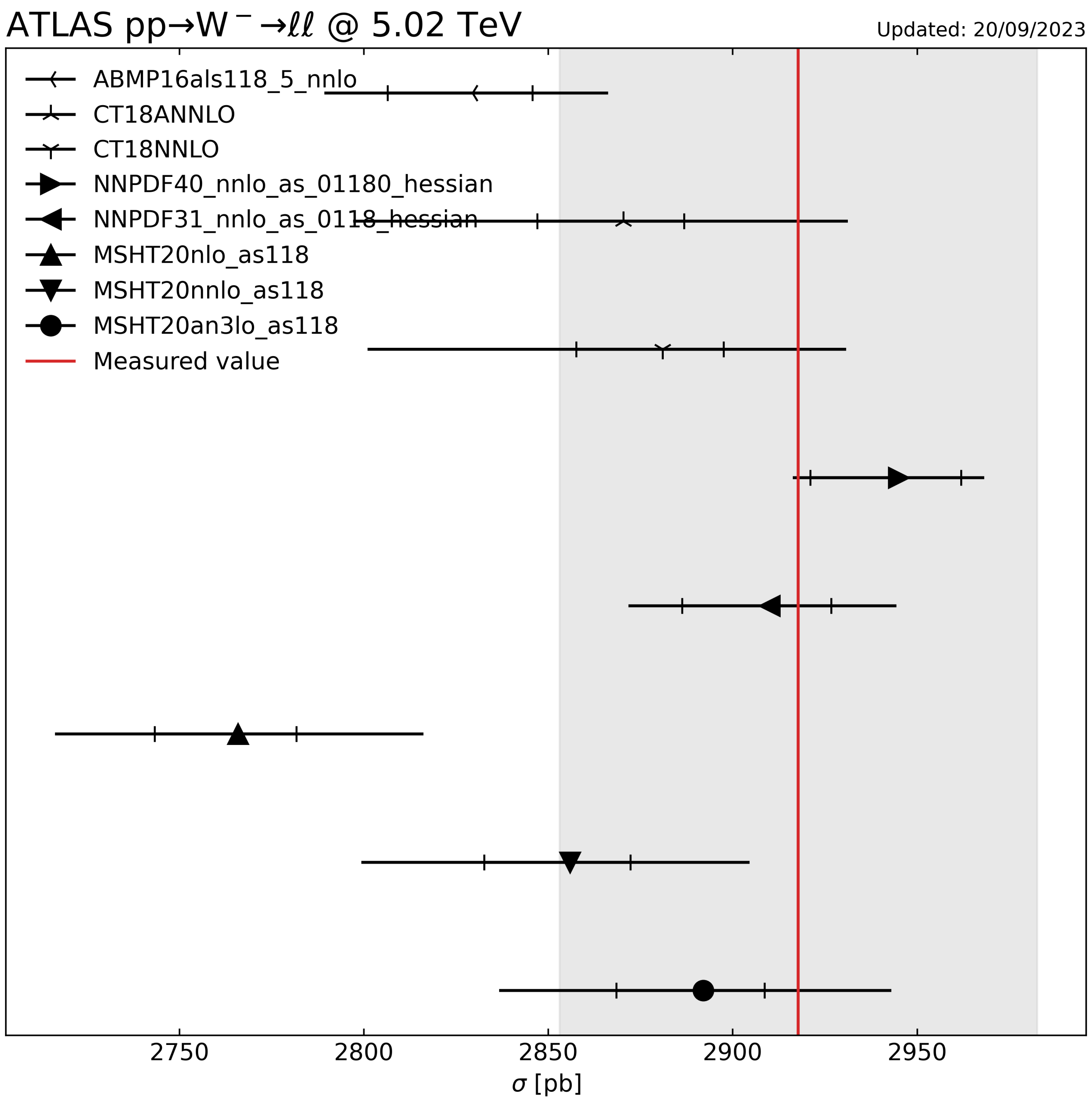

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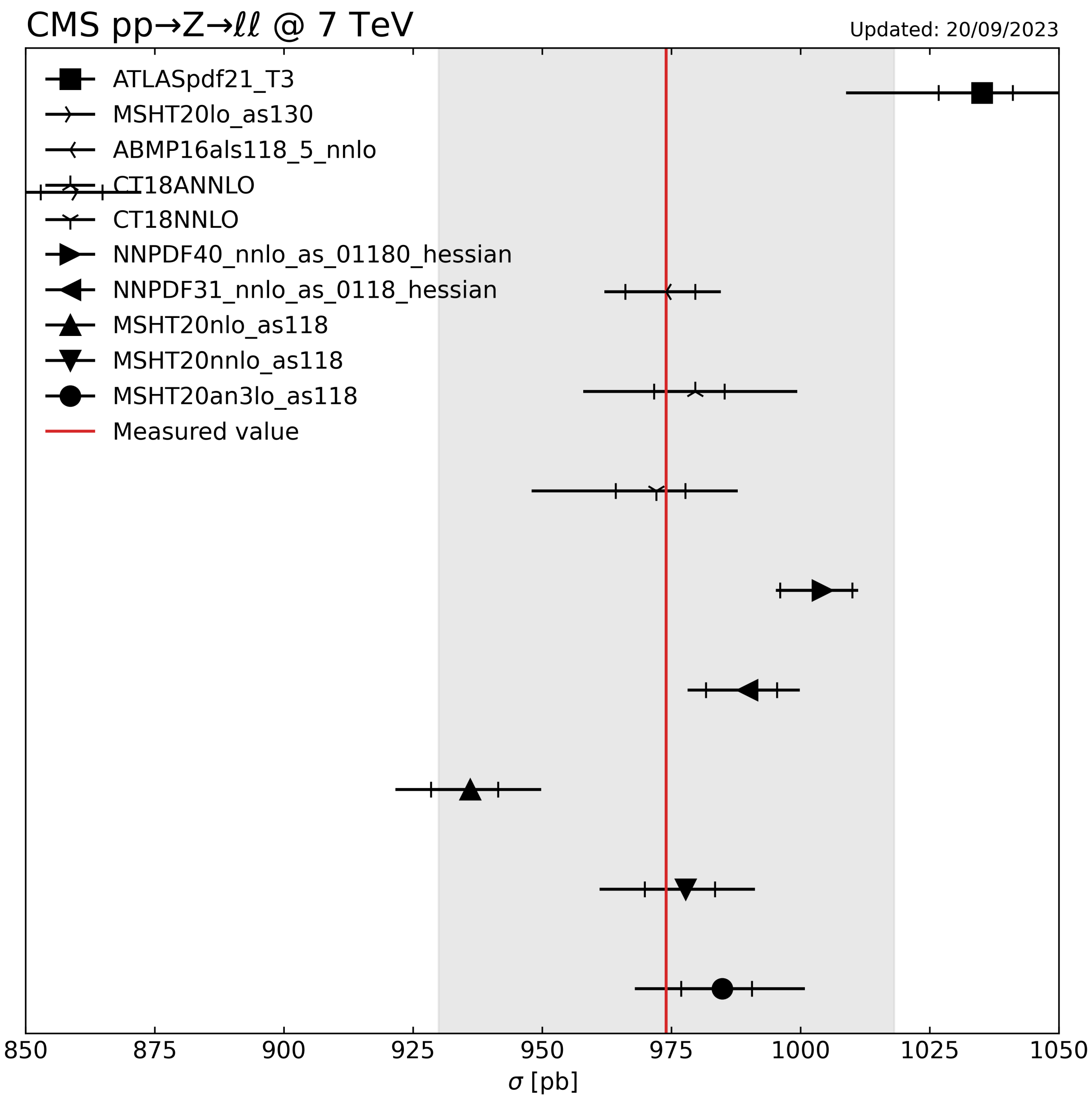

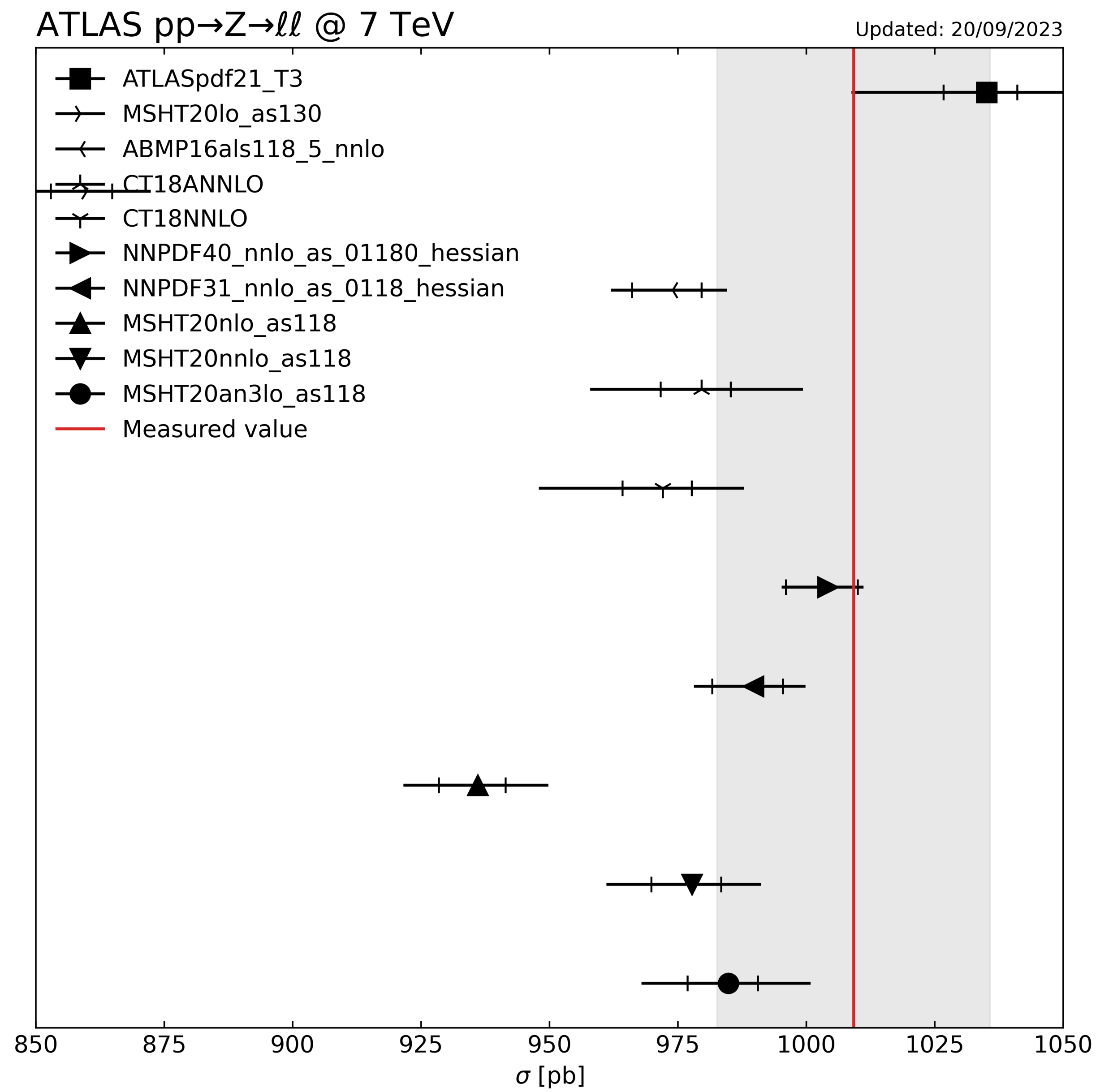

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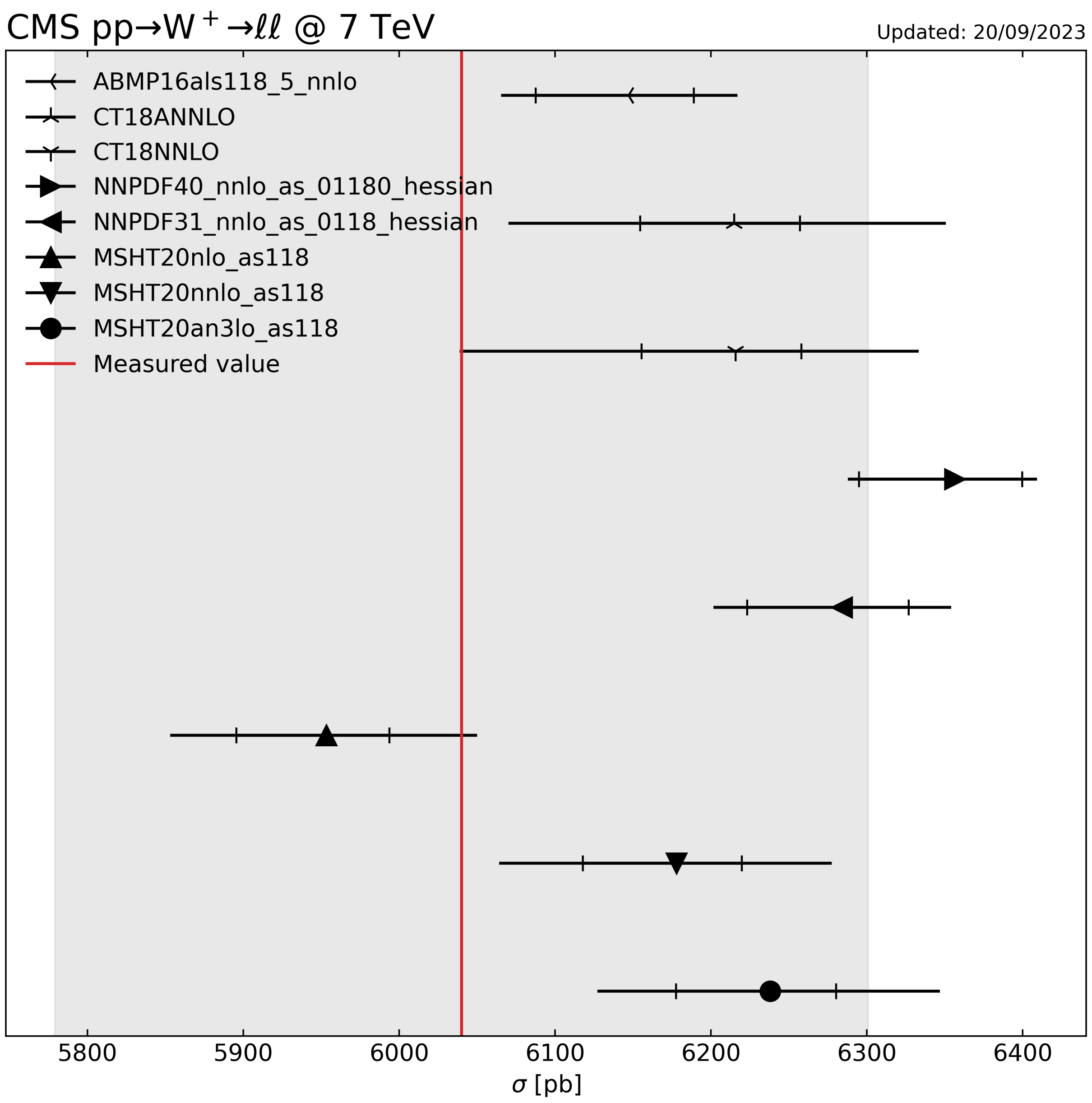
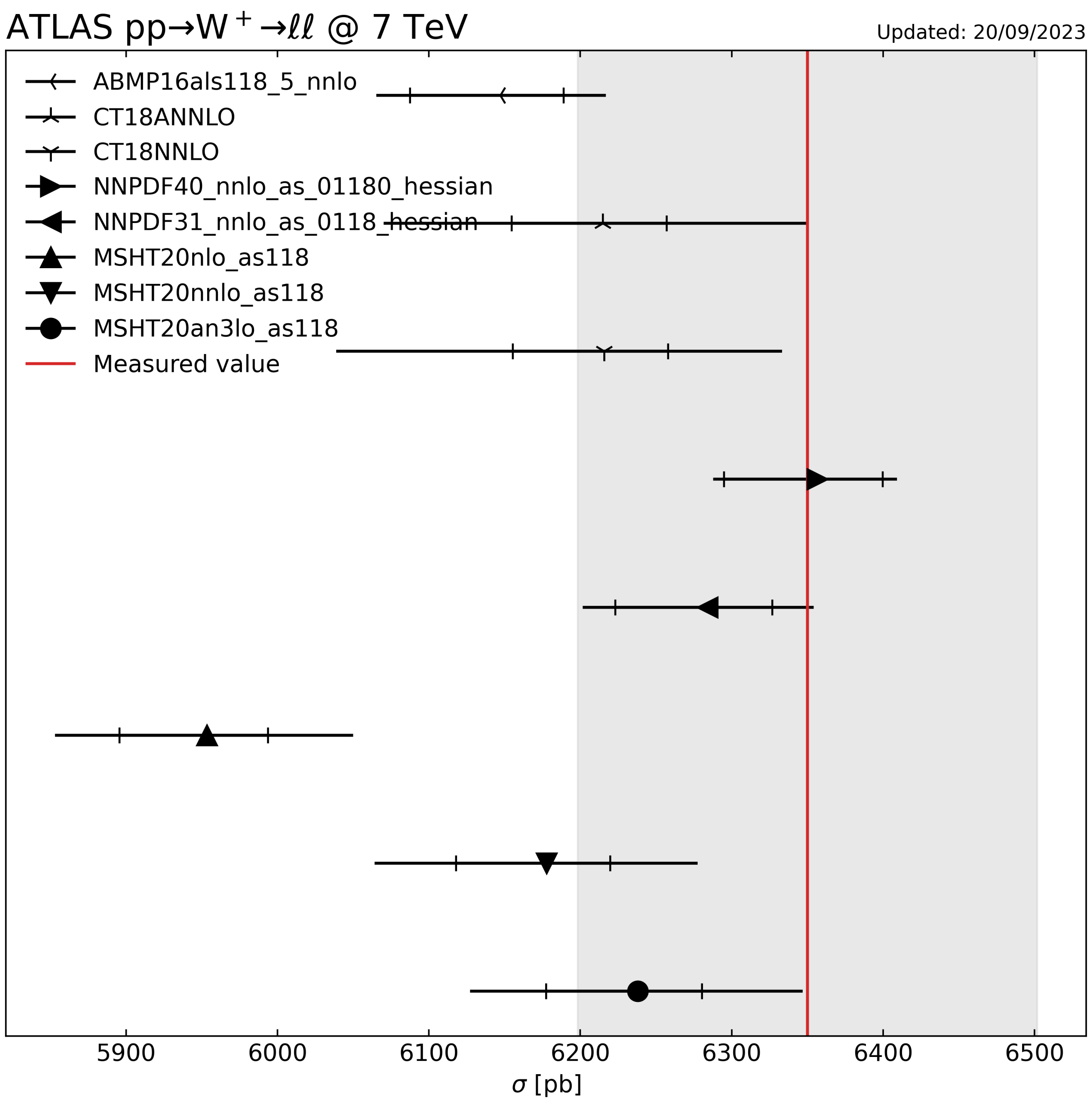
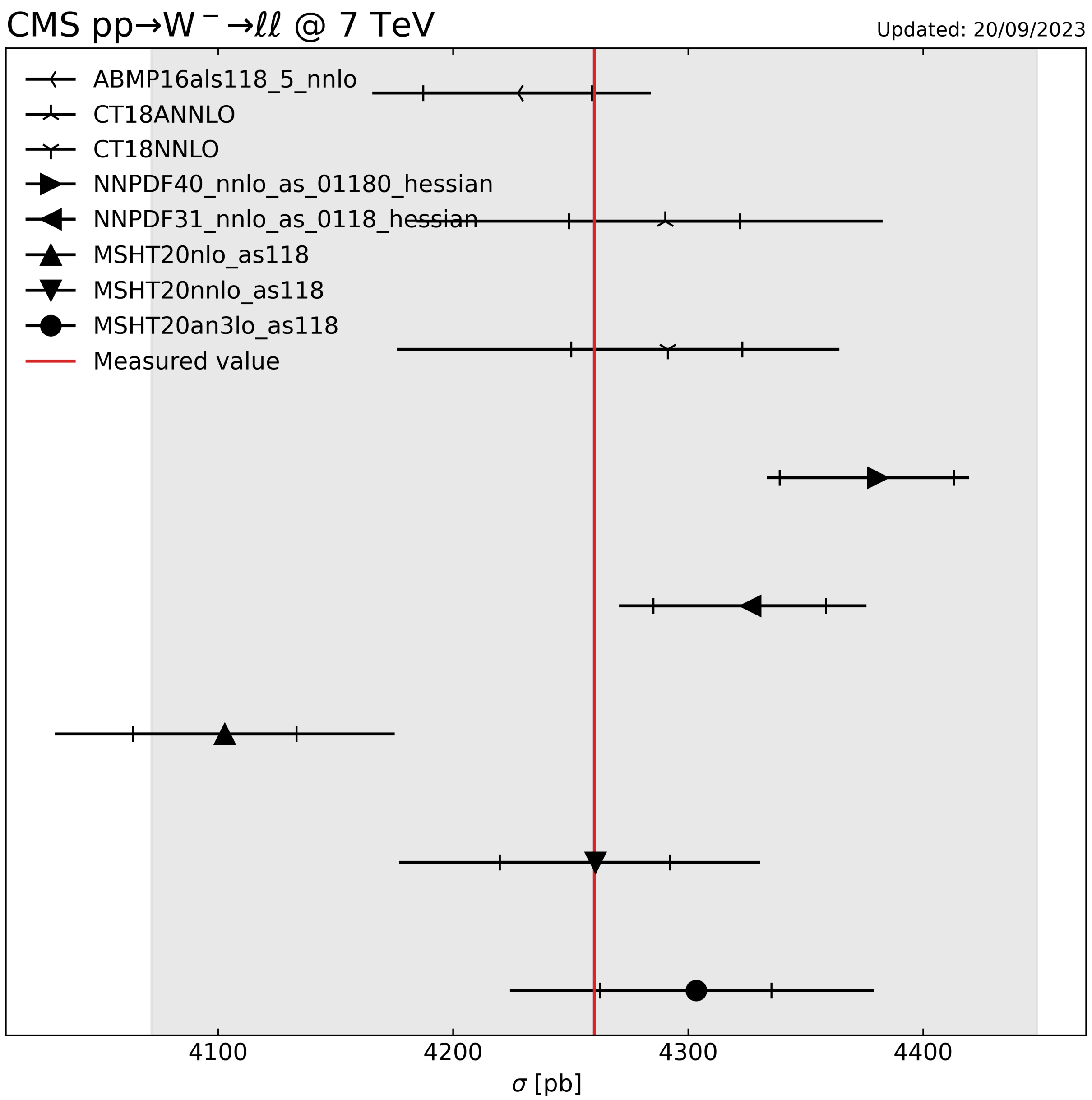
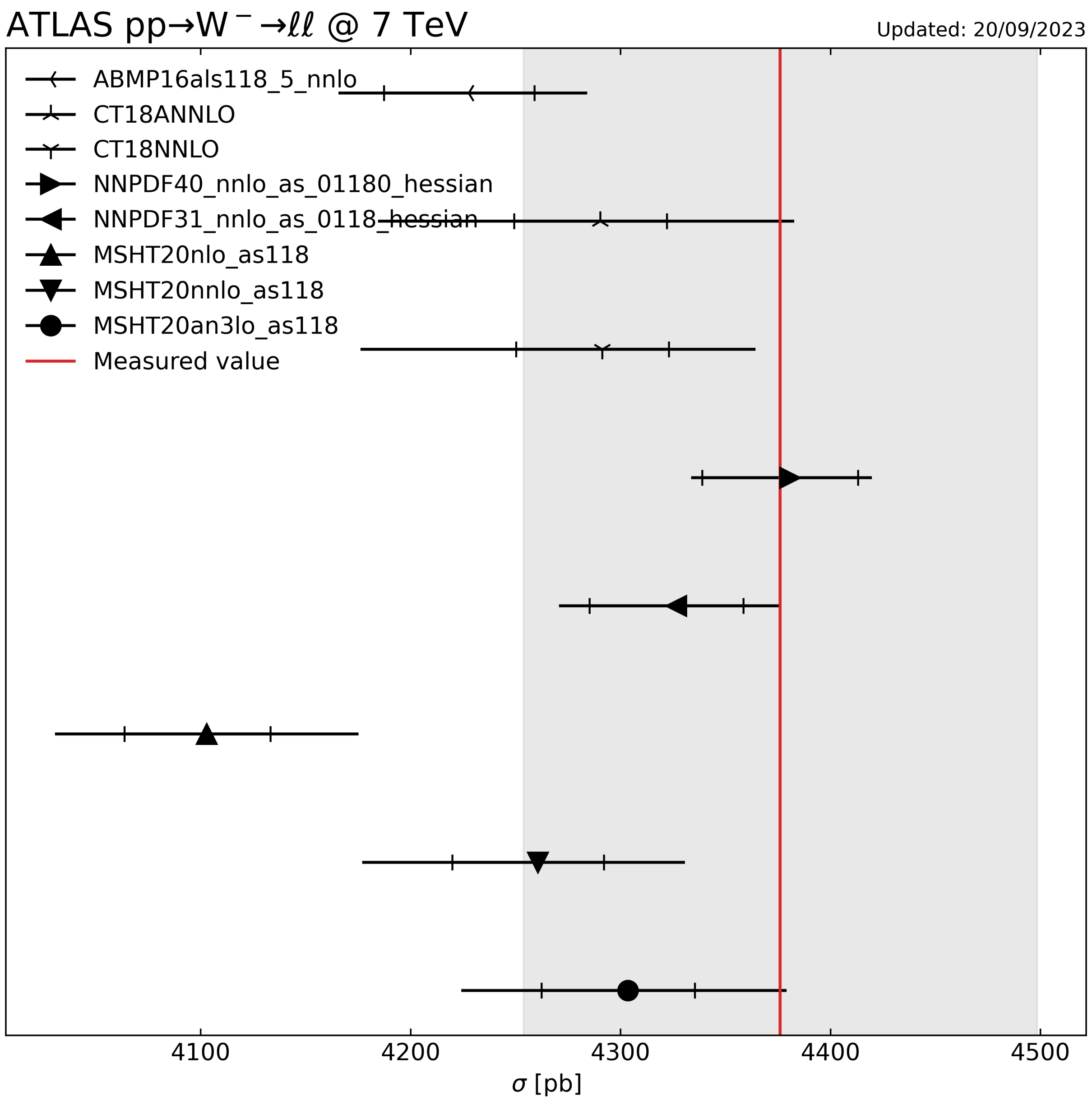


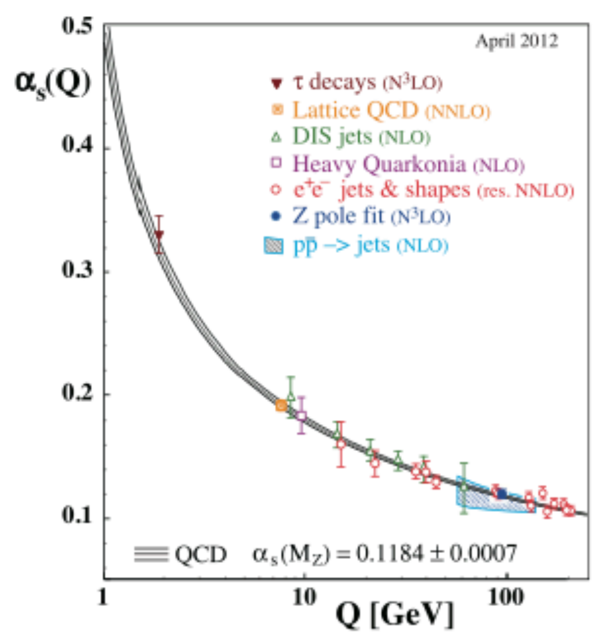


Other selected results...





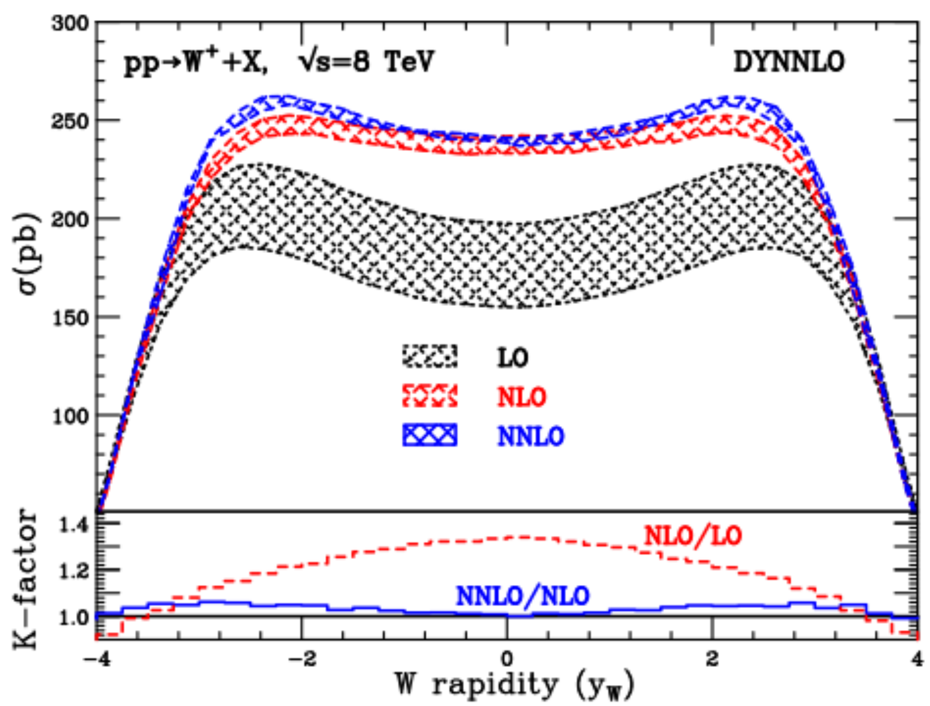
Fixed-order calculations



The QCD coupling

$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$

(for $Q \sim m_H$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

- Perturbation theory at **NNLO & beyond**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\,\hat{\sigma}^{(1)} + \alpha_S^2\,\hat{\sigma}^{(2)} + \cdots$$

- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & robust uncertainty.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

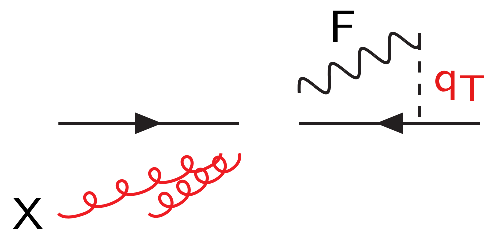
Drell-Yan process	LO	NLO	NNLO
	Drell,Yan (1974)	Altarelli,Ellis,Greco Martinelli,(1980-84)	Hamberg,van Neerven, Matsuura (1991) Melnikov,Petriello (2006) Catani,Cieri,de Florian, G.F.,Grazzini (2009)

N^{3}LO
Duhr,Mistlberger (2021)
Camarda,GF,Cieri(2021)

The q_T-subtraction method at NNLO (and beyond)

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(M, q_T) + X$$

F is one or more colourless particles (vector bosons, photons, Higgs bosons,...) [Catani,Grazzini('07)].



- **Observation:** at LO the q_T of the F is exactly zero.

$$d\sigma^{F}_{N^nLO}|_{q_T\neq 0} = d\sigma^{F+\text{jets}}_{N^{n-1}LO} \quad ,$$

for $q_T \neq 0$ the N^nLO IR divergences cancelled with the $N^{n-1}LO$ subtraction method.

- The only remaining N^nLO singularities are associated with the $q_T \to 0$ limit.
- **Key point**: treat the N^nLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from q_T resummation formalism [Catani,de Florian,Grazzini('00)].

$$\begin{aligned} d\sigma^{F}_{N^nLO} \quad &\xrightarrow{q_T\to 0} \quad d\sigma^{F}_{LO} \otimes \Sigma(q_T/M)dq_T^2 \\ &= \quad d\sigma^{F}_{LO} \otimes \sum_{n=1}^{\infty}\sum_{k=1}^{2n}\left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)}\frac{M^2}{q_T^2}\ln^{k-1}\frac{M^2}{q_T^2}\, d^2q_T \end{aligned}$$

Fiducial power corrections within the $\mathbf{q_T}$ subtraction

[Camarda,Cieri,G.F.('21)]

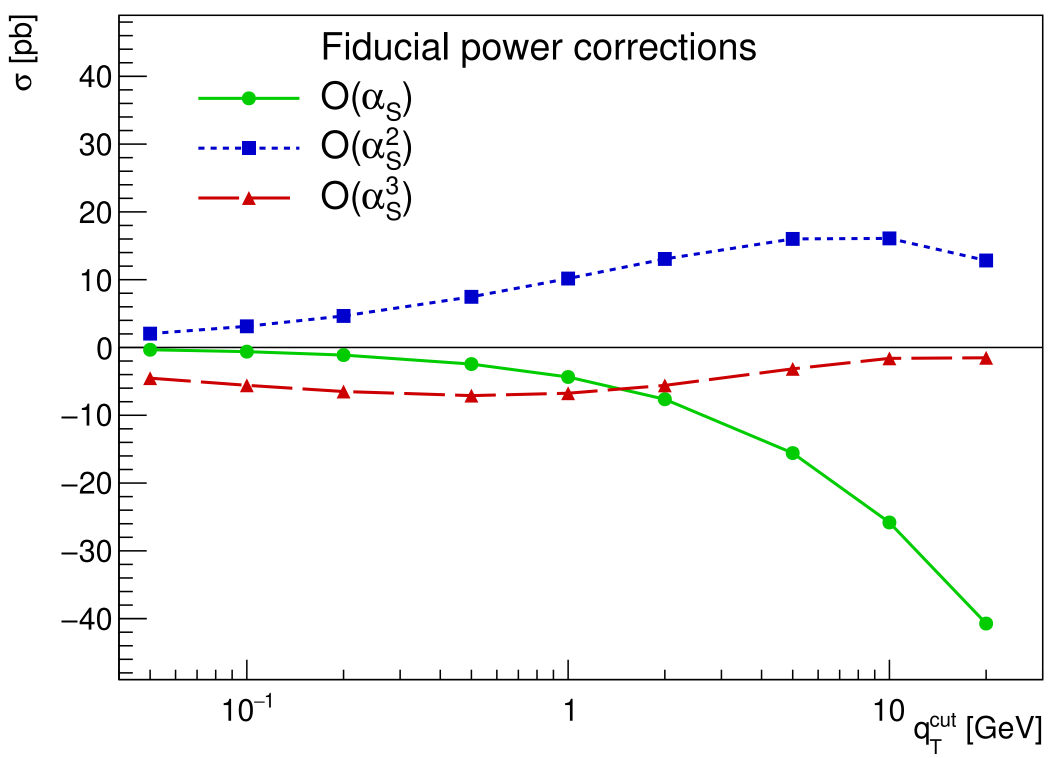
$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\mathrm{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \mathcal{O}\left((q_T^{cut}/M)^p \right)$$

- $d\sigma^{F+\mathrm{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity $\mathcal{O}\left((q_T^{cut}/M)^p\right)$.
- Typical cuts on the p_T and rapidities of the final state particles leads to linear $(p = 1)$ "fiducial" power corrections (FPC) $\left((q_T^{cut}/M)^p\right)$ [Alekhin et al.('21)].
- The limit $q_T^{cut} \to 0$ leads to large cancellations and large numerical integration uncertainties.
- **Key point**: FPC are absent in resummed calculations when q_T recoil is correctly taken into account.

$$\int_{cuts} d\sigma^{F+\mathrm{jets}} \xrightarrow{q_T \to 0} \int_{cuts} d\widetilde{\sigma}^{CT} + \mathcal{O}\left((q_T/M)^2 \right)$$

where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm with the Born amplitude $\hat{\sigma}^{(0)}$ evaluated with the recoil kinematics [Catani,de Florian,G.F.,Grazzini('15)].

Fiducial power corrections up to N^{3}LO



- Flip sign of the FPC with order. Alternating-sign "unphysical" factorial growth of the FO expansion due to symmetric cuts [Salam,Slade('21)].
- Unphysical behaviour can be removed within resummed perturbative predictions. *However* the goal of having precise FO calculations is very relevant.
- No reduction of FPC with higher orders. At N^{3}LO with $q_T^{cut} = 0.05\,\text{GeV}$ -0.4% (+0.3% from α_S^2 and a -0.7% α_S^3.
- Our method is crucial when *local* calculations are not available or when large numerical uncertainties are associated to the $q_T \to 0$ limit (e.g. at N^3LO).