# Twisting and untwisting in supergravity, conformal field theory, and holography


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This talk is an overview of some joint work (past, present, and future) with numerous collaborators.

Here is an incomplete list:

Ilka Brunner, Martin Cederwall, Kevin Costello, Tudor Dimofte, Richard Eager, Chris Elliott, Owen Gwilliam, Fabian Hahner, John Huerta, Simon Jonsson, Simone Noja, Jakob Palmkvist, Natalie Paquette, Surya Raghavendran, Johannes Walcher, Brian R. Williams.

I owe thanks to all of them, and to the many other people I've been fortunate enough to talk with and learn from (some of whom are in the audience).

As befits an overview talk, I'll try and focus on what I find exciting and what I see as important *conceptual* points. I will privilege narrative at the expense of detail; apologies in advance.

Here's a rough plan of the talk:

1. A few words on the "derived" approach to field theory. This will be in the background everywhere.[†]
2. Reminders on twisted holography and a few of the theories that appear there. The worldsheet approach.
3. Twisted theories are to complex geometry as supersymmetric theories are to superspace geometry.
4. What can we hope to learn about supersymmetric physics by studying twists? (More than expected.)
5. Some lessons about M-theory and fivebranes.

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[†]Really, this is a unifying perspective on many things we already do in QFT: BRST, on-shell symmetries, higher-form symmetries, ...

## *1 – Derived geometry is not so scary*

The big idea in derived geometry is that it is generally the wrong thing to do to *explicitly* impose a constraint or pass to a quotient. This loses information. Instead, one should work with simple, *unstructured* objects and produce a *cochain-level* model of the object one wants to study.

Example from topology: The singular cochains of a manifold $X$ specify the manifold. (The boundary operator is a recipe for gluing simplices together to reproduce $X$.) The singular cohomology is *far* from being a perfect invariant.

Example from physics: Instead of doing the path integral over the (complicated) space of gauge equivalence classes of fields, we add ghosts, together with a BRST differential that implements the gauge quotient cohomologically.

If we are interested in a structured object $O$, we should replace it by a cochain complex $L^\bullet$ of *unstructured* objects that is quasi-isomorphic to $O$. Such a replacement is called a *resolution*.

Often, we do not actually require the higher cohomology of the derived replacement $L^\bullet$ to vanish! More on this later.

What I mean by "object," and by "structured," will depend on the context. But the vagueness is a matter of exposition; these terms are precisely defined in a wide range of mathematical settings.

The unstructured objects in quantum field theory are smooth sections of vector bundles: they are (locally) free modules over smooth functions on spacetime, with no constraints and no gauge equivalences. (Not "free fields!")

In the derived model, all symmetries, deformations, constraints, and so on are treated on the same footing. (Everything is a “local $L_\infty$ algebra.”)

A simple example: the field in perturbative abelian Yang–Mills theory is a gauge equivalence class of one-forms, $[A] \in \Omega^1/d\Omega^0$.

The derived replacement (BRST model) is easy:

$$\text{“connections:”} \qquad \Omega^0 \xrightarrow{\ d\ } \Omega^1.$$

Form degree determines ghost number.[†]

The degree-zero cohomology encodes gauge transformations at infinity, which *do* act!

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[†] I will use a slightly perverse convention: shifted by one.

## *Derived models of field theories*

Of course, Yang–Mills theory isn't just about any connection. We need to include dynamics: our structured object is actually an equivalence class of one-forms satisfying $d \star dA = 0$.

$$
\begin{array}{lcccc}
 & 0 & 1 & 2 & 3 \\
\hline
\text{“YM connections:”} & \Omega^0 \xrightarrow{d} & \Omega^1 \xrightarrow{d\star d} & \Omega^{d-1} \xrightarrow{d} & \Omega^d \\
\downarrow & \downarrow & \downarrow & & \\
\text{“connections:”} & \Omega^0 \xrightarrow{d} & \Omega^1. & &
\end{array}
$$

This is a simple example of the *Batalin–Vilkovisky formalism*. The symmetry between degrees $k$ and $3-k$ reflects a *shifted symplectic structure*, which is a derived incarnation of Hamiltonian dynamics.

## *Derived symmetries and currents*

Just like the equations of motion in a field theory, the failure of a symmetry to act fully locally is encoded by *constraints* on the symmetry generators. The obstruction arises from the spacetime derivatives in the kinetic term.

For example, for a typical "global" symmetry—including any higher-form symmetry—the generator of the symmetry is required to be a locally constant function.

Locally constant functions are resolved by the de Rham complex, so the derived model is

$$\text{“flat connections:”} \qquad \Omega^\bullet \otimes \mathfrak{g}.$$

In degree one, we see the modulus corresponding to coupling to a flat background.

In holomorphic theories (such as holomorphic twists, or two-dimensional chiral CFTs), only antiholomorphic derivatives appear in the kinetic term. So any global symmetry enhances to a *holomorphic* one; the derived model is given by the Dolbeault complex,

$$\text{“holomorphic connections:”} \qquad \Omega^{0,\bullet} \otimes \mathfrak{g}.$$

Nothing in this discussion is particular to global symmetries at all. Spacetime symmetries work in precisely the same fashion. In a holomorphic theory defined without additional geometric data, the natural symmetry is by:

$$\text{“holomorphic vector fields:”} \qquad \Omega^{0,\bullet}(T).$$

In degree one, we see $H^{0,1}(T^{1,0})$: these are *Beltrami differentials*, or deformations of complex structure.

Costello and Gwilliam worked out a powerful generalization of Noether's theorem for any local $L_\infty$ algebra $\mathscr{L}$. They construct a *current algebra*, which I will call $\mathrm{Cur}(\mathscr{L})$. To a field theory $T$ with a symmetry by $\mathscr{L}$, they then associate a map of $P_0$ factorization algebras

$$\mathrm{Cur}(\mathscr{L}) \to \mathrm{Obs}(T).$$

(Here "$P_0$ factorization algebra" should be parsed, roughly, as "algebra of observables and OPEs.")

The normal Noether current (a degree-zero observable) is dual to the modulus in degree one. Its conservation law is dual to the symmetry in degree zero. Constraints in degree two are dual to improvement transformations.

We've now seen that local $L_\infty$ algebras control *both* field theories and their symmetries.

Correspondingly, we've seen *two* ways of producing a factorization algebra from a local $L_\infty$ algebra:

- For any local $L_\infty$ algebra, we can form its *factorization algebra of currents*, $\mathrm{Cur} = C_\bullet(\mathscr{L}_c)$.
- If $\mathscr{L}$ is equipped with the additional structure that makes it a BV theory (roughly, a shifted Poisson bracket), it has a *factorization algebra of observables*, $\mathrm{Obs} = C^\bullet(\mathscr{L})$.

One expects that these two factorization algebras are *Koszul dual* (although the details of Koszul duality in this setting have not been worked out). In physical terms: "The universal object that couples to a theory is its currents."

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One should check that the two factorization algebras *are* Koszul dual in the sense of Lurie.

An additional thing that one learns from the derived approach is that there are *higher operations* in topological field theories, arising (roughly) from topological descent.

The observables of a $d$-dimensional topological field theory form an $E_d$-algebra: roughly, one gets a *locally* constant family of products for every picture of embeddings of little $d$-disks. But this operad has higher cohomology!

One can prove that $E_d$ algebras are equivalent to $(d-1)$-shifted Poisson algebras.

In the case $d = 2$, this structure returns the 1-shifted bracket of Lian–Zuckerman and Getzler in string theory, or the BV algebra structure on polyvector fields in the $B$-model. We will meet this example shortly, together with other examples for $d = 1, 2, 3$.

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Kontsevich; Tamarkin; Beem–Ben-Zvi–Bullimore–Dimofte–Neitzke; ...

## 2 – Twisted holography via topological strings

Twisted holography goes back to the foundational work of Costello–Li, and has been an actively developing area over the last few years.

One main aim is to construct simpler models of holographic dualities in holomorphic or topological field theories. Such simpler models are, on the one hand, tractable to formulate rigorously, and, on the other hand, valuable sources of structural insight.

These simpler models are related to physical examples of holography by *twisting:* passing to the invariants of particular supercharges. They should thus be viewed as lifts of typical computations where BPS quantities are matched to the level of full-fledged field theories.

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Costello–Gaiotto; Budzik–Gaiotto; Costello–Paquette; Oh–Zhou; Moosavian–Zhou. . .

Much of what is understood about twisted holography was, in fact, computed *without* any direct twists of supergravity theories. The original conjectures of Costello and Li, for example, all originate in the $B$-model of topological string theory.

Let $\mathrm{Coh}(X)$ denote (an appropriate version of) the derived category of coherent sheaves on a Calabi–Yau manifold $X$ (generally noncompact). This is a Calabi–Yau category.

On general grounds, the moduli problem of deformations of a category is described by its Hochschild cochains, and the moduli problem of deformations compatible with the Calabi–Yau structure is described by cyclic cochains. By the Deligne conjecture, there is an $E_2$ algebra structure!

By the Hochschild–Kostant–Rosenberg theorem, the Hochschild cohomology of $\mathrm{Coh}(X)$ is described by holomorphic polyvector fields on $X$:

$$\mathrm{PV}^{\bullet,\bullet} = \left(\Omega^{0,\bullet}(\wedge^{\bullet} T),\ \bar{\partial}\right).$$

The cyclic cohomology is described by taking derived $U(1)$ invariants. Explicitly,

$$HC^{\bullet} = \left(\mathrm{PV}^{\bullet,\bullet}[\![t]\!],\ \bar{\partial} + t\partial_{\Omega}\right),$$

where $t$ is a formal parameter of degree two (generator of the cohomology of $BU(1)$) and $\partial_{\Omega}$ is the divergence operator defined by the holomorphic volume form.

(This is closed string field theory.)

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Bershadsky–Cecotti–Ooguri–Vafa, Costello–Li, ...

Costello and Li's version of BCOV theory is *this* formal moduli problem. They show that there is a shifted Poisson bracket on $HC^\bullet$ that makes it into a field theory.

One can see that a piece of the theory, in degrees zero and one, consists of the fields

$$\text{“divergence-free holomorphic VFs:”} \quad \mathrm{PV}^{1,\bullet} \xrightarrow{\partial_\Omega} t \cdot \mathrm{PV}^{0,\bullet}.$$

So BCOV theory is (among other things) a theory of deformations of the complex-structure moduli of the Calabi–Yau on which it is defined.

It's natural to conjecture—as Costello and Li did—that BCOV theory on $\mathbb{C}^5$ is the twist of type IIB string theory with respect to a holomorphic target-space supercharge, and that its “minimal” sector is the holomorphic twist of IIB supergravity.

*What about open strings?*

In a sense, this is even easier. We know that the open-string states between branes form the morphism spaces in $\mathrm{Coh}(X)$. So, for a brane $\Gamma$, the fields of the brane worldvolume theory should be given by $\mathrm{End}(\Gamma)$.

For a rank-one $B$-brane $\Gamma$ supported on $\mathbb{C}^k \subset \mathbb{C}^5$, we obtain

$$\mathrm{End}(\Gamma) = \Omega^{0,\bullet}(\mathbb{C}^k) \otimes \wedge^\bullet(\mathbb{C}^{5-k}).$$

These are the fields of *holomorphic Chern–Simons theory*—and, not by coincidence, the fields of the holomorphic twist of maximal super Yang–Mills theory, the worldvolume theory of a single $\mathrm{D}(2k-1)$-brane. In general, we tensor with $\mathfrak{gl}(N)$; the theory has an $E_1$ (associative) structure, corresponding to superparticle models.

The typical form of a twisted holography statement is something like the following: Consider a bulk theory living on $X$, and place a stack of $N$ branes on $Y \subset X$.

There then exists a coupling map describing the interactions. Generally, this map takes the form

$$\mathrm{Obs}_{\mathrm{grav}}\big|_Y^! \to \mathrm{Obs}_{\mathrm{brane}}^N,$$

where the left-hand side describes gravitational observables at the support of the brane *after* backreaction is taken into account.

The conjecture is that this map becomes an equivalence in the large $N$ limit, so that the brane theory and the gravitational theory are Koszul dual. (This is a version of the normal statement that CFT *observables* couple to supergravity *fields*.)

This map has been worked out in several different examples. In the IIB examples studied so far, the equivalence is related to a theorem of Loday–Quillen–Tsygan.

I would like to focus on an example for $M$-theory in the omega background, worked out by Costello. Here the bulk theory is effectively on $\mathbb{R} \times \mathbb{C}^2$, with a stack of fivebranes along $\mathbb{C}$. (Six directions have been localized away.)

Costello argues using dualities that the bulk theory is a Poisson–Chern–Simons theory: its fields are $\Omega^\bullet(\mathbb{R}) \otimes \Omega^{0,\bullet}(\mathbb{C}^2)$, and the Lie structure comes from the Poisson bracket defined by a holomorphic volume form. (The Poisson structure is not accidental; remember $E_3 \ldots$)

From the AGT correspondence, or from work of Beem–Rastelli–van Rees, one expects that the observables of the theory on $N$ fivebranes, in this limit, are the $W_N$ current algebra.

For $N = 2$, this is just Virasoro currents. And for $N = \infty$, it is again the currents of a local Lie algebra $\mathscr{L}$: namely, differential operators on $\mathbb{C}$.

The currents of a local Lie algebra should be dual to its observables, so we expect $\mathscr{L}$ to correspond to the fields of Poisson–Chern–Simons theory. And this is indeed the case: $z$ maps to itself, and $w$ maps to $\partial_z$.

Some obvious questions:

- How might one go about proving such conjectures for twists of supergravity theories?
- There's a lot of nice structure here; is it an artifact of the twist? How much of it functions in a parallel way in the physical theory?
- All of the moduli problems that we've seen were easy to describe or geometrically natural. Isn't real supergravity a lot more baroque and complicated?

In the remaining time, I will try to address each a bit.

## *3 – The moduli problem of deformations of superspace*

The key idea here is to recall that superspace is not just a smooth supermanifold. It is equipped with an extra geometric structure, witnessing the fact that the supersymmetry algebra is *not* abelian: $Q$'s commute to $P$'s.†

This structure can be thought of as a *nonintegrable odd distribution of maximal dimension*, which specifies the supersymmetry covariant derivatives. On flat superspace,

$$Q_a = \frac{\partial}{\partial\theta^a} + \gamma^\mu_{ab}\theta^b \frac{\partial}{\partial x^\mu}, \qquad D_a = \frac{\partial}{\partial\theta^a} - \gamma^\mu_{ab}\theta^b \frac{\partial}{\partial x^\mu},$$

when acting on the left and on the right, respectively.

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† "Flat superspace has torsion."

An almost-complex structure is defined by precisely the same kind of data: a distribution

$$\overline{T} \subset T_{\mathbb{C}}$$

in the (complexified) tangent bundle. The structure is integrable precisely when its “torsion” vanishes.

So any structure in almost-complex geometry that I can define using that data has an analogue in superspace.

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A very similar perspective was taken by Berkovits and Howe. See also the work of Tanaka.

To reconstruct holomorphic functions, I need to build the Dolbeault complex, which I can do using the "Hodge filtration." Differential forms on flat superspace are generated by coordinates $x^\mu, \theta^a$ and one-forms $dx^\mu, d\theta^a$. But we need to work with respect to the left-invariant frame

$$\lambda^a = d\theta^a, \quad v^\mu = dx^\mu + \theta^a \gamma^\mu_{ab} d\theta^b.$$

In this basis, the de Rham differential becomes

$$d = \lambda^a \lambda^b \gamma^\mu_{ab} \frac{d}{dv^\mu} + \lambda^a \left( \frac{d}{d\theta^a} - \gamma^\mu_{ab} \theta^b \frac{d}{dx^\mu} \right) + v^\mu \frac{d}{dx^\mu}.$$

The middle term is the analogue of $\bar{\partial}$; it squares to zero up to the "pure spinor constraint" $\lambda^a \gamma^\mu_{ab} \lambda^b = 0$, which is imposed by the first term $\gamma$ of the differential. We can make a smaller model on the cohomology of $\gamma$...

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Berkovits, Cederwall, Howe, Nilsson, ...

But what is the multiplet $A^\bullet$ of "holomorphic" functions?

| *Superspace* | *Structure sheaf* | *On-shell?* | $\dim_{\mathbb{C}}$ | *CY?* |
|---|---|---|---|---|
| 3d $\mathscr{N}=1$ | vector | | 1 | |
| 4d $\mathscr{N}=1$ | vector | | 2 | |
| 6d $\mathscr{N}=(1,0)$ | vector | | 3 | |
| 6d $\mathscr{N}=(2,0)$ | abelian tensor | ✓* | 1 | |
| $P^1 \times P^2$ | $T$ | ✓* | 1 | |
| 10d $\mathscr{N}=(1,0)$ | vector | ✓ | 5 | ✓† |
| 10d $\mathscr{N}=(2,0)$ | supergravity (IIB) | ✓* | 1 | |
| $\wedge^2(T)\oplus T^*$ | min. BCOV | ✓* | 1 | |
| 11d $\mathscr{N}=1$ | supergravity | ✓ | 2 | ✓ |
| $\mathrm{Gr}(2,5)$ | $E(5\|10)$ | ✓ | 2 | ✓ |

The star refers to *presymplectic* on-shell (BV) theories, which include self-dual fields. The dagger refers to a subtlety that will not be important here ("Gorenstein, but not maximally Cohen–Macaulay.")

From this, it's easy to compute the multiplet (indeed, the local $L_\infty$ algebra) analogous to "holomorphic vector fields" above. These are those vector fields on superspace that are compatible with the odd distribution.

Computing this in examples, we rediscover the *conformal supergravity multiplet* in every case! This is unsurprising: the distribution is invariant under rescaling.

This multiplet naturally acts on any supersymmetric field theory whose superspace definition does not require additional data—in particular, on free pure spinor theories.

## *4 – Twisting and untwisting*

The key point can be put as follows: *On flat superspace, twisting is the odd version of dimensional reduction*. In a precise sense, a theory and its twists have the same structure.

After all, dimensional reduction takes invariants of a vector field generating an even translation. Twisting takes invariants of a supercharge; in a superfield formalism, this is just a vector field generating an odd translation.

From this perspective, it's not surprising that superfield formulations or actions do not change dramatically under twisting. (When I dimensionally reduce, I write *the same* action functional, restricted to a smaller space of fields.)

One rigorous formulation of this point is the following:

## Theorem (IAS–Williams)

*The construction outlined above commutes with twisting. In particular, the twist of $A^\bullet$ by a square-zero supercharge $Q$ is obtained by applying $A^\bullet$ to the algebra of residual supertranslations in the desired twist.*

As a corollary, the holomorphic twist of the type IIB supergravity multiplet is (free) minimal BCOV theory—just as conjectured by Costello–Li.

Because of the computational efficiency of the formalism, checking this—which would be deeply unwieldy in components—becomes a task you can do in five minutes. And it's not just about free theories. . .

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This statement is true up to taking potentials for various field strengths in BCOV.

- *Baulieu:* Holomorphically twisted ten-dimensional super Yang–Mills theory is holomorphic Chern–Simons theory on $\mathbb{C}^5$.
- *Berkovits; Schwarz; Witten:* Ten-dimensional super Yang–Mills theory is holomorphic Chern–Simons theory on superspace.
- *Costello:* Maximally twisted eleven-dimensional supergravity should be Poisson–Chern–Simons theory on $\mathbb{C}^2 \times \mathbb{R}^7$.
- *Raghavendran–IAS–Williams:* Holomorphically twisted eleven-dimensional supergravity on $\mathbb{C}^5 \times \mathbb{R}$ should be the exceptional Lie superalgebra $E(5|10)$.
- *Cederwall; Hahner–IAS: Untwisted* eleven-dimensional supergravity is Poisson–Chern–Simons theory on superspace. So is $E(5|10)$.

## *5 – Membranes and fivebranes*

The payoff of all of this is that we can use the structures in the simplest twisted holography models, *mutatis mutandis*, in more complicated twisted settings—or directly in the physical theory.

Moreover, we have concrete proposals for theories whose twists are known—even in cases where the theory itself is mysterious. . .

As an example of the first kind: Our construction of eleven-dimensional supergravity as a Poisson–Chern–Simons theory provides a 2-shifted Poisson algebra structure on its fields—otherwise known as an $E_3$-algebra structure.

This is a new and concrete piece of evidence for a first-quantized origin.

Going further, it's easy to write down an object whose maximal twist agrees with Virasoro currents: namely, the currents of "holomorphic" vector fields on superspace. And it's natural to guess that this object is (closely related to?) the theory on a stack of two fivebranes.

Here is the result:

$$\begin{array}{llllll} 0: & \mathrm{Vect} & \Pi S_+(R) & \Omega^0(\mathrm{ad}_R) & & \\ & \downarrow & \downarrow & \downarrow & & \\ 1: & \mathrm{Met}_0 & \Pi\,\mathrm{RS}(R) & \Omega^1(\mathrm{ad}_R) \oplus \Omega^3_+(\mathbf{5}) & \Pi S_-(\mathbf{16}) & \Omega^0(\mathbf{14}). \end{array}$$

And here it is for holomorphic $\mathscr{N} = (2,0)$ supersymmetry:

$$0: \quad \mathrm{Vect}_{\mathrm{hol}} \quad \Omega^1_{\mathrm{hol}} \otimes \Pi R' \quad \Omega^0_{\mathrm{hol}}(\mathrm{ad}_{R'}).$$

The first of these, $\mathscr{L}_{(2,0)}$, is a derived version of the conformal supergravity multiplet of Bergshoeff, Sezgin, and van Proeyen. The second (on flat space) is the exceptional simple Lie superalgebra $E(3|6)$ constructed by Kac.

The connection to 5d super Yang–Mills theory is *not* automatic. I will check this at the holomorphic level by giving a map from the dimensional reduction of $E(3|6)$ currents to the perturbative holomorphic twist of the $\mathfrak{sl}(2)$ theory. The dimensional reduction to 5d is

$$
\mathscr{L}^{\mathrm{red}}_{(2,0)}: \quad \frac{\begin{array}{ccc} \mathrm{Vect}_{\mathrm{hol}} & \Omega^1_{\mathrm{hol}} \otimes \Pi R' & \Omega^0_{\mathrm{hol}}(\mathrm{ad}_{R'}). \\ \Omega^0_{\mathrm{hol}} \cdot \partial_w & (\Omega^0_{\mathrm{hol}} \cdot dw) \otimes \Pi R' & \end{array}}{\begin{array}{ccc} (X^\mu, x) & (\psi^a_\mu, \xi^a) & \rho^{ab} \end{array}}
$$

Inside of $\mathrm{Cur}(\mathscr{L}^{\mathrm{red}}_{(2,0)})$, these generators are shifted down by one. But we need to take *compactly supported* sections, meaning that the nontrivial generators in cohomology are overall in degree $+2$.

Perturbative holomorphic super Yang–Mills theory with $\mathfrak{g} = \mathfrak{su}(2)$ is also described (in BV) by a local $L_\infty$ algebra of the form

$$\mathscr{E}: \qquad \begin{array}{ccc} \Omega^0_{\text{hol}} \otimes \mathfrak{g} & \Omega^0_{\text{hol}} \otimes \Pi R' \otimes K^{1/2} & \Omega^3_{\text{hol}} \otimes \mathfrak{g}. \\ \hline \alpha & \phi^a & \beta \end{array}$$

Overall, $\alpha$ and $\beta$ are observables of odd parity, and the $\phi^a$ are of even parity. Each observable has degree $+1$. $\mathfrak{su}(2)$ has only one quadratic Casimir invariant, so the gauge-invariant observables (at lowest order in holomorphic derivatives) are generated by quadratic expressions in $\beta$, $\phi^a$, and the holomorphic derivatives of $\alpha$.

The map can be written down very explicitly:

$$\rho^{ab} \mapsto \mathrm{tr}\Big(\phi^a \phi^b\Big),$$
$$\xi^a \mapsto \mathrm{tr}\big(\phi^a \beta\big), \qquad \psi^a_\mu \mapsto \mathrm{tr}\big(\phi^a \partial_\mu \alpha\big),$$
$$X^\mu \mapsto \mathrm{tr}\big(\beta \vee \partial_\mu \alpha\big), \qquad x \mapsto \mathrm{tr}(\partial_1 \alpha \partial_2 \alpha).$$

At a hand-waving level, and ignoring the central extension, one can already see how the relevant components of the Poisson brackets match up (recalling that $\alpha$ is conjugate to $\beta$ and $\phi_1$ to $\phi_2$).

More properly, one might frame this as a map from $\mathscr{L}^{\mathrm{red}}_{(2,0)}$ to (shifted) local functionals in holomorphic Yang–Mills.

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At an intuitive level: compare to Drinfeld–Sokolov, Feigin–Frenkel. . .

To fully work out the structure of this algebra, one needs to understand possible central extensions of $\mathscr{L}_{(2,0)}$. One expects a unique local central extension, closely connected to the superconformal invariant studied in the literature. (In progress with Williams.)

We expect to construct higher-rank examples as higher analogues of $W_n$ algebras. $W_\infty$ is understood: it is (the Koszul dual, so the currents, of) eleven-dimensional supergravity!

In forthcoming work with Hahner, Raghavendran, and Williams, we give the superspace description of the higher-spin currents that generate the $W_n$ algebra. At the holomorphic level, we identify the corresponding $E(3|6)$ modules, and show that they arise from the sequence of line bundles $\mathscr{O}(0,n)$ on the pure spinor space $P^1 \times P^2$.

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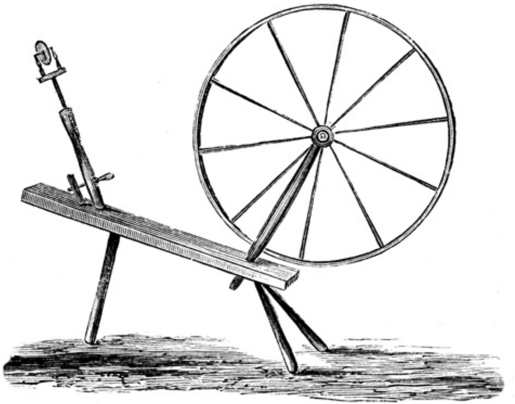
Butter–Novak–Tartaglino-Mazzucchelli; Raghavendran–Williams; IAS–Williams

To make sense of the full algebraic structure, we should better understand the factorization structure on higher $W_n$ algebras. We expect to recover analogues of the Gelfand–Dickey Poisson bracket in this higher setting. (In progress with Raghavendran and Williams.)

Is there an $E_3$ algebra whose factorization homology on the two-sphere reproduces the 2-shifted Poisson structure of eleven-dimensional supergravity? Can one construct such a thing by studying ABJM theory?

There is lots to do here!



*Thanks for your attention!*

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