# Theoretical concepts and measurement prospects for BSM trilinear couplings: a case study for scalar top quarks

**Based on**

**arXiv:2212.11213 (EPJC) in collaboration with Henning Bahl and Georg Weiglein**


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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

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# Introduction

- Numerous reasons to expect Beyond-the-Standard-Model physics (e.g. baryon asymmetry, hierarchy problem, dark matter, …)

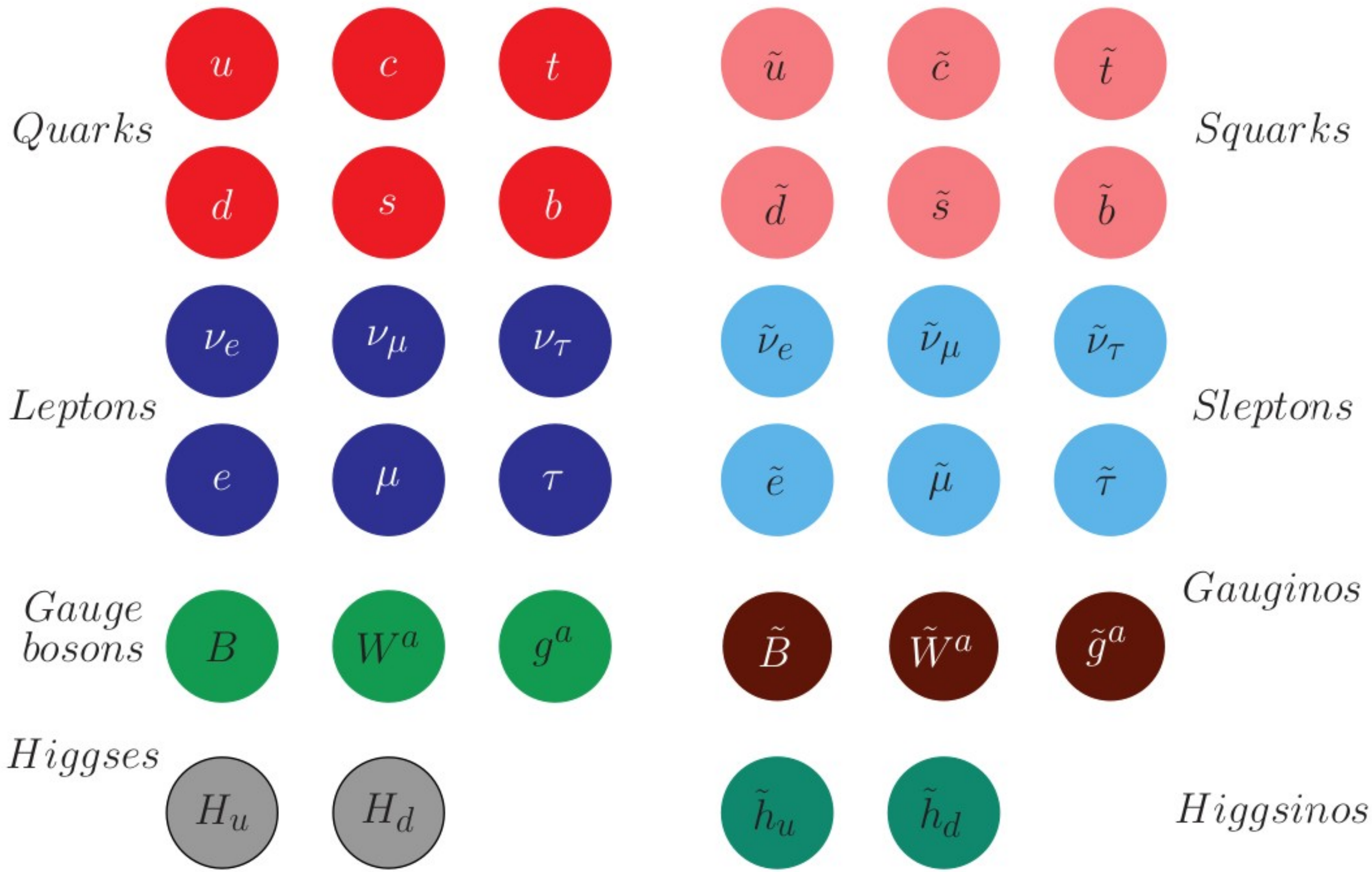
- BSM theories commonly involve **additional scalars**, e.g.
  - ***Extended Higgs sectors*** → bottom-up extensions of the SM, supersymmetric models (MSSM, NMSSM, ...)
  - ***Scalar partners*** → SUSY, ...

- **BSM scalar trilinear couplings** (not generated via EWSB) especially interesting
  → *how could we access such trilinear couplings via experiments?*
  → *how should we define them theoretically?*

- *In this talk*: Minimal Supersymmetric Standard Model (MSSM) as an example
  & trilinear coupling between scalar tops and Higgs boson
  $X_t$ ≡ *stop mixing parameter*

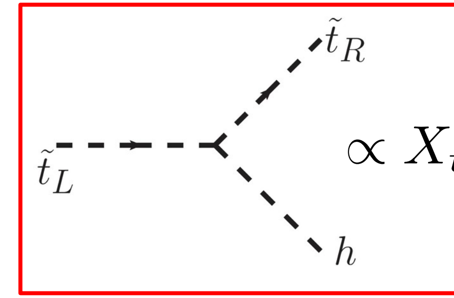
NB: most of conclusions expected to apply for other BSM models too!

# The Minimal Supersymmetric Standard Model




→ ***Stops $\tilde{t}_L$, $\tilde{t}_R$ ≡ scalar partners of top quarks***

# Stop sector and stop mixing parameter




- Stop mass matrix (in gauge eigenstate basis $\tilde{t}_L$, $\tilde{t}_R$):

$$\mathbf{M}_{\tilde{t}} = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + \cos(2\beta)(\frac{1}{2} - \frac{2}{3}s_W^2)M_Z^2 & m_t\, X_t^* \\ m_t\, X_t & m_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3}\cos(2\beta)s_W^2 M_Z^2 \end{pmatrix}$$

- $m_{\tilde{t}L}$, $m_{\tilde{t}R}$: stop soft SUSY-breaking masses; $X_t \equiv A_t - \mu^* \cot\beta$: stop mixing parameter; $A_t$: soft trilinear stop coupling; μ: Higgsino mass parameter; tanβ: ratio of VEVs of Higgs doublets

- Diagonalise the stop mass matrix (→ mass eigenstate basis $\tilde{t}_1$, $\tilde{t}_2$)

$$\mathbf{U}_{\tilde{t}}\mathbf{M}_{\tilde{t}}\mathbf{U}_{\tilde{t}}^\dagger = \mathrm{diag}(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)$$

with $m_{\tilde{t}_{1,2}}^2 = m_t^2 + \frac{1}{2}\left\{ m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 \mp \sqrt{\left[m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2 + M_Z^2 c_{2\beta}\left(\frac{1}{2} - \frac{4}{3}s_W^2\right)\right]^2 + 4m_t^2|X_t|^2}\right\}$

and $\cos(2\theta_{\tilde{t}}) = \dfrac{m_{\tilde{t}_R}^2 - m_{\tilde{t}_L}^2 - M_Z^2 c_{2\beta}(\frac{1}{2} - \frac{4}{3}s_W^2)}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}$ *(stop mixing angle)*

- In the following, we assume $X_t$ to be real for simplicity

# Experimental probes of $X_t$

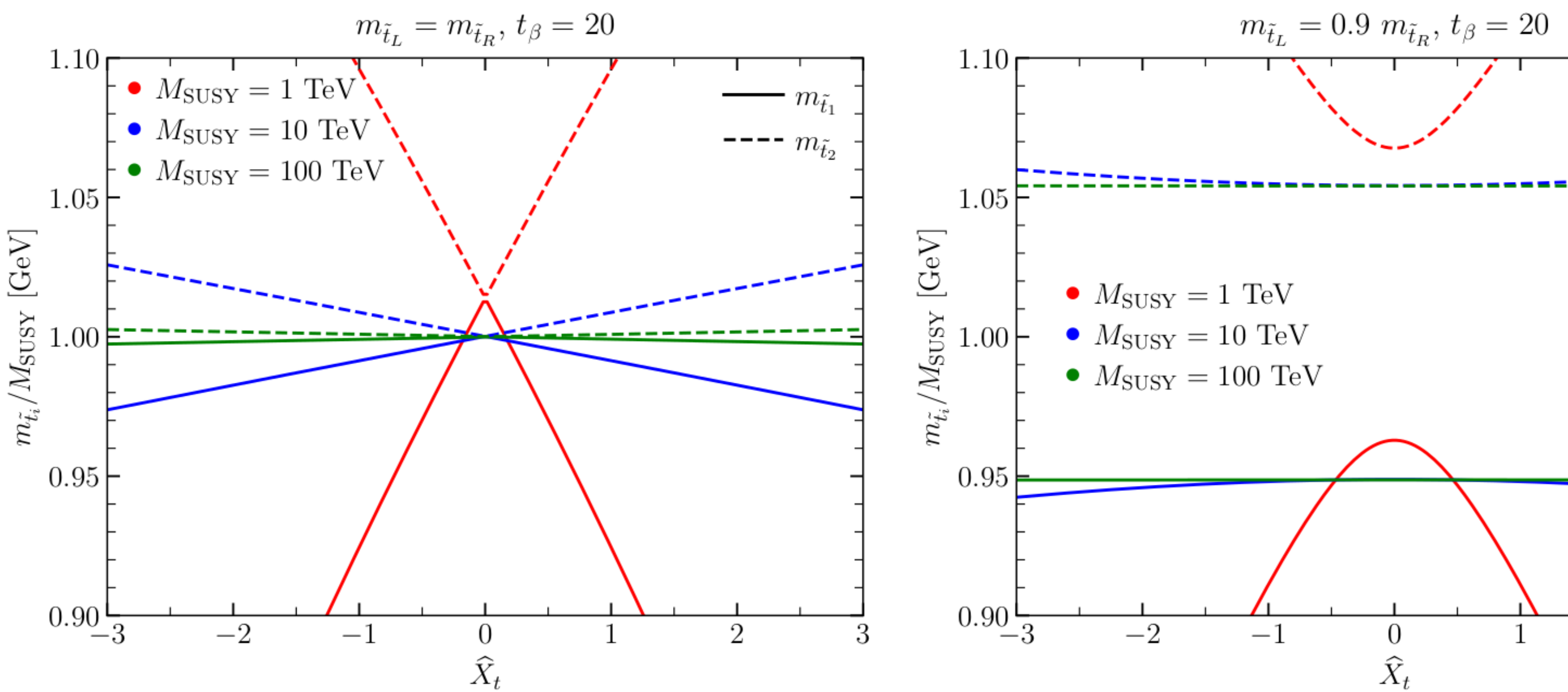
# Accessing $X_t$ via stop mass measurements

*Stop mass eigenvalues*

$$m^2_{\tilde{t}_{1,2}} = m_t^2 + \frac{1}{2}\left\{m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R} \mp \sqrt{\left[m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R} + M_Z^2 c_{2\beta}\left(\frac{1}{2} - \frac{4}{3}s_W^2\right)\right]^2 + 4m_t^2|X_t|^2}\right\}$$

$$M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$$

$$\widehat{X}_t = X_t / M_{\text{SUSY}}$$




[Bahl, JB, Weiglein '22]

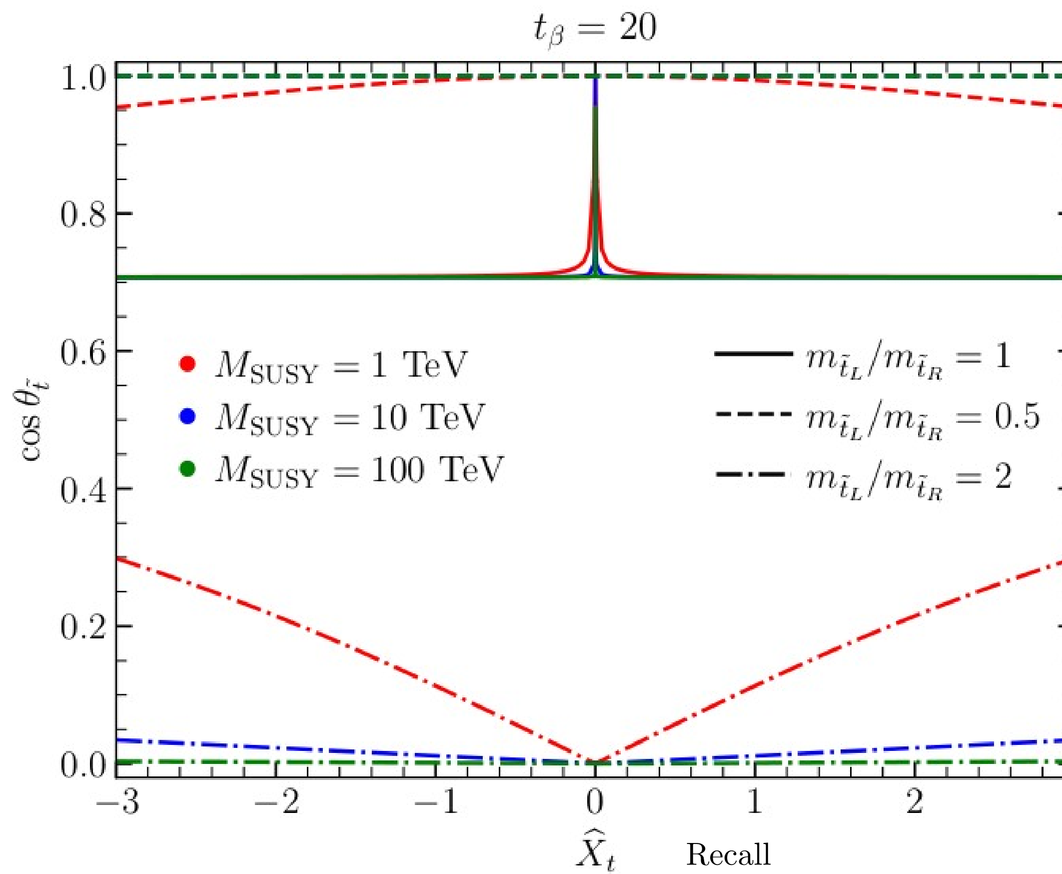
- Assumption on relation between soft masses is necessary ✗
- Not possible in general to disentangle $X_t$ from measurement of $m_{\tilde{t}1}$, $m_{\tilde{t}2}$ only
- Sensitivity lost as stop masses increase ✗

# Accessing $X_t$ via a measurement of the stop mixing angle

*Stop mixing angle*

$$\cos(2\theta_{\tilde t}) = \frac{m^2_{\tilde t_R} - m^2_{\tilde t_L} - M_Z^2 c_{2\beta}(\frac{1}{2} - \frac{4}{3}s_W^2)}{m^2_{\tilde t_2} - m^2_{\tilde t_1}}$$

- Measurement of stop mixing angle already challenging (see e.g. [Rolbiecki, Tattersall, Moortgat-Pick '09])
- But supposing it can be done (+ measurement of $m_{\tilde t_1}$, $m_{\tilde t_2}$), can we derive $X_t$?
- Again sensitivity lost as stop masses increase, as well as if $m_{\tilde t_L} \sim m_{\tilde t_R}$ ✗

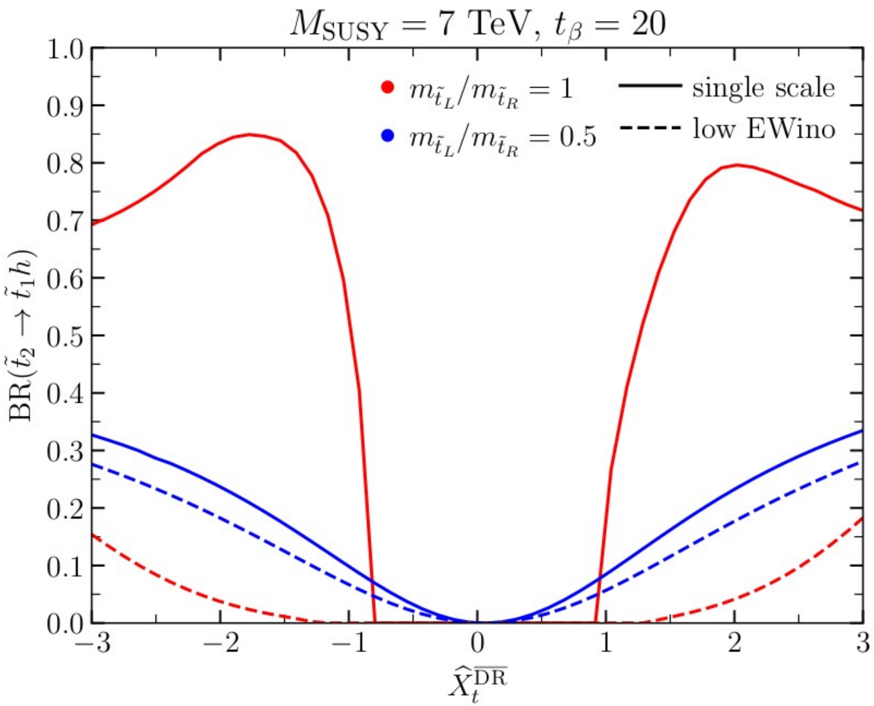


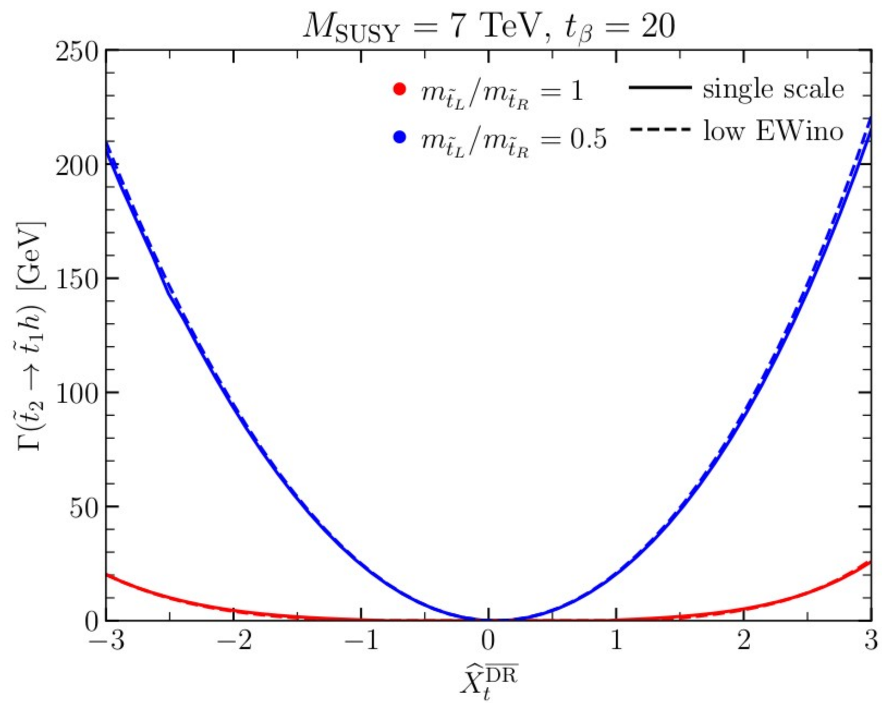

Recall
$M_{\rm SUSY} = \sqrt{m_{\tilde t_L} m_{\tilde t_R}}$
$\widehat X_t = X_t/M_{\rm SUSY}$

[Bahl, JB, Weiglein '22]

# Accessing $X_t$ via stop decays

- Stop decays, like $\tilde{t}_2 \rightarrow \tilde{t}_1 h$, can depend on $X_t$, from leading order
- With SUSY-HIT [Djouadi, Mühlleitner, Spira '06], investigate 2 scenarios
  1) Single scale: all SUSY-breaking masses = $M_{SUSY}$ = 7 TeV → $\tilde{t}_2$ decays to $\tilde{t}_1$ h only
  2) Set instead $M_1 = M_2 = \mu = M_{SUSY}/2$ → light electroweakinos → other decays of $\tilde{t}_2$ open




[Bahl, JB, Weiglein '22]

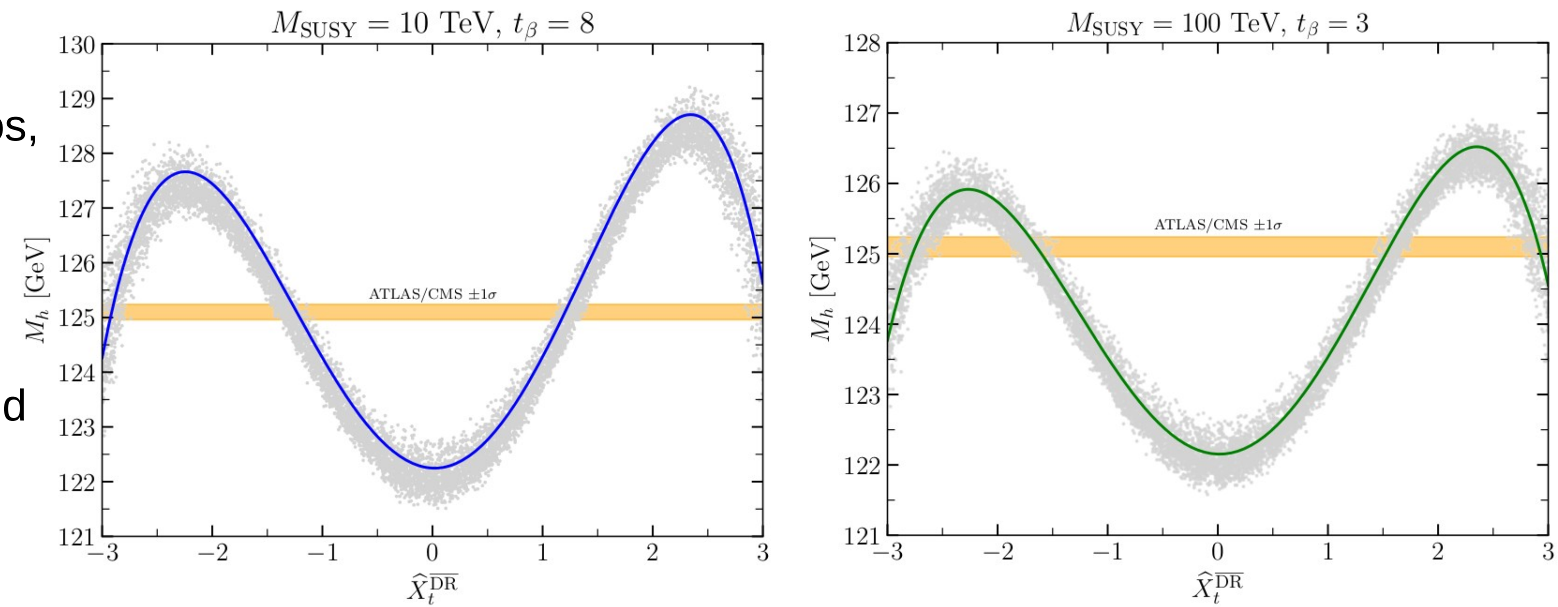
- Usefulness depends highly on sparticle spectrum:
  If $m_{\tilde{t}L} \sim m_{\tilde{t}R}$ or if other decay channels are open (e.g. to quark+electroweakino), it becomes more difficult to extract $X_t$ from stop decay

# Accessing $X_t$ via the Higgs boson mass

- $X_t$ enters prediction for $M_h$ from 1L:

$$M_h^2 \simeq \underbrace{m_h^2}_{M_Z^2 c_{2\beta}^2} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\rm SUSY}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

- **Blue/green lines**: common mass scenarios, i.e. all non-SM masses $= M_{SUSY}$ and $A_{f\neq t} = 0$
- **Grey points**: scan over SUSY parameters (masses and trilinears) between $M_{SUSY}/2$ and $2\,M_{SUSY}$




[Bahl, JB, Weiglein '22] *with* `FeynHiggs 2.18.1`

- Significant dependence of $M_h$ on $X_t$, no matter if $m_{\tilde{t}L} \sim m_{\tilde{t}R}$ or not, and even for high SUSY scale, at 10 or 100 TeV!
- If stop masses and tanβ known → **$X_t$ can be extracted from $M_h$** ✓

# How to define $X_t$ theoretically: choices of renormalisation schemes

# Renormalisation of the stop/top sector

- One choice of parameters: $m_t$, $m_{\tilde{t}L}$, $m_{\tilde{t}R}$, $X_t$
- Define counter terms: $m_t \to m_t + \delta^{(1)} m_t, \quad m^2_{\tilde{t}_{L/R}} \to m^2_{\tilde{t}_{L/R}} + \delta^{(1)} m^2_{\tilde{t}_{L/R}}, \quad X_t \to X_t + \delta^{(1)} X_t$
- Stop mass matrix counterterm (gauge eigenstate basis):

$$\delta^{(1)}\mathbf{M}_{\tilde{t}} = \begin{pmatrix} \delta^{(1)} m^2_{\tilde{t}_L} + \delta^{(1)} m_t^2 & X_t^* \, \delta^{(1)} m_t + m_t \, \delta^{(1)} X_t^* \\ X_t \, \delta^{(1)} m_t + m_t \, \delta^{(1)} X_t & \delta^{(1)} m^2_{\tilde{t}_R} + \delta^{(1)} m_t^2 \end{pmatrix}$$

- Rotate to mass eigenstate basis: $\mathbf{U}_{\tilde{t}} \, \delta^{(1)}\mathbf{M}_{\tilde{t}} \, \mathbf{U}^\dagger_{\tilde{t}} = \begin{pmatrix} \delta^{(1)} m^2_{\tilde{t}_1} & \delta^{(1)} m^2_{\tilde{t}_{12}} \\ (\delta^{(1)} m^2_{\tilde{t}_{12}})^* & \delta^{(1)} m^2_{\tilde{t}_2} \end{pmatrix}$
- Relate counterterms in gauge eigenstate basis to those in mass eigenstate basis (easier to impose conditions on):

$$\begin{aligned}
\delta^{(1)} X_t &= \frac{1}{m_t} \Big[ \mathbf{U}_{\tilde{t}_{11}} \mathbf{U}^*_{\tilde{t}_{12}} \left( \delta^{(1)} m^2_{\tilde{t}_1} - \delta^{(1)} m^2_{\tilde{t}_2} \right) \\
&\quad + \delta^{(1)} m^2_{\tilde{t}_{12}} \mathbf{U}_{\tilde{t}_{21}} \mathbf{U}^*_{\tilde{t}_{12}} + \delta^{(1)} m^2_{\tilde{t}_{21}} \mathbf{U}_{\tilde{t}_{11}} \mathbf{U}^*_{\tilde{t}_{22}} - X_t \delta^{(1)} m_t \Big] , \\
\delta^{(1)} m^2_{\tilde{t}_L} &= \delta^{(1)} m^2_{\tilde{t}_1} |\mathbf{U}_{\tilde{t}_{11}}|^2 + \delta^{(1)} m^2_{\tilde{t}_2} |\mathbf{U}_{\tilde{t}_{12}}|^2 \\
&\quad + \delta^{(1)} m^2_{\tilde{t}_{12}} \mathbf{U}_{\tilde{t}_{21}} \mathbf{U}^*_{\tilde{t}_{11}} + \delta^{(1)} m^2_{\tilde{t}_{21}} \mathbf{U}_{\tilde{t}_{11}} \mathbf{U}^*_{\tilde{t}_{21}} - 2 m_t \, \delta^{(1)} m_t , \\
\delta^{(1)} m^2_{\tilde{t}_R} &= \delta^{(1)} m^2_{\tilde{t}_1} |\mathbf{U}_{\tilde{t}_{12}}|^2 + \delta^{(1)} m^2_{\tilde{t}_2} |\mathbf{U}_{\tilde{t}_{22}}|^2 \\
&\quad + \delta^{(1)} m^2_{\tilde{t}_{12}} \mathbf{U}_{\tilde{t}_{22}} \mathbf{U}^*_{\tilde{t}_{12}} + \delta^{(1)} m^2_{\tilde{t}_{21}} \mathbf{U}_{\tilde{t}_{12}} \mathbf{U}^*_{\tilde{t}_{22}} - 2 m_t \, \delta^{(1)} m_t .
\end{aligned}$$

# Process-dependent/-independent OS renormalisation schemes

- For stop/top masses, simple interpretation of OS scheme in terms of **physical masses**

$$\delta^{(1)} m^2_{\tilde{t}_i} = \mathrm{Re}\Sigma^{(1)}_{\tilde{t}_i\tilde{t}_i}(m^2_{\tilde{t}_i}), \quad \delta^{(1)} m_t = \mathrm{Re}\Sigma^{(1)}_{tt}(m^2_t)$$

$\Sigma^{(1)}$: *1L self-energy*

- For $X_t$, no unique/straightforward choice
- **Process-dependent** definition, e.g. with $\tilde{t}_2 \to \tilde{t}_1 h$ process

→ difficult to access processes involving $X_t$ experimentally (c.f. previous discussion)

→ depends on sparticle spectrum / only reliable in parts of parameter space

- **Process-independent**, like

$$\delta^{(1)} m^2_{\tilde{t}_{12}} = \frac{1}{2}\,\mathrm{Re}\left[\Sigma^{(1)}_{\tilde{t}_1\tilde{t}_2}(m^2_{\tilde{t}_1}) + \Sigma^{(1)}_{\tilde{t}_1\tilde{t}_2}(m^2_{\tilde{t}_2})\right]$$

from which one can obtain $\delta^{(1)} X_t, \quad \delta^{(1)} m_{\tilde{t}_{L,R}}$

→ but not related to physical observable directly

→ potentially gauge dependent

# $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ / mixed renormalisation schemes

- **$\overline{\text{DR}}$**: set finite parts of all counterterms to 0
  - No direct physical interpretation of parameters
  - But, convenient e.g. with high-scale SUSY scenarios
  - Can be plagued by unphysical non-decoupling effects if gluinos are much heavier than stops

- **$\overline{\text{MDR}}$**: keep idea of $\overline{\text{DR}}$ scheme, but define finite part of counterterms to absorb unphysical large corrections

$$\left(m_{\tilde{t}_{L,R}}^{\overline{\text{MDR}}}\right)^2(Q) = \left(m_{\tilde{t}_{L,R}}^{\overline{\text{DR}}}\right)^2(Q)\left[1+\frac{\alpha_s}{\pi}C_F\frac{|M_3|^2}{m^2_{\tilde{t}_{L,R}}}\left(1+\ln\frac{Q^2}{|M_3|^2}\right)\right]$$

$$X_t^{\overline{\text{MDR}}}(Q) = X_t^{\overline{\text{DR}}}(Q) - \frac{\alpha_s}{\pi}C_F M_3\left(1+\ln\frac{Q^2}{|M_3|^2}\right)$$

[Bahl, Sobolev, Weiglein ‘19]

- **Mixed**: renormalise stop and top masses OS, but keep $X_t$ in $\overline{\text{DR}}$/$\overline{\text{MDR}}$ scheme (possible problems with 1/ε * ε pieces at higher orders)

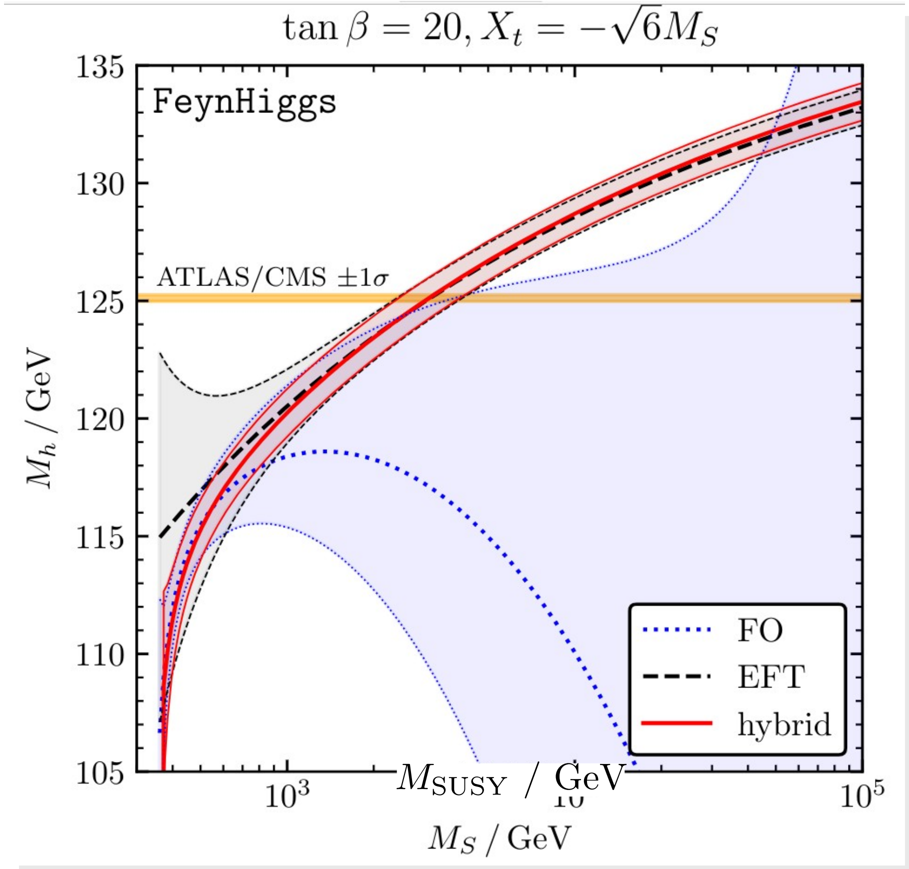
# What renormalisation scheme to use for $X_t$ in Higgs mass calculations

# Renormalisation of $X_t$ for different types of Higgs mass calculations

$$M_h^2 \simeq m_h^2 + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

**3 types of calculations for $M_h$:**

- **Fixed order**: (process-independent) OS scheme of $X_t$ possible and convenient
- **EFT**: if $X_t$ in OS scheme, large $\log(M_{SUSY}/m_t)$ pieces remain, which are resummed by running of $X_t$
  → $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ scheme preferable for $X_t$
- **Hybrid**: use OS for fixed-order part; $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ for EFT part
- Both in EFT and hybrid approaches
  → $X_t^{\overline{\text{DR}}}$ must be extracted from physical input or related to $X_t^{\text{OS}}$
  → **large logs!**




[KUTS report, Slavich, Heinemeyer et al. '20]

# OS to $\overline{\text{DR}}$ conversion of $X_t$ and large logarithms

- OS → $\overline{\text{DR}}$ conversion of $X_t$:

$$X_t^{\text{OS}} = X_t^{\overline{\text{DR}}}(M_{\text{SUSY}})\,\frac{m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}})}{m_t^{\text{OS}}} - \frac{1}{m_t^{\text{OS}}}\delta^{(1)}(m_t X_t)\big|_{\text{fin}}$$

**both terms contain large logs!**

- First from $m_t$:

$$m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}}) = m_t^{\text{OS}} \underbrace{- m_t^{\text{OS}}\left[\left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}\right)\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + \text{no large logs}\right] + \dots}_{=\delta^{(1)} m_t^{\text{OS}}(Q=M_{\text{SUSY}})\big|_{\text{fin}}}$$

*sub-leading*

→ resum the large logs by using $m_t^{\overline{\text{DR}},\text{MSSM}}(Q=M_{\text{SUSY}})$ or $m_t^{\overline{\text{MS}},\text{SM}}(Q=M_{\text{SUSY}})$ ✓

- What about the 2[nd] term? → we consider $O(\alpha_t)$ [with $\alpha_t \equiv y_t^2/4\pi$] and we expand in powers of $v/M_{\text{SUSY}}$, as done for EFT calculations

# OS to $\overline{\text{DR}}$ conversion of $X_t$ and large logarithms

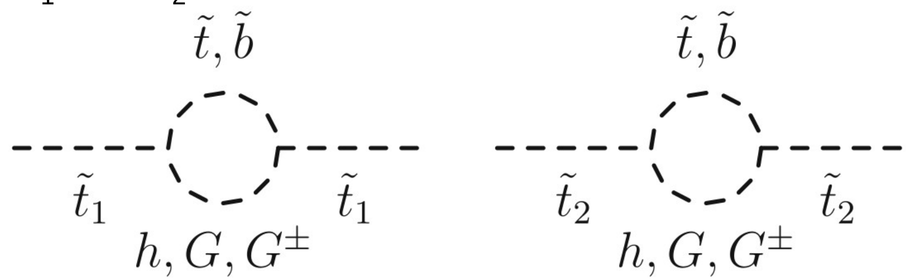
$$X_t^{\text{OS}} = X_t^{\overline{\text{DR}}}(M_{\text{SUSY}})\,\frac{m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}})}{m_t^{\text{OS}}} - \frac{1}{m_t^{\text{OS}}}\delta^{(1)}(m_t X_t)\big|_{\text{fin}}$$

- **Case 1**: $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{\text{SUSY}}$ and for $v/M_{\text{SUSY}} \ll 1$ (as in EFT setting)

At $O(\alpha_t)$:

$$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{3\alpha_t}{16\pi}\, m_t X_t\, |\widehat{X}_t|^2 \ln\frac{M^2_{\text{SUSY}}}{m_t^2} + \text{no large logs}$$

- Caused by diagrams in $\tilde{t}_1$ and $\tilde{t}_2$ mass counterterms of the form




- **Same type of diagrams as in external-leg corrections!**
  - IR divergence for $m_{\tilde{t}2} \to m_{\tilde{t}1}$, cured by real Higgs radiation (NB: Higgs massless in limit $v/M_{\text{SUSY}} \ll 1$)
  - For $m_{\tilde{t}2} \neq m_{\tilde{t}1}$, IR div. regulated by squared-mass difference → large log remains
  - Can't be resummed by standard EFT techniques, but size of 2L corrections much smaller than 1L [Bahl, JB, Weiglein '21]

# OS to $\overline{\text{DR}}$ conversion of $X_t$ and large logarithms

$$X_t^{\text{OS}} = X_t^{\overline{\text{DR}}}(M_{\text{SUSY}})\,\frac{m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}})}{m_t^{\text{OS}}} - \frac{1}{m_t^{\text{OS}}}\delta^{(1)}(m_t X_t)\big|_{\text{fin}}$$

- **Case 1**: $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{\text{SUSY}}$ and for $v/M_{\text{SUSY}} \ll 1$ (as in EFT setting)

At $O(\alpha_t)$: $$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{3\alpha_t}{16\pi}\, m_t X_t\, |\widehat{X}_t|^2 \ln\frac{M_{\text{SUSY}}^2}{m_t^2} + \text{no large logs}$$

- **Case 2**: $m_{\tilde{t}L} \neq m_{\tilde{t}R}$ and for $v/M_{\text{SUSY}} \ll 1$

At $O(\alpha_t)$: $$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{\alpha_t}{8\pi}\, m_t X_t\, |\widehat{X}_t|^2 \left(\frac{2m_{\tilde{t}_L}}{m_{\tilde{t}_R}}\ln\frac{m_{\tilde{t}_L}^2}{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|} + \frac{m_{\tilde{t}_R}}{m_{\tilde{t}_L}}\ln\frac{m_{\tilde{t}_R}^2}{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|}\right)$$

Once again large logs, also regulated by squared-mass difference $|m_{\tilde{t}2}^2 - m_{\tilde{t}1}^2| \sim |m_{\tilde{t}L}^2 - m_{\tilde{t}R}^2|$

# OS to $\overline{\text{DR}}$ conversion of $X_t$ and large logarithms

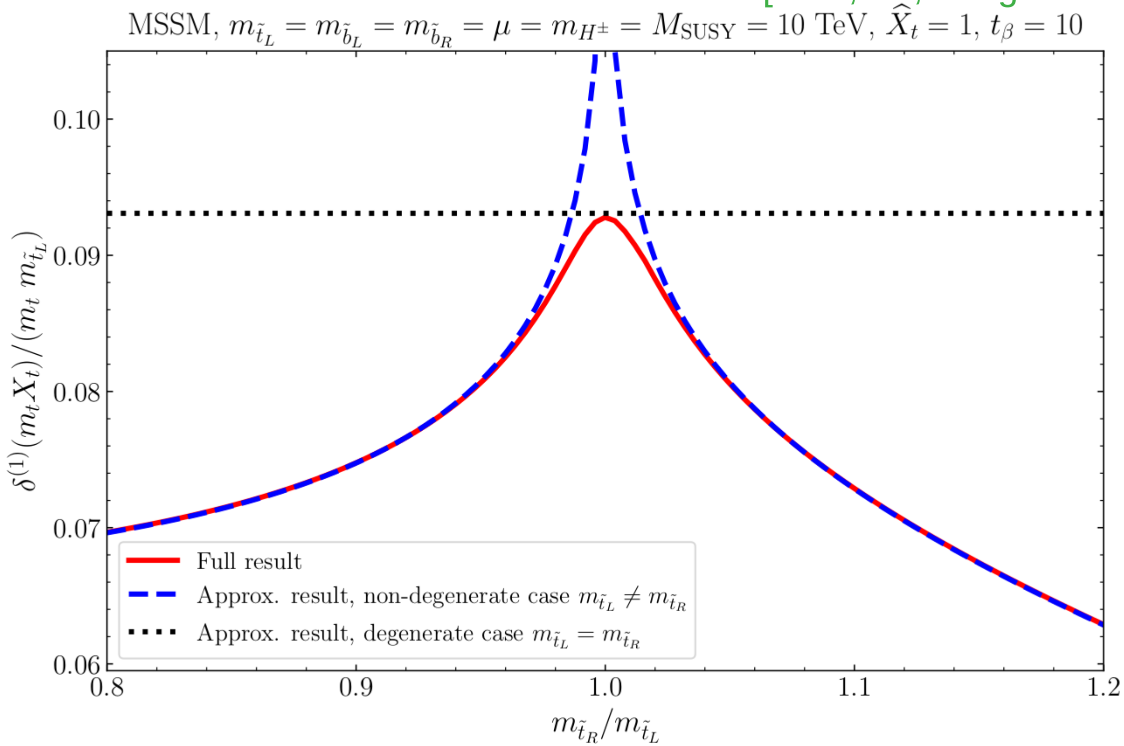
[Bahl, JB, Weiglein '22]

- Case 1: $m_{\tilde{t}L} = m_{\tilde{t}R}$

$$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{3\alpha_t}{16\pi}\, m_t X_t\, |\widehat{X}_t|^2 \ln \frac{M_{\text{SUSY}}^2}{m_t^2}$$
$$+\text{no large logs}$$

- Case 2: $m_{\tilde{t}L} \neq m_{\tilde{t}R}$

$$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{\alpha_t}{8\pi}\, m_t X_t\, |\widehat{X}_t|^2$$
$$\times \left( \frac{2 m_{\tilde{t}_L}}{m_{\tilde{t}_R}} \ln \frac{m_{\tilde{t}_L}^2}{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|} + \frac{m_{\tilde{t}_R}}{m_{\tilde{t}_L}} \ln \frac{m_{\tilde{t}_R}^2}{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|} \right)$$




→ no transition between the two expanded cases $m_{\tilde{t}L} = m_{\tilde{t}R}$ and $m_{\tilde{t}L} \neq m_{\tilde{t}R}$

→ full result is well behaved (c.f. [Bahl, JB, Weiglein '21]) but one is then mixing orders in EFT expansion in $v/M_{\text{SUSY}}$

*Our suggestion:*

→ **keep $X_t$ in $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ scheme even in fixed-order calculation**, to avoid conversion

# Summary

- **Experimental probes of $X_t$:**
  - **$M_h$ seems the best avenue to determine $X_t$** (once stop masses and tanβ are known)
    → sensitivity to $X_t$ no matter the stop mass hierarchy, and even to high SUSY scales

  - Stop decays also an option, but highly dependent on sparticle spectrum (i.e. what decay channels are open) → only useful for parts of parameter space

- **Renormalisation scheme choices for $X_t$:**
  - Choice of scheme for $X_t$ in $M_h$ calculation crucial, as $M_h$ is best way to access $X_t$
  - **No ideal choice**, but given that $\overline{\text{DR}}/\overline{\text{MDR}}$ is preferable for EFT and hybrid
    → use also **$\overline{\text{DR}}/\overline{\text{MDR}}$ for $X_t$ (mixed scheme) in fixed-order part of hybrid calculation**, to avoid large log in conversion (would however reappear in extraction of $X_t$ from experimental input + issues with mixed OS/$\overline{\text{DR}}$ schemes at higher orders)

- Results in principle applicable more broadly to BSM trilinear couplings

# Thank you very much for your attention!

**Contact**

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# Stop sector and stop mixing parameter – details

- Stop mass matrix (in gauge eigenstate basis $\tilde{t}_L$, $\tilde{t}_R$):

$$\mathbf{M}_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}_L} + m_t^2 + \cos(2\beta)(\frac{1}{2} - \frac{2}{3}s_W^2)M_Z^2 & m_t\ X_t^* \\ m_t\ X_t & m^2_{\tilde{t}_R} + m_t^2 + \frac{2}{3}\cos(2\beta)s_W^2 M_Z^2 \end{pmatrix}$$

- $m_{\tilde{t}L}$, $m_{\tilde{t}R}$: stop soft SUSY-breaking masses; $X_t \equiv A_t - \mu^* \cot\beta$: stop mixing parameter
- Diagonalise the stop mass matrix

$$\mathbf{U}_{\tilde{t}}\mathbf{M}_{\tilde{t}}\mathbf{U}_{\tilde{t}}^\dagger = \mathrm{diag}(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2})$$

with $m^2_{\tilde{t}_{1,2}} = m_t^2 + \dfrac{1}{2}\left\{ m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R} \mp \sqrt{\left[m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R} + M_Z^2 c_{2\beta}\left(\dfrac{1}{2} - \dfrac{4}{3}s_W^2\right)\right]^2 + 4m_t^2|X_t|^2}\right\}$

and $\mathbf{U}_{\tilde{t}} = \begin{pmatrix} c_{\tilde{t}} & s_{\tilde{t}}e^{-i\phi_{X_t}} \\ -s_{\tilde{t}}e^{i\phi_{X_t}} & c_{\tilde{t}} \end{pmatrix}$ where $\phi_{X_t} = \arg(X_t)$

$$\cos(2\theta_{\tilde{t}}) = \frac{m^2_{\tilde{t}_R} - m^2_{\tilde{t}_L} - M_Z^2 c_{2\beta}(\frac{1}{2} - \frac{4}{3}s_W^2)}{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}$$

*(stop mixing angle)*

- In the following, we assume $X_t$ to be real for simplicity ($\to \phi_{X_t}=0$)

# Accessing $X_t$ via stop decays

- Decay $\tilde{t}_2 \rightarrow \tilde{t}_1\, h$ depends on $X_t$ at tree level

$$d\Gamma_{\tilde{t}_2\to\tilde{t}_1 h} = \frac{1}{64\pi^2}\frac{\sqrt{(m^2_{\tilde{t}_2}-(m_{\tilde{t}_1}+m_h)^2)(m^2_{\tilde{t}_2}-(m_{\tilde{t}_1}-m_h)^2)}}{m^3_{\tilde{t}_2}}|\underbrace{\mathcal{M}(\tilde{t}_2\to\tilde{t}_1 h)}_{\propto X_t}|^2 d\cos\theta$$

- Limit $m_{\tilde{t}1}$, $m_h$ << $m_{\tilde{t}2}$

$$d\Gamma_{\tilde{t}_2\to\tilde{t}_1 h} \xrightarrow{m_h,m_{\tilde{t}_1}\ll m_{\tilde{t}_2}} \frac{1}{64\pi^2}\frac{1}{m_{\tilde{t}_2}}|\mathcal{M}(\tilde{t}_2\to\tilde{t}_1 h)|^2 d\cos\theta \propto \frac{|X_t|^2}{m_{\tilde{t}_2}} \underset{X_t\sim\mathcal{O}(M_{\mathrm{SUSY}})}{\sim} \mathcal{O}(M_{\mathrm{SUSY}})$$

- Limit $m_h$ << $m_{\tilde{t}1}$, $m_{\tilde{t}2}$

$$d\Gamma_{\tilde{t}_2\to\tilde{t}_1 h} \xrightarrow{m_h\ll m_{\tilde{t}_1}\sim m_{\tilde{t}_2}} \frac{1}{64\pi^2}\frac{|m^2_{\tilde{t}_2}-m^2_{\tilde{t}_1}|}{m^3_{\tilde{t}_2}}|\mathcal{M}(\tilde{t}_2\to\tilde{t}_1 h)|^2 d\cos\theta \simeq$$

$$\simeq \frac{1}{64\pi^2}\frac{2m_t|X_t|}{m^3_{\tilde{t}_2}}|\mathcal{M}(\tilde{t}_2\to\tilde{t}_1 h)|^2 d\cos\theta \propto \frac{m_t|X_t|^3}{m^3_{\tilde{t}_2}} \underset{X_t\sim\mathcal{O}(M_{\mathrm{SUSY}})}{\sim} \mathcal{O}(m_t)$$

*(phase space suppression)*

# Accessing $X_t$ via the Higgs boson mass

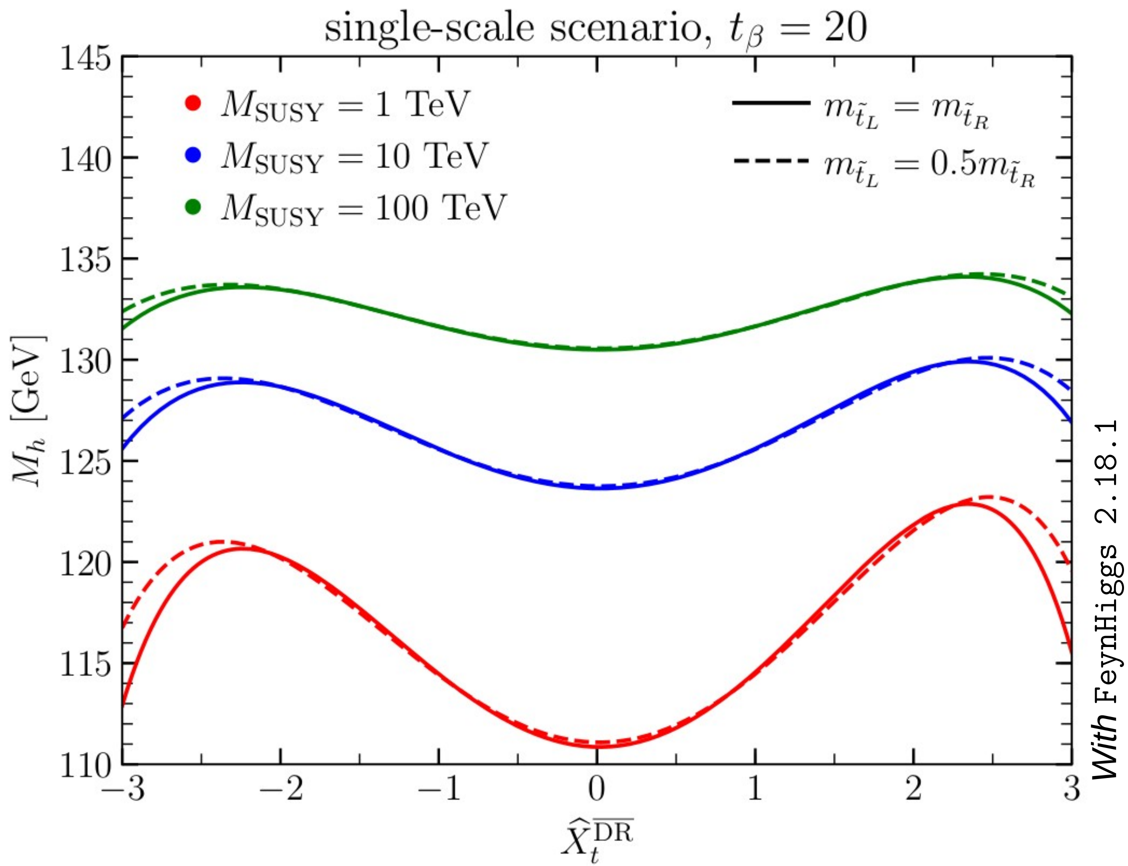
- Another observable where $X_t$ enters is $M_h$, from 1L

$$M_h^2 \simeq \underbrace{m_h^2}_{M_Z^2 c_{2\beta}^2} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

- Single scale scenario (all soft SUSY-breaking masses = $M_A$ = $\mu$ = $M_{SUSY}$)

- Significant dependence of $M_h$ on $X_t$, no matter if $m_{\tilde{t}L} \sim m_{\tilde{t}R}$ or not, and even for high SUSY scale (10 or 100 TeV)

- $X_t$ could be extracted from $M_h$, if stop masses are known



# Renormalisation of the stop/top sector – alternative choice

- Alternative choice of parameters: $m_t$, $m_{\tilde{t}L}$,$m_{\tilde{t}R}$, $\theta_t$, $\phi_{Xt}$
- Counter terms:

$$m^2_{\tilde{t}_{L/R}} \to m^2_{\tilde{t}_{L/R}} + \delta^{(1)} m^2_{\tilde{t}_{L/R}}, \quad \theta_t \to \theta_t + \delta^{(1)}\theta_t, \quad \phi_{X_t} \to \phi_{X_t} + \delta^{(1)}\phi_{X_t}, \quad m_t \to m_t + \delta^{(1)} m_t$$

- Reexpress stop mass matrix → obtain counterterm matrix elements:

$$\begin{aligned}
\delta^{(1)}\mathbf{M}_{\tilde{t}_{11}} &= \cos^2\theta_{\tilde{t}}\,\delta^{(1)}m^2_{\tilde{t}_1} + \sin^2\theta_{\tilde{t}}\,\delta^{(1)}m^2_{\tilde{t}_2} + (m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1})\sin 2\theta_{\tilde{t}}\,\delta^{(1)}\theta_{\tilde{t}},\\
\delta^{(1)}\mathbf{M}_{\tilde{t}_{12}} &= (\delta^{(1)}m^2_{\tilde{t}_1} - \delta^{(1)}m^2_{\tilde{t}_2})\sin\theta_{\tilde{t}}\cos\theta_{\tilde{t}}\,e^{-i\phi_{X_t}}\\
&\quad + (m^2_{\tilde{t}_1} - m^2_{\tilde{t}_8 2})(\delta^{(1)}\theta_{\tilde{t}}\ \cos 2\theta_{\tilde{t}} - i\delta^{(1)}\phi_{X_t}\sin\theta_{\tilde{t}}\ \cos\theta_{\tilde{t}})\ e^{-i\phi_{X_t}},\\
\delta^{(1)}\mathbf{M}_{\tilde{t}_{21}} &= (\delta^{(1)}m^2_{\tilde{t}_1} - \delta^{(1)}m^2_{\tilde{t}_2})\sin\theta_{\tilde{t}}\cos\theta_{\tilde{t}}\,e^{i\phi_{X_t}}\\
&\quad + (m^2_{\tilde{t}_1} - m^2_{\tilde{t}_{22}})(\delta^{(1)}\theta_{\tilde{t}}\ \cos 2\theta_{\tilde{t}} + i\delta^{(1)}\phi_{X_t}\sin\theta_{\tilde{t}}\ \cos\theta_{\tilde{t}})\ e^{i\phi_{X_t}},\\
\delta^{(1)}\mathbf{M}_{\tilde{t}_{22}} &= \cos^2\theta_{\tilde{t}}\,\delta^{(1)}m^2_{\tilde{t}_2} + \sin^2\theta_{\tilde{t}}\,\delta^{(1)}m^2_{\tilde{t}_1} + (m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})\sin 2\theta_{\tilde{t}}\,\delta^{(1)}\theta_{\tilde{t}}\ .
\end{aligned}$$

- Obtain for the off-diagonal mass counterterm in mass eigenstate basis

$$\delta^{(1)} m^2_{\tilde{t}_{12}} = e^{-i\phi_{X_t}}(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})(\delta^{(1)}\theta_{\tilde{t}} - i\delta^{(1)}\phi_{X_t}\sin\theta_{\tilde{t}}\ \cos\theta_{\tilde{t}})$$

# On-shell renormalisation schemes – details

- For stop/top masses, simple interpretation of OS scheme in terms of **physical masses**

$$\delta^{(1)} m^2_{\tilde{t}_i} = \mathrm{Re}\Sigma^{(1)}_{\tilde{t}_i\tilde{t}_i}(m^2_{\tilde{t}_i}), \quad \delta^{(1)} m_t = \mathrm{Re}\Sigma^{(1)}_{tt}(m^2_t)$$

$\Sigma^{(1)}$*: 1L self-energy*

- For $X_t$, no unique/straightforward choice!
- **Process-dependent** definition, e.g. with $\tilde{t}_2 \to \tilde{t}_1 h$ process

  → difficult to access processes involving $X_t$ experimentally (c.f. previous discussion)

  → depends on sparticle spectrum / only reliable in parts of parameter space

- **Process-independent**, like

$$\delta^{(1)} m^2_{\tilde{t}_{12}} = \frac{1}{2}\,\mathrm{Re}\left[\Sigma^{(1)}_{\tilde{t}_1\tilde{t}_2}(m^2_{\tilde{t}_1}) + \Sigma^{(1)}_{\tilde{t}_1\tilde{t}_2}(m^2_{\tilde{t}_2})\right]$$

  from which one can obtain $\delta^{(1)} X_t, \quad \delta^{(1)} m_{\tilde{t}_{L,R}}$ with relations shown before

  → but not related to physical observable directly

  → potentially gauge dependent

# $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ / mixed renormalisation schemes – details

- **$\overline{\text{DR}}$**: set finite parts of all counterterms to 0
  - No direct physical interpretation of parameters
  - But, convenient e.g. with high-scale SUSY scenarios
  - Can be plagued by unphysical non-decoupling effects if gluinos are much heavier than stops ✗

- **$\overline{\text{MDR}}$**: keep idea of $\overline{\text{DR}}$ scheme, but define finite part of counterterms to absorb unphysical large corrections

$$\left(m_{\tilde{t}_{L,R}}^{\overline{\text{MDR}}}\right)^2(Q) = \left(m_{\tilde{t}_{L,R}}^{\overline{\text{DR}}}\right)^2(Q)\left[1+\frac{\alpha_s}{\pi}C_F\frac{|M_3|^2}{m_{\tilde{t}_{L,R}}^2}\left(1+\ln\frac{Q^2}{|M_3|^2}\right)\right]$$

$$X_t^{\overline{\text{MDR}}}(Q) = X_t^{\overline{\text{DR}}}(Q) - \frac{\alpha_s}{\pi}C_F M_3\left(1+\ln\frac{Q^2}{|M_3|^2}\right)$$

[Bahl, Sobolev, Weiglein ‘19]

- **Mixed**: renormalise stop and top masses OS, but keep $X_t$ in $\overline{\text{DR}}$/$\overline{\text{MDR}}$ scheme (possible problems with 1/ε * ε pieces at higher orders ✗)

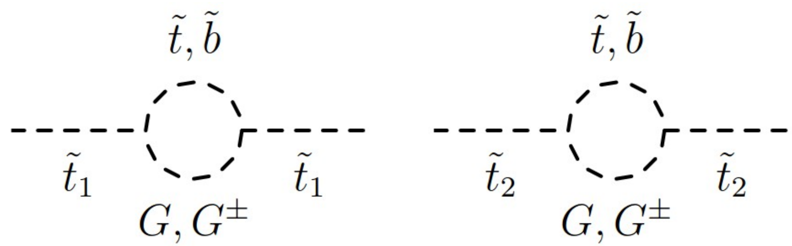
# OS to $\overline{\text{DR}}$ conversion of $X_t$ and large logarithms II

$$X_t^{\text{OS}} = X_t^{\overline{\text{DR}}}(M_{\text{SUSY}})\,\frac{m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}})}{m_t^{\text{OS}}} - \frac{1}{m_t^{\text{OS}}}\delta^{(1)}(m_t X_t)\big|_{\text{fin}}$$

- **Case 1**: $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{SUSY}$ and for $v/M_{SUSY} << 1$ (as in EFT setting)

At O($\alpha_t$): $$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{3\alpha_t}{16\pi}\, m_t X_t\, |\widehat{X}_t|^2 \ln\frac{M_{\text{SUSY}}^2}{m_t^2} + \text{no large logs}$$

- Caused by diagrams in $\tilde{t}_1, \tilde{t}_2$ mass counterterm of the form




- **Same type of diagrams as in external-leg corrections!**
  - IR divergence for $m_{\tilde{t}2} \rightarrow m_{\tilde{t}1}$, cured by real Higgs radiation (NB: Higgs massless in limit $v/M_{SUSY} << 1$)
  - Large log remains for $m_{\tilde{t}2} \neq m_{\tilde{t}1}$, regulated by squared mass difference
  - Can't be resummed by standard EFT techniques, but size of 2L corrections much smaller than 1L [Bahl, JB, Weiglein '21]

# OS to $\overline{\mathrm{DR}}$ conversion of $X_t$ and large logarithms II

$$X_t^{\mathrm{OS}} = X_t^{\overline{\mathrm{DR}}}(M_{\mathrm{SUSY}})\,\frac{m_t^{\overline{\mathrm{DR}},\mathrm{MSSM}}(M_{\mathrm{SUSY}})}{m_t^{\mathrm{OS}}} - \frac{1}{m_t^{\mathrm{OS}}}\delta^{(1)}(m_t X_t)\big|_{\mathrm{fin}}$$

- **Case 1**: $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{SUSY}$ and for $v/M_{SUSY} << 1$ (as in EFT setting)

At O($\alpha_t$): $$\delta^{(1)}(m_t X_t)\Big|_{\mathrm{fin}} = \frac{3\alpha_t}{16\pi}\, m_t X_t\, |\widehat{X}_t|^2 \ln\frac{M_{\mathrm{SUSY}}^2}{m_t^2} + \text{no large logs}$$

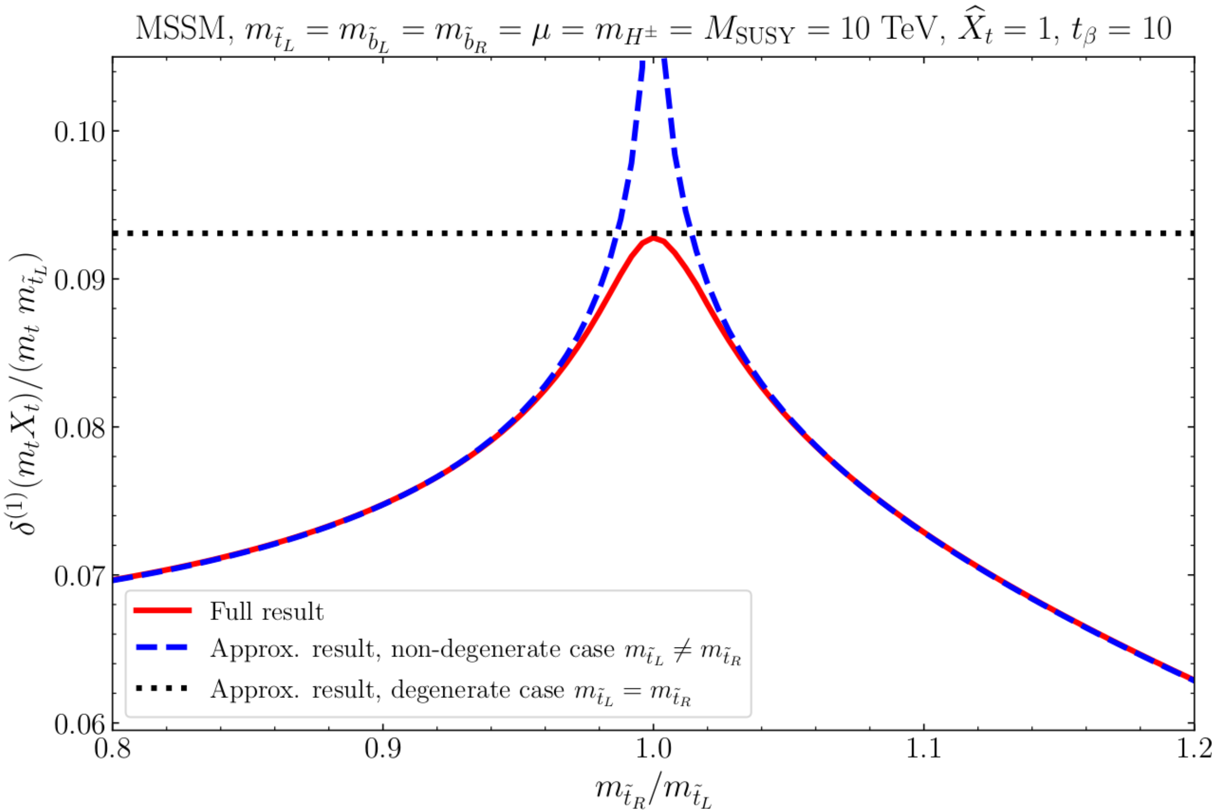
- **Case 2**: $m_{\tilde{t}L} \neq m_{\tilde{t}R}$ and for $v/M_{SUSY} << 1$

At O($\alpha_t$): $$\delta^{(1)}(m_t X_t)\Big|_{\mathrm{fin}} = \frac{\alpha_t}{8\pi}\, m_t X_t\, |\widehat{X}_t|^2 \left(\frac{2m_{\tilde{t}_L}}{m_{\tilde{t}_R}}\ln\frac{m_{\tilde{t}_L}^2}{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|} + \frac{m_{\tilde{t}_R}}{m_{\tilde{t}_L}}\ln\frac{m_{\tilde{t}_R}^2}{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|}\right)$$

- Once again large logs, again regulated by squared mass difference $|m_{\tilde{t}2}^2 - m_{\tilde{t}1}^2| \sim |m_{\tilde{t}L}^2 - m_{\tilde{t}R}^2|$

# OS to $\overline{\text{DR}}$ conversion of $X_t$ and large logarithms III

[Bahl, JB, Weiglein '22]




- Case 1: $m_{\tilde{t}L} = m_{\tilde{t}R}$

$$X_t^{\overline{\text{DR}}}(M_{\text{SUSY}}) = X_t^{\text{OS}} \left\{ 1 + \left[ \frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}\left(1 - \frac{|X_t|^2}{M_{\text{SUSY}}^2}\right)\right] \ln \frac{M_{\text{SUSY}}^2}{m_t^2} \right\} + \cdots$$

- Case 2: $m_{\tilde{t}L} \neq m_{\tilde{t}R}$

$$X_t^{\overline{\text{DR}}}(M_{\text{SUSY}}) = X_t^{\text{OS}} \left\{ 1 + \left[ \frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}\right] \ln \frac{M_{\text{SUSY}}^2}{m_t^2} \right\} + \cdots$$

→ no transition between the two expanded cases $m_{\tilde{t}L} = m_{\tilde{t}R}$ and $m_{\tilde{t}L} \neq m_{\tilde{t}R}$

→ full result is well behaved, but one is then mixing orders in EFT expansion (in $v/M_{\text{SUSY}}$)

→ **keep $X_t$ in $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ scheme even in fixed-order calculation**, to avoid conversion