Cosmological Correlators at Finite Coupling

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based on work to appear

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Punch line

We solved* $O(N)$ model in dS

@ large N, finite λ

* late-time 4pt

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We solved* $O(N)$ model in dS
@ large N, finite λ

- Confirm & extend results from stochastic approach
  - Starobinsky, Yokoyama
  - Gorbenko, Senatore, ...
- Nontrivial check of positivity of spectral density
  - Hogervorst, Penedones, Vaziri
  - Di Pietro, Gorbenko, SK
- Exploration of non-perturbative analyticity

Why should we care about non-perturbative QFT in dS ?

Our universe has $\Lambda > 0$

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- Not exactly de Sitter
- $\exists$ gravity, inflaton
- Most of phenomenological models are perturbative

Nonperturbative QFT in dS

Pheno

- ∃ important questions for which non-perturbative QFT effects are crucial.

  "rare events": * primordial black hole formation
  * probability of eternal inflation

- Intuition from perturbation theory is sometimes misguiding.

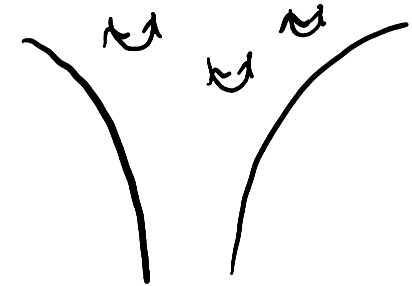
  * IR problem for light scalar.

Formal

- new setup for bootstrapping massive QFTs

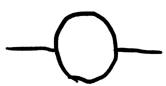
IR Problem

- Particle production due to expansion



$$ds^2 = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2} = -dt^2 + e^{2t} d\vec{x}^2$$

- Secular divergence @ late-time for light scalar

 $\sim \lambda \log\left(\frac{k}{e^t}\right) \xrightarrow[t\to\infty]{} \infty$

- Some even suggested QFT for light scalar in dS is ill-defined.

Stochastic Approach Starobinsky, ---

- Similar divergence arises for free heavy scalar if we expand correlators in mass

$$\langle \phi \, \phi \rangle \sim m^2 \log\left(\frac{k}{e^t}\right) + \cdots \rightsquigarrow$$ artifact of expansion

- Suggest a clever resummation for light scalar

$\rightsquigarrow$ Stochastic approach

Stochastic Approach

Starobinsky, ---

- Similar divergence arises for free heavy scalar if we expand correlators in mass

$$\langle \phi \, \phi \rangle \sim m^2 \log\left(\frac{k}{\mathcal{E}t}\right) + \cdots \rightsquigarrow \text{artifact of expansion}$$

- Suggest a clever resummation for light scalar

  $\rightsquigarrow$ Stochastic approach

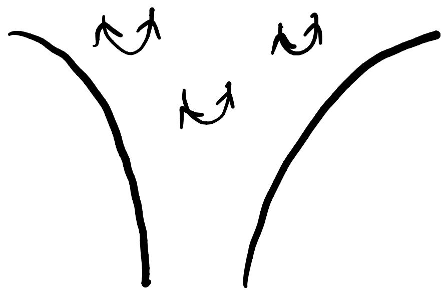
- Separate modes into long $(k>H)$ & short $(k<H)$

  $\rightsquigarrow$ Fokker-Planck equation for long modes

$$\frac{\partial P}{\partial t} = \cdots + \text{UV noise} \leftrightarrow \text{perturbation theory}$$

$\rightsquigarrow$ No secular divergence

Lesson from $O(N)$



$\longrightarrow$ Different vacuum with no secular div.

Unitarity & Analyticity

Conformal Partial Waves for de Sitter

- Important obs. : late-time correlator

$$\langle \Omega | \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 |_{\eta=0} | \Omega \rangle \qquad ds^2 = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2}$$

- Isometry of dS_{d+1} ~ Euclidean conformal group $SO(d+1,1)$

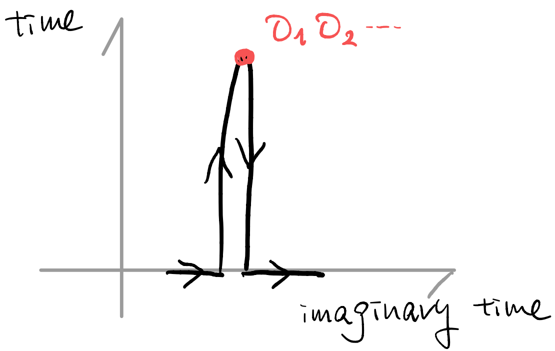
- $\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle$

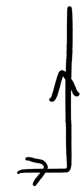
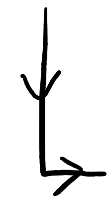
$$= \sum_J \int_{\frac{d}{2}+i\mathbb{R}} d\Delta \; \underbrace{\rho_J(\Delta)}_{\text{Spectral density}} \; \underbrace{\mathcal{F}_{\Delta,J}(x_1,x_2,x_3,x_4)}_{\text{Conformal partial wave}} + \cdots$$

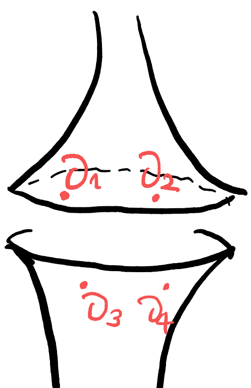
- $\rho_J(\Delta) \geq 0 \quad \Leftarrow$ Unitarity

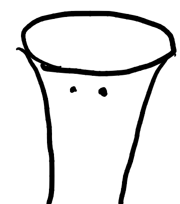
Positivity : A rough sketch

- late-time correlator : In-in observable



$\langle \Omega | =$  $| \Omega \rangle =$ 

- 

$=$ (kinematical) $\times$ $\Big|$  $\Big|^2 \geq 0$

Analyticity

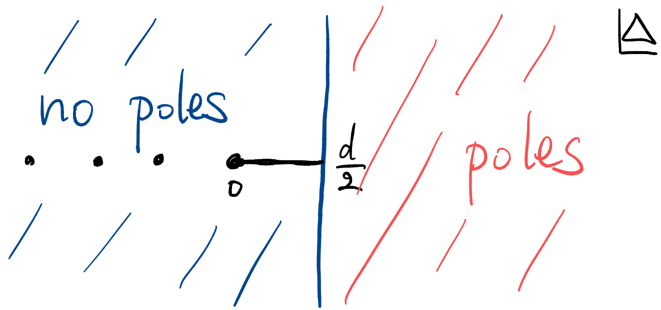
$$\langle O_1 O_2 O_3 O_4 \rangle = \sum_J \int d\Delta \;\; \rho_J(\Delta) \;\; F_{\Delta,J}$$

- In all the examples we know,

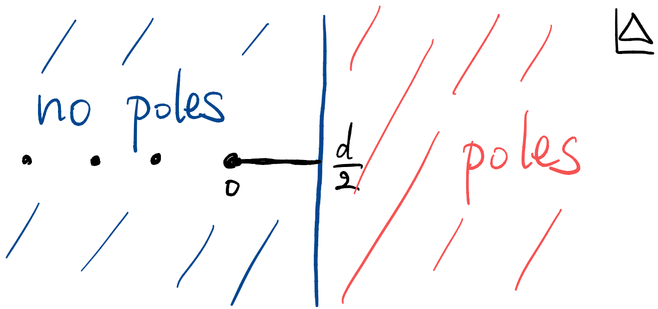
  $\rho_J(\Delta)$: meromorphic (no branch cut)

- Analytic continuation from S_{d+1} to dS_{d+1}

⇝ Spectral amplitude $f_J(\Delta)$ $\left(\rho_J(\Delta) = \frac{1}{2}\left[f_J(\Delta) + f_J(d-\Delta) \right] \right)$



Poles ~ Resonance



$$\xrightarrow{s \sim -\Delta^2}$$

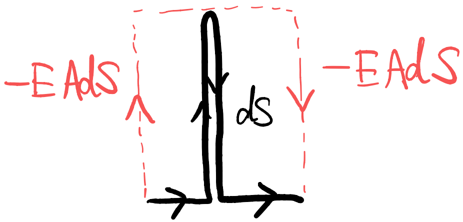
dS flat

- Poles in dS ~ poles in 2nd sheet in S-matrix
  $\rightsquigarrow$ resonance
-  particles tend to decay in dS

Arguments for analyticity

- Many perturbative examples

- Rotation to ("minus" EAdS)^2 · Hartle, Hertog · Stanford, Harlow (Polyakov, Maldacena, McFadden, Skenderis, ...)

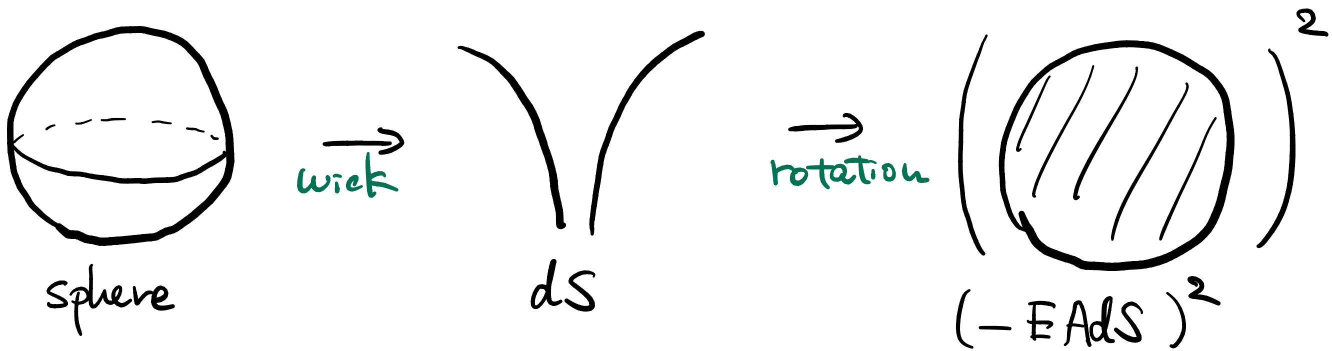


· Taronna, Sleight · Di Pietro, Gorbenko, SK

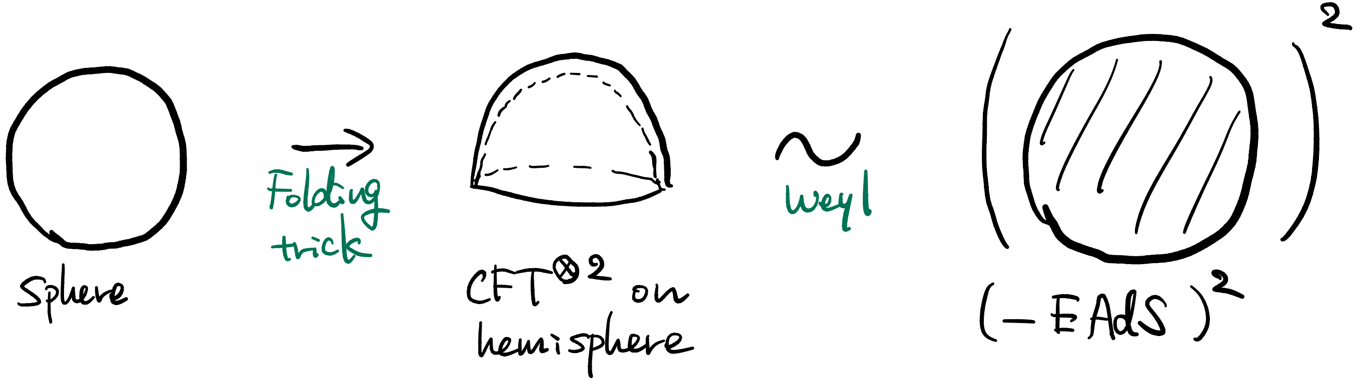
- CFT in dS · Hogervorst, Penedones, Vaziri, Loparco, Sun

poles on real axis

Relation btwn various geometries



- For CFT in dS, there is a shortcut



$O(N)$ model in dS

Sample of results

Sketch of Derivation

- $S \sim \int \sum_i (\partial\phi^i)^2 + m^2 \sum_i \phi_i^2 + \frac{\lambda}{2N} \left(\sum_i \phi_i^2\right)^2$

- Hubbard-Stratonovich

$$S \sim \int \sum_i (\partial\phi^i)^2 + m^2 \sum_i \phi_i^2 - \frac{N}{2\lambda}\sigma^2 + \sigma \sum_i \phi_i^2$$

- Quadratic in ϕ^i $\rightsquigarrow$ integrate out ϕ^i

- $S_{eff}(\sigma) = N\left[-\frac{\sigma^2}{2\lambda} - \log\det(\partial^2 + m^2 + \sigma)\right]$

$\rightsquigarrow$ saddle point σ_* $\rightsquigarrow$ $M_{phys}^2 = m^2 + \sigma_*$

- Correlators from fluctuations around $S_{eff}(\sigma^*)$

Sample of results

- IR problem

$m_{bare}^2 \ll \sqrt{\lambda}\, m_{Hubble}^2$ → $M_{phys}^2 = \sqrt{\lambda}\, m_{Hubble}^2 + \cdots$

perturbation is "ill-defined"

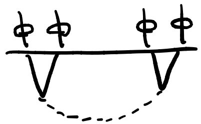
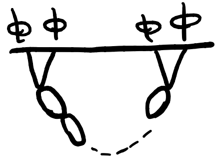
$(\sim m^2 + \sigma_*)$

Agrees with stochastic approach

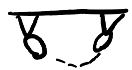
- No symmetry breaking

Similarity with 2d flat space

Sample of results 2

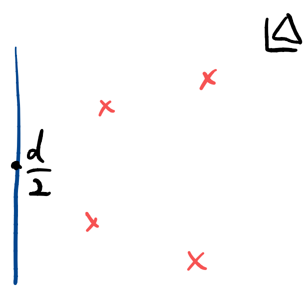
$-$  $\delta\sigma$ $= \sum_n$  $\sim \int d\nu' \, \frac{1}{\frac{1}{\lambda} + B(\nu')} \, \mathcal{F}_{\text{partial wave}}$

$$\hat{B}(\nu')|_{d=2} = \frac{i}{8\pi\nu'}\left[\pi - i\coth(\pi\nu)\left(\psi\left(-i\nu + \frac{i\nu'}{2} + \frac{1}{2}\right) - \psi\left(i\nu + \frac{i\nu'}{2} + \frac{1}{2}\right)\right)\right],$$

$-$ ρ  $+$ ρ  ≥ 0

- Both are $\mathcal{O}(1)$
- ρ  can be negative

$-$ $\rho(\Delta)$: meromorphic



Future Directions

Conclusion

- Large N theory in dS is useful for checks of positivity, analyticity, stochastic

Future

- Large N theories as solvable toy models for nonperturbative physics in cosmology?
  - PBH, reheating
- CFT + perturb in dS → E AdS?
- Bounding $(\partial\phi)^4$ using positivity...?
  - Sum rule [Bonifacio, Mazac, Pal]

Conclusion

- Large N theory in dS is useful for checks of positivity, analyticity, stochastic approach

Future

- Large N theories as solvable toy models for non-perturbative physics in cosmology?
  - Primordial blackholes
  - eternal inflation
- Bounding $(\partial\phi)^4$ using positivity?
  - Spin truncated sum rule for $\Delta = \frac{d}{2} + i\mathbb{R}$ Bonifacio, Mazac, Pal
- Positivity for $\Delta = \frac{d}{2} + i\mathbb{R}$ for CFT bootstrap?
  - 2pt on $\mathbb{RP}^d$
  - Thermal 2pt + KMS

Back up slides

Spin truncated sum rule

- $\left(\prod_k \int_{S^d} dx_k \, Y^{(J_k)}(x_k) \right) \langle \mathcal{O}(x_1)\, \mathcal{O}(x_2)\, \mathcal{O}(x_3)\, \mathcal{O}(x_4) \rangle$

spherical harmonics on S^d

- $J_1 = J_2 = 0$, $J_3 = J_4 = 1$

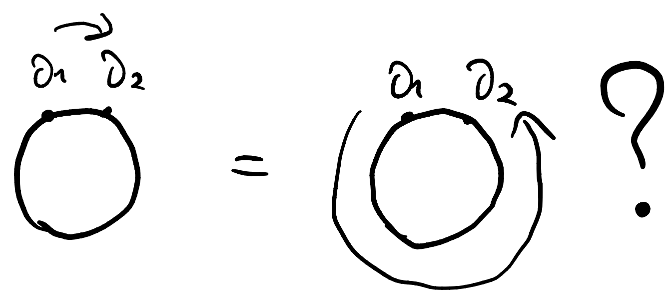
$$\rightsquigarrow \int_{\frac{d}{2}+i\mathbb{R}} d\Delta \, \frac{\Gamma \cdots \Gamma}{\Gamma \cdots \Gamma} \, C_{J=0}(\Delta) = 0$$

Positivity for $\Delta = \frac{d}{2} + i\mathbb{R}$

- Euclidean positivity ($\rho_{\frac{d}{2}+i\mathbb{R}} \geq 0$) is expected to hold for CFT with positive path-integral measure
  - * 3d Ising
  - * $O(N)$
  - * percolation

- Positive expansion in

RP_d



Thermal