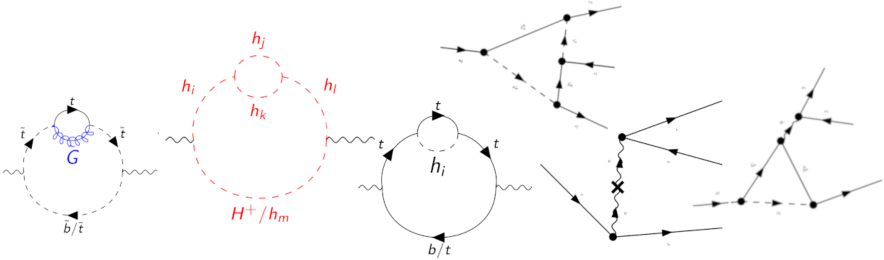
Precise W-boson mass predictions in the complex NMSSM

$\Delta\rho$ and M_W including two-loop SUSY corrections
based on [2308.04059]



Thi Nhung Dao, **Martin Gabelmann**, Margarete Mühlleitner | DESY Theory-Workshop, September 2023

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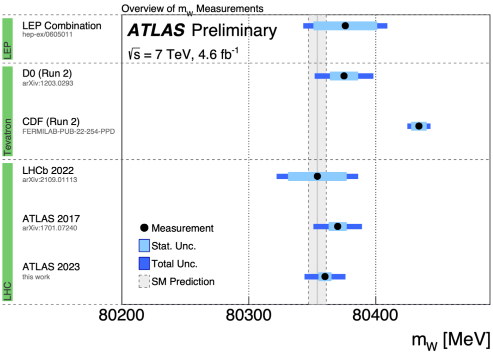
DESY.

The W boson mass...

- two most-recent most-precise individual measurements
- huge discrepancy of about $\propto 7\sigma$
- also w/o CDF: small upwards tendency



[ATLAS-CONF-2023-004]

BSM perspective:

despite what the solution for the M_W-miracle might be, it will most-likely involve a very precise number $\rightarrow$ precision predictions for concrete BSM scenarios required!

Getting started: how to "predict" M_W?

- in most models (incl. SM): M_W cannot be "predicted"
- actual prediction: relation between M_W and other observables
- e.g. in the SM the tree-level relation [Ross, Veltman '75]

$$\rho = \frac{G_{CC}}{G_{NC}} = \frac{M_W^2}{c_w^2 M_Z^2} = 1\,, \quad (c_w = \cos\theta_w,\ s_w = \sin\theta_w)$$

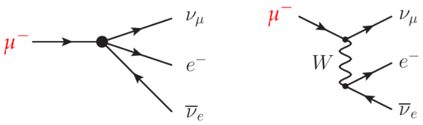
is perturbed by higher-order terms.

Strategy: use relations that involve both, theoretically and experimentally, well-defined observables (e.g. OS pole-masses) that have smallest uncertainties.
$\rightarrow$ use $\rho^{(0)} = 1$ to eliminate presence of θ_w in the relation of *some other* set of observables.

Relation between $M_W \leftrightarrow M_Z$, G_F and α_{QED}

consider muon decay: $G_F/\sqrt{2}\,(\bar{e}\gamma_\rho\nu_e)\,(\bar{\mu}\gamma^\rho\nu_\mu)$

$$G_F = \frac{\pi\alpha_{\text{QED}}}{\sqrt{2}M_W^2\left(1 - M_W^2/M_Z^2\right)}(1+\Delta r)$$



[fig.: Illana, Cano]

- G_F: from muon life time
- $\alpha_{\text{QED}}(0)$: fine-structure constant (including $\Delta\alpha$ resummation)
- M_W and M_Z: pole-masses
- Δr: contains higher-order corrections $\Delta r \equiv \Delta r(M_W, M_Z, \alpha, \ldots)$

Solve (iteratively) for M_W:

$$M_W = M_Z\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha_{\text{QED}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r)}\right)$$

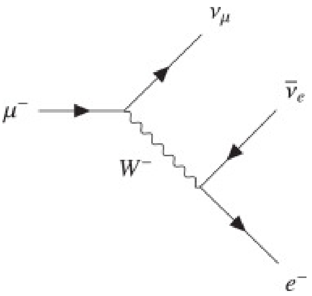
Instead of *predicting* M_W we are asking "which (OS) value of M_W do we need to get the muon decay right?"

Higher-order corrections to the muon decay: Δr and $\Delta\rho$

$$\Delta r^{(1)} = 2\frac{\delta^{(1)} Z_e}{e} + \frac{\Sigma_W^{(1),T}(0) - \delta^{(1)} M_W^2}{M_W^2} - \frac{\delta^{(1)} s_w^2}{s_w^2} + \delta_{\text{vertex+box}}$$
$$= \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\text{remainder}}$$

ingredients:

- $\delta^{(1)} Z_e$: photon and photon-Z self-energies
- $\delta^{(1)} M_{W/Z}^2$, $\Sigma_{W/Z}^{(1),T}(0)$, $\delta^{(1)} s_W^2$: (transverse) W/Z self-energies
- $\delta_{\text{vertex+box}}$: vertex/box diagrams



dominant:

- $\Delta\alpha$: light fermion contributions (SM-like)
- $\Delta\rho = \frac{\Sigma_Z^{(1),T}(0)}{M_Z^2} - \frac{\Sigma_W^{(1),T}(0)}{M_W^2}$ ← sensitivity to BSM physics $\Delta\rho^{\text{fit.}} \approx 3.8 \pm 2.0 \times 10^{-4}$ [PDG '22]
  (also part of EW fits, "T" parameter)
- this work: Δr at full one-loop. $\Delta\rho$ at two-loops.

The CP-Violating NMSSM

The Complex Next-to-Minimal Supersymmetric Standard Model

- Singlet extension of minimal SUSY (MSSM).
- Theoretically well-motivated (solves μ- and little-hierarchy-problem).
- Rich phenomenology in the Higgs boson sector:

$$H_d = \begin{pmatrix} \frac{v_d + h_d + i a_d}{\sqrt{2}} \\ h_d^- \end{pmatrix},\ H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{v_u + h_u + i a_u}{\sqrt{2}} \end{pmatrix},\ S = \frac{e^{i\varphi_s}}{\sqrt{2}}(v_S + h_s + i a_s)$$

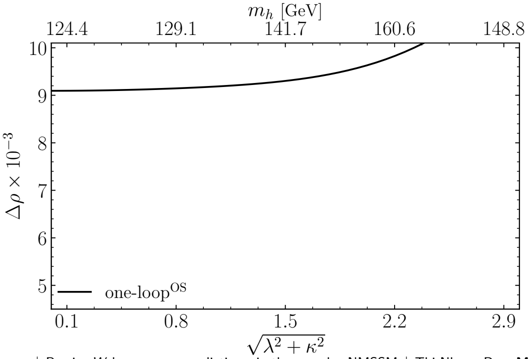
mix to

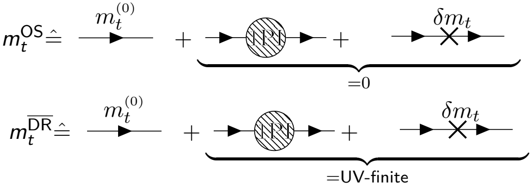
$$h_1, h_2, h_3, h_4, h_5, G^0 \text{ (mass ordered) and } h^\pm, G^\pm$$

- LHC measurements: h_1, h_2 **or** h_3 play the role of the Higgs boson h measured at LHC (h_1 or $h_{2,3}$ are "SM-like"). MSSM: no CPV at tree-level and always $h_1 = h$.
- extended electroweakino-sector: $\chi^0_{1,2,3,4,5}$, $\chi^\pm_{1,2}$
- two important parameters λ and κ: NMSSM $\xrightarrow{\lambda,\kappa\to 0}$ MSSM

$\Delta\rho$ at one-loop in the NMSSM: uncertainties?

- all one-loop contributions are well-known (in CP-conserving case) [Stål, Weiglein, Zeune '15]
- how to estimate the uncertainty without calculating higher-orders?
  $\rightarrow$ using different renormalisation conditions
- $\overline{\text{DR}}$ or OS renormalisation of top & stop sector [Graf et al. '12]
  $\rightarrow$ estimate uncertainty due to missing higher-orders
- one-loop: huge uncertainty $\rightarrow$ higher-orders required





$$\Delta\rho(m_t^{\text{OS}}) \overset{\text{all-orders}}{=} \Delta\rho(m_t^{\overline{\text{DR}}})$$

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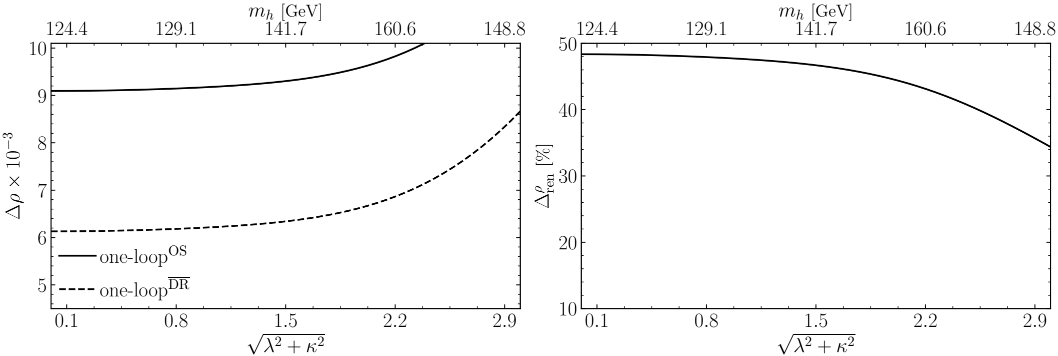
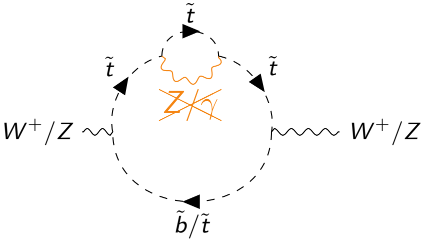
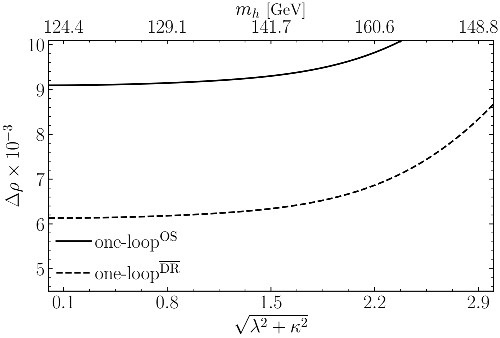
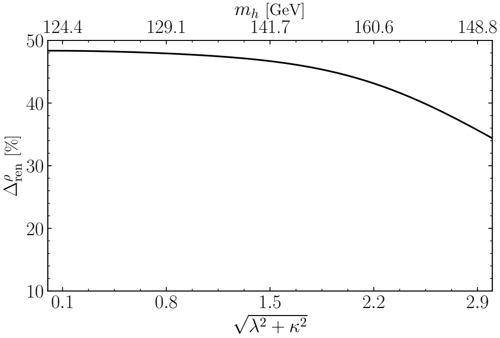


$\Delta\rho$ beyond one-loop

> approximation: gaugeless limit $g_1, g_2 \to 0$

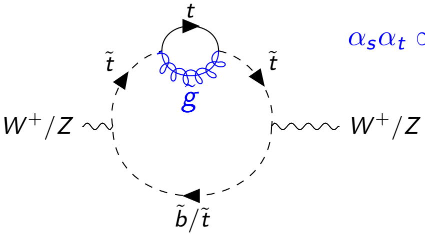




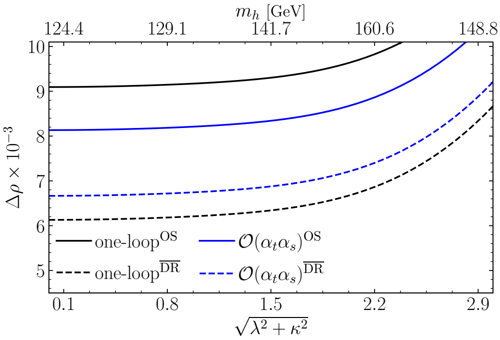


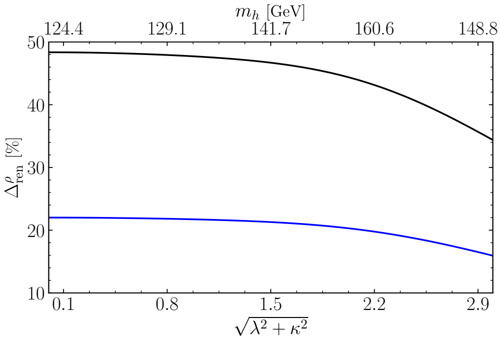
$\Delta\rho$ beyond one-loop

- approximation: gaugeless limit $g_1, g_2 \to 0$
- leading QCD $\mathcal{O}(\alpha_s\alpha_t)$ mixed OS/$\overline{\text{DR}}$



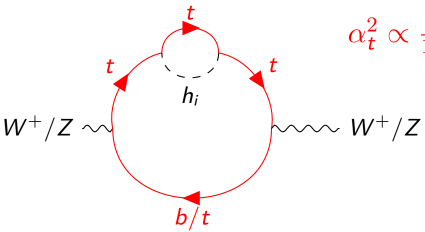
$$\alpha_s\alpha_t \propto \frac{g_3^2 y_t^2}{16\pi^2}$$



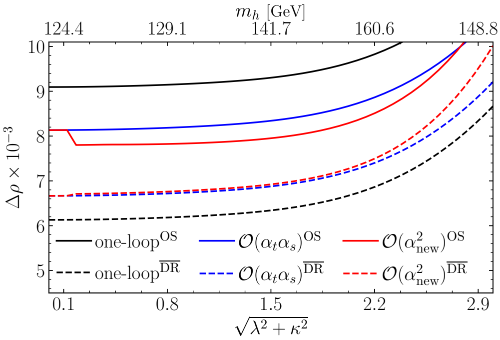


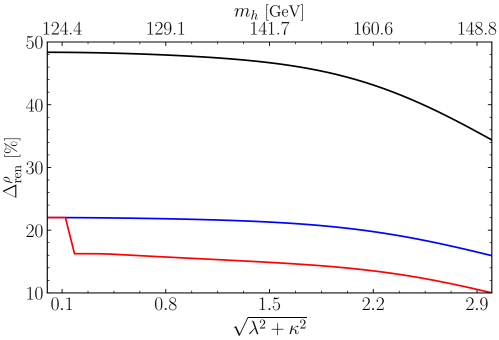
$\Delta\rho$ beyond one-loop

- approximation: gaugeless limit $g_1, g_2 \to 0$
- leading QCD $\mathcal{O}(\alpha_s\alpha_t)$ mixed OS/$\overline{\text{DR}}$
- $\mathcal{O}(\alpha_t^2)$ very sensitive to top mass



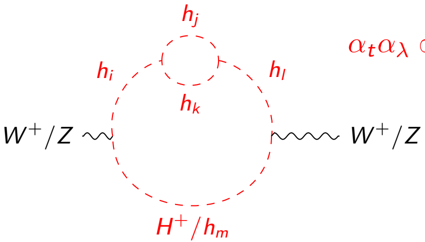
$\alpha_t^2 \propto \frac{y_t^4}{16\pi^2}$



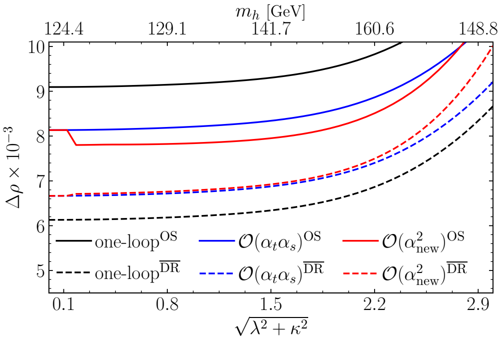


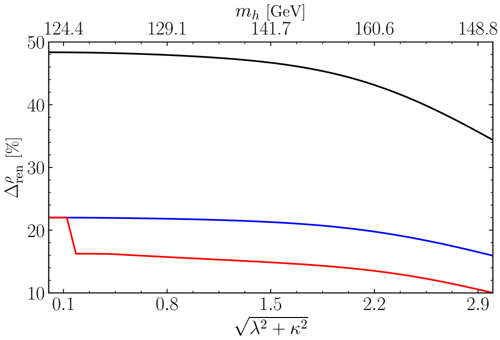
$\Delta\rho$ beyond one-loop

- approximation: gaugeless limit $g_1, g_2 \to 0$
- leading QCD $\mathcal{O}(\alpha_s\alpha_t)$ mixed OS/$\overline{\text{DR}}$
- $\mathcal{O}(\alpha_t^2)$ very sensitive to top mass
- $\mathcal{O}((\alpha_t+\alpha_\lambda+\alpha_\kappa)^2)$ can be large for large λ, κ
  technical difficulty: intermediate infrared-divergences (backup slides)



$\alpha_t\alpha_\lambda \propto \frac{y_t^2\lambda^2}{16\pi^2}$



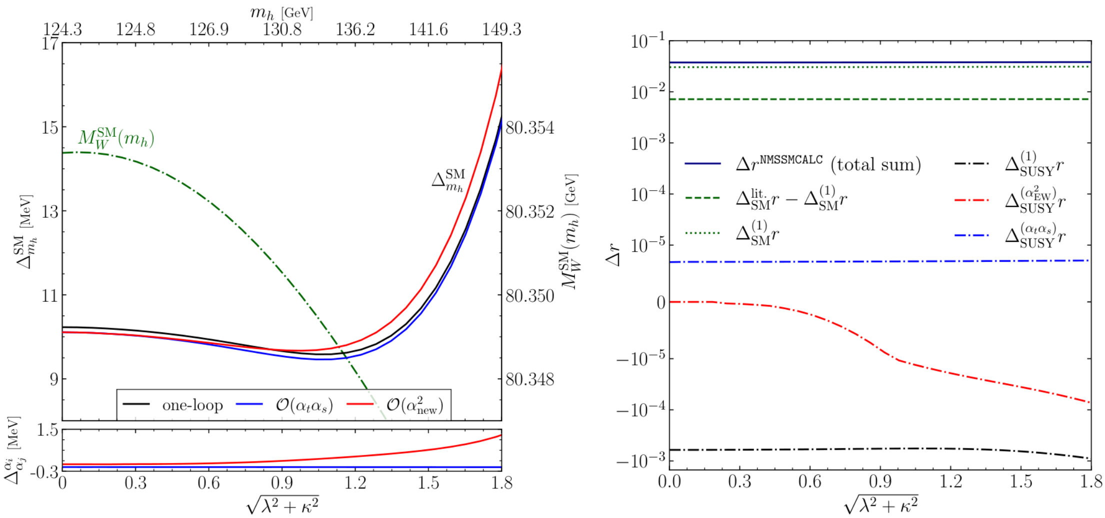


M_W via Δr: Combination with known higher-order SM-results

Reminder: $$M_W = M_Z\left(\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha_{\text{QED}}}{\sqrt{2}G_F M_Z^2}\left(1+\Delta r(M_W, M_Z, M_t, \underbrace{M_{H_i}, M_{\chi^\pm}, M_{\tilde{f}}, \ldots}_{\text{SUSY sector}})\right)}\right)$$

- $$\Delta r = \Delta^{(4)} r\big|_{\text{SM}} + \Delta r^{(2)}\big|_{\text{SUSY}}$$
  $$\approx \Delta r^{(4)}\big|_{\text{SM}} + \Delta r^{(1)}\big|_{\text{SUSY}} - \frac{c_w^2}{s_w^2}\Delta^{(2)}\rho\big|_{\text{SUSY}}$$
- $$\Delta^{(n)} X\big|_{\text{SUSY}} = \Delta^{(n)} X\big|_{\text{NMSSM}} - \Delta^{(n)} X\big|_{\text{SM}}\,, \quad X = \rho, r$$
- $\Delta^{(4)} r\big|_{\text{SM}}$ with full two-loop and partial three & four loop results **obtained in the OS scheme** [Awramik, Chakraborti, Chen, Chetyrkin, Czakon, Degrassi, Djouad, Fleischer, Freitas, Giardino, Gambino, Heinemeyer, Hollik, Jegerlehner, Kühn, Kniehl, Saha, Sirlin, Steinhauser, Tarasov, Weiglein, ...]

NMSSM-specific two-loop corrections to M_W and Δr



Implementation in NMSSMCALC [(see itp.kit.edu/∼maggie/NMSSMCALC for references)]

NMSSMCALC is more than a *spectrum generator* for the CP-violating NMSSM:

- takes parameter point using SLHA and calculates:
- Higgs boson masses ($m_{H_i^0}$ and $m_{H^\pm}$) up to two-loop $\mathcal{O}(\alpha_s\alpha_t + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$
- Higgs boson self-couplings $\lambda^{\text{eff}}_{hhh}$ up to two-loop $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$
- $H_i^{0,\pm} \to X_j X_k$ decays at NLO including SUSY QCD+EW corrections (or using $\lambda^{\text{eff}}_{hhh}$)
- electric dipole moments (EDMs) of e and various bound-states
- $(g-2)_e$ and $(g-2)_\mu$
- **New:** M_W and $\Delta\rho$
- also for inverse seesaw scenario (NMSSMCALC-nuSS)

example: compile and run NMSSMCALC

```
wget https://www.itp.kit.edu/\~maggie/NMSSMCALC/nmssmcalc.tar
tar xf nmssmcalc.tar
cd nmssmcalc-C
make
./run inp.dat
```

Summary

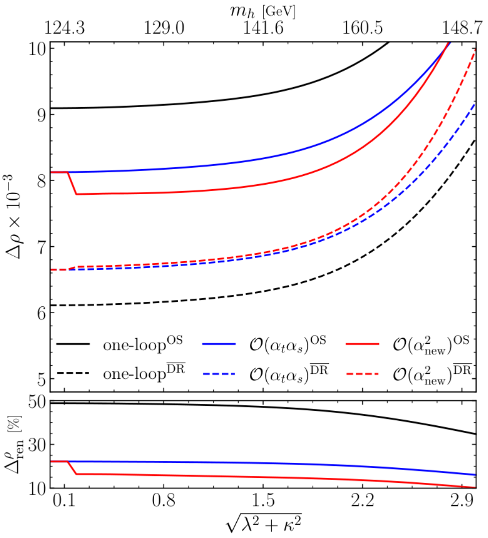
- W boson mass M_W and $\Delta\rho$ are important precision observables!
- studied two-loop corrections to $\Delta\rho$ in the NMSSM
  - uncertainties significantly reduced
- combined with full one-loop correction to Δr (muon decay)
  $\rightarrow$ precise M_W prediction

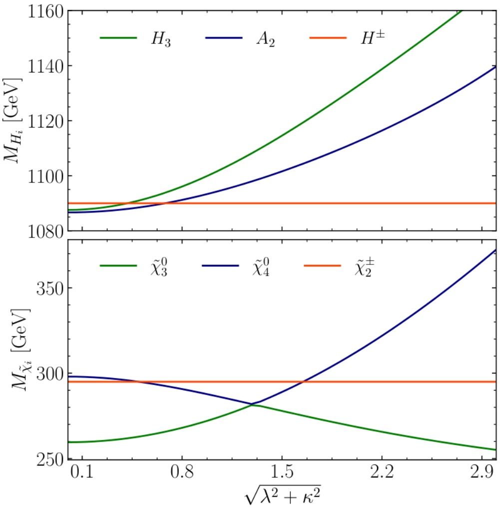
Outlook

- more detailed pheno studies
- influence of ren. scheme of charged Higgs mass (OS / $\overline{\text{DR}}$ scheme)
- uncertainty estimate for M_W
- two-loop corrections w/ sleptons (can have large one-loop shifts)

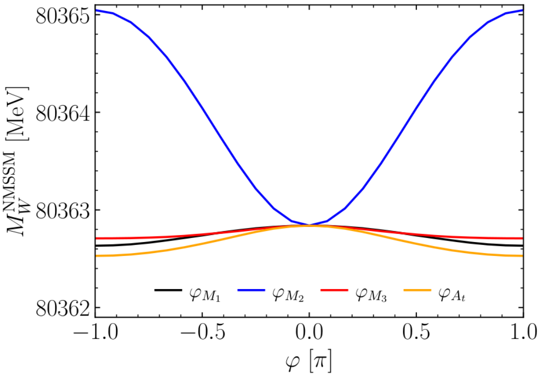
Backup

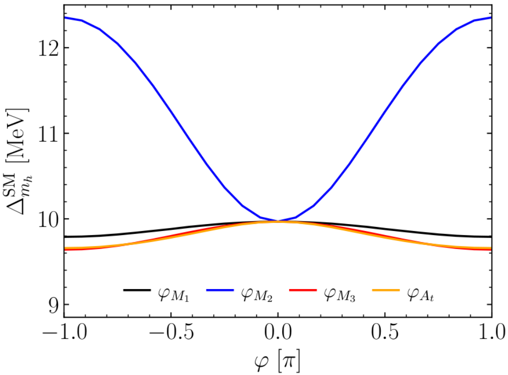
$SU(2)_L$ mass splittings $\rightarrow \Delta\rho$ (custodial symmetry)





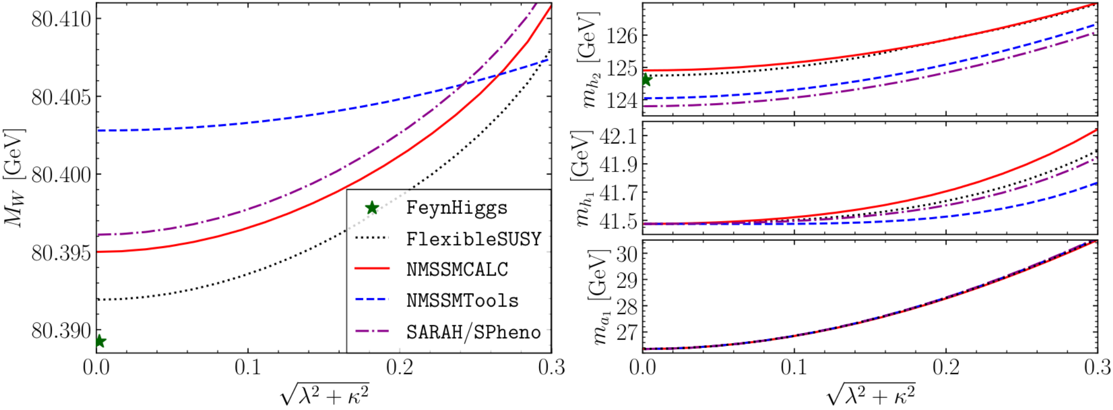
CP-violating effects





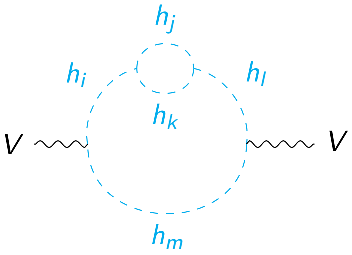
- Phases in gluino- (φ_{M_3}) and stop-sector (φ_{A_t}) typically have a small effect if the particles have large ($\gtrsim 1\,\mathrm{TeV}$) masses
- electroweakinos ($\varphi_{M_{1,2}}$) can be light and can have sizeable effects

Comparison with other tools / the MSSM (FeynHiggs)



→ agreement within SM uncertainty band

Comparison with MSSM/previous Results

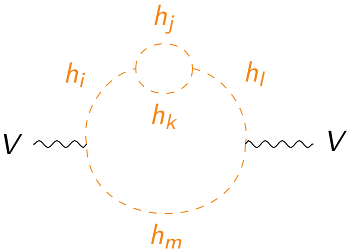


$\xrightarrow{g_1, g_2 \to 0} 0$

In the MSSM, Higgs-self couplings are given by gauge couplings:

$$V_{\text{MSSM}}^{\text{quartic}} \propto \boldsymbol{g_1}^2(|H_u|^2 - |H_d|^2)^2 + \boldsymbol{g_2}^2(H_u\sigma_a H_u + H_d\sigma_a H_d)^2 \xrightarrow{g_1, g_2 \to 0} 0$$

Comparison with MSSM/previous Results



$\xrightarrow{g_1, g_2 \to 0} \neq 0$

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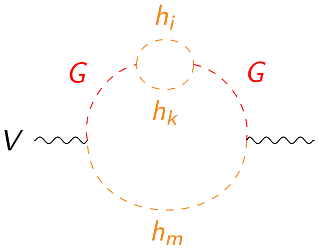
In the NMSSM, there are additional non-zero self-couplings:

$$V_{\text{NMSSM}}^{\text{quartic}} \propto V_{\text{MSSM}}^{\text{quartic}} + |\boldsymbol{\lambda} H_u H_d + \boldsymbol{\kappa} S^2|^2 \xrightarrow{g_1, g_2 \to 0} \neq 0$$

$\to$ Many new **two-loop diagrams with Higgs self-couplings**.

Comparison with MSSM/previous Results



$$\propto \log m_G^2 \xrightarrow{m_G^2 \to 0} -\infty$$

In the MSSM, Higgs-self couplings are given by gauge couplings:

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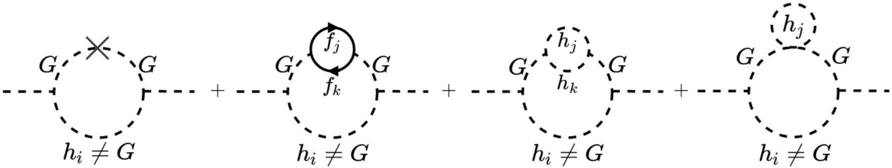
→ Many new **two-loop diagrams with Higgs self-couplings**.
Massless Goldstones → appearance of intermediate IR divergences (final result IR-finite).

IR-finite two-loop self-energies

Example of an IR-finite subset with intermediate IR-divergences:



Careful isolation of divergences using mass regulator or dimensional regularisation shows:

> IR-divergence of first diagram cancels against the other three

> cancellation happens only if $M^{\text{1-loop}}_{\text{Goldstone}} \equiv 0$

> $\rightarrow$ working at the *tree-level* minimum is sufficient [this work] or alternatively using an OS-condition for the Goldstone mass [Braathen, Goodsell, '16]

Parameter point P1 for the shown plots

$$\begin{aligned}
\texttt{P1}: \quad & m_{\tilde{t}_R} = 2002\ \text{GeV}\,,\ m_{\tilde{Q}_3} = 2803\ \text{GeV}\,,\ m_{\tilde{b}_R} = 2765\ \text{GeV}\,,\\
& m_{\tilde{L}_{1,2}} = 565\ \text{GeV}\,,\ m_{\tilde{e}_R,\tilde{\mu}_R} = 374\ \text{GeV}\,,\\
& m_{\tilde{L}_3} = 575\ \text{GeV}\,,\ m_{\tilde{\tau}_R} = 981\ \text{GeV}\,,\\
& |A_{u,c,t}| = 2532\ \text{GeV}\,,\ |A_{d,s,b}| = 1885\ \text{GeV}\,,\ |A_{e,\mu,\tau}| = 1170\ \text{GeV}\,,\\
& |M_1| = 133\ \text{GeV},\ |M_2| = 166\ \text{GeV}\,,\ |M_3| = 2300\ \text{GeV}\,,\\
& \lambda = 0.301\ \text{GeV},\ \kappa = 0.299\ \text{GeV}\,,\ \tan\beta = 4.42\ \text{GeV}\,,\\
& \mu_{\text{eff}} = 254\ \text{GeV},\ \text{Re}A_\kappa = -791\ \text{GeV}\,,\ M_{H^\pm} = 1090\ \text{GeV}\,,\\
& \varphi_{A_{e,\mu,\tau}} = 0\,,\ \varphi_{A_{d,s,b}} = \pi\,,\ \varphi_{A_{u,c,t}} = \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0\,.
\end{aligned}$$

H_1	H_2	H_3	H_4	H_5
125.4	230.6	770.5	1088.0	1090.1
h_u	h_s	a_s	a	h_d

$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	$\tilde{\chi}_5^0$	$\tilde{\chi}_1^+$	$\tilde{\chi}_2^+$
113.9	145.07	261.95	295.74	509.49	132.93	294.98

Parameter point BP3 for the tool-comparison

$$\texttt{BP3}: \quad m_{\tilde{t}_R} = 2144\ \text{GeV}\,,\ m_{\tilde{Q}_3} = 1112\ \text{GeV}\,,\ m_{\tilde{b}_R} = 1539\ \text{GeV}\,,$$
$$m_{\tilde{L}_{1,2}} = 131.9\ \text{GeV}\,,\ m_{\tilde{e}_R,\tilde{\mu}_R} = 103.6\ \text{GeV}\,,$$
$$m_{\tilde{L}_3} = 205.2\ \text{GeV}\,,\ m_{\tilde{\tau}_R} = 238.6\ \text{GeV}\,,\ |A_{u,c,t}| = 3971.2\ \text{GeV}\,,$$
$$|A_{d,s,b}| = 1210.3\ \text{GeV}\,,\ |A_{e,\mu}| = 3643\ \text{GeV}\,,\ |A_\tau| = 2052.4\ \text{GeV}\,,$$
$$|M_1| = 178.3\ \text{GeV},\ |M_2| = 128.6\ \text{GeV}\,,\ |M_3| = 1757.6\ \text{GeV}\,,$$
$$\lambda = 0.1229\ \text{GeV},\ \kappa = 0.0128\ \text{GeV}\,,\ \tan\beta = 8.7199\ \text{GeV}\,,$$
$$\mu_{\text{eff}} = 212\ \text{GeV},\ \text{Re}A_\kappa = -10.48.\ \text{GeV}\,,\ \text{Re}A_\lambda = 2245\ \text{GeV}\,,$$
$$\varphi_{A_{e,\mu,\tau}} = 0\,,\ \varphi_{A_{u,c,t}} = \pi\,,\ \varphi_{A_{d,s,b}} = \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0\,.$$

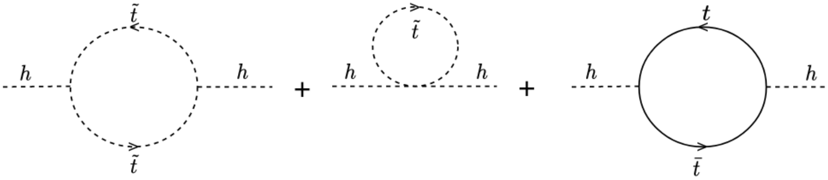
The SM-like neutral Higgs boson mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{NMSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z < 125\,\text{GeV}$
- NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass. At one-loop, the leading contributions to $\delta^{(1)} m_h^2$ from the top/stop sector are:



$$\approx \frac{1}{(4\pi)^2} \frac{m_t^4}{v^2} \log \frac{m_t^2}{M_{\tilde{t}}^2}$$

- $M_{\tilde{t}} = m_t + m_{\text{SUSY}} \Rightarrow$ in the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0$
- but we need $\delta m_h^2 \approx \mathcal{O}(20 - 40\,\text{GeV})$! $\rightarrow$ higher-orders required